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On the Combined Analysis of Sudoku Square Designs with Some Common Treatments

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Abstract Experiments are repeatedly carried out at several times, locations (environments) or season (period) with one or more treatments in common to each experiment, to investigate the interaction between the treatments and location (enviroment) of the experiment or treatments and times or season (period). The analysis of such experiments are carry out using combined analysis of technique. This paper proposed combined analysis of experiments conducted, using Sudoku Square designs of odd order when some treatments are common to all the experiments.

Keywords Combined analysis, Sudoku Square design, Multi-experiments

1. Introduction

Experiment may be conducted with the aid of randomized block designs or Latin square designs or balanced incomplete designs over different environments. The reason may be due to lack of space that would accomodate all the experimental plots or with the underlying condition that the experiments must be carried out at different locations (environment) or different seasons, whatever may be the reason, the major aim is to do joint analysis of the data obtained from these multi-environments instead of doing the analysis separately or individually (Albassan and Ali, 2014) and one of the advantages of multi-environment analysis is that it increases the accuracy of evaluation (hence the accuracy of selection). The joy derived accuracy is a function of several factors or treatments (Cullis *et al.,* 2010). The other reasons are estimation of consistency of treatments effects for a particular environments over large population of environments (Blouin *et al*., 2011).

Gomes and Gumarie (1958) considers the intra-block analysis of a group of experiments in complete randomized block, where treatments applied are all different but some treatments are common to the whole experiment, some examples were given, separate tables of analysis and the table of the combined analysis were also given. They further gave the condition that must be met before combined analysis can be carried out, as such, if the residual variance estimates from separate analyses are not too different a joint analysis can be carried out for the whole set of the experiments.

Pavate (1961) used the same approach and obtained the combined analysis of balancded incomplete block design with some common treatments applied to all the experiments of the multi-environment, methods of obtaining adjusted treatment sumof square was explained and illustration was given.

Giri (1963) discusses combination of the method used by Pavate (1961), he said Pavate method can be used to analyzed data from combined data of set of Youden Squares when some treatments are common while that of Gomes and Gumarie can be used for combined analysis of a set of latin square when some treatments are common to all the experiments. Mcintosh (1983) gave a clue for the analysis of combined experiments, the tables indicate sources of variation, degree of freedom and F-ratio for factors and split-plot experiment combined over location and / or year. F-ratios for the fixed, mixed and random model were as well presented. Paul (1989) discusses method used for the combined analysis when treatment s are used to represents levels of quantitative factor but differ among experiments. Multiple regresion analysis was used when a continuous variable represents treatment levels, classification variables represents experiments and product of the continuous and classification variables present differences among experiments. Analyses for experiments combined across years and location are presented by Moore and Dixon (2015). This is similar to Mcintosh (1983) but the F tests are specific based on the alternative about assumption about mixed interactions, i.e the fixed effects do not sum to zero in a mixed interaction. Although, there is little research on the combined analysis of Sudoku square designs, until recent time, Danbaba (2016) presented a paper on the combined analysis of Sudoku Square Designs, of odd order with same

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experimental treatments. A combined analysis of data from multi-environments experiment was presented using Sudoku square designs and modified linear model and the tables of analysis were as well presented.

Sudoku is the abbreviation of the japanese longer phrase "suji wa dokushin ni kagiru" which means the digit must occur only once (Berthier, 2007; Gordon and Longo, 2006). Sudoku puzzle is a very popular game, the objective of the game is to complete a 9 × 9 grid with digits from 1 to 9. Each digit must appear once only in each row, each column and each of the 3 × 3 boxes see Bailey et al., (2008).

Sudoku square design consists of treatments which are arranged in a square array such that each row, column or sub-square of the design contains each of the treatments exactly once (Hui-Dong and Ru-Gen, 2008). Away from the construction process of Sudoku square, mathematical models and statistical method of data analysis were presented by Hui-Dong and Rui-Gen, (2008). He observed that sudoku sqauare design go beyond Latin sqaure design with one additional source of variation i.e box effect.

