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Charateristic of Walter's B Visco-Elastic Nanofluid Layer Heated from Below

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Abstract The onset of convection in a horizontal layer of Walter's B' visco-elastic nanofluid is studied. A linear stability analysis based upon normal mode analysis is used to find solution of the fluid layer confined between two free boundaries. The onset criterion for stationary and oscillatory convection was derived analytically and graphically. The effects of the concentration Rayleigh number, Prandtl number, capacity ratio, Lewis number and kinematics visco elasticity Parameter on the stability of the system are investigated both on stationary and oscillatory convection. The sufficient conditions for the non-existence of oscillatory convection were also obtained.

Keywords Thermal Instability, Nanofluid, Walter's B' Visco- Elastic, Oscillatory Convection

1. Introduction

Nanofluid have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid powered engines, domestic refrigerator, chiller, heat exchanger and nuclear reactor, in grinding, in machining, in space, defense and ships, and in boiler flue gas temperature reduction. Nanofluid is a fluid colloidal mixture of nano (<100 nm) sized particles, in base fluid. Nanoparticles materials may be taken as oxide ceramics (Al_2O_3, CuO), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) etc. and base fluids are water, ethylene or tri-ethylene- glycols and other coolants, oil and other lubricants, bio-fluids, polymer solutions, other common fluids. The term 'nanofluid' was coined by Choi [1]. Since Choi proposed his theory on nanofluids a continuous effort has ensued to look for the causes of the so-called anomalous increase in thermal conductivity of nanofluids. The presence of nanoparticles in the fluid significantly increases the effective thermal conductivity of the mixture. Buongiorno [2] noted that the nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. He also discussed the effect of seven slip mechanisms: Inertia, Brownian diffusion, Thermophoresis, Diffusiophoresis, Magnus effect, Fluid drainage and Gravity setting. He concludes that in the absence of turbulent eddies Brownian diffusion and thermophoresis dominate the other slip mechanisms. Xuan and Li [3] investigated convective heat transfer and flow features of Cu-water nanofluid. They observed that the suspended nanoparticles remarkably enhance heat transfer process and the nanofluid has larger heat transfer coefficient than that of the original base liquid under the same Reynolds number. The heat transfer feature of a nanofluid increases with volume fraction of nanoparticles. A detail account of the thermal instability of Newtonian fluids has been discussed in detail by Chanderasekher [4]. The Bénard problem (the onset of convection in a horizontal layer uniformly heated from below) for a nanofluid was studied by many authors [5-16].

The above study deals with nanofluid as Newtonian fluid. There is growing importance of non-Newtonian fluids in geophysical fluid dynamics, chemical technology, petroleum, biological and material industries. The study of non-Newtonian nanofluid is desirable. Bhatia and Steiner [17] studied the thermal instability of visco-elastic fluids. An experimental demonstration by Toms et al. [18] has revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd [19]. There are many visco-elastic fluids which cannot be characterized by Maxwell's constitutive relations. Two such classes of elastico-viscous fluids are Rivlin-Ericksen and Walters' (Model B') fluids. Walters [20] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5g of polymer per litre with density 0.98g per litre behaves very nearly as the Walters' (Model B') fluid. Walters' (Model B') visco- elastic fluid forms the basis for the manufacture of many important polymers and useful products. Sharma et al. [21] have studied the stability of two superposed Walters' (Model B') liquids whereas

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thermosolutal convection problem in the presenceof magnetic field for Walters' (Model B') fluid has been investigated by Sunil et al. [22]. The interest for investigations of visco-elastic nanofluids is also motivated by a wide range of engineering applications which include ground pollutions by chemicals which are non-Newtonian like lubricants and polymers and in the treatment of sewage sludge in drying beds. Recently, polymers are used in agriculture, communications appliances and in bio medical applications. In the present study, we investigated the thermal instability of a visco-elastic (Model B') nanofluid fluid.

