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Design of Unknown Inputs Observer for a Class of Discrete-Time Takagi-Sugeno Descriptor Models

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Abstract This paper addresses the problem of simultaneous estimation of unmeasurable states and unknown inputs (UIs) for a class of discrete-time nonlinear descriptor models (DNDMs) described by Takagi-Sugeno (T-S) structure with unmeasurable premise variables. The UIs affect both state and output of the system. The main idea of the proposed design of fuzzy unknown inputs observer (FUIO) is based on the separation between dynamic and static relations in T-S descriptor model. First, the method permitting to separate dynamic equations from static equations is developed. Next, based on the augmented fuzzy model which contains the dynamic equations and the UIs, a new FUIO design in explicit structure is given. The exponential convergence of the state estimation error is studied by using the Lyapunov theory and the stability conditions are given in terms of linear matrix inequalities (LMIs). Finally, an application to a DNDM of a single-link flexible joint robot is presented in order to illustrate the validity and applicability of the proposed method.

Keywords Discrete-time nonlinear descriptor model, Discrete-time Takagi-Sugeno descriptor model, Unmeasurable premise variables, Unknown inputs, Fuzzy unknown input observer, LMI

1. Introduction

Many industrial processes as e.g. circuit systems, robotics, chemical processes, biological systems and so on, are naturally modelled as systems of differential and algebraic equations also called descriptor models or singular models or implicit models. Known as a generalization of standard models, such descriptor models constitute a powerful modeling tool allowing to describe the dynamic behaviour of processes. They represent physical phenomenas that can not be described by standard models. We may cite [1], [2], [3] for some real applications of implicit models. The numerical simulation of such models usually combines an ODE numerical method together with an optimization algorithm.

On the other hand, ordinary T-S approach [4], [5] known as an interesting alternative for the analysis and controller/observer synthesis for nonlinear systems, finds its success on the fact that once the T-S fuzzy models are obtained, some analysis and design tools developed in the linear case can be used, which facilitates observer or/and controller synthesis for complex nonlinear systems see for example [6], [7] and the references therein. Moreover, notice that in [8], [9], a fuzzy implicit model is defined by extending the T-S fuzzy model [4].

In this paper, the aim is to consider the problem of FUIO design for a class of DTSDMs. Due to its important role in the area of fault detection and design of fault tolerant control strategy, the field of the FUIO design for nonlinear systems has attracted much attention from researchers during these last two decades. Indeed, many research works on fuzzy observer and its application to fault detection can be found in the literature. They relate to explicit and implicit nonlinear systems in both continuous-time and discrete-time cases. Concerning the continuous-time case, we may cite [10], [11], [12], [13], [14] for explicit models and [8], [9], [15], [16], [17], [18], [19], [20] for implicit models. Likewise, in discrete-time case, several works exist for explicit or implicit structures see e.g. [21], [22], [23], [24], [25], [26], [27]. It should be noted that, generally an interesting way to solve the various FUIO raised previously is to write the convergence conditions on the LMI form [28].

Based on T-S fuzzy approach with unmeasurable premise variables, the main contribution of the paper consists in an observer design allowing the simultaneous estimation of the unknown states and unknown inputs for a class of DNDMs subject to unknown inputs affecting states and outputs of the system simultaneously. The idea is based on the separation between dynamic and static equations in the considering DTSDM and the use of an augmented system

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structure formed by dynamic equations and unknown inputs. The global exponential stability of the state estimation error of the augmented system is studied by using the Lyapunov theory and the stability conditions are given in term of LMIs. Besides, the proposed FUIO is given without the use of an optimization algorithm.

The paper is organized as follows. The considered class of DNDMs subject to unknown inputs described by T-S fuzzy structure with unmeasurable premise variables is presented in Section 2. The main contribution about FUIO design permitting simultaneous estimation of unknown states and unknown inputs is stated in Section 3. To show the good performance of the proposed FUIO, an application of a single-link flexible joint robot is given in Section 4. Finally, a conclusion is given in section 5.

In this paper, some notations used are fair standard. For example, $X>0$ means the matrix X is symmetric and positive definite. X^T denotes the transpose of X. The symbol I (or 0) represents the identity matrix (or zero matrix) with appropriate dimension.

$$\sum_{i,j=1}^{q}\mu_i\mu_j=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i\mu_j \text{ and } \begin{pmatrix} X & * \\ Y & Z \end{pmatrix}=\begin{pmatrix} X & Y^T \\ Y & Z \end{pmatrix}.$$

2. System Description

In this paper, the aim is to consider the problem of FUIO design for a class of DNDMs subject to UIs described by T-S structure with unmeasurable premise variables. For this objective, the following class of DNDMs subject to UIs is adopted:

$$\begin{cases} Mx_{k+1} = A(x_k)x_k + B(x_k)u_k + C(x_k)d_k \\ y_k = D(x_k)x_k + E(x_k)u_k + F(x_k)d_k \end{cases} \tag{1}$$