Subramani and Ponnuswamy (2009) came up with an extension of the work done by Hui-Dong and Rui-Gen in 2008, with additional terms in the models and in the sources of variation namely Row-block effect, and column-block effect. They considered Sudoku of order $k = m^2$ and gave detail analysis with illustrative example and the application of Sudoku square to agricultural experiment. If we compare the models by Hui-Dong and Rui-Gen, (2008) and that of Subramani and Ponnuswamy, we observed that Hui-Dong and Rui-Gen did not includes row-block and column-block effects into their analysis but Subramani and Ponnuswamy went ahead to include row-block and column-block effects into their analysis and proposed four differents models with their respective ANOVA tables.

	CB 1			CB 2			CB 3		
A	B	C	D	E	F	G	H	I	
D	E	F	G	H	I	A	B	C	RB 1
G	H	I	A	B	C	D	E	F	
B	C	A	E	F	D	H	I	G	
E	F	D	H	I	G	B	C	A	RB 2
H	I	G	B	C	A	E	F	D	
C	A	B	F	D	E	I	G	H	
F	D	E	I	G	H	C	A	B	RB 3
I	G	H	C	A	B	F	D	E	

Figure 1. Typical example of Sudoku square with 3 × 3 square region

This paper proposed combined analysis of multi-environment experiments carried out using Sudoku Square design of odd order when treatments are not the same in all Sudoku square but some treatments are common to the whole set of experiments.

2. Analyses

If we assumed that there g multi-environment experiments each carried –out using Sudoku square design, each with $k = z + c$ treatments, of which $c$treatments are common to all Sudoku square design multi-environment experiments which connects them out of k treatments and z treatments are not common but for the sake of this research it will be called ''uncommon''treatments in each design. The total number of different treatments in each design is then $gz + c$. The "uncommon" treatments have k replications because uncommon treatments are limited to a particular Sudoku square design in an experimental environment and the common ones are replicated gk times that is, g environments x k replications. We carry-out analysis of variance for each experiment in the usual way and obtained tests of significant and estimate of effects with errors has suggested by Gomes and Gumarie (1958). If the residual variance estimates are not significantly different a combined analysis can be carried out for the whole set of experiments.

Sudoku square design assume that having g multi-environment experiments each with k treatments, each occurs only once in each row, column or sub-block or square) such that $k = m^2$ where k and m (number of row-block and column-block) are odd see (Subramani and Ponnuswamy 2009). If for example fig1. is 9×9 Latin square or 3×3 Sudoku square design. Considering the fig 1., the number of row-block and column-block is 3 which is an odd number.

In this study, (Hui-Dong andRui-Gen, 2008) and (Subramani and Ponnuswamy, 2009) models were modified and combined analysis for each were discussed. For all the models considered in this study, it is assumed that all effects are fixed and only $k = m^2(z + c)$ number of treatments is discussed.

Model 1

Hui-Dong and Rui-Gen (2008) proposed linear model for Sudoku square design and being modified to includes the multi-environment experiment as follows.

$$Y_{ijlmx} = \mu + \theta_x + \alpha_{ix} + \beta_{jx} + \delta_{lx} + \gamma_{mx} + (\alpha\theta)_{ix} + \varepsilon_{(i,j)lmx} \begin{cases} i = 1,2,\cdots k \\ j = 1,2,\cdots k \\ l = 1,2,\cdots k \\ m = 1,2,\cdots k \\ x = 1,2,\cdots g \end{cases} \qquad (1)$$

where Y_{ijlmx} is a obseved value of the plot in the lth row and mth column, subjected to theith treatment, jth box of the xth experimental environment; μ is the grand mean, $\alpha_i, \beta_j, \delta_l, \gamma_m, \theta_x, (\alpha\theta)_{ix}$ are the main effects of the ith treatment, jth box, lth row, mth column, xth experimental environment and interaction between common treatments and environment respectively and $\varepsilon_{(i,j)lmx}$ is the random error.

