A Class of Modified Ratio Estimators Using Deciles of an Auxiliary Variable

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Abstract   In the past, a number of modified ratio estimators are suggested for estimation of the population mean of the study variable using Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Population Correlation Coefficient, Median, Quartile and their linear combinations of the auxiliary variable. However no attempt is made to use the deciles, which are more generalized version of quartiles and Median. Hence an attempt is made in this paper to use the deciles in the modified ratio estimators for estimation of population mean of the study variable when the population deciles of the auxiliary variable are known. The biases and the mean squared errors of the proposed estimators are derived and are compared with that of existing modified ratio estimators for certain known populations. Further we have also derived the conditions for which the proposed estimators perform better than the existing modified ratio estimators. From the numerical study it is observed that the proposed modified ratio estimators perform better than the existing modified ratio estimators.

Keywords   Bias, Mean Squared Error, Population Mean, Simple Random Sampling

1. Introduction

Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i = 1,2,3,\dots,N$ giving a vector $Y = \{Y_1, Y_2, \dots, Y_N\}$. The problem is to estimate the population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample selected from the population U. If the population parameters of the auxiliary variable X such as Population Mean, Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Median, Quartiles etc., are known, a number of modified ratio estimators proposed in the literature. Before discussing further about the existing modified ratio estimators and the proposed modified ratio estimators the notations to be used in this paper are described below:

N − Population size
n − Sample size
f = n/N, Sampling fraction
Y − Study variable
X − Auxiliary variable
$\overline{X}, \overline{Y}$ − Population means
x, y - Sample totals
$\overline{x}, \overline{y}$ − Sample means

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S_X, S_y − Population standard deviations
C_X, C_v − Coefficient of variations
ρ − Coefficient of correlation
$\beta_1 = \frac{N\sum_{i=1}^{N}(X_i - \overline{X})^3}{(N-1)(N-2)S^3}$, Coefficient of skewness of the auxiliary variable
$\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i - \overline{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Coefficient of kurtosis of the auxiliary variable
$B(.)$ − Bias of the estimator
$MSE(.)$ − Mean squared error of the estimator
$\widehat{\overline{Y}}_i(\widehat{\overline{Y}}_{pi})$ − Existing (proposed) modified ratio estimator of $\overline{Y}$

The Ratio estimator for estimating the population mean $\overline{Y}$ of the study variable Y is defined as

$$\widehat{\overline{Y}}_R = \frac{\overline{y}}{\overline{x}}\overline{X} = \widehat{R}\,\overline{X} \text{ where}$$

$$\widehat{R} = \frac{\overline{y}}{\overline{x}} = \frac{y}{x} \text{ is the estimate of } R = \frac{\overline{Y}}{\overline{X}} = \frac{Y}{X} \quad (1)$$

The Ratio estimator given in (1) is used for improving the precision of the estimate of the population mean compared to simple random sampling whenever a positive correlation exist between X and Y. Further improvements are achieved by introducing a large number of modified ratio estimators with the use of known Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness etc. A list of existing modified ratio estimators, which are to be compared with that of proposed estimators are given in Table 1 together with their biases, mean squared errors and constants.

Table 1.   Existing modified ratio estimators with their biases, mean squared errors and their constants