2. Mathematical Formulations

Consider an infinite horizontal layer of Walter's B' elastico-viscous nanofluid of thickness'd' bounded by plane z = 0 and z = d and heated from below. Each boundary wall is assumed to be impermeable and perfectly thermal conducting. Fluid layer is acted upon by gravity force **g**(0, 0, -g). The temperature T and volumetric fraction φ of nano particles at z = 0 taken to be T_0 and φ_0 at z = 0 and T_1 and φ_1 at z = d, ($T_0 > T_1$). The reference temperature is taken to be T_1. Thermo physical properties of the nano fluid are constant for the analytical formulation but these properties are not constant and strongly depend upon volume fraction of the nano particles.

Thus the governing equations for Walter's B' elastico-viscous nanofluid are

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$z\rho\frac{d\mathbf{q}}{dt} = -\nabla p + \rho\mathbf{g} + \left(\mu - \mu'\frac{\partial}{\partial t}\right)\nabla^2\mathbf{q}, \tag{2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\nabla)$ stands for convection derivative, q(u, v, w) is the velocity vector, p is the hydrostatic pressure, μ is viscosity, μ' kinematic visco-elasticity and g(0, 0, -g) is acceleration due to gravity.

The ρ density of the nanofluid can be written as Buongiorno[2]

$$\rho = \varphi\rho_p + (1-\varphi)\rho_f, \tag{3}$$

where φ is the volume fraction of the nanoparticles, ρ_p density of nano particles and ρ_f density of base fluid.

Using equation (3), equation of motion for Walter's B' elastico-viscous nanofluid is given as

$$\begin{aligned}\rho\frac{d\mathbf{q}}{dt} &= -\nabla p + \left(\varphi\rho_p + (1-\varphi)\left\{\rho\left(1-\alpha\left(T-T_0\right)\right)\right\}\right)\mathbf{g} \\ &+ \left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2\mathbf{q}.\end{aligned} \tag{4}$$

where α is the coefficient of thermal expansion.

The continuity equation for the nanoparticles is

$$\frac{\partial\varphi}{\partial t} + \mathbf{q}.\nabla\varphi = D_B\nabla^2\varphi + \frac{D_T}{T_1}\nabla^2 T. \tag{5}$$

where D_B is the Brownian diffusion coefficient, given by Einstein-Stokes equation and D_T is the thermoporetic diffusion coefficient of the nanoparticles given as:

The energy equation in nanofluid is

$$\begin{aligned}&\rho c\left(\frac{\partial T}{\partial t} + \vec{q}.\nabla T\right) \\ &= k\nabla^2 T + (\rho c)_p\left(D_B\nabla\varphi.\nabla T + \frac{D_T}{T_1}\nabla T.\nabla T\right)\end{aligned} \tag{6}$$

where ρc is heat capacity of fluid, $(\rho c)_p$ is heat capacity of nano particles and k is thermal conductivity.

We introduce non-dimensional variables as

$$(x', y', z',) = \left(\frac{x, y, z}{d}\right),\ (u', v', w',) = \left(\frac{u, v, w}{\kappa}\right)d,$$

$$t' = \frac{t\kappa}{d^2},\ p' = \frac{p}{\rho\kappa^2}d^2,\ \varphi' = \frac{(\varphi-\varphi_0)}{(\varphi_1-\varphi_0)},\ T' = \frac{(T-T_1)}{(T_0-T_1)}.$$

where $\kappa = \frac{k}{\rho c}$ is thermal diffusivity of the fluid.

