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Variance Estimation Using Quartiles and their Functions of an Auxiliary Variable

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Abstract In this paper we have proposed a class of modified ratio type variance estimators for estimation of population variance of the study variable using Quartiles and their functions of the auxiliary variable are known. The biases and mean squared errors of the proposed estimators are obtained and also derived the conditions for which the proposed estimators perform better than the traditional ratio type variance estimator and existing modified ratio type variance estimators. Further we have compared the proposed estimators with that of traditional ratio type variance estimator and existing modified ratio type variance estimators for certain known populations. From the numerical study it is observed that the proposed estimators perform better than the traditional ratio type variance estimator and existing modified ratio type variance estimators.

Keywords Inter-Quartile Range, Natural Populations, Simple Random Sampling, Semi-Quartile Average, Semi-Quartile Range

1. Introduction

Consider a finite population $U = \{U_1, U_2, \ldots, U_N\}$ of N distinct and identifiable units. Let Y be a real variable with value Y_i measured on $U_i, i = 1,2,3,\ldots,N$ giving a vector $Y = \{Y_1, Y_2, \ldots, Y_N\}$. The problem is to estimate the populations mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ or population variance $S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ on the basis of a random sample selected from the population U. Estimating the finite population variance has great significance in various fields such as Industry, Agriculture, Medical and Biological Sciences. For example in matters of health, variations in body temperature, pulse beat and blood pressure are the basic guides to diagnosis where prescribed treatment is designed to control their variation. In this paper we intend to suggest some estimators for population variance. When there is no additional information on the auxiliary variable available, the simplest estimator of population variance is the simple random sample variance without replacement. It is common practice to use the auxiliary variable for improving the precision of the estimate of a parameter. In this paper, we consider the auxiliary information to improve the efficiency of the estimation of population variance $S_y^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ in simple random sampling. When the information on an auxiliary variable X is known, a number of estimators such as ratio, product and linear regression estimators are proposed in the literature.

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When the correlation between the study variable and the auxiliary variable is positive, ratio method of estimation is quite effective. On the other hand, when the correlation is negative, Product method of estimation can be employed effectively. Among the estimators mentioned above, the ratio estimator and its modifications are widely used for the estimation of the variance of the study variable. Estimation of population variance is considered by Isaki[10] where ratio and regression estimators are proposed. Prasad and Singh[14] have considered a ratio type estimator for estimation of population variance by improving Isaki[10] estimator with respect to bias and precision. Arcos et al.[4] have introduced another ratio type estimator, which has also improved the Isaki[10] estimator, which is almost unbiased and more precise than the other estimators.

Before discussing further about the traditional ratio type variance estimator, existing modified ratio type variance estimators and the proposed modified ratio type variance estimators, the notations to be used in this paper are described below:

- $N -$ Population size
- $n -$ Sample size
- $\gamma = 1/n$
- $Y -$ Study variable
- $X -$ Auxiliary variable
- $\bar{X}, \bar{Y} -$ Population means
- $\bar{x}, \bar{y} -$ Sample means
- $S_y^2, S_x^2 -$ Population variances
- $s_y^2, s_x^2 -$ Sample variances
- $C_x, C_y -$ Coefficient of variations
- $\rho -$ Coefficient of correlation
- $Q_1 -$ First (lower) quartile

- Q_3 −Third (upper) quartile
- Q_r −Inter-quartile range
- Q_d −Semi-quartile range
- Q_a −Semi-quartile average
- $B(.)$ − Bias of the estimator
- $MSE(.)$ − Mean squared error of the estimator
- $\hat{S}_R^2$ − Traditional ratio type variance estimator of S_y^2
- $\hat{S}_{KCi}^2$ − Existing modified ratio type variance estimator of S_y^2
- $\hat{S}_{JGi}^2$ − Proposed modified ratio type variance estimator of S_y^2

Isaki[10] suggested a ratio type variance estimator for the population variance S_y^2 when the population variance S_x^2 of the auxiliary variable X is known together with its bias and mean squared error and are as given below:

