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Copula Approach for Modeling Oil and Gold Prices and Exchange Rate Co-movements in Iran

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Abstract Modeling dependence between financial returns is regarded as a difficult task. It has been shown that, gold and oil prices and exchange rate are skewed and leptokurtic which illustrate tail dependence and asymmetric distributional properties. In this study a new methodology based on copula functions and application of GARCH models with focus on fitting these models to mention financial time series during 2001 to 2008, is used, to show their co-movement. This co-movement is very important in economic policies.

Keywords Co-movements, Copula, Exchange rate, GARCH, Gold price, Oil price

1. Introduction

Oil and gold have the vital role in economics. Sharpincrease in these prices are followed by high inflation, high risk for investment and economics problems. Analysis of financial time series co-movement is very important in monetary policies. In these time series, linear correlation may fail for showing dependence structure between oil and gold and exchange rate. However there would be no cynicism over the non-normality and skewness of many financial variables. This empirical fact often rules out the use of the multivariate normal distribution.

Most financial data exhibit skewness and are often modeled with GARCH models; see Christian Francq and Jean-Micheal Zakoian (2010). In these models the conditional distribution is normal or student-t. To overcome the distributional constraint, copula – GARCH models are used; see Jondeau and Rockinger (2006), Fortin and Kuzmics (2002), and Patton (2006). Copula provides grounding to join different margins in a dependence structure.

The aim of this survey is to examine how oil and gold prices and exchange rate move together during8 years from 2001 to 2008 in Iran. We examine data for two periods before and after 2007, when prices started to fluctuate noticeably after asignificant upward trend. To illustrate this co-movement, copula GARCH models are used.

The copula has an appealing feature which is called tail dependence. Tail dependence is a measure of probability that two variables are in lower or upper joint tails of bivariate distribution. In modeling co-movement, tail dependence shows how variables go up and down with each other.

Copula-GARCH model requires the marginal distribution for the residuals of fitted time series models. Since the residuals of some autoregressive moving-average (ARMA) models are dependent, generalized autoregressive conditional heteroscedasticity (GARCH) model introduced by Angel (1982), is considered.

In section 2, we note GARCH model, correlation and copula. In section 3, descriptive statistics for pre-2007 and post-2007 and also correlation coefficients are showed. In section 4, appropriate time series models are fitted.

In section 5, marginal distribution for residuals will be found. In the last section, the best copula for residuals is fitted.

2. The Model Specification

2.1. GARCH Model

If an autoregressive moving average model (ARMA) be assumed for the variance of error, the model is called generalized autoregressive conditional heteroscedasticity (GARCH). In that case, the GARCH (p, q) (where p is the order of GARCH terms, q is the order of arch terms) is given by

$$\varepsilon_t = \delta_t \times \eta_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2$$

$$= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

Where σ_t^2 is the variance of error at time t and ε_t is a white noise process and η_t is a sequence of (i.i.d) variables with unit variance and α_i $(i = 1, \dots, q)$, β_i $(i = 1, \dots, p)$ are nonnegative constants and α_0 is a positive constant.

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ARMA(P,Q)- GARCH (p,q) model:

When residuals of an ARMA model are correlated, ARMA GARCH would be beneficial which leads to uncorrelated residuals. In this model,

$$\begin{cases} X_t - \sum_{i=1}^{P} a_i X_{t-i} = \varepsilon_t - \sum_{i=1}^{Q} b_i \varepsilon_{t-i} \\ \varepsilon_t = \sigma_t \eta_t \\ \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{cases},$$

Where X_t is an ARMA process and a_i, b_i are ARMA coefficients.

2.2. Correlation and Copula

Correlation measures are used to measure dependence of variables, such as Pearson correlation coefficient, Spearman and Kendal. Pearson correlation coefficient is based on linear association between two variables which has some disadvantages. For instance the relationship may be nonlinear and also may be asymmetric during co-movements hence this correlation would not be beneficial. Spearman correlation coefficient is a nonlinear measure associated with dependence of ranks and is useful with analyzing data in extreme observation. Kendal correlation coefficient measures dependence between ranked data. Left or right tail dependence measures tendency of variables to go up and down at the same time, which would be beneficial in economics and risk management. These types of correlations have association with copula functions.