Estimator	Bias-B(.)	Mean squared error MSE(.)	Constant R_i
$\widehat{Y}_1 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{\bar{x}}\overline{X}$ Kadilar and Cingi[2]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_1^2$	$\dfrac{(1-f)}{n}\left(R_1^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_1 = \dfrac{\overline{Y}}{\overline{X}}$
$\widehat{Y}_2 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x} + C_x)}(\overline{X} + C_x)$ Kadilar and Cingi[2]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_2^2$	$\dfrac{(1-f)}{n}\left(R_2^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_2 = \dfrac{\overline{Y}}{\overline{X} + C_x}$
$\widehat{Y}_3 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x} + \beta_2)}(\overline{X} + \beta_2)$ Kadilar and Cingi[2]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_3^2$	$\dfrac{(1-f)}{n}\left(R_3^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_3 = \dfrac{\overline{Y}}{\overline{X} + \beta_2}$
$\widehat{Y}_4 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}\beta_2 + C_x)}(\overline{X}\beta_2 + C_x)$ Kadilar and Cingi[2]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_4^2$	$\dfrac{(1-f)}{n}\left(R_4^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_4 = \dfrac{\overline{Y}\beta_2}{\overline{X}\beta_2 + C_x}$
$\widehat{Y}_5 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}C_x + \beta_2)}(\overline{X}C_x + \beta_2)$ Kadilar and Cingi[2]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_5^2$	$\dfrac{(1-f)}{n}\left(R_5^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_5 = \dfrac{\overline{Y}C_x}{\overline{X}C_x + \beta_2}$
$\widehat{Y}_6 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x} + \beta_1)}(\overline{X} + \beta_1)$ Yan and Tian[18]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_6^2$	$\dfrac{(1-f)}{n}\left(R_6^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_6 = \dfrac{\overline{Y}}{\overline{X} + \beta_1}$
$\widehat{Y}_7 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}\beta_1 + \beta_2)}(\overline{X}\beta_1 + \beta_2)$ Yan and Tian[18]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_7^2$	$\dfrac{(1-f)}{n}\left(R_7^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_7 = \dfrac{\overline{Y}\beta_1}{\overline{X}\beta_1 + \beta_2}$
$\widehat{Y}_8 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x} + \rho)}(\overline{X} + \rho)$ Kadilar and Cingi[3]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_8^2$	$\dfrac{(1-f)}{n}\left(R_8^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_8 = \dfrac{\overline{Y}}{\overline{X} + \rho}$
$\widehat{Y}_9 = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}C_x + \rho)}(\overline{X}C_x + \rho)$ Kadilar and Cingi[3]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_9^2$	$\dfrac{(1-f)}{n}\left(R_9^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_9 = \dfrac{\overline{Y}C_x}{\overline{X}C_x + \rho}$
$\widehat{Y}_{10} = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}\rho + C_x)}(\overline{X}\rho + C_x)$ Kadilar and Cingi[3]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_{10}^2$	$\dfrac{(1-f)}{n}\left(R_{10}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{10} = \dfrac{\overline{Y}\rho}{\overline{X}\rho + C_x}$
$\widehat{Y}_{11} = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}\beta_2 + \rho)}(\overline{X}\beta_2 + \rho)$ Kadilar and Cingi[3]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_{11}^2$	$\dfrac{(1-f)}{n}\left(R_{11}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{11} = \dfrac{\overline{Y}\beta_2}{\overline{X}\beta_2 + \rho}$
$\widehat{Y}_{12} = \dfrac{\bar{y} + b(\overline{X} - \bar{x})}{(\bar{x}\rho + \beta_2)}(\overline{X}\rho + \beta_2)$ Kadilar and Cingi[3]	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\overline{Y}}R_{12}^2$	$\dfrac{(1-f)}{n}\left(R_{12}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{12} = \dfrac{\overline{Y}\rho}{\overline{X}\rho + \beta_2}$

It is to be noted that "the existing modified ratio estimators" means the list of modified ratio estimators to be considered in this paper unless otherwise stated. It does not mean to the entire list of modified ratio type estimators available in the literature. For a more detailed discussion on the ratio estimator and its modifications one may refer to Cochran[1], Kadilar and Cingi[2, 3], Koyuncu and Kadilar[4], Murthy[5], Prasad[6], Rao[7], Singh[9], Singh and Tailor[10, 12], Singh

et.al[11], Sisodia and Dwivedi[13], Subramani and Kumarapandiyan[14, 15 and 16], Upadhyaya and Singh[17], Yan and Tian[18] and the references cited there in.