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \tag{1}$$

$$B(\hat{S}_R^2) = \gamma S_y^2 [(\beta_{2(x)} - 1) - (\lambda_{22} - 1)]$$

$$MSE(\hat{S}_R^2) = \gamma S_y^4 [(\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) - 2(\lambda_{22} - 1)]$$

where $\beta_{2(y)} = \frac{\mu_{40}}{\mu_{20}^2}, \beta_{2(x)} = \frac{\mu_{04}}{\mu_{02}^2}, \lambda_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}}$ and

$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \overline{Y})^r (x_i - \overline{X})^s$

The Ratio type variance estimator given in (1) is used to improve the precision of the estimate of the population variance compared to simple random sampling when there exists a positive correlation between X and Y. Further improvements are also achieved on the classical ratio estimator by introducing a number of modified ratio estimators with the use of known parameters like Co-efficient of Variation, Co-efficient of Kurtosis. The problem of constructing efficient estimators for the population variance has been widely discussed by various authors such as Agarwal and Sithapit[1], Ahmed et al.[2], Al-Jqrarha and Al-Haj Ebrahem[3], Arcos et al.[4], Bhushan[5], Cochran[6], Das and Tripathi[7], Garcia and Cebrain[8], Gupta and Shabbir[9], Isaki[10], Kadilar and Cingi[11,12], Murthy[13], Prasad and Singh[14], Reddy[15], Singh and Chaudhary[16], Singh et al.[17,19], Upadhyaya and Singh[23] and Wolter[24].

Motivated by Singh et al.[18], Sisodia and Dwivedi[20], and Upadhyaya and Singh[22], Kadilar and Cingi[11] suggested four ratio type variance estimators using known values of Co-efficient of variation C_x and Co-efficient of Kurtosis $\beta_{2(x)}$ of an auxiliary variable X together with their biases and mean squared errors as given in the Table 1.

The existing modified ratio type variance estimators discussed above are biased but have minimum mean squared errors compared to the traditional ratio type variance estimator suggested by Isaki[10]. The list of estimators given in Table 1 uses the known values of the parameters like S_x^2, C_x, β_2 and their linear combinations. Subramani and Kumarapandiyan[21] used Quartiles and their functions of the auxiliary variable like Inter-quartile range, Semi-quartile range and Semi-quartile average to improve the ratio estimators in estimation of population mean. Further we know that the value of quartiles and their functions are unaffected by the extreme values or the presence of outliers in the population values unlike the other parameters like the variance, coefficient of variation and coefficient of kurtosis. The points discussed above have motivated us to introduce a modified ratio type variance estimators using the known values of the quartiles and their functions of the auxiliary variable.

Now briefly we will discuss about quartiles and its functions. The median divides the data into two equal sets. The first (lower) quartile is the middle value of the first set, where 25% of the values are smaller than Q_1 and 75% are larger. The third (upper) quartile is the middle value of the second set, where 75% of the values are smaller than the third quartile Q_3 and 25% are larger. It should be noted that the median will be denoted by the notation Q_2, the second quartile. The inter-quartile range is another range used as a measure of the spread. The difference between upper and lower quartiles, which is called as the inter-quartile range, also indicates the dispersion of a data set. The formula for inter-quartile range is:

$$Q_r = (Q_3 - Q_1) \tag{2}$$

The semi-quartile range is another measure of spread. It is calculated as one half of the differences between the quartiles Q_3 and Q_1. The formula for semi-quartile range is:

$$Q_d = (Q_3 - Q_1)/2 \tag{3}$$

Subramani and Kumarapandiyan[21] suggested another new measure called as Semi-quartile average denoted by the notation Q_a and defined as:

$$Q_a = (Q_3 + Q_1)/2 \tag{4}$$

Table 1. Existing Modified ratio type variance estimators with their biases and mean squared errors