By Sklar'stheorem, it can be possible to construct joint distribution, $F_{XY}(x,y)$, for two continuous random variable (X,Y), with marginal distributions $F_X(x)$, $F_Y(y)$ through copula function C by $F_{XY}(x,y) = C(F_X(x), F_Y(y))$.

A copula is a cumulative distribution function with uniform marginal distributions U and V, $C(u,v) = \Pr[U \le u, V \le v]$ where $u = F_X(x)$ and $v = F_Y(y)$. Note that the copula connects margins to a multivariate distribution function without any constrains on marginal distributions.

The most striking feature of copula could be regarded as tail dependence by defining

$$\lambda_U = \lim_{u\to 1} \Pr[X \ge F_X^{-1}(u) | Y \ge F_Y^{-1}(u)] = \lim_{u\to 1} \frac{1-2u+C(u,u)}{1-u}$$

$$\lambda_L = \lim_{u\to 0} \Pr[X \le F_X^{-1}(u) | Y \le F_Y^{-1}(u)] = \lim_{u\to 0} \frac{C(u,u)}{u}$$

If $\lambda_U \in (0,1]$, Chas uppertail dependence, and if $\lambda_U = 0$ then C has not.

There are some advantages in analyzing dependence structures via copula functions. First; copulas are more flexible in modeling and estimating marginal distribution using parametric multivariate distribution function. Second; copulas are invariant under monotone transformation. Third; copulas provide information not only about the strength of dependence but also about the dependence structure.

Here we mention two types of most applied copulas in finance.

Normal copula

If (X,Y) have bivariate standard normal distribution with correlation θ, then the copula for $U_1 = F_1(X)$ and $U_2 = F_2(Y)$ is:

$$C_\theta(u_1,u_2) = \phi_2(\phi^{-1}(u_1), \phi^{-1}(u_2); \theta) = \int_{-\infty}^{\phi^{-1}(u_1)} \int_{-\infty}^{\phi^{-1}(u_2)} \frac{1}{2\pi(1-\theta^2)^{\frac{1}{2}}} \times \left\{\frac{-(s^2 - 2\theta st + t^2)}{2(1-\theta^2)}\right\} ds\, dt,$$

In which ϕ is the normal standard distribution function and ϕ_2 is the bivariate normal distribution function with correlation parameter θ.

T-student copula

Let (X,Y) are bivariate t-distributed withparameter θ_1 as the degree of freedom and θ_2 as correlation parameter. Then the copula for $U_1 = F_1(X)$ and $U_2 = F_2(Y)$ is:

$$c(u_1,u_2) = \int_{-\infty}^{t_{\theta_2}^{-1}(u_1)} \int_{-\infty}^{t_{\theta_2}^{-1}(u_2)} \frac{1}{2\pi(1-\theta_2^2)^{\frac{1}{2}}} \times \left\{1 + \frac{(s^2 - 2\theta_2 st + t^2)}{\theta_1(1-\theta_2^2)}\right\}^{\frac{-\theta_1+2}{2}} ds\, dt,$$

In which $t_{\theta_2}^{-1}(u_1)$ is the inverse of t-student distribution with degree of freedom θ_2.

3. Data Description

Data are considered in two periods, before and after 2007 when a noticeable fluctuation is occurred.

Descriptive statistics are presented in Table 1 for two periods.

All returns showed excess kurtosis. Kurtosis was generally greater inpre-2007 than post-2007, it can be interpreted as heaviness of tails in this period. For gold prices, skewness increased. Normal assumption was rejected for all the series by Jarque-Bra test.