There after dropping the dashes (') for simplicity.

$$\nabla.\mathbf{q} = 0, \tag{7}$$

$$\frac{1}{P_r}\frac{\partial\mathbf{q}}{\partial t} = -\nabla p + (1-nF)\nabla^2\mathbf{q} - Rm\hat{e}_z + RaT\hat{e}_z - Rn\varphi\hat{e}_z \tag{8}$$

$$\frac{\partial\varphi}{\partial t} + \mathbf{q}.\nabla\varphi = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T \tag{9}$$

$$\frac{\partial T}{\partial t} + \mathbf{q}.\nabla T = \nabla^2 T + \frac{N_B}{Le}\nabla\varphi.\nabla T + \frac{N_A N_B}{Le}\nabla T.\nabla T \tag{10}$$

where non-dimensional parameters are (**Table 1**):

In the spirit of the Boussinesq approximations, equation (8) was linearized by neglecting the term proportional to the product of φ and T. Neglecting this value is valid in the case of small temperature gradients in the dilute suspension of nanoparticles.

We assume that temperature and volumemetric fraction of the nanoparticles are constants on boundaries. Thus the dimensionless boundary conditions are:

$$w = 0,\ T = 1,\ \varphi = 0 \ \text{ at } \ z = 0 \ \text{ and}$$
$$w = 0,\ T = 0\ \ \varphi = 1 \ \text{ at } \ z = 1. \tag{12}$$

2.1. Basic Solutions

The basic state was assumed to be quiescent and is given by:

$$u = v = w = 0,\ p = p(z),\ T = T_b(z)\ \ \varphi = \varphi_b(z). \tag{13}$$

Approximation for the solution is given by:

$$T_b = 1 - z, \quad \varphi_b = z. \tag{14}$$

Table 2. List of non-dimension less parameters

S.No	Parameters	Relations
1.	Prandtl number	$P_r = \frac{\mu}{\rho\kappa}$
2.	Lewis number	$Le = \frac{\kappa}{D_B}$
3.	Rayleigh number	$Ra = \frac{\rho g \alpha d^3 (T_0 - T_1)}{\mu\kappa}$
4.	Basic-density Rayleigh number	$Rm = \frac{\rho_p \varphi_0 + \rho(1-\varphi_0) g d^3}{\mu\kappa}$
5.	Nanoparticle Rayleigh number	$Rn = \frac{(\rho_p - \rho)(\varphi_1 - \varphi_0) g d^3}{\mu\kappa}$
6.	Kinematic visco-elasticity parameter	$F = \frac{\mu'}{\rho d^2}$
7.	Modified diffusivity ratio	$N_A = \frac{D_T (T_0 - T_1)}{D_B T_1 (\varphi_1 - \varphi_0)}$
8.	Modified particle-density increment.	$N_B = \frac{(\rho c)_p (\varphi_1 - \varphi_0)}{(\rho c)_f}$

2.2. Perturbation Solutions

To study the stability of the system, we superimposed infinitesimal perturbations on the basic state, which are of the forms

$$\begin{aligned} q\,(u,v,w) &= 0 + q'\,(u,v,w), T \\ &= T_{b,} + T', \varphi = \varphi_b + \varphi', p \\ &= p_b + p', \text{with } T_b = 1\text{-}z, \varphi_b = z. \end{aligned} \tag{15}$$

(There after dropping the dashes for simplicity.)

Using the equation (15) in the equations (7) – (10) and linearize by neglecting the product of the prime quantities we obtained following equations

$$\nabla.\mathbf{q} = 0, \tag{16}$$

$$\frac{1}{P_r}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + (1 - nF)\nabla^2 \mathbf{q} + RaT\hat{e}_z - Rn\varphi\hat{e}_z \tag{17}$$

$$\frac{\partial \varphi}{\partial t} + w = \frac{1}{Le}\nabla^2\varphi + \frac{N_A}{Le}\nabla^2 T \tag{18}$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{Le}\left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z}\right) - 2\frac{N_A N_B}{Le}\frac{\partial T}{\partial z}. \tag{19}$$

And boundary conditions

$$\begin{aligned} w = 0, \quad T = 0 \quad \varphi = 0 \text{ at } \quad z = 0, \\ w = 0, \quad T = 0 \quad \varphi = 0 \text{ at } \quad z = 1. \end{aligned} \tag{20}$$

It will be noted that the parameter Rm is not involved in these and subsequent equations. It is just a measure of the basic static pressure gradient.