where $x_k^T=\begin{bmatrix} X_k^{1T} & X_k^{2T}\end{bmatrix}\in R^n$ is the state vector with $X_k^1\in R^{n_1}$ is the vector of difference variables, $X_k^2\in R^{n_2}$ is the vector of algebraic variables with $n_1+n_2=n$, $u_k\in R^m$ is the control input, $d_k\in R^r$ is the unknown control input, $y_k\in R^p$ is the measured output. $A(x_k)\in R^{n\times n}$, $B(x_k)\in R^{n\times m}$, $C(x_k)\in R^{n\times r}$, $D(x_k)\in R^{p\times n}$, $E(x_k)\in R^{p\times m}$, $F(x_k)\in R^{p\times r}$ are nonlinear matrices functions. $M\in R^{n\times n}$ such that $rank(M)=n_1$ is a real known constant matrix with:

$$M=\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

By the sector nonlinearity approach [6], the DNDM (1) can be exactly represented by the following T-S fuzzy descriptor model:

$$\begin{cases} Mx_{k+1} = \sum_{i=1}^{q}\mu_i(x_k)\left(A_ix_k+B_iu_k+C_id_k\right) \\ y_k = \sum_{i=1}^{q}\mu_i(x_k)\left(D_ix_k+E_iu_k+F_id_k\right) \end{cases} \tag{3}$$

where $A_i\in R^{n\times n}$, $B_i\in R^{n\times m}$, $C_i\in R^{n\times r}$, $D_i\in R^{p\times n}$, $E_i\in R^{p\times m}$, $F_i\in R^{p\times r}$, are real known constant matrices with:

$$\begin{gathered} A_i=\begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix};\quad B_i=\begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix}; \\ \mathrm{C}_i=\begin{pmatrix} C_{1i} \\ C_{2i} \end{pmatrix};\ \mathrm{D}_i=\begin{pmatrix} D_{1i} & D_{2i} \end{pmatrix} \end{gathered} \tag{4}$$

where constant matrices A_{22i} are supposed invertible. q is the number of sub-models.

The $\mu_i(x_k)$ $(\mathrm{i}=1,...,\mathrm{q})$ are the weighting functions that ensure the transition between the contribution of each sub model:

$$\begin{cases} Mx_{k+1} = A_ix_k+B_iu_k+C_id_k \\ y_k = D_ix_k+E_iu_k+F_id_k \end{cases} \tag{5}$$

They verify the so-called convex sum properties:

$$\sum_{i=1}^{q}\mu_i(x_k)=1;\quad 0\le\mu_i(x_k)\le 1 \quad i=1,\dots,q \tag{6}$$

Before giving the main result, let us make the following assumption [1], [16]:

Assumption 1: Suppose that:

- (M,A_i) is regular, i.e. $\det(zM-A_i)\neq 0\ \ \forall z\in\mathbb{C}$
- All sub-models (5) are impulse observable and detectable.

In order to investigate the Fuzzy UIO design for system (3), we proceed as mentioned above to the separation of the dynamic equations from static equations of the model (3). Indeed, from (2)-(4), sub-model (5) can be rewritten as follows:

$$\begin{cases} X_{k+1}^1 = A_{11i}X_k^1+A_{12i}X_k^2+B_{1i}u_k+C_{1i}d_k \\ 0 = A_{21i}X_k^1+A_{22i}X_k^2+B_{2i}u_k+C_{2i}d_k \\ y_k = D_{1i}X_k^1+D_{2i}X_k^2+E_iu_k+F_id_k \end{cases} \tag{7}$$

Since A_{22i} is invertible, it follows:

$$X_k^2 = J_iX_k^1+K_iu_k+L_id_k \tag{8}$$

where

$$\begin{cases} J_i = -A_{22i}^{-1}\,A_{21i} \\ K_i = -A_{22i}^{-1}\,B_{2i} \\ L_i = -A_{22i}^{-1}\,\mathrm{C}_{2i} \end{cases} \tag{9}$$

Thus, combining (7) and (8) we have:

$$\begin{cases} X_{k+1}^1 = M_i X_k^1 + N_i u_k + P_i d_k \\ X_k^2 = J_i X_k^1 + K_i u_k + L_i d_k \\ y_k = R_i X_k^1 + S_i u_k + T_i d_k \end{cases} \tag{10}$$

where

$$\begin{cases} M_i = A_{11i} + A_{12i} J_i \\ N_i = B_{1i} + A_{12i} K_i \\ P_i = C_{1i} + A_{12i} L_i \\ R_i = D_{1i} + D_{2i} J_i \\ S_i = E_i + D_{2i} K_i \\ T_i = F_i + D_{2i} L_i \end{cases} \tag{11}$$

The weighting functions $\mu_i(x_k)$ can be rewritten as:

$$\mu_i(x_k) = \mu_i(X_k^1, X_k^2 = J_i X_k^1 + K_i u_k + L_i d_k) = \mu_i(\eta_k) \tag{12}$$

with $\eta_k^T = \begin{bmatrix} (X_k^1)^T & u_k^T & d_k^T \end{bmatrix}$.