Estimator	Bias - $\mathbf{B}(.)$	Mean squared error $\mathbf{MSE}(.)$
$\hat{S}_{KC1}^2 = s_y^2\left[\frac{S_x^2 + C_x}{s_x^2 + C_x}\right]$ Kadilar and Cingi[11]	$\gamma S_y^2 A_{KC1}\left[A_{KC1}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_{KC1}^2\left(\beta_{2(x)} - 1\right) - 2A_{KC1}(\lambda_{22} - 1)\right]$
$\hat{S}_{KC2}^2 = s_y^2\left[\frac{S_x^2 + \beta_{2(x)}}{s_x^2 + \beta_{2(x)}}\right]$ Kadilar and Cingi[11]	$\gamma S_y^2 A_{KC2}\left[A_{KC2}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_{KC2}^2\left(\beta_{2(x)} - 1\right) - 2A_{KC2}(\lambda_{22} - 1)\right]$
$\hat{S}_{KC3}^2 = s_y^2\left[\frac{S_x^2\beta_{2(x)} + C_x}{s_x^2\beta_{2(x)} + C_x}\right]$ Kadilar and Cingi[11]	$\gamma S_y^2 A_{KC3}\left[A_{KC3}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_{KC3}^2\left(\beta_{2(x)} - 1\right) - 2A_{KC3}(\lambda_{22} - 1)\right]$
$\hat{S}_{KC4}^2 = s_y^2\left[\frac{S_x^2 C_x + \beta_{2(x)}}{s_x^2 C_x + \beta_{2(x)}}\right]$ Kadilar and Cingi[11]	$\gamma S_y^2 A_{KC4}\left[A_{KC4}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + A_{KC4}^2\left(\beta_{2(x)} - 1\right) - 2A_{KC4}(\lambda_{22} - 1)\right]$

where $A_{KC1} = \frac{S_x^2}{S_x^2 + C_x}$, $A_{KC2} = \frac{S_x^2}{S_x^2 + \beta_{2(x)}}$, $A_{KC3} = \frac{S_x^2 \beta_{2(x)}}{S_x^2 \beta_{2(x)} + C_x}$ and $A_{KC4} = \frac{S_x^2 C_x}{S_x^2 C_x + \beta_{2(x)}}$

2. Proposed Estimators Using Quartiles and Their Functions

In this section we have suggested a class of modified ratio type variance estimators using the quartiles and their functions of the auxiliary variable like Inter-quartile range, Semi-quartile range and Semi-quartile average

The proposed class of modified ratio type variance estimators $\hat{S}_{JGi}^2$, $i = 1,2, \ldots ,5$ for estimating the population variance S_y^2 together with the first degree of approximation, the biases and mean squared errors and the constants are given below:

Table 2. Proposed modified ratio type variance estimators with their biases and mean squared errors

Estimator	Bias - $\mathbf{B}(.)$	Mean squared error $\mathbf{MSE}(.)$
$\hat{S}_{JG1}^2 = s_y^2 \left[\frac{S_x^2 + Q_1}{s_x^2 + Q_1}\right]$	$\gamma S_y^2 A_{JG1} \left[A_{JG1}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{JG1}^2\left(\beta_{2(x)} - 1\right) - 2A_{JG1}(\lambda_{22} - 1)\right]$
$\hat{S}_{JG2}^2 = s_y^2 \left[\frac{S_x^2 + Q_3}{s_x^2 + Q_3}\right]$	$\gamma S_y^2 A_{JG2} \left[A_{JG2}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{JG2}^2\left(\beta_{2(x)} - 1\right) - 2A_{JG2}(\lambda_{22} - 1)\right]$
$\hat{S}_{JG3}^2 = s_y^2 \left[\frac{S_x^2 + Q_r}{s_x^2 + Q_r}\right]$	$\gamma S_y^2 A_{JG3} \left[A_{JG3}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{JG3}^2\left(\beta_{2(x)} - 1\right) - 2A_{JG3}(\lambda_{22} - 1)\right]$
$\hat{S}_{JG4}^2 = s_y^2 \left[\frac{S_x^2 + Q_d}{s_x^2 + Q_d}\right]$	$\gamma S_y^2 A_{JG4} \left[A_{JG4}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{JG4}^2\left(\beta_{2(x)} - 1\right) - 2A_{JG4}(\lambda_{22} - 1)\right]$
$\hat{S}_{JG5}^2 = s_y^2 \left[\frac{S_x^2 + Q_a}{s_x^2 + Q_a}\right]$	$\gamma S_y^2 A_{JG5} \left[A_{JG5}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]$	$\gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{JG5}^2\left(\beta_{2(x)} - 1\right) - 2A_{JG5}(\lambda_{22} - 1)\right]$