The six unknown's u, v, w, p, T and φ *c*an be reduced to three by operating equation (17) with $e_z.\text{curl curl},$ we get

$$\frac{1}{P_r}\frac{\partial}{\partial t}\nabla^2 w - (1 - nF)\nabla^4 w = Ra\nabla_H^2 T - Rn\nabla_H^2\varphi, \tag{21}$$

where $\nabla_H^2,$ is two-dimensional Laplacian operator.

3. Normal Modes

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form

$$[w,\theta,\varphi] = [W(z),\Theta(z),\Phi(z)]\exp(ik_x x + ik_y y + nt) \tag{22}$$

where $k_{x,}\, k_y$ are wave numbers in x and y direction and n is growth rate of disturbances.

Using equation (22), equations (18), (19) and (21) becomes

$$(D^2 - a^2)\left((1 - nF)(D^2 - a^2) - \frac{n}{P_r}\right)W - a^2 Ra\Theta + a^2 Rn\Phi = 0 \tag{23}$$

$$W\text{-}\frac{N_A}{Le}(D^2 - a^2)\,\Theta - \frac{1}{Le}(D^2 - a^2 - n)\,\Phi = 0 \tag{24}$$

$$W + \left(D^2 - a^2 - n + \frac{N_A}{Le}D - \frac{2N_A N_B}{Le}D\right)\Theta - \frac{N_B}{Le}D\Phi = 0. \tag{25}$$

Where $D = \frac{d}{dz}$ and $a^2 = k^2_x + k^2_y$ is dimensionless the resultant wave number.

The boundary conditions for free-free boundaries surfaces are thus

$$\begin{aligned} W = 0, \quad D^2W = 0, \Theta = 0, \Phi = 0 \text{ at } \quad z = 0 \quad \text{and} \\ W = 0, \quad D^2W = 0, \Theta = 0, \Phi = 0 \text{ at } \quad z = 1. \end{aligned} \tag{26}$$

4. Linear Stability Analysis

The solution to be W, Θ and Φ is of the form

$$W = W_0 \sin\pi\ z, \Theta = \Theta_0 \sin\pi z, \Phi = \Phi_0 \sin\pi z \tag{27}$$

satisfying boundary conditions (26).

Substituting solution (27) in equations (23) - (25), integrating each equation from z = 0 to z = 1 and performing some integrations by parts, we obtain Eigen equation

$$\mathrm{Ra}=\frac{1}{a^{2}}\left((J+n)\left(J^{2}+\left(J^{2}F-\frac{J}{P_{r}}\right)n\right)\right)-\frac{\left(\frac{N_{A}}{Le}J+J\right)+n}{\frac{J}{Le}+n}\mathrm{Rn}. \quad (28)$$

Setting $n = i\omega$, (where ω is real and dimensional frequency) in equation (28), we get

$$\mathrm{Ra}=\Delta_{1}+i\omega\Delta_{2} \quad (29)$$

where

$$\Delta_{1}=\frac{J}{a^{2}}\left(J^{2}-\left(\frac{1}{P_{r}}-JF\right)\omega^{2}\right)-\frac{\frac{J^{2}}{Le^{2}}\left((N_{A}+Le)+\omega^{2}\right)}{\left(\frac{J}{Le}\right)^{2}+\omega^{2}}\mathrm{Rn} \quad (30)$$

and

$$\Delta_{2}=\frac{J^{2}}{a^{2}}\left(1-JF+\frac{J}{P_{r}}\right)-\frac{\frac{J}{Le}-J\left(\frac{N_{A}}{Le}+1\right)}{\left(\frac{J}{Le}\right)^{2}+\omega^{2}}\mathrm{Rn}. \quad (31)$$

either $\omega = 0$ (exchange of stability, steady state) or $\Delta_2 = 0$ (ω # 0 overstability or oscillatory onset).