So, by aggregation of the resulting sub-models (10), the following global fuzzy model is obtained:

$$\begin{cases} X_{k+1}^1 = \sum_{i=1}^{q} \mu_i(\eta_k) \left(M_i X_k^1 + N_i u_k + P_i d_k \right) \\ X_k^2 = \sum_{i=1}^{q} \mu_i(\eta_k) \left(J_i X_k^1 + K_i u_k + L_i d_k \right) \\ y_k = \sum_{i=1}^{q} \mu_i(\eta_k) \left(R_i X_k^1 + S_i u_k + T_i d_k \right) \end{cases} \tag{13}$$

Assumption 2: Suppose that d_k is considered as a constant unknown control input per time interval i.e.:

$$d_{k+1} = d_k \quad k \in \begin{bmatrix} T_1 & T_2 \end{bmatrix}; \quad \forall\ T_1, T_2 \in R^+ \tag{14}$$

Let us define the augmented state vector $\xi_k^{1T} = \begin{bmatrix} X_k^{1T} & d_k^T \end{bmatrix}$ and $\xi_k^2 = X_k^2$. Thus, the system (13) can be represented as:

$$\begin{cases} \xi_{k+1}^1 = \sum_{i=1}^{q} \mu_i(\theta_k) \left(\tilde{M}_i \xi_k^1 + \tilde{N}_i u_k \right) \\ \xi_k^2 = \sum_{i=1}^{q} \mu_i(\theta_k) \left(\tilde{J}_i \xi_k^1 + K_i u_k \right) \\ y_k = \sum_{i=1}^{q} \mu_i(\theta_k) \left(\tilde{R}_i \xi_k^1 + S_i u_k \right) \end{cases} \tag{15}$$

where

$$\begin{cases} \theta_k = \begin{bmatrix} \xi_k^{1T} & u_k^T \end{bmatrix}^T \\ \tilde{M}_i = \begin{pmatrix} M_i & P_i \\ 0 & I \end{pmatrix} \\ \tilde{N}_i = \begin{pmatrix} N_i \\ 0 \end{pmatrix} \\ \tilde{J}_i = \begin{pmatrix} J_i & L_i \end{pmatrix} \\ \tilde{R}_i = \begin{pmatrix} R_i & T_i \end{pmatrix} \end{cases} \tag{16}$$

3. Main Result

Based on the transformation of the T-S descriptor system (3) into the equivalent form (15), the proposed FUIO permitting the estimate of unmeasurable state and unknown inputs takes the following form:

$$\begin{cases} \hat{\xi}_{k+1}^1 = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k) \left(\tilde{M}_i \hat{\xi}_k^1 + \tilde{N}_i u_k - G_i(\hat{y}_k - y_k) \right) \\ \hat{\xi}_k^2 = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k) \left(\tilde{J}_i \hat{\xi}_k^1 + K_i u_k \right) \\ \hat{y}_k = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k) \left(\tilde{R}_i \hat{\xi}_k^1 + S_i u_k \right) \end{cases} \tag{17}$$

where $(\hat{\xi}_k^1,\ \hat{\xi}_k^2)$, $\hat{y}_k$ and $\hat{\theta}_k$ denote the estimated augmented state vector, the output vector and the decision variable vector respectively. $G_i,\ i = 1,\ldots,q$ are the observer gains which are determined such that $(\hat{\xi}_k^1,\ \hat{\xi}_k^2)$ asymptotically converges to $(\xi_k^1,\ \xi_k^2)$.

In order to establish the conditions for the asymptotic convergence of the observer (17), we define the state estimation error:

$$e_k = \begin{pmatrix} e_k^1 \\ e_k^2 \end{pmatrix} = \begin{pmatrix} \hat{\xi}_k^1 - \xi_k^1 \\ \hat{\xi}_k^2 - \xi_k^2 \end{pmatrix} \tag{18}$$

It follows from (15) and (17) that the dynamics of state estimation error e_k is given by the differential and algebraic equations:

$$\begin{cases} e_{k+1}^1 = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k) \left(\tilde{M}_i \hat{\xi}_k^1 + \tilde{N}_i u_k - G_i(\hat{y}_k - y_k) \right) \\ \qquad - \sum_{i=1}^{q} \mu_i(\theta_k) \left(\tilde{M}_i \xi_k^1 + \tilde{N}_i u_k \right) \\ e_k^2 = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k) \left(\tilde{J}_i \hat{\xi}_k^1 + K_i u_k \right) \\ \qquad - \sum_{i=1}^{q} \mu_i(\theta_k) \left(\tilde{J}_i \xi_k^1 + K_i u_k \right) \end{cases} \tag{19}$$

which are equivalent to the following equations:

$$\begin{cases} e_{k+1}^1 = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k)\left(\tilde{M}_i e_k^1 - G_i(\hat{y}_k - y_k)\right) \\ \qquad -\sum_{i=1}^{q}\left(\mu_i(\theta_k) - \mu_i(\hat{\theta}_k)\right)\left(\tilde{M}_i \xi_k^1 + \tilde{N}_i u_k\right) \\ e_k^2 = \sum_{i=1}^{q} \mu_i(\hat{\theta}_k)\, \tilde{J}_i e_k^1 \\ \qquad -\sum_{i=1}^{q}\left(\mu_i(\theta_k) - \mu_i(\hat{\theta}_k)\right)\left(\tilde{J}_i \xi_k^1 + K_i u_k\right) \end{cases} \tag{20}$$

Thus, from (20) to prove the convergence of the estimation error e_k toward zero, it suffices to prove that e_k^1 converges to zero. So, using the fact that:

$$\begin{cases} \sum_{i=1}^{q}(\mu_i(\theta_k) - \mu_i(\hat{\theta}_k))\, \tilde{M}_i = \sum_{i,j=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\, \Delta\tilde{M}_{ij} \\ \sum_{i=1}^{q}(\mu_i(\theta_k) - \mu_i(\hat{\theta}_k))\, \tilde{N}_i = \sum_{i,j=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\, \Delta\tilde{N}_{ij} \end{cases} \tag{21}$$

where $\Delta\tilde{M}_{ij} = \tilde{M}_i - \tilde{M}_j$ and $\Delta\tilde{N}_{ij} = \tilde{N}_i - \tilde{N}_j$.