Table 1(a). Descriptive statistics (pre-2007)

	Oil	Gold	Exchange-rate
Mean	47.24	122571.9	8793.7
Max	99.35	262927	9437
Min	22.24	64628	7989
Skewness	0.57	0.72	-0.40
Kurtosis	2.43	2.91	1.68
Jarque-Bra statistics	5.048	6.47	7.20

Table 1(b). Descriptive statistics (post-2007)

	Oil	Gold	Exchange-rate
Mean	76.95	231348.8	9750.7
Max	128.19	262927	10174
Min	40.03	216391	9195
Skewness	0.43	1.0464	-0.55
Kurtosis	1.85	3.25	1.8832
Jarque-Bra statistics	1.71	3.87	2.0698

Table 2 summarizes correlation estimates for oil and gold and ex-change rate for pre and post 2007. The positive linear correlation coefficient values between oil and gold prices and exchange rate indicate that these three series moved together in a same direction, in both periods. This high correlation for overall sample, pointing to great sensitivity of these variables. Kendall's tau and Spearman's rho presented similar evidence. The co-movements between these prices were in general negative in post-2007 exception between oil and gold prices.

Table 2(a). Correlation coefficient estimates (oil and gold)

	Pearson	Kendall	Spearman
Overallsample	0.81	0.72	0.87
Pre-2007	0.946	0.81	0.941
Post-2007	0.584	0.442	0.654

Table 2(b). Correlation coefficient estimates (oil and exchange rate)

	Pearson	Kendall	Spearman
Overall sample	0.660	0.661	0.816
Pre-2007	0.920	0.830	0.955
Post-2007	0.87	- 0.515	-0.71

Table 2(c). Correlation coefficient estimates (gold and exchange rate)

	Pearson	Kendall	Spearman
Overall sample	0.8171	0.833	0.945
Pre-2007	0.886	0.881	0.97
Post-2007	-0.60	- 0.315	-0.485

4. Modeling Oil-gold Prices and Exchange Rate Co-movements

The best ARMA (P,Q) -GARCH (p,q) models were estimated for the periods before and after 2007 by identifying the orders and using AIC values. As we know, after fitting ARMA models, the residuals and their squares should be uncorrelated.

We examine serial correlation via Portmanteau (L-jung statistic) or Box-Peirce test; see Box, G.E.P. and Pierce, D.A. (1970). If the correlation between residuals and their squares is remained, a suitable GARCH model should be examined. In this case, the residuals became uncorrelated.

In this survey, as these tables show for first period, ARIMA (1,1,1) for oil prices, ARIMA (1,1,2) for gold prices and GARCH (1,1) for exchange rate were suitable with the least AIC (Table 3).

Results for second period were; ARIMA (1,2,1) for gold prices, ARIMA (1,1,1) for oil prices and GARCH (1,0) for exchange rate. The relative test results are presented in table (4).

Table 3(a). Standardized Residuals Tests (oil-pre 2007)

		statistic	p-value
Port Test	$R(Lag = 5)$	5.396	0.21
Port Test	$R(Lag = 10)$	11.975	0.09
Port Test	$R(Lag = 15)$	19.356	0.03
Port Test	$R^2(Lag = 5)$	5.031	0.258
Port Test	$R^2(Lag = 10)$	10.091	0.192
Port Test	$R^2(Lag = 15)$	13.83	0.192

Table 3(b). Standardized Residuals Tests (gold-pre 2007)

		statistic	p-value
Port Test	$R(Lag = 5)$	0.735	0.95
Port Test	$R(Lag = 10)$	7.85	0.94
Port Test	$R(Lag = 15)$	11.16	0.79
Port Test	$R^2(Lag = 5)$	6.880	0.115
Port Test	$R^2(Lag = 10)$	8.01	0.348
Port Test	$R^2(Lag = 15)$	10.12	0.470

Table 3(c). Standardized Residuals Tests (exchange rate-pre 2007)

		statistic	p-value
Jurque-Bra Test	R	7.122	0.028
Ljung-Box Test	$R(Lag = 10)$	14.026	0.1717
Ljung-Box Test	$R(Lag = 15)$	19.17	0.205
Ljung-Box Test	$R^2(Lag = 10)$	6.54	0.76
Ljung-Box Test	$R^2(Lag = 15)$	7.783	0.93
Lm Arch Test	R	0.487	0.745