4.1. Stationary Convection

For stationary convection $\omega = 0$ ($n = 0$), equation (29) reduces to

$$(\mathrm{Ra})_{s}=\frac{(\pi^{2}+a^{2})^{3}}{a^{2}}-\mathrm{Rn}(Le+N_{A}). \quad (32)$$

We find that for the stationary convection the kinematic visco-elasticity parameter F vanishes with n and the Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid.

Figure 3 represent the variation of stationary Rayleigh number with wave number for different value of Lewis number Le and it is found that stationary Rayleigh number increases with Lewis number, thus Lewis number stabilize stationary convection. Figure 1 represent the variation of stationary Rayleigh number with wave number for different value of concentration Rayleigh number Rn and it is found that stationary Rayleigh number decreases with increase in the value concentration Rayleigh number Rn which imply that concentration Rayleigh number destabilize the stationary convection. The negative value of Rn indicates a bottom-heavy distribution while positive value of Rn indicates a top-heavy distribution of nano particles.

It is also observed that stationary convection is possible for both bottom- and top- heavy nanoparticles distribution and stationary Rayleigh number is smaller for top-heavy than that of bottom-heavy distribution of nano particles.

Figure 2 represents the variation of stationary Rayleigh number with wave number for the different values of modified diffusivity ratio N_A and it is noted that stationary Rayleigh number increases with increase in the value modified diffusivity ratio N_A, thus modified diffusivity ratio N_A has stabilize the stationary convection.

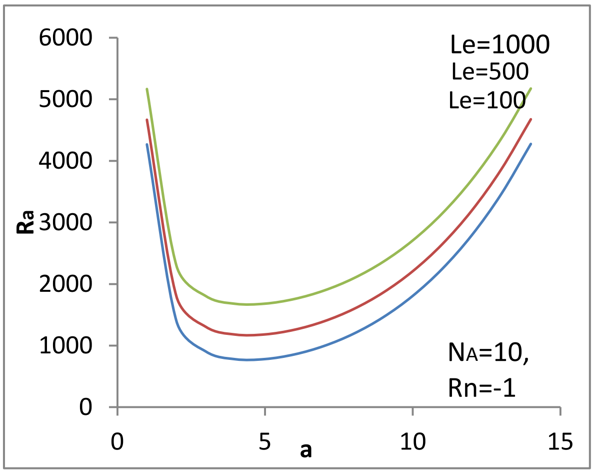

Figure 1. Variation of stationary Rayleigh number Ra with wave number - a for different values concentration Rayleigh number

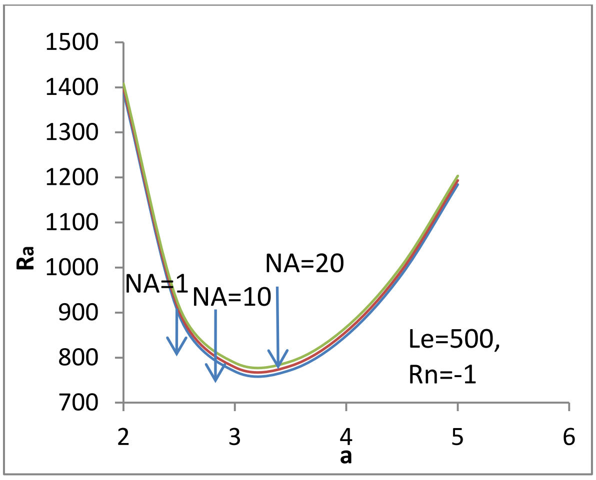

Figure 2. Variation of stationary Rayleigh number Ra with wave number - a for different values of modified diffusivity ratio