Hence, the first equation of (20) becomes:

$$\begin{aligned} e_{k+1}^1 = &\sum_{i=1}^{q} \mu_i(\hat{\theta}_k)\left(\tilde{M}_i e_k^1 - G_i(\hat{y}_k - y_k)\right) \\ &- \sum_{i,j=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\left(\Delta\tilde{M}_{ij}\xi_k^1 + \Delta\tilde{N}_{ij} u_k\right) \end{aligned} \tag{22}$$

Since $\sum_{i=1}^{q}\mu_i(\theta_k) = 1$, equality (22) can be written as follows:

$$\begin{aligned} e_{k+1}^1 = &\sum_{i,j=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\left(\tilde{M}_j e_k^1 - G_j(\hat{y}_k - y_k)\right) \\ &- \sum_{i,j=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\left(\Delta\tilde{M}_{ij}\xi_k^1 + \Delta\tilde{N}_{ij} u_k\right) \end{aligned} \tag{23}$$

Similarly, y_k and $\hat{y}_k$ can be written as follows:

$$\begin{cases} y_k = \sum_{i,h=1}^{q} \mu_i(\theta_k)\mu_h(\hat{\theta}_k)\left((\tilde{R}_h + \Delta\tilde{R}_{ih})\xi_k^1 + (S_h + \Delta S_{ih})u_k\right) \\ \hat{y}_k = \sum_{i,h=1}^{q} \mu_i(\theta_k)\mu_h(\hat{\theta}_k)\left(\tilde{R}_h \xi_k^1 + S_h u_k\right) \end{cases} \tag{24}$$

where $\Delta\tilde{R}_{ih} = \tilde{R}_i - \tilde{R}_h$ and $\Delta S_{ih} = S_i - S_h$.

By substituting (24) in (23), we obtain:

$$e_{k+1}^1 = \sum_{i,j,h=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\mu_h(\hat{\theta}_k)\left(\Gamma_{jh} e_k^1 + \Phi_{ijh}\xi_k^1 + \Omega_{ijh} u_k\right) \tag{25}$$

where

$$\begin{cases} \Gamma_{jh} = \tilde{M}_j - G_j\tilde{R}_h \\ \Phi_{ijh} = G_j\Delta\tilde{R}_{ih} - \Delta\tilde{M}_{ij} \\ \Omega_{ijh} = G_j\Delta S_{ih} - \Delta\tilde{N}_{ij} \\ i, j, h \in \{1,\dots,q\} \end{cases} \tag{26}$$

Let $\tilde{\xi}_k^1 = \left((e_k^1)^T \quad (\xi_k^1)^T\right)^T$, then from (15) and (25) we have:

$$\tilde{\xi}_{k+1}^1 = \sum_{i,j,h=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\mu_h(\hat{\theta}_k)\left(\mathrm{A}_{ijh}\tilde{\xi}_k^1 + \mathrm{B}_{ijh} u_k\right) \tag{27}$$

where

$$\begin{cases} \mathrm{A}_{ijh} = \begin{pmatrix} \Gamma_{jh} & \Phi_{ijh} \\ 0 & \tilde{M}_i \end{pmatrix} \\ \mathrm{B}_{ijh} = \begin{pmatrix} \Omega_{ijh} \\ \tilde{N}_i \end{pmatrix} \end{cases} \tag{28}$$

Thus, in order to prove the convergence of the FUIO (17), the aim is to determine the observer gains G_i, $i = 1,\dots,q$ to ensure the stability of the system (27). Therefore, the convergence conditions of (17) can be formulated by the following Theorem.

Theorem 1: Under above Assumptions 1 and 2, the state estimation error between the DTSDM (3) and its FUIO (17) converges exponentially asymptotically towards zero, if given $0 < \lambda < 1$ there exist matrices $Q_1 > 0$ and $Q_2 > 0$, W_i, $i = 1,\dots,q$ such that the following LMIs hold:

$$\Lambda_{ijh} = \begin{pmatrix} \Lambda_{11} & * & * & * \\ \Lambda_{21} & \Lambda_{22} & * & * \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} & * \\ \Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} \end{pmatrix} < 0, \tag{29}$$

