

# Group Delay Enhancement for Slow and Fast Light in Silicon Microring Resonator Structures

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**Abstract** Single - and double-waveguide coupled resonators are designed and analyzed according to Silicon On Insulator (SOI) technology. In the last few decades much study has been done on the Microring Resonators (MRRs). In order to enhance group delay, many different structures have been proposed and constructed. In this theoretical study, by variation of coupling coefficient in terms of altering distance gap between ring and waveguide, we've successfully simulated and estimated high group delay for slow and fast light in microring with radius of  $\sim 50\mu\text{m}$ . Compared with previously reported experimental results, time delay is improved significantly from 32 ps to 166 ps, which is increased considerably. Also, fast light is observed in single waveguide structure with a maximum time advance of 114 ps. For double waveguide structure, by changing the coupling coefficient, time delay is also improved significantly. Another port of this structure is monitored, simultaneous slow and fast light achieved at single frequency with certain coupling condition.

**Keywords** Microring resonator, Slow and fast light, Silicon waveguide, Transmission and group delay

## 1. Introduction

In recent years, there has been a growing interest towards finding applications in photonic devices. Slow and fast light techniques can be used to obtain significant adjustments for group velocity of light pulses through a medium. Generating slow and fast light signals, have become so attractive in research topics during recent years. In order to control the delay or advancement of optical signals, much of the studies have been concentrated on development of these techniques. For example a number of theoretical and experimental works in this area have been reported in [1].

Slow and fast light devices have been intensely studied because of their specific properties for affecting the flow of light and light-matter interactions. By using optical data transmission through photonic integration, significant saving in power consumption, cost and volume are possible. Photonic techniques overcome the restriction of conventional electronics technology i.e. bandwidth. Many applications such as optical buffer, optical memory, tuneable optical delay line, all optical switch, data synchronization, synthetic aperture radars, cryptography and nonlinear optics have been demonstrated recently [2-4].

Slow and fast light can be generated with different mechanisms. For example: Coherent Population Oscillation (CPO) [5], Stimulated Raman Scattering (SRS) [6],

Stimulated Brillouin Scattering (SBS) in optical filters [7, 8] and Electromagnetically Induced Transparency (EIT) in atomic vapour [9] based on material resonances. Another mechanism for controlling the velocity of light is to utilize dispersive structures, such as Bragg grating [10], photonic crystal waveguides and cavities [11], microsphere resonators and micro-ring resonators [12]. The above optical devices based on dispersive structure are able to work in room temperature, more compact and compatible with modern advance fabrication technology.

Silicon on Insulator (SOI) technology, has become a favourable platform for highly compact photonic devices due to compatibility with CMOS technology and high index contrast between core and cladding. The process of fabrication can be taken from electronic industry. SOI based microring resonators (MRRs) have received intense attention. A typical MRR consists of a bus waveguide and a microring which coupled by a Directional Coupler (DC) or a Multimode Interference coupler (MMI). In DC, by altering the distance between bus and microring, coupling is changed but the loss penalty is observed [13]. MMI based MRRs characteristics have a larger resonance bandwidth and a lower Q factor respect to DC ones.

MRRs have extreme miniaturization, compact size, simple structure and large delay among several resonance structures. These devices are interesting and useful for integrated optics and photonic applications. MRRs have been merged in optical sensing, filtering, switching and wavelength division multiplexing by add-dropping filters [14-16]. Add-drop filters using MRRs due to ease of fabrication and capability for on chip design have been used recently. Slow and fast

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light can be achieved in a single device by changing the spacing gap, input polarization and wavelength. MRRs similar to Fabry-Perot are explained as filters and can be used for higher order filters with expanded free spectral range, switching, fast and slow light. MRRs can be used in different structures, such as elliptic-resonators [17]. Also MRRs may be constructed in series or in parallel. There are two main series structure which have been suggested theoretically and experimentally: Coupled Resonator Optical Waveguide Resonators (CROWs) [18] and Side Coupled Integrated Spaced Sequence of Resonators (SCISSORs) [19]. Both these structures have the potential to alter the propagation of light. SCISSORs show smoother spectral while CROWs demonstrate a noisy characteristic because of fluctuation in the coupling strength.

The rest of the paper is organized as follows. In section 2, we will describe in brief the theoretical viewpoints of coupled mode theory and its applications in ring resonators. In section 3 simulation results demonstrates the effect of design parameters such as coupling and loss in the characteristics of the device. Finally, the conclusion are drawn in section 4.

## 2. Wave Coupling Theory in Ring Resonators

In this section, transmission and group delay characteristics for single- and double-waveguide coupled resonator are studied in brief. These general transmission functions are derived from coupled mode theory or transfer matrix method [18, 20] that described in details in the next sections. Using MATLAB code in order to calculate magnitude and effective phase shift of coupled structures. Group delay also is obtained and is proportional to derivative of the effective phase shift respect to radian-frequency. It can be calculated via MATLAB simulation.