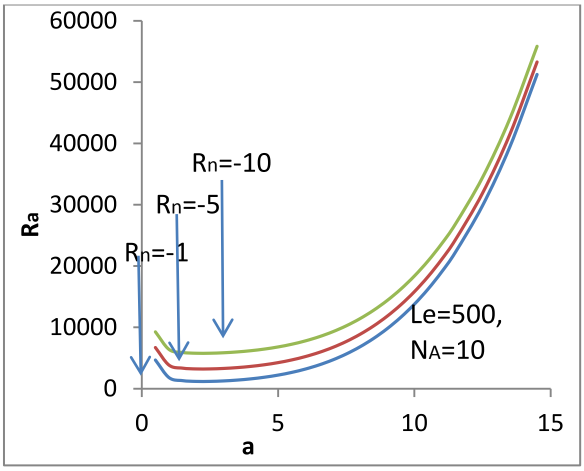

Figure 3. Variation of stationary Rayleigh number Ra with wave number- a for different values of Lewis number Le

4.2. Oscillatory Convection

For oscillatory convection ω # 0, we must have $\Delta_2 = 0$, which gives

$$\omega^2 = \frac{a^2}{J}\left(\frac{\frac{1}{Le}-\left(\frac{N_A}{Le}+1\right)}{1-JF+\frac{1}{P_r}}\right)Rn - \frac{J^2}{Le^2}. \quad (33)$$

Equation (33) gives the frequency of oscillatory mode. If there is no positive of ω^2 then oscillatory instability is not possible. If there exist positive values of ω^2, then thermal oscillatory Rayleigh number is obtain by putting the positive values of ω^2 in equation (28).

Thus thermal oscillatory Rayleigh number given by

$$(Ra)_{osc} = \frac{J}{a^2}\left(J^2 - \left(\frac{1}{P_r} - JF\right)\omega^2\right) - \frac{\frac{J^2}{Le^2}(Le+N_A)+\omega^2}{\left(\frac{J}{Le}\right)^2+\omega^2}Rn, (34)$$

where ω^2 is given by equation (33).

If $Rn < 0$ and $1 > (N_A + Le)$ and $JF < \left(1+\frac{1}{P_r}\right)$. then ω^2 is negative and hence oscillatory convection cannot occur.

Thus for $Rn < 0$ and $1 > (N_A + Le)$ and $JF < \left(1+\frac{1}{P_r}\right)$. are sufficient conditions for the non-existence oscillatory convection, the violation of which does not necessarily imply the occurrence of oscillatory convection.

5. Results and Discussions

Expression for stationary thermal Rayleigh number is given in equation (32) and for oscillatory thermal Rayleigh number is given in equation (34). We have discussed our results graphically.

Figure 4 shows the variation of oscillatory Rayliegh number R_a with wave number a for different value of kinematic visco-elasticity parameter F and it is found that oscillatory Rayliegh number Ra decreases with increases in the values of kinematic visco-elasticity parameter F, thus kinematic visco-elasticity parameter F have destabilzing effect on the oscillatory convection.

Figure 5 shows the variation of oscillatory Rayliegh number Ra with wave number a for different value of modified diffusivity ratio N_A and it is found that the oscillatory Rayliegh number Ra slightly decreases as values of modified diffusivity ratio N_A inceases, thus N_A destabilze the oscillatory convection.

Figure 6 shows the variation of oscillatory Rayliegh number with wave number a for different values of Prandtl number P_r and it is found that the oscillatory Rayliegh number Ra slightly decreases as values of Prandtl number P_r inceases, thus Prandtl number P_r has destabilzing effect on oscillating convection.

Figure 7 shows the variation of oscillatory Rayliegh number with wave number a for different values of concentration Rayleigh number Rn and it is observed that oscillatory Rayliegh number slightly decreases with increases in the values of concentration Rayleigh number Rn, (for bottom-heavy distribution of nanoparticles), thus concentration Rayleigh number Rn destabilze the oscillatory convection. It was also observed that oscillatory convection was not possible for top-heavy distribution of nano particles.