$$\forall (i, j, h) \in \{1,\dots,q\}^3$$

where

$$\begin{cases} \Lambda_{11} = \tilde{M}_j^T Q_1 \tilde{M}_j - \tilde{M}_j^T W_j \tilde{R}_h - \tilde{R}_h^T W_j^T \tilde{M}_j - \lambda^2 Q_1 \\ \Lambda_{21} = \Delta\tilde{R}_{ih}^T W_j^T \tilde{M}_j - \Delta\tilde{M}_{ij}^T Q_1 \tilde{M}_j + \Delta\tilde{M}_{ij}^T W_j \tilde{R}_h \\ \Lambda_{22} = \Delta\tilde{M}_{ij}^T Q_1 \Delta\tilde{M}_{ij} + \tilde{M}_i^T Q_2 \tilde{M}_i - \Delta\tilde{R}_{ih}^T W_j^T \Delta\tilde{M}_{ij} \\ \qquad - \Delta\tilde{M}_{ij}^T W_j \Delta\tilde{R}_{ih} - \lambda^2 Q_2 \\ \Lambda_{31} = \Delta S_{ih}^T W_j^T \tilde{M}_j - \Delta\tilde{N}_{ij}^T Q_1 \tilde{M}_j + \Delta\tilde{N}_{ij}^T W_j \tilde{R}_h \\ \Lambda_{32} = \Delta\tilde{N}_{ij}^T Q_1 \Delta\tilde{M}_{ij} + \tilde{N}_i^T Q_2 \tilde{M}_i \\ \qquad - \Delta S_{ih}^T W_j^T \Delta\tilde{M}_{ij} - \Delta\tilde{N}_{ij}^T W_j \Delta\tilde{R}_{ih} \\ \Lambda_{33} = \Delta\tilde{N}_{ij}^T Q_1 \Delta\tilde{N}_{ij} - \Delta S_{ih}^T W_j^T \Delta\tilde{N}_{ij} \\ \qquad - \Delta\tilde{N}_{ij}^T W_j \Delta S_{ih} + \tilde{N}_i^T Q_2 \tilde{N}_i \\ \Lambda_{41} = W_j \tilde{R}_h \\ \Lambda_{42} = -W_j \Delta\tilde{R}_{ih} \\ \Lambda_{43} = -W_j \Delta S_{ih} \\ \Lambda_{44} = -Q_1 \end{cases} \tag{30}$$

The observer gains in (17) are derived from:

$$G_i = Q_1^{-1} W_i \tag{31}$$

Proof of Theorem 1: Considering the following quadratic Lyapunov function:

$$V_k = (\tilde{\xi}_k^1)^T \; Q \; \tilde{\xi}_k^1 \tag{32}$$

with

$$Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} > 0 \tag{33}$$

The convergence of (27) is exponentially ensured if the following condition is guaranteed (see [29] as cited in [6]):

$$\Delta V_k = V_{k+1} - V_k < (\lambda^2 - 1) V_k \qquad 0 < \lambda < 1 \tag{34}$$

From (27) and (32), inequality (34) becomes:

$$\Delta V_k = \sum_{i,j,h=1}^{q} \mu_i(\theta_k)\mu_j(\hat{\theta}_k)\mu_h(\hat{\theta}_k) \begin{pmatrix} \tilde{\xi}_k^1 \\ u_k \end{pmatrix}^T \Sigma_{ijh} \begin{pmatrix} \tilde{\xi}_k^1 \\ u_k \end{pmatrix} < 0 \tag{35}$$

where

$$\Sigma_{ijh} = \begin{pmatrix} \mathrm{A}_{ijh}^T \; Q \; \mathrm{A}_{ijh} - \lambda^2 Q & * \\ \mathrm{B}_{ijh}^T \; Q \; \mathrm{A}_{ijh} & \mathrm{B}_{ijh}^T \; Q \; \mathrm{B}_{ijh} \end{pmatrix} \tag{36}$$

By using (28), Σ_{ijh} can be reduced as follows:

$$\Sigma_{ijh} = \begin{pmatrix} \Sigma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{pmatrix} \tag{37}$$

where

$$\begin{cases} \Sigma_{11} = \Gamma_{jh}^T \; Q_1 \; \Gamma_{jh} - \lambda^2 Q_1 \\ \Sigma_{21} = \Phi_{ijh}^T \; Q_1 \; \Gamma_{jh} \\ \Sigma_{22} = \Phi_{ijh}^T \; Q_1 \; \Phi_{ijh} + \tilde{M}_i^T \; Q_2 \; \tilde{M}_i - \lambda^2 Q_2 \\ \Sigma_{31} = \Omega_{ijh}^T \; Q_1 \; \Gamma_{jh} \\ \Sigma_{32} = \Omega_{ijh}^T \; Q_1 \; \Phi_{ijh} + \tilde{N}_i^T \; Q_2 \; \tilde{M}_i \\ \Sigma_{33} = \Omega_{ijh}^T \; Q_1 \; \Omega_{ijh} + \tilde{N}_i^T \; Q_2 \; \tilde{N}_i \end{cases} \tag{38}$$

The inequality (35) is satisfied if:

$$\Sigma_{ijh} < 0 \qquad \forall i, j, h \in \{1, \ldots, q\} \tag{39}$$

Then from (26), (33), we can establish the LMI conditions (29) of Theorem 1 by using the Schur complement [28] and the following change of variables:

$$W_i = Q_1 G_i \tag{40}$$

Thus, from the Lypunov stability theory, if the LMI conditions (29) given in Theorem 1 are satisfied, the system (27) is exponentially asymptotically stable. This completes the proof of Theorem 1.