where $A_{JG1} = \frac{S_x^2}{S_x^2 + Q_1}$, $A_{JG2} = \frac{S_x^2}{S_x^2 + Q_3}$, $A_{JG3} = \frac{S_x^2}{S_x^2 + Q_r}$, $A_{JG4} = \frac{S_x^2}{S_x^2 + Q_d}$ and $A_{JG5} = \frac{S_x^2}{S_x^2 + Q_a}$

3. Efficiency of the Proposed Estimators

As we mentioned earlier the bias and mean squared error of the traditional ratio type variance estimator are given below:

$$B\left(\hat{S}_R^2\right) = \gamma S_y^2 \left[\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right] \tag{5}$$

$$MSE\left(\hat{S}_R^2\right) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) - 2(\lambda_{22} - 1)\right] \tag{6}$$

For want of space; for the sake of convenience to the readers and for the ease of comparisons, the biases and mean squared errors of the existing modified ratio type variance estimators given in Table 1 are represented in single class as given below:

$$B\left(\hat{S}_{KCi}^2\right) = \gamma S_y^2 A_{KCi}\left[A_{KCi}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]; i = 1,2,3 \text{ and } 4 \tag{7}$$

$$MSE\left(\hat{S}_{KCi}^2\right) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{KCi}^2\left(\beta_{2(x)} - 1\right) - 2A_{KCi}(\lambda_{22} - 1)\right];\ i = 1,2,3 \text{ and } 4 \tag{8}$$

where $A_{KC1} = \frac{S_x^2}{S_x^2 + C_x}$, $A_{KC2} = \frac{S_x^2}{S_x^2 + \beta_{2(x)}}$, $A_{KC3} = \frac{S_x^2 \beta_{2(x)}}{S_x^2 \beta_{2(x)} + C_x}$ and $A_{KC4} = \frac{S_x^2 C_x}{S_x^2 C_x + \beta_{2(x)}}$

In the same way, the biases and mean squared errors of the proposed modified ratio type variance estimators given in Table 2 are represented in a single class as given below:

$$B\left(\hat{S}_{JGj}^2\right) = \gamma S_y^2 A_{JGj}\left[A_{JGj}\left(\beta_{2(x)} - 1\right) - (\lambda_{22} - 1)\right]; j = 1,2,3,4 \text{ and } 5 \tag{9}$$

$$MSE\left(\hat{S}_{JGj}^2\right) = \gamma S_y^4 \left[\left(\beta_{2(y)} - 1\right) + A_{JGj}^2\left(\beta_{2(x)} - 1\right) - 2A_{JGj}(\lambda_{22} - 1)\right];\ j = 1,2,3,4 \text{ and } 5 \tag{10}$$

where $A_{JG1} = \frac{S_x^2}{S_x^2 + Q_1}$, $A_{JG2} = \frac{S_x^2}{S_x^2 + Q_3}$, $A_{JG3} = \frac{S_x^2}{S_x^2 + Q_r}$, $A_{JG4} = \frac{S_x^2}{S_x^2 + Q_d}$ and $A_{JG5} = \frac{S_x^2}{S_x^2 + Q_a}$