Figure 8 shows the variation of oscillatory Rayliegh number with wave number a for different values of Lewis number Le and it is found that the oscillatory Rayliegh number decreases as values of Lewis number Le inceases, thus Lewis number Le has destabilzing effect on on oscillating convection.

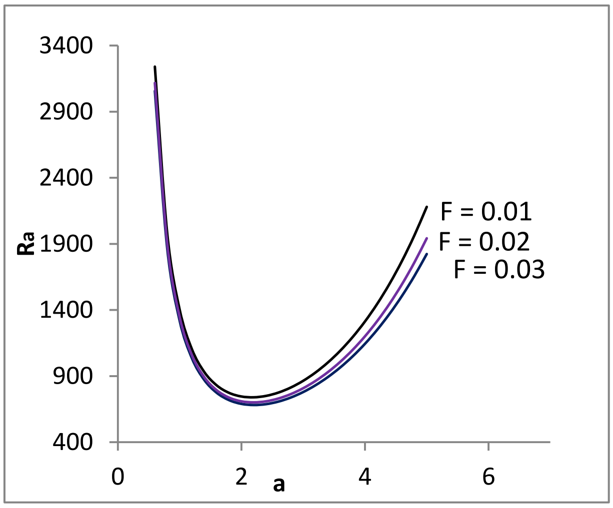

Figure 4. Variation of oscillatory Rayleigh number Ra with wave number - a for different concentration Rayleigh number

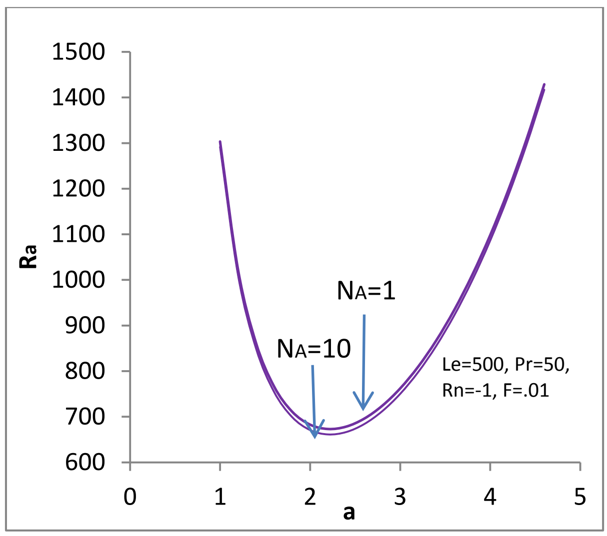

Figure 5. Variation oscilltory Rayleigh number with wave number - a for different value of kinematic visco-elasticity parameter F

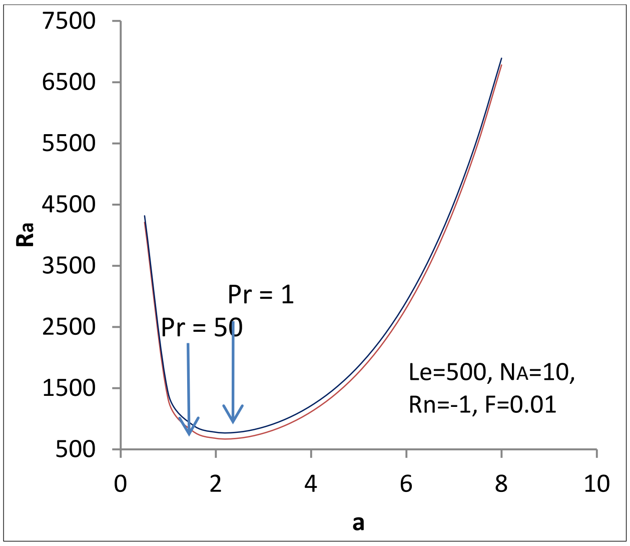

Figure 6. Variation of oscillatory Rayliegh number with wave number - a for different modified diffusivity ratio