4. Application to Single-Link Flexible Joint Robot

In this section, the proposed FUIO design (17) is applied to a single-link flexible joint robot in order to estimate on-line these unknown states and its UI simultaneously. The following DNDM that we consider here is obtained by Euler discretisation of the model given in [19]. It takes the form:

$$\begin{cases} M x_{k+1} = A(x_k) x_k + B u_k + C d_k \\ \quad y_k = D x_k \end{cases} \tag{41}$$

where $x_k = (x_{1k}, x_{2k}, x_{3k}, x_{4k}, x_{5k}, x_{6k})^T$ is the state vector with x_{1k} and x_{2k} are the angles of rotations of the motor and the link respectively. x_{3k} and x_{4k} are their angular velocities. x_{5k} and x_{6k} are their angular accelerations. u_k is the control variable, y_k is the output measurement vector and d_k is the unknown input variable.

$$A(x_k) = \begin{pmatrix} 1 & 0 & \tau & 0 & 0 & 0 \\ 0 & 1 & 0 & \tau & 0 & 0 \\ 0 & 0 & 1 & 0 & \tau & 0 \\ 0 & 0 & 0 & 1 & 0 & \tau \\ \frac{-k_1\tau}{J_m} & \frac{k_1\tau}{J_m} & \frac{-\beta\tau}{J_m} & 0 & -\tau & 0 \\ \frac{k_1\tau}{J_L} & \nu & 0 & 0 & 0 & -\tau \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{k_2\tau}{J_m} \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} \tau \\ 0 \\ \tau \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

where $\tau > 0$ is sampling time and

$$\nu = -\frac{k_1\tau}{J_L} - \frac{mgb\tau}{J_L}\frac{\sin(x_{2k})}{x_{2k}} \tag{42}$$

To express the model (41) process as a T-S model with unmeasurable premise variable, we consider the sector of nonlinearities of the term $\nu \in [\nu_{\min}, \nu_{\max}]$ of the matrix $A(x_k)$. Then, we can transform the nonlinear term under the following shape:

$$\nu = \Lambda_1 \nu_{\max} + \Lambda_2 \nu_{\min} \tag{43}$$

where

$$\begin{cases} \Lambda_1 = \dfrac{\nu - \nu_{\min}}{\nu_{\max} - \nu_{\min}} \\ \Lambda_2 = \dfrac{\nu_{\max} - \nu}{\nu_{\max} - \nu_{\min}} \end{cases} \tag{44}$$

Hence, the global T-S fuzzy model which is a particular case of the system (3) is inferred as:

$$\begin{cases} Mx_{k+1} = \sum_{i=1}^{2} \mu_i(x_k)\left(A_i x_k + Bu_k + Cd_k\right) \\ y_k = Dx_k \end{cases} \tag{45}$$

with

$$A_1 = \begin{pmatrix} 1 & 0 & \tau & 0 & 0 & 0 \\ 0 & 1 & 0 & \tau & 0 & 0 \\ 0 & 0 & 1 & 0 & \tau & 0 \\ 0 & 0 & 0 & 1 & 0 & \tau \\ \frac{-k_1\tau}{J_m} & \frac{k_1\tau}{J_m} & \frac{-\beta\tau}{J_m} & 0 & -\tau & 0 \\ \frac{k_1\tau}{J_L} & \nu_{\max} & 0 & 0 & 0 & -\tau \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 1 & 0 & \tau & 0 & 0 & 0 \\ 0 & 1 & 0 & \tau & 0 & 0 \\ 0 & 0 & 1 & 0 & \tau & 0 \\ 0 & 0 & 0 & 1 & 0 & \tau \\ \frac{-k_1\tau}{J_m} & \frac{k_1\tau}{J_m} & \frac{-\beta\tau}{J_m} & 0 & -\tau & 0 \\ \frac{k_1\tau}{J_L} & \nu_{\min} & 0 & 0 & 0 & -\tau \end{pmatrix}$$

The weighting functions are given by:

$$\begin{cases} \mu_1(x_k) = \Lambda_1 \\ \mu_2(x_k) = \Lambda_2 \end{cases} \tag{46}$$

Based on the theory developed in Sections 2 and 3, the aim is to design a FUIO for DTSDM (45). More precisely, based on the on-line measurements of x_{1k}, x_{3k} and x_{4k}, we shall show that the previous result (17) can be used to the on-line estimation of the unknown states x_{2k}, x_{5k}, x_{6k} and the UI d_k simultaneously. To this end, let:

$X_k^1 = \begin{bmatrix} x_{1k} & x_{2k} & x_{3k} & x_{4k} \end{bmatrix}^T$ and $X_k^2 = \begin{bmatrix} x_{5k} & x_{6k} \end{bmatrix}^T$

$M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$ with $rank(M) = 4$.

$$A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix} = \begin{pmatrix} A_i(1:4,1:4) & A_i(1:4,5:6) \\ A_i(5:6,1:4) & A_i(5:6,5:6) \end{pmatrix}$$

with i=1,2.

$$B_i = B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} B(1:4) \\ B(5:6) \end{pmatrix}; \; C_i = C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \mathrm{C}(1:4) \\ \mathrm{C}(5:6) \end{pmatrix}.$$

$\mathrm{D}_i = D = \begin{pmatrix} D_1 & D_2 \end{pmatrix} = \begin{pmatrix} D(1:4) & D(5:6) \end{pmatrix}$ with $\mathrm{D}_2 = 0$.

Notice that in this case $A_{22i} = \begin{pmatrix} -\tau & 0 \\ 0 & -\tau \end{pmatrix}$ are invertible.

This shows that model (45) is a particular case of model (3) with $E_i = 0$ and $\mathrm{F}_i = 0$.