The list of estimators given in Table 1 uses the known values of the parameters like $\bar{X}$, C_x, β_1, β_2, ρ and their linear combinations. However, it seems, no attempt is made to use the known values of the population deciles of the auxiliary variable to improve the ratio estimator. Further we know that the parameters like the mean, coefficient of variation, coefficient of skewness and coefficient of kurtosis are affected by the extreme values or the presence of outliers in the population value, while deciles are unaffected and are robustness to extreme values. The points discussed above have motivated us to introduce a class of modified ratio estimators using the known value of the population deciles of the auxiliary variable and it is observed that the proposed estimators perform better than the existing estimators. The proposed modified ratio estimators using known values of population deciles of the auxiliary variable

are presented in section 2 where as the conditions in which the proposed estimators perform better than the existing modified ratio estimators are derived in section 3. The efficiencies of the proposed modified ratio estimators and the existing modified ratio estimators are assessed for certain natural populations in section 4 whereas the conclusion part is presented in section 5.

2. Proposed Modified Ratio Type Estimators

In this section, we have suggested a class of modified ratio type estimators using the population deciles, denoted by D_j ; $j = 1, 2, 3, \ldots, 10$ of the auxiliary variable. The proposed modified ratio type estimators $\widehat{\overline{Y}}_{pj}$, $j = 1, 2, \ldots, 10$ for estimating the population mean $\bar{Y}$ together with the first degree of approximation, the biases and mean squared errors and the constants are given below:

Table 2. Proposed modified ratio estimators with their biases, mean squared errors and their constants

Estimator	Bias-$B(.)$	Mean squared error $MSE(.)$	Constant R_i
$\widehat{\overline{Y}}_{p1} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_1)}(\bar{X} + D_1)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p1}^2$	$\dfrac{(1-f)}{n}\left(R_{p1}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p1} = \dfrac{\bar{Y}}{\bar{X} + D_1}$
$\widehat{\overline{Y}}_{p2} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_2)}(\bar{X} + D_2)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p2}^2$	$\dfrac{(1-f)}{n}\left(R_{p2}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p2} = \dfrac{\bar{Y}}{\bar{X} + D_2}$
$\widehat{\overline{Y}}_{p3} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_3)}(\bar{X} + D_3)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p3}^2$	$\dfrac{(1-f)}{n}\left(R_{p3}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p3} = \dfrac{\bar{Y}}{\bar{X} + D_3}$
$\widehat{\overline{Y}}_{p4} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_4)}(\bar{X} + D_4)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p4}^2$	$\dfrac{(1-f)}{n}\left(R_{p4}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p4} = \dfrac{\bar{Y}}{\bar{X} + D_4}$
$\widehat{\overline{Y}}_{p5} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_5)}(\bar{X} + D_5)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p5}^2$	$\dfrac{(1-f)}{n}\left(R_{p5}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p5} = \dfrac{\bar{Y}}{\bar{X} + D_5}$
$\widehat{\overline{Y}}_{p6} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_6)}(\bar{X} + D_6)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p6}^2$	$\dfrac{(1-f)}{n}\left(R_{p6}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p6} = \dfrac{\bar{Y}}{\bar{X} + D_6}$
$\widehat{\overline{Y}}_{p7} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_7)}(\bar{X} + D_7)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p7}^2$	$\dfrac{(1-f)}{n}\left(R_{p7}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p7} = \dfrac{\bar{Y}}{\bar{X} + D_7}$
$\widehat{\overline{Y}}_{p8} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_8)}(\bar{X} + D_8)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p8}^2$	$\dfrac{(1-f)}{n}\left(R_{p8}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p8} = \dfrac{\bar{Y}}{\bar{X} + D_8}$
$\widehat{\overline{Y}}_{p9} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_9)}(\bar{X} + D_9)$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p9}^2$	$\dfrac{(1-f)}{n}\left(R_{p9}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p9} = \dfrac{\bar{Y}}{\bar{X} + D_9}$
$\widehat{\overline{Y}}_{p10} = \dfrac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + D_{10})}(\bar{X} + D_{10})$	$\dfrac{(1-f)}{n}\dfrac{S_x^2}{\bar{Y}}R_{p10}^2$	$\dfrac{(1-f)}{n}\left(R_{p10}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{p10} = \dfrac{\bar{Y}}{\bar{X} + D_{10}}$