From the expressions given in (6) and (10) we have derived the condition for which the proposed estimators $\hat{S}_{JGj}^2; j = 1,2,3,4$ and 5 are more efficient than the traditional ratio type variance estimator and it is given below:

$$MSE\left(\hat{S}_{JGj}^2\right) < MSE\left(\hat{S}_R^2\right) \text{ if } \lambda > 1 + \frac{\left(A_{JGj} + 1\right)\left(\beta_{2(x)} - 1\right)}{2}; j = 1,2,3,4 \text{ and } 5 \tag{11}$$

From the expressions given in (8) and (10) we have derived the conditions for which the proposed estimators $\hat{S}_{JGj}^2; j = 1,2,3,4$ and 5 are more efficient than the existing modified ratio type variance estimators given in Table 1, $\hat{S}_{KCi}^2; i = 1, 2, 3$ and 4 and are given below:

$$MSE\left(\hat{S}_{JGj}^2\right) < MSE\left(\hat{S}_{KCi}^2\right) \text{ if } \lambda > 1 + \frac{\left(A_{JGj} + A_i\right)\left(\beta_{2(x)} - 1\right)}{2}\ ; i = 1, 2, 3 \text{ and } 4\ ; j = 1,2,3,4 \text{ and } 5 \tag{12}$$

4. Numerical Study

The performance of the proposed modified ratio type variance estimators are assessed with that of traditional ratio type estimator and existing modified ratio type variance estimators listed in Table 1 for certain natural populations. The populations 1 and 2 are the real data set taken from the Report on Waste 2004 drew up by the Italian bureau for the environment protection-APAT. Data and reports are available in the website http://www.osservatorionazionalerifiuti.it[25]. In the data set, for each of the Italian provinces, three variables are considered: the total amount (tons) of recyclable-waste collection in Italy in 2003 (Y), the total amount of recyclable-waste collection in Italy in 2002 (X_1) and the number of inhabitants in 2003 (X_2). The population 3 is taken from Murthy[13] given in page 228 and population 4 is taken from Singh and Chaudhary[16] given in page 108. The population parameters and the constants computed from the above populations are given below:

Table 3. Parameters and Constants of the Populations

Parameters	Population 1	Population 2	Population 3	Population 4
N	103	103	80	70
n	40	40	20	25
$\bar{Y}$	626.2123	62.6212	51.8264	96.7000
$\bar{X}$	557.1909	556.5541	11.2646	175.2671
ρ	0.9936	0.7298	0.9413	0.7293
S_y	913.5498	91.3549	18.3569	60.7140
C_y	1.4588	1.4588	0.3542	0.6254
S_x	818.1117	610.1643	8.4563	140.8572
C_x	1.4683	1.0963	0.7507	0.8037
$\beta_{2(x)}$	37.3216	17.8738	2.8664	7.0952
$\beta_{2(y)}$	37.1279	37.1279	2.2667	4.7596
λ_{22}	37.2055	17.2220	2.2209	4.6038
Q_1	142.9950	259.3830	5.1500	80.1500
Q_3	665.6250	628.0235	16.9750	225.0250
Q_r	522.6300	368.6405	11.8250	144.8750
Q_d	261.3150	184.3293	5.9125	72.4375
Q_a	404.3100	443.7033	11.0625	152.5875
A_{KC1}	0.9999	0.9999	0.9896	0.9999
A_{KC2}	0.9999	0.9999	0.9615	0.9996
A_{KC3}	0.9999	0.9999	0.9964	0.9999
A_{KC4}	0.9999	0.9999	0.9493	0.9996
A_{JG1}	0.9997	0.9994	0.9328	0.9960
A_{JG2}	0.9990	0.9983	0.8082	0.9888
A_{JG3}	0.9992	0.9990	0.8581	0.9928
A_{JG4}	0.9996	0.9995	0.9236	0.9964
A_{JG5}	0.9993	0.9988	0.8660	0.9924

The biases and mean squared errors of the existing and proposed modified ratio type variance estimators for the populations given above are given in the following Tables:

Table 4. Biases of the existing and proposed modified ratio type variance estimators