Consequently, apply the theory developed in the Section 2 (see (7) to (15)), model (45) is written as follows:

$$\begin{cases} \xi_{k+1}^1 = \sum_{i=1}^{2} \mu_i(\theta_k)\left(\tilde{M}_i \xi_k^1 + \tilde{N}_i u_k\right) \\ \xi_k^2 = \sum_{i=1}^{2} \mu_i(\theta_k)\left(\tilde{J}_i \xi_k^1 + K_i u_k\right) \\ y_k = \tilde{R}\xi_k^1 \end{cases} \tag{47}$$

where

$$\begin{cases} \xi_k^1 = \left[X_k^{1T} \quad d_k \right]^T \\ \xi_k^2 = \mathrm{X}_k^2 \\ \theta_k = \mathrm{x}_{2k} \\ \tilde{R} = \left[D_1 \quad 0 \right] \end{cases} \tag{48}$$

$K_i, \tilde{M}_i, \tilde{N}_i, \tilde{J}_i, \tilde{R}_i$ are given in the above equations (9) and (16).

Then using Section 3, the FUIO for model (45) (see its equivalent model (47)) permitting to estimate simultaneously x_{2k}, x_{5k}, x_{6k} and the UI d_k takes de following form:

$$\begin{cases} \hat{\xi}_{k+1}^1 = \sum_{i=1}^{2} \mu_i(\hat{\theta}_k)\left(\tilde{M}_i \hat{\xi}_k^1 + \tilde{N}_i u_k - G_i(\hat{y}_k - y_k) \right) \\ \hat{\xi}_k^2 = \sum_{i=1}^{2} \mu_i(\hat{\theta}_k)\left(\tilde{J}_i \hat{\xi}_k^1 + K_i u_k \right) \\ \hat{y}_k = \tilde{R} \hat{\xi}_k^1 \end{cases} \tag{49}$$

where the observer gains G_i are given by equation (31).

For all the results of computer simulations discussed below, we use the physical parameters whose definitions and numerical values are given in [19] and we assume $\tau = 0.012\,\mathrm{sec}$.

The expression of the unknown input signal is defined as in Figure 1.

Thus, by Theorem 1 with $\lambda = 0.95$ the following observer gains G_1 and G_2 are obtained:

$$G_1 = \begin{pmatrix} 0.7159 & -0.0541 & 0.0000 \\ -0.0227 & 0.4795 & 0.0120 \\ -0.3505 & 0.8821 & -0.0000 \\ 0.6012 & 0.3961 & 1.0921 \\ 3.7251 & -0.2957 & 0.0000 \end{pmatrix},$$

$$G_2 = \begin{pmatrix} 0.7159 & -0.0541 & 0.0000 \\ -0.0227 & 0.4795 & 0.0120 \\ -0.3505 & 0.8821 & -0.0000 \\ 0.2015 & 0.6913 & 1.0679 \\ 3.7251 & -0.2957 & 0.0000 \end{pmatrix}$$

Simulation results with initial conditions:

$Z_1(0) = [0 \quad 0.3142 \quad 0 \quad 0 \quad 4]^T$,

$Z_2(0) = [16.3645 \quad -16.6906]^T$

$\hat{Z}_1(0) = [0 \quad 0.6283 \quad 0 \quad 0.1 \quad 6]^T$,

$\hat{Z}_2(0) = [31.6479 \quad -32.3426]^T$

are given in Figures 1 to 7.

These simulation results show the performances of the proposed FUIO (17) with the gains G_1, G_2 where the dashed lines denote the state variables and unknown input estimated by the fuzzy observer. They show that the observer gives a good estimation of unknown states and unknown input of the considered robot.