3. Efficiency of the Proposed Estimators

For want of space; for the sake of convenience to the readers and for the ease of comparisons, the modified ratio estimators given in Table 1and the proposed modified ratio estimators given in Table 2 are represented into two classes as given below:

Class 1: The biases, the mean squared errors and the constants of the 12 modified ratio estimators $\widehat{\overline{Y}}_1$ to $\widehat{\overline{Y}}_{12}$ listed in the Table 1 are represented in a single class (say, Class 1), which will be very much useful for comparing with that of proposed modified ratio estimators and are given below:

$$B\left(\widehat{\overline{Y}}_i\right) = \frac{(1-f)}{n} \frac{S_x^2}{\overline{Y}} R_i^2$$

$$MSE\left(\widehat{\overline{Y}}_i\right) = \frac{(1-f)}{n}\left(R_i^2 S_x^2 + S_y^2(1-\rho^2)\right); \qquad (2)$$

$$i = 1, 2, 3, \ldots, 12$$

where $R_1 = \frac{\overline{Y}}{\overline{X}}, R_2 = \frac{\overline{Y}}{\overline{X}+C_x}, R_3 = \frac{\overline{Y}}{\overline{X}+\beta_2},$

$R_4 = \frac{\overline{Y}\beta_2}{\overline{X}\beta_2+C_x}, R_5 = \frac{\overline{Y}C_x}{\overline{X}C_x+\beta_2}, R_6 = \frac{\overline{Y}}{\overline{X}+\beta_1},$

$R_7 = \frac{\overline{Y}\beta_1}{\overline{X}\beta_1+\beta_2}, \ R_8 = \frac{\overline{Y}}{\overline{X}+\rho}, R_9 = \frac{\overline{Y}C_x}{\overline{X}C_x+\rho},$

$R_{10} = \frac{\overline{Y}\rho}{\overline{X}\rho+C_x}, R_{11} = \frac{\overline{Y}\beta_2}{\overline{X}\beta_2+\rho}$ and $R_{12} = \frac{\overline{Y}\rho}{\overline{X}\rho+\beta_2}$

Class 2: The biases, the mean squared errors and the constants of the 10 proposed modified ratio estimators $\widehat{\overline{Y}}_{p1}$ to $\widehat{\overline{Y}}_{p10}$ listed in the Table 2 are represented in a single class (say, Class 2), which will be very much useful for comparing with that of existing modified ratio estimators (given in Class1) are given below:

$$B\left(\widehat{\overline{Y}}_{pj}\right) = \frac{(1-f)}{n} \frac{S_x^2}{\overline{Y}} R_{pj}^2$$

$$MSE\left(\widehat{\overline{Y}}_{pj}\right) = \frac{(1-f)}{n}\left(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)\right); \qquad (3)$$

$$j = 1, 2, 3, \ldots, 10$$

where $R_{p1} = \frac{\overline{Y}}{\overline{X}+D_1}, R_{p2} = \frac{\overline{Y}}{\overline{X}+D_2},$

$R_{p3} = \frac{\overline{Y}}{\overline{X}+D_3}, R_{p4} = \frac{\overline{Y}}{\overline{X}+D_4}, R_{p5} = \frac{\overline{Y}}{\overline{X}+D_5},$

$R_{p6} = \frac{\overline{Y}}{\overline{X}+D_6}, R_{p7} = \frac{\overline{Y}}{\overline{X}+D_7}, R_{p8} = \frac{\overline{Y}}{\overline{X}+D_8},$

$R_{p9} = \frac{\overline{Y}}{\overline{X}+D_9}$ and $R_{p10} = \frac{\overline{Y}}{\overline{X}+D_{10}}$