Table 4(a). Standardized Residuals Tests (oil-post 2007)

		statistic	p-value
Box-Peirce Test	$R(Lag = 5)$	0.1522	0.696
Box-Peirce Test	$R(Lag = 10)$	4.5474	0.919
Box-Peirce Test	$R(Lag = 15)$	5.259	0.989
Box-Peirce Test	$R^2(Lag = 5)$	0.0249	0.874
Box-Peirce Test	$R^2(Lag = 10)$	4.081	0.9435
Box-Peirce Test	$R^2(Lag = 15)$	5.791	0.963

Table 4(b). Standardized Residuals Tests (gold-post 2007)

		statistic	p-value
Box-Peirce Test	$R(Lag = 5)$	1.062	0.302
Box-Peirce Test	$R(Lag = 10)$	9.48	0.486
Box-Peirce Test	$R(Lag = 15)$	11.41	0.721
Box-Peirce Test	$R^2(Lag = 5)$	2.4675	0.7814
Box-Peirce Test	$R^2(Lag = 10)$	5.403	0.862
Box-Peirce Test	$R^2(Lag = 15)$	3.284	0.999

Table 4(c). Standardized Residuals Tests (exchange rate-post 2007)

		statistic	p-value
Jurque-Bra Test	R	0.279	0.8696
Ljung-Box Test	$R(Lag = 10)$	6.170	0.800
Ljung-Box Test	$R(Lag = 15)$	8.71	0.892
Ljung-Box Test	$R^2(Lag = 10)$	9.28	0.505
Ljung-Box Test	$R^2(Lag = 15)$	11.50	0.715
Lm Arch Test	R	7	0.85

5. Marginal Distributions of Residuals

In this section, marginal distribution of residuals will be found. The goodness of fit for the margins were done by Kolmogorov-Smirnov test. The conditional normal distribution for the selected GARCH models for exchange rate was not rejected by Jurque-Bra test at the level for pre and p-value: 0.8 for post 2007. The margins for fitted ARMA models were t-student. The results are presented in Table (5).

Table 5. Marginal Distributions

	Marginal Distribution	Statistic	Df	P-value
Oil (pre)	T-student	2.52	9	0.013
Gold (pre)	T-student	2.27	2	0.025
Oil (post)	T-student	0.152	3	0.79
Gold (post)	T-student	0.134	4	0.68
Exchange rate (pre)	Normal			0.05
Exchange rate (post)	Normal			0.8

6. Copula Models for Describing Co-movements

In this section, several copula function are considered to capture different patterns of dependence: tail dependence, symmetric and asymmetric tail dependence. The performance of the different copula models were evaluated by AIC and Cramer –Von statistic for goodness of fit. The results for oil and gold prices and exchange rate dependence are presented in Tables 6.

Table 6(a). Fitted copula (pre-2007)

Copula	Parameter	Cramer-Von	p-value
Frank	0.1826	0.1469	0.023(AIC=1.08)
Gumbel	0.1088	0.108	0.023(AIC=-298)
Gausian	0.1250	0.125	0.023(AIC=-284)

Table 6(b). Fitted copula (post-2007)

Copula	Parameter	Cramer-Von	p-value
Frank	-0.552	0.598	0.023(AIC=1.16)
Gumbel	-0.49	0.635	0.023(AIC=-283)
Gaussian	1.12	1.045	0.00491(AIC=-231)

All the fitted distributions were not rejected for pre-2007. The best copula model, selected by the least AIC, was Gumbel copula. For post-2007, Gumbeland Frank were not rejected, and Gumbel copula is selected. For pre-2007, as table shows the dependence structure was positive and for post-2007 was negative. The estimated parameters indicated weak dependence for pre-2007.

7. Conclusions

Looking at the marginal distributions of the residuals of fitted time series to exchange rate and oil and gold prices we conclude that exchange rate is well described by normal and oil and gold prices are well described by t-student distribution.

To model dependency, comparing different copula model, Gumbel copula for the first period and Normal copula for the second period were the best.

We found a considerable decrease in oil-gold and exchange rate after 2007.

Looking at asymmetric tail dependence, the parameter estimate for Gumbel copula in pre-2007 is not very high which shows that the upper and lower tail dependence are not significant.

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