Where SSEE is the sum of square of the experimental environment, SSt is the total treatment sum of square for all the multi-environment experiments, SSr is the total row sumof squres for all the multi-environment experiments, SSc the total column sumof squares for all the multi-environment experiments, SSs is the total sub-block sumof squares for all the multi-environment experiments, SSI is the total interaction sumof squares between common treatment and the multi-environment experiments and Sse the total error sumof squares for all the multi-environment experiments.

If we let S_x^r , S_x^c, S_x^s , S_x^t , S_x^{ct} and S_x^e be the xth environment row, column, sub-square, treatment, common treatment and error sum of square respectively.

$$SSr = \sum_{x=1}^{g} S_x^r, \quad SSc = \sum_{x=1}^{g} S_x^c, \quad SSs = \sum_{x=1}^{g} S_x^s$$

$$SSt = \sum_{x=1}^{g} S_x^t, \quad SSct = \sum_{x=1}^{g} S_x^{ct}, \quad SSe = \sum_{x=1}^{g} S_x^e$$

The alternative method of the above can be obtained as follows

$$S_1 = y_{....}^2 / gk^2, \quad S_2 = \frac{\sum_x y_{....x}^2}{k^2}, \quad S_3 = \frac{\sum_l \sum_x y_{..l.x.}^2}{k},$$

$$S_4 = \frac{\sum_m \sum_x y_{..mx.}^2}{k} \quad S_5 = \frac{\sum_i y_{i...}^2}{gk}, \quad S_6 = \frac{\sum_x \sum_i y_{..ix.}^2}{k},$$

$$S_7 = \frac{\sum_j \sum_x y_{..ix.}^2}{k}, \quad S_8 = \sum_i \sum_j \sum_g \sum_m y_{ijlmx}^2,$$

$$S_9 = \frac{\sum_i y_{....ct}^2}{gk}$$

The computational analysis of the sum of the squares for the remaining models (2-5) are almost the same with model 1 and therefor not discussed in this paper for the detail computation analysis check Subramani and Ponnuswamy (2012) for separate analysis.

Table 1. ANOVA Table for the combined analysis of the set of Sudoku square design for model 1

Source	df	Sum of Square
Experiments	$g(k\text{-}1)$	SSEE
Treatments	$gz + c - 1$	SSt
Rows	$g(k\text{-}1)$	SSr
Columns	$g(k\text{-}1)$	SSc
Sub-block(Boxes)	$g(k\text{-}1)$	SSs
Env.experiment × treatment	$(g\text{-}1)(c\text{-}1)$	SSI
Error	By Subtraction	Sse
Total	$gk^2 - 1$	SST

SSI are obtained by adding all Env.experiment × treatment as suggested by Pavate the same goes to other models.

Model 2: The modification of Type I of Subramani & Ponnuswamy linear model is as follows. We will assume that row, column, and treatments effect as in the latin square, also in addition to the assumption, Row block, column block, square effects, environmental experiments and interaction between common treatments and environmental experiments.

$$Y_{ij\,(klpq)} = \mu + \alpha_{ix} + \beta_{jx} + \tau_{kx} + \gamma_{lx} + C_{px} + S_{qx} + \theta_x + \left(\tau_{k'}\theta\right)_{ix} + \varepsilon_{(i,j)klpqx} \quad (2)$$

$i, j = 1,2 \cdots m$ and $k, l, p, q = 1,2, \cdots m^2 x = 1,2, \cdots g$.

Where μ is general mean, α_{ix} is ith row-block effect in xth environmental experiment, β_{jx} is jth column-block effect in xth environmental experiment, τ_{kx} is kth treatment effect in xth environmental experiment, γ_{lx} is the lth row effect in xth environmental experiment, C_{px} is pth the column effect in xth environmental experiment, S_{qx} is the qth sub-square effect in xth environmental experiment, θ_x is the xth environmental experiment, $\left(\tau_{l'}\theta\right)_{ix}$ is the interaction between $k'th$ common treatment and xth environmental experiment and $\varepsilon_{(i,j)klpqx}$ error component with mean zero and constant variance.