From the expressions given in (2) and (3) we have derived the conditions for which the proposed estimator $\widehat{\overline{Y}}_{pj}$ is more efficient than the existing modified ratio type estimators given in Class 1, $\widehat{\overline{Y}}_i ; i = 1, 2, 3, \ldots, 12$ and are given below.

$$MSE\left(\widehat{\overline{Y}}_{pj}\right) < MSE\left(\widehat{\overline{Y}}_i\right) \text{if } R_{pj} < R_i ; \qquad (4)$$

$$i = 1, 2, 3, \ldots, 12: j = 1, 2, 3, \ldots, 10$$

4. Empirical Study

The performances of the proposed modified ratio estimators listed in Table 2 are assessed with that of existing modified ratio estimators listed in Table 1 for certain natural populations. In this connection, we have considered three natural populations for the assessment of the performances of the proposed modified ratio estimators with that of existing modified ratio estimators. They are: The population 1 is the closing price of the industry ACC in the National Stock Exchange from 2, January 2012 to 27, February 2012[19]; population 2 and population 3 are taken from Singh and Chaudhary[8] given in page 177. The population parameters and the constants computed from the above populations are given below in Table 3, where as the constants, the biases and the mean squared errors of the existing and proposed modified ratio estimators for the above populations are respectively given from the Tables 4 to 6.

Table 3. Parameters and Constants of the Populations

Parameters	Population 1	Population 2	Population 3
N	40	34	34
n	20	20	20
$\overline{Y}$	5141.5363	856.4117	856.4117
$\overline{X}$	1221.6463	208.8823	199.4412
ρ	0.9244	0.4491	0.4453
S_y	256.1464	733.1407	733.1407
C_y	0.0557	0.8561	0.8561
S_x	102.5494	150.5059	150.2150
C_x	0.0839	0.7205	0.7531
β_2	-1.5154	0.0978	1.0445
β_1	0.3761	0.9782	1.1823
D_1	1111.8150	70.3000	60.6000
D_2	1119.4800	76.8000	83.0000
D_3	1139.2000	108.2000	102.7000
D_4	1159.8400	129.4000	111.2000
D_5	1184.2250	150.0000	142.5000
D_6	1252.5500	227.2000	210.2000
D_7	1307.1000	250.4000	264.5000
D_8	1345.7200	335.6000	304.4000
D_9	1366.7850	436.1000	373.2000
D_{10}	1389.3000	564.0000	634.0000

Table 4. The constants of the existing and proposed modified ratio type estimators

Estimators	Constant	Population 1	Population 2	Population 3
$\widehat{\overline{Y}}_1$ Kadilar and Cingi[2]	R_1	4.2087	4.1000	4.2941
$\widehat{\overline{Y}}_2$ Kadilar and Cingi[2]	R_2	4.2084	4.0859	4.2779
$\widehat{\overline{Y}}_3$ Kadilar and Cingi[2]	R_3	4.2139	4.0981	4.2717
$\widehat{\overline{Y}}_4$ Kadilar and Cingi[2]	R_4	4.2089	3.9598	4.2786
$\widehat{\overline{Y}}_5$ Kadilar and Cingi[2]	R_5	4.2718	4.0973	4.2644
$\widehat{\overline{Y}}_6$ Yan and Tian[18]	R_6	4.2074	4.0809	4.2688
$\widehat{\overline{Y}}_7$ Yan and Tian[18]	R_7	4.2226	4.0980	4.2751
$\widehat{\overline{Y}}_8$ Kadilar and Cingi[3]	R_8	4.2055	4.0912	4.2845
$\widehat{\overline{Y}}_9$ Kadilar and Cingi[3]	R_9	4.1711	4.0878	4.2814
$\widehat{\overline{Y}}_{10}$ Kadilar and Cingi[3]	R_{10}	4.2084	4.0687	4.2579
$\widehat{\overline{Y}}_{11}$ Kadilar and Cingi[3]	R_{11}	4.2108	4.0115	4.2849
$\widehat{\overline{Y}}_{12}$ Kadilar and Cingi[3]	R_{12}	4.2143	4.0957	4.2441
$\widehat{\overline{Y}}_{p1}$ (Proposed estimator)	R_{p1}	2.2034	3.0676	3.2934
$\widehat{\overline{Y}}_{p2}$ (Proposed estimator)	R_{p2}	2.1962	2.9978	3.0322
$\widehat{\overline{Y}}_{p3}$ (Proposed estimator)	R_{p3}	2.1778	2.7009	2.8345
$\widehat{\overline{Y}}_{p4}$ (Proposed estimator)	R_{p4}	2.1590	2.5316	2.7569
$\widehat{\overline{Y}}_{p5}$ (Proposed estimator)	R_{p5}	2.1371	2.3863	2.5046
$\widehat{\overline{Y}}_{p6}$ (Proposed estimator)	R_{p6}	2.0781	1.9639	2.0906
$\widehat{\overline{Y}}_{p7}$ (Proposed estimator)	R_{p7}	2.0332	1.8647	1.8459
$\widehat{\overline{Y}}_{p8}$ (Proposed estimator)	R_{p8}	2.0027	1.5729	1.6998
$\widehat{\overline{Y}}_{p9}$ (Proposed estimator)	R_{p9}	1.9864	1.3278	1.4955
$\widehat{\overline{Y}}_{p10}$ (Proposed estimator)	R_{p10}	1.9692	1.1081	1.0276