Estimator	Bias $\mathbf{B}(.)$			
	Population 1	Population 2	Population 3	Population 4
$\hat{S}^2_R$ Isaki[10]	2422.3488	135.9935	10.8762	364.4211
$\hat{S}^2_{KC1}$ Kadilar and Cingi[11]	2420.6810	135.9827	10.4399	364.3702
$\hat{S}^2_{KC2}$ Kadilar and Cingi[11]	2379.9609	135.8179	9.2918	363.9722
$\hat{S}^2_{KC3}$ Kadilar and Cingi[11]	2422.3041	135.9929	10.7222	364.4139
$\hat{S}^2_{KC4}$ Kadilar and Cingi[11]	2393.4791	135.8334	8.8117	363.8627
$\hat{S}^2_{JG1}$ Proposed Estimator	2259.9938	133.4494	8.1749	359.3822
$\hat{S}^2_{JG2}$ Proposed Estimator	1667.7818	129.8456	3.9142	350.4482
$\hat{S}^2_{JG3}$ Proposed Estimator	1829.6315	132.3799	5.5038	355.3634
$\hat{S}^2_{JG4}$ Proposed Estimator	2125.7591	134.1848	7.8275	359.8641
$\hat{S}^2_{JG5}$ Proposed Estimator	1963.6570	131.6458	5.7705	354.8875

Table 5. Mean squared errors of the existing and proposed modified ratio type variance estimators

Estimator	Mean Squared Error $\mathbf{MSE}(.)$			
	Population 1	Population 2	Population 3	Population 4
$\hat{S}^2_R$ Isaki[10]	670393270	35796612	3925.1627	1415946
$\hat{S}^2_{KC1}$ Kadilar and Cingi[11]	670384403	35796605	3850.1552	1415839
$\hat{S}^2_{KC2}$ Kadilar and Cingi[11]	670169790	35796503	3658.4051	1414994
$\hat{S}^2_{KC3}$ Kadilar and Cingi[11]	670393032	35796611	3898.5560	1415931
$\hat{S}^2_{KC4}$ Kadilar and Cingi[11]	670240637	35796512	3580.8342	1414762
$\hat{S}^2_{JG1}$ Proposed Estimator	669558483	35795045	3480.5516	1405276
$\hat{S}^2_{JG2}$ Proposed Estimator	667000531	35792872	2908.9467	1386468
$\hat{S}^2_{JG3}$ Proposed Estimator	667623576	35794395	3098.4067	1396798
$\hat{S}^2_{JG4}$ Proposed Estimator	668911625	35795495	3427.1850	1406294
$\hat{S}^2_{JG5}$ Proposed Estimator	668182833	35793951	3133.3256	1395796

From the values of Table 4, it is observed that the biases of the proposed modified ratio type variance estimators are less than the biases of the traditional and existing modified ratio type variance estimators. Similarly from the values of Table 5, it is observed that the mean squared errors of the proposed modified ratio type variance estimators are less than the mean squared errors of the traditional and existing modified ratio type variance estimators.

5. Conclusions

Estimating the finite population variance has great significance in various fields such as Industry, Agriculture, Medical and Biological sciences, etc. In this paper we have proposed a class of modified ratio type variance estimators using the quartiles and their functions of the auxiliary variable like Inter-quartile range, Semi-quartile range and Semi-quartile average. The biases and mean squared errors of the proposed modified ratio type variance estimators are obtained and compared with that of traditional ratio type variance estimator and existing modified ratio type variance estimators. Further we have derived the conditions for which the proposed estimators are more efficient than the traditional and existing estimators. We have also assessed the performance of the proposed estimators for some known natural populations. It is observed that the biases and mean squared errors of the proposed estimators are less than the biases and mean squared errors of the traditional and existing modified estimators for certain known populations. Hence we strongly recommend that the proposed modified ratio type variance estimator may be preferred over the traditional ratio type variance estimator and existing modified ratio type variance estimators for the use of practical applications.

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