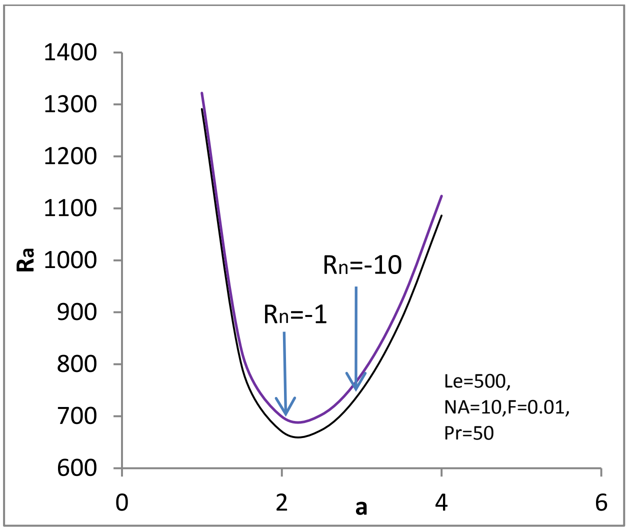

Figure 7. Variation of oscillatory Rayliegh number with wave number- a for different Prandtl number

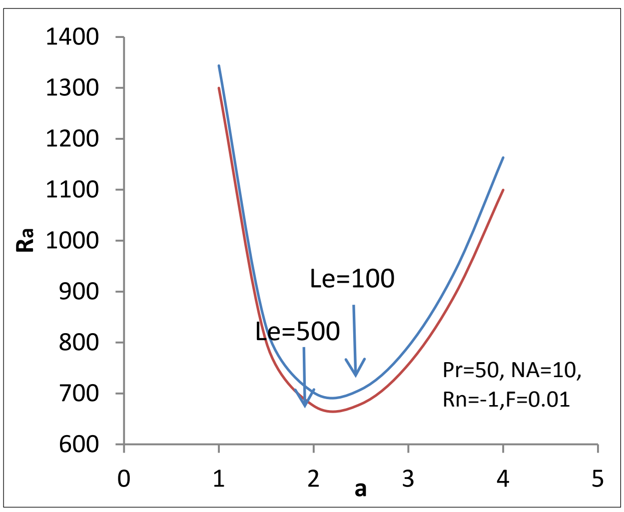

Figure 8. Variation of oscillatory Rayliegh number with wave number - a for different Lewis number Le

6. Conclusions

We studied linear instability of Walter's B' elastico-viscous nanofluid heated from below by employing a model that incorporate the effects of Brownian motion, thermophoresis and visco elasticity. An expression for Rayleigh number, for the stationary convection and oscillatory convection is obtained. We draw following conclusion:

(i) The critical cell size is not a function of any thermophysical properties of nanofluid.

(ii) The effect of Lewis number Le and modified diffusivity ratio N_A is to stabilizes the stationary convection and destabilize s the oscillatory convection.

(iii) The concentration Rayleigh number Rn destabilizes both stationary and oscillatory convection.

(iv) The oscillatory convection is possible only for the bottom-heavy nanoparticles distribution while stationary convection is possible for both bottom and top-heavy distribution of nanoparticles.

(v) The Prandtl number P_r destabilizes the oscillatory convection and no has effect on stationary convection.

(vi) Kinematic visco-elasticity parameter F destabilizes the oscillatory convection and no has effects on stationary convection.

(vii) A comparison between nano fluid and ordinary fluid made and it is found that nano fluid is more stable than ordinary fluid.

(viii) It was also found that Rayleigh number in stationary convection has higher value in than that of Rayleigh number in oscillatory convection.

The sufficient conditions for the non-existence of oscillatory convection are

$$\mathrm{Rn} < 0 \text{ and } 1 > \left(\mathrm{N_A} + \mathrm{L}e\right) \text{ and } \mathrm{JF} < \left(1 + \frac{1}{\mathrm{P_r}}\right).$$

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