Table 5. The biases of the existing and proposed modified ratio type estimators

Estimator	Bias $B(.)$		
	Population 1	**Population 2**	**Population 3**
$\widehat{\overline{Y}}_1$ Kadilar and Cingi[2]	0.9058	9.1539	10.0023
$\widehat{\overline{Y}}_2$ Kadilar and Cingi[2]	0.9056	9.0911	9.9272
$\widehat{\overline{Y}}_3$ Kadilar and Cingi[2]	0.9080	9.1454	9.8983
$\widehat{\overline{Y}}_4$ Kadilar and Cingi[2]	0.9058	8.5387	9.9303
$\widehat{\overline{Y}}_5$ Kadilar and Cingi[2]	0.9331	9.1420	9.8646
$\widehat{\overline{Y}}_6$ Yan and Tian[18]	0.9052	9.0688	9.8847
$\widehat{\overline{Y}}_7$ Yan and Tian[18]	0.9118	9.1452	9.9143
$\widehat{\overline{Y}}_8$ Kadilar and Cingi[3]	0.9044	9.1147	9.9578
$\widehat{\overline{Y}}_9$ Kadilar and Cingi[3]	0.8896	9.0995	9.9432
$\widehat{\overline{Y}}_{10}$ Kadilar and Cingi[3]	0.9056	9.0149	9.8348
$\widehat{\overline{Y}}_{11}$ Kadilar and Cingi[3]	0.9066	8.7630	9.9597

$\widehat{\overline{Y}}_{12}$ Kadilar and Cingi[3]	0.9081	9.1349	9.7711
$\widehat{\overline{Y}}_{p1}$ (Proposed estimator)	0.2483	5.1243	5.8836
$\widehat{\overline{Y}}_{p2}$ (Proposed estimator)	0.2466	4.8938	4.9874
$\widehat{\overline{Y}}_{p3}$ (Proposed estimator)	0.2425	3.9725	4.3582
$\widehat{\overline{Y}}_{p4}$ (Proposed estimator)	0.2383	3.4902	4.1230
$\widehat{\overline{Y}}_{p5}$ (Proposed estimator)	0.2335	3.1010	3.4027
$\widehat{\overline{Y}}_{p6}$ (Proposed estimator)	0.2208	2.1003	2.3709
$\widehat{\overline{Y}}_{p7}$ (Proposed estimator)	0.2114	1.8934	1.8484
$\widehat{\overline{Y}}_{p8}$ (Proposed estimator)	0.2051	1.3472	1.5673
$\widehat{\overline{Y}}_{p9}$ (Proposed estimator)	0.2018	0.9601	1.2133
$\widehat{\overline{Y}}_{p10}$ (Proposed estimator)	0.1983	0.6686	0.5728