Table 2. ANOVA Table for the combined analysis of the set of Sudoku square design for model 2

Source	df	Sum of Square
Experiments	$g(k\text{-}1)$	SSEE
Treatments	$gz + c - 1$	SSt
Rows	$g(k\text{-}1)$	SSr
Columns	$g(k\text{-}1)$	SSc
Sub-squares	$g(k\text{-}1)$	SSs
Row-block	$g(m - 1)$	SSrb
Column-block	$g(m - 1)$	SScb
Env.experiment × treatment	$(g\text{-}1)(c\text{-}1)$	SSI
Error	By Subtraction	Sse
Total	$gk^2 - 1$	SST

Model 3 : The modification of Type II of Subramani & Ponnuswamy linear model is as follows. The model assumed that row effects are nested in the row block effect and the column effects are nested in the column block effects. In addition to, environmental experiments and interaction between common treatments and environmental experiments.

$$Y_{ij\,(k,l,p,q)} = \mu + \alpha_{ix} + \beta_{jx} + \tau_{kx} + \gamma(\alpha)_{l(i)x} + C(\beta)_{p(j)x} + s_{qx} + \theta_x + \left(\tau_{k'}\theta\right)_{ix} + \varepsilon_{i,j\,(k,l,p,q)} \quad (3)$$

$i, j\ l, p = 1 \cdots m \quad k, q = 1 \cdots m^2 \quad x = 1,2, \cdots g$

Where μ is general mean, α_{ix} is ith row-block effect in xth environmental experiment, β_{jx} is jth column-block effect in xth environmental experiment, τ_{kx} is kth treatment effect in xth environmental experiment, $\gamma(\alpha)_{l(i)x}$ is the lth row effect nested in ith row-block in xth environmental experiment, $C(\beta)_{p(j)x}$ is the pth column effect in jth column-blockin xth environmental experiment, S_{qx} is the qth sub-square effect in xth environmental experiment, θ_x is the xth environmental experiment, $\left(\tau_{k'}\theta\right)_{ix}$ is the interaction between $k'th$ common treatment and xth environmental experiment and $\varepsilon_{(i,j)klpqx}$ error component with mean zero and constant variance.

Table 3. ANOVA Table for the combined analysis of the set of Sudoku square design for model 3

Source	df	Sum of Square
Experiments	$g(k$-1)	SSEE
Treatments	$gz + c - 1$	SSt
Rows within row-block	$gm(m - 1)$	SSrrb
Columns within column block	$gm(m - 1)$	SSccb
Sub-squares	$g(k$-1)	SSss
Row-block	$g(m - 1)$	SSrb
Column-block	$g(m - 1)$	SScb
Env.experiment × treatment	(g-1)(c-1)	SSI
Error	By Subtraction	Sse
Total	$gk^2 - 1$	SST

Model 4: The modification of Type III of Subramani & Ponnuswamy linear model is as follows. The model assumes that the horizontal square effects are nested in the row block and vertical Square effects are nested in the column block effects. In addition to, environmental experiments and interaction between common treatments and environmental experiments.

$$Y_{ij\,(k,l,p,q,r)} = \mu + \alpha_{ix} + \beta_{jx} + \tau_{kx} + \gamma_{lx} + c_{px} + s(\alpha)_{q(i)x} + \pi(\beta)_{r(j)x} + \theta_x + \left(\tau_{k'}\theta\right)_{ix} + \varepsilon_{i,j\,(k,l,p,q)} \quad (4)$$

$$i, j, q, r = 1 \cdots m \qquad k, l, p = 1 \cdots m^2, x = 1,2, \cdots g$$

Where μ is general mean, α_{ix} is ith row-block effect in xth environmental experiment, β_{jx} is jth column-block effect in xth environmental experiment, τ_{kx} is kth treatment effect in xth environmental experiment, γ_{lx} is the lth row effect in xth environmental experiment, C_{px} is pth the column effect in xth environmental experiment, $s(\alpha)_{q(i)x}$ is the qth horizontal square effect nested in ith row-block in xth environmental experiment, $\pi(\beta)_{r(j)x}$ is the rth vertical square effect nested in jth column-block in xth environmental experiment, S_{qx} is the qth sub-square effect in xth environmental experiment, θ_x is the xth environmental experiment, $\left(\tau_{k'}\theta\right)_{ix}$ is the interaction between $k'th$ common treatment and xth environmental experiment and $\varepsilon_{(i,j)klpqx}$ error component with mean zero and constant variance.