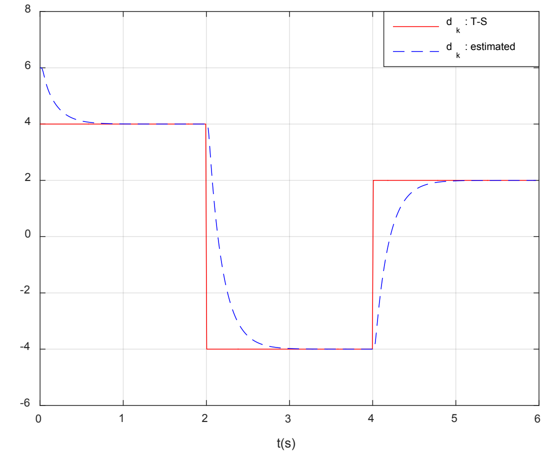

Figure 1. Unknown input d_k and its estimate

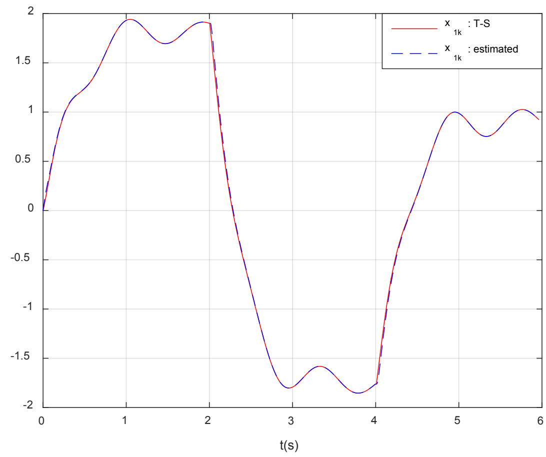

Figure 2. State variables x_{1k} and its estimate

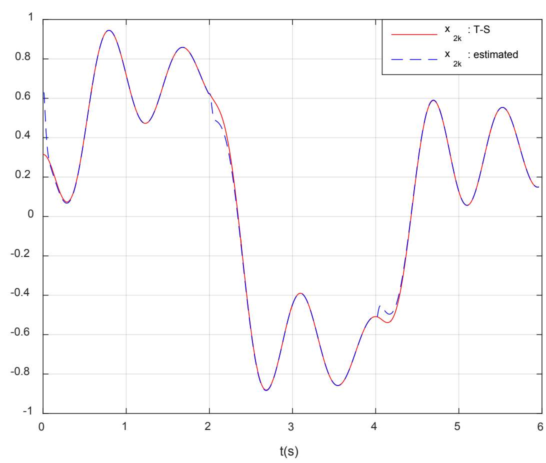

Figure 3. State variables x_{2k} and its estimate

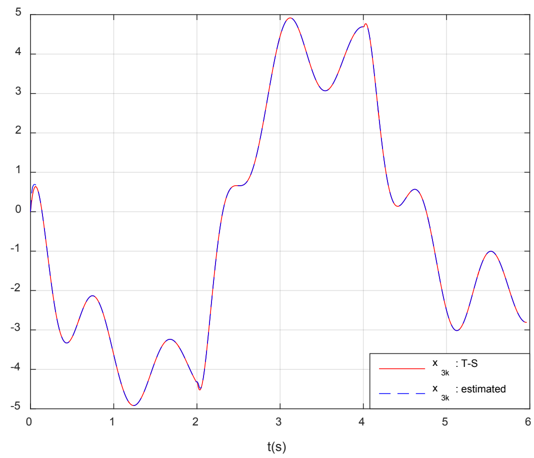

Figure 4. State variables x_{3k} and its estimate

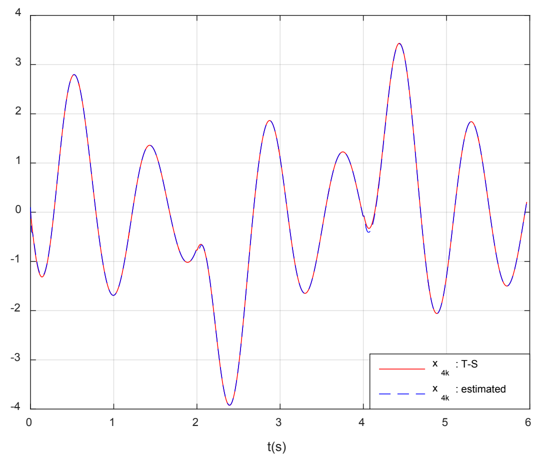

Figure 5. State variables x_{4k} and its estimate

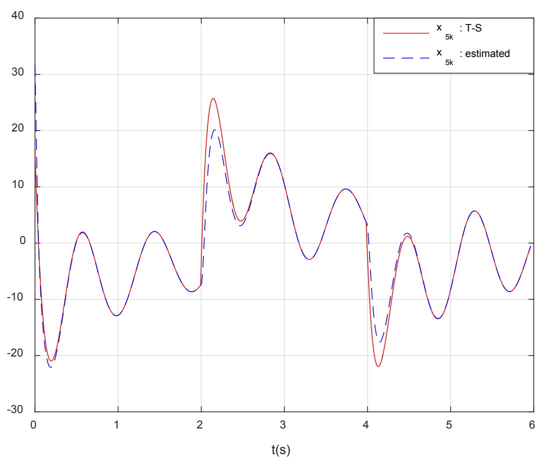

Figure 6. State variables x_{5k} and its estimate

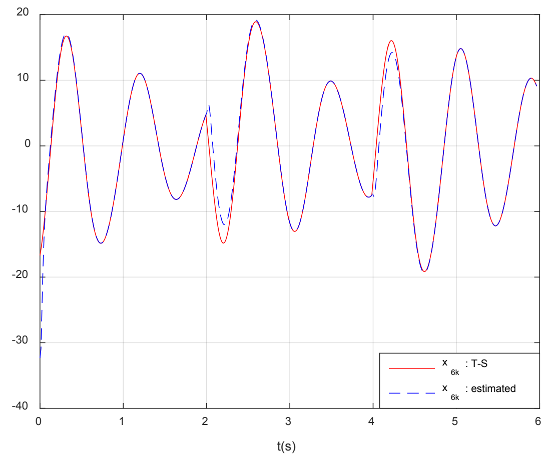

Figure 7. State variables x_{6k} and its estimate

5. Conclusions

In order to estimate simultaneously the unknown state and unknown inputs for a class of DNDMs described by T-S fuzzy structure with unmeasurable premise variables, a new FUIO design without the use of an optimization algorithm is proposed in this paper. The main idea is based on the separation between dynamic and static relations in the DTSDM and the use of an augmented system structure formed by dynamic equations and unknown inputs. The global exponential stability of the state estimation error is studied by using the Lyapunov theory and the conditions ensuring this stability are expressed in term of LMIs. In order to demonstrate the good performance of the proposed result, a DNDM of a single-link flexible joint robot is considered. The effectiveness of the proposed FUIO for the on-line estimation of unknown states and unknown inputs of the used model is verified by numerical simulation.

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