Table 6.   The mean squared errors of the existing and proposed modified ratio type estimators

Estimator	Mean Squared Error MSE(.)		
	Population 1	Population 2	Population 3
$\widehat{\overline{Y}}_1$ Kadilar and Cingi[2]	4954.6195	16673.4489	17437.6451
$\widehat{\overline{Y}}_2$ Kadilar and Cingi[2]	4953.9796	16619.6435	17373.3111
$\widehat{\overline{Y}}_3$ Kadilar and Cingi[2]	4966.1946	16666.1389	17348.6192
$\widehat{\overline{Y}}_4$ Kadilar and Cingi[2]	4955.0419	16146.6142	17376.0389
$\widehat{\overline{Y}}_5$ Kadilar and Cingi[2]	5095.3661	16663.3064	17319.7468
$\widehat{\overline{Y}}_6$ Yan and Tian[18]	4951.7534	16600.5393	17336.9770
$\widehat{\overline{Y}}_7$ Yan and Tian[18]	4985.4911	16665.9758	17362.2582
$\widehat{\overline{Y}}_8$ Kadilar and Cingi[3]	4947.5796	16639.8457	17399.5196
$\widehat{\overline{Y}}_9$ Kadilar and Cingi[3]	4871.7809	16626.8702	17387.0811
$\widehat{\overline{Y}}_{10}$ Kadilar and Cingi[3]	4953.9273	16554.4002	17294.1864
$\widehat{\overline{Y}}_{11}$ Kadilar and Cingi[3]	4959.2739	16338.6465	17401.1397
$\widehat{\overline{Y}}_{12}$ Kadilar and Cingi[3]	4967.1427	16657.1867	17239.6579
$\widehat{\overline{Y}}_{p1}$ (Proposed estimator)	1574.0848	13222.4476	13910.3747
$\widehat{\overline{Y}}_{p2}$ (Proposed estimator)	1565.7404	13025.0205	13142.8285
$\widehat{\overline{Y}}_{p3}$ (Proposed estimator)	1544.6447	12236.0545	12604.0041
$\widehat{\overline{Y}}_{p4}$ (Proposed estimator)	1523.1237	11822.9995	12402.5396
$\widehat{\overline{Y}}_{p5}$ (Proposed estimator)	1498.4082	11489.7024	11785.7032
$\widehat{\overline{Y}}_{p6}$ (Proposed estimator)	1433.0071	10632.6335	10902.0814
$\widehat{\overline{Y}}_{p7}$ (Proposed estimator)	1384.5527	10455.5071	10454.5930
$\widehat{\overline{Y}}_{p8}$ (Proposed estimator)	1352.0995	9987.7330	10213.7973
$\widehat{\overline{Y}}_{p9}$ (Proposed estimator)	1335.0072	9656.1852	9910.6498
$\widehat{\overline{Y}}_{p10}$ (Proposed estimator)	1317.1938	9406.5679	9362.1020

From the values of Table 5, it is observed that the biases of the proposed modified ratio estimators are less than the biases of all the 12 existing modified ratio estimators. Similarly from the values of Table 6, it is observed that the mean squared errors of the proposed modified ratio estimators are less than the mean squared errors of all the 12 existing modified ratio estimators.

5. Conclusions

In this paper we have proposed a class of modified ratio type estimators using known values of population deciles of the auxiliary variable. The biases and mean squared errors of the proposed estimators are obtained and compared with that of existing modified ratio estimators. Further we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators. We have also assessed the performances of the proposed estimators for some known populations given in page 177 of Singh and Chaudhary [8]. It is observed that the biases and mean squared errors of the ten proposed estimators based on the deciles are less than the biases and mean squared errors of the existing modified ratio estimators for all the three known populations considered for the numerical study. Hence we strongly recommend that the proposed modified estimators with use of deciles, particularly the higher order deciles may be preferred over the existing modified ratio estimators for the use of practical applications.

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