Model 5: The modification of Type IV of Subramani & Ponnuswamy linear model is as follows. In the model below, it is assumed that the row effects and horizontal square effects are nested in the row block and the column effects and the vertical square effects are nested in the column block effect

$$Y_{ij\,(k,l,p,q,r)} = \mu + \alpha_{ix} + \beta_{jx} + \tau_{kx} + \gamma(\alpha)_{l(i)x} + c(\beta)_{p(j)x} + s(\alpha)_{q(i)x} + \pi(\beta)_{r(j)x} + \theta_x + \left(\tau_{k'}\theta\right)_{ix} + \varepsilon_{i,j\,(k,l,p,q,r)} \quad (5)$$

where $i, j, l, p, q, r = 1 \cdots m \;\; and \;\; k = 1 \cdots m^2$.

Where μ is general mean, α_{ix} is ith row-block effect in xth environmental experiment, β_{jx} is jth column-block effect in xth environmental experiment, τ_{kx} is kth treatment effect in xth environmental experiment, $\gamma(\alpha)_{l(i)x}$ is the lth row effect nested in ith row-block in xth environmental experiment, $C(\beta)_{p(j)x}$ is the pth column effect in jth column-block in xth environmental experiment $s(\alpha)_{q(i)x}$ is the qth horizontal square effect nested in ith row-block in xth environmental experiment, $\pi(\beta)_{r(j)x}$ is the rth vertical square effect nested in jth column-block in xth environmental experiment, S_{qx} is the qth sub-square effect in xth environmental experiment, θ_x is the xth environmental experiment, $\left(\tau_{k'}\theta\right)_{ix}$ is the interaction between $k'th$ common treatment and xth environmental experiment and $\varepsilon_{(i,j)klpqx}$ error component with mean zero and constant variance.

Table 4. ANOVA Table for the combined analysis of the set of Sudoku square design for model 4

Source	df	Sum of Square
Experiments	$g(k$-1)	SSEE
Treatments	$gz + c - 1$	SSt
Rows	$g(k - 1)$	SSr
Columns	$g(k - 1)$	SSb
Row-block	$g(m - 1)$	SSrb
Column-block	$g(m - 1)$	SScb
Horizontal sq.withinrow-block	$gm(m - 1)$	SShrb
Vertical sq.within column-block	$gm(m - 1)$	SSvcb
Env.experiment × treatment	(g-1)(c-1)	SSI
Error	By Subtraction	Sse
Total	$gk^2 - 1$	SST

Table 5. ANOVA Table for the combined analysis of the set of Sudoku square design for model 5

Source	df	Sum of Square
Experiments	$g(k$-1)	SSEE
Treatments	$gz + c - 1$	SSt
Rows within row-block	$gm(m - 1)$	SSrrb
Columns within column-block	$gm(m - 1)$	SScb
Horizontal sq.withinrow-block	$gm(m - 1)$	SShrb
Vertical sq.within column-block	$gm(m - 1)$	SSvcb
Row-block	$g(m - 1)$	SSrb
Column-block	$g(m - 1)$	SScb
Env.experiment × treatment	(g-1)(c-1)	SSI
Error	By Subtraction	Sse
Total	$gk^2 - 1$	SST

3. Conclusions

The methods of combining analysis of Sudoku square design of odd order has been presented when experiments were carried-out in a g enviroments when treatments are not same in all experiments, but some treatments common in all the experiments. Five Sudoku design linear models were modified by adding environmental effects and interaction between common treatments and environmental effects terms to the models. The sum of square for rows, columns and sub-squares of g environments are obtained by summing all the individual sum of squares for rows, columns and sub-square respectively. Similarly, the same are done to obtain sum of squares for row-blocks, column-blocks, rows within row-block, column within column-block, horizontal square within row-block and vertical square within column-block respectively.

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