### 2.1. Single Waveguide Coupled Resonator

The relation between incident, circulating and output fields of ring resonator is established from coupled mode theory. Using the notation of Fig. 1, the coupling can be depicted by [20, 21]

$$E_2 = tE_1 + i\sqrt{1-t^2}E_3 \quad (1)$$

$$E_4 = i\sqrt{1-t^2}E_1 + tE_3 \quad (2)$$

Where  $t^2$  is the power self-coupling coefficient. The relation  $r^2 + t^2 = 1$  is between two real qualities. When light in the bus waveguide is coupled to the ring, it experiences round trip phase shift  $\phi$  in the circulation. Also the amplitude of transmission decreased caused by loss. The resulting expression is

$$E_3 = a \exp(i\phi) E_4 \quad (3)$$

Where  $a^2$  is the power loss factor and  $\phi = kL$  denotes

the accumulated phase shift over the ring waveguide.  $K = 2\pi nL/\lambda$ ,  $L$  is the ring cavity length,  $n$  is the effective refractive index of ring waveguide and  $\lambda$  is the wavelength of input light in vacuum. By solving these 3 lately equations at the same time, the normalized transmission response of ring resonator is obtained as

$$t = \frac{E_2}{E_1} = \exp(i(\pi + \phi)) \frac{a - t \exp(-i\phi)}{1 - ta \exp(i\phi)} \quad (4)$$

The transfer function resonances with  $c/nL$  period in the spectrum. The transmission power spectra can be calculated theoretically

$$T = \left| \frac{E_2}{E_1} \right|^2 = \frac{\tau^2 - 2ta \cos(\phi) + t^2}{1 - 2ta \cos(\phi) + a^2 t^2} \quad (5)$$

In the case of  $a=t$  (critically coupling) and  $\phi=0$ , input light is coupled to the ring and transmission is zero.

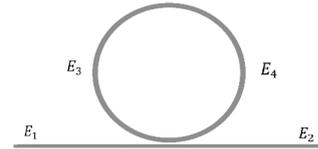


Figure 1. Schematic diagram of single waveguide coupled resonator

The phase of the transmitted light  $\Phi$  and the group delay  $\tau_d$ , respectively, denoted as  $\Phi(\omega) = \arg(E_2/E_1)$  and  $\tau = d\Phi/d\omega$ . Table 1. Summarizes characteristics of single waveguide coupled resonators. Fast ( $d\Phi/d\omega < 0$ ) and slow ( $d\Phi/d\omega > 0$ ) light can be realized under certain conditions.

Table 1. Dispersive Properties of Coupled Resonator

| Condition | Coupling         | Dispersion response |
|-----------|------------------|---------------------|
| $t=a$     | Critical coupled | -                   |
| $t<a$     | Over coupled     | Slow light          |
| $t>a$     | Under coupled    | Fast light          |

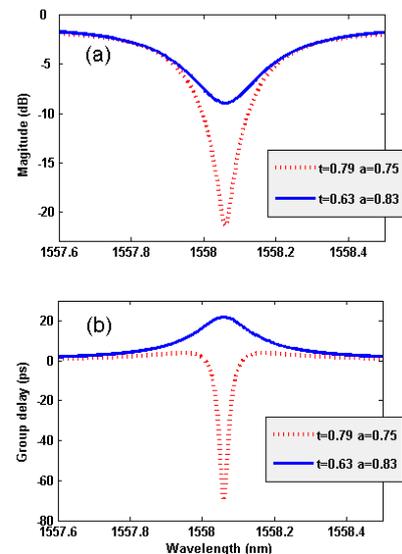


Figure 2. Theoretical results. (a) Magnitude, and (b) group delay of single waveguide coupled resonator for two different coupling conditions

Using coupling and loss parameters in [22], the transmission and group delay can be simulated from above general equations (Fig. 2). As can be seen these results agree with the content of Table 1.

### 2.2. Double Waveguide Coupled Resonator

The schematic diagram of double-waveguide coupled resonator is depicted in Fig.3. The transmission response of port 2 can be calculated from transfer matrix method and expressed as [20]

$$t = \frac{E_2}{E_1} = \frac{t_1 - t_2 a \exp(i\phi)}{1 - t_1 t_2 a \exp(i\phi)} \quad (6)$$

Where  $t_1$  and  $t_2$ , respectively, are self-coupling coefficients of coupler 1 and coupler 2. The intensity transmission factor and group delay can be obtained

$$T = \left| \frac{E_2}{E_1} \right|^2 = \frac{t_1^2 - 2a t_1 t_2 \cos(\phi) + a^2 t_2^2}{1 - 2a t_1 t_2 \cos(\phi) + a^2 t_1^2 t_2^2} \quad (7)$$

$$\tau_d = \frac{d\phi}{d\omega} = \frac{d}{d\omega}(\arg(t)) \quad (8)$$

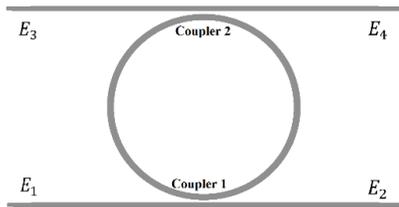


Figure 3. Schematic diagram of double waveguide coupled resonator

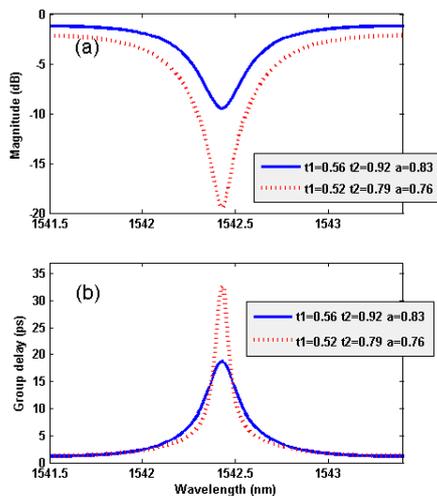


Figure 4. Theoretical results. (a) Magnitude, and (b) group delay of double waveguide coupled resonator for different coupling conditions

Similar to previous section, parameters extracted from [22] are used and transmission spectra and group delay are drawn (Fig.4). This structure can also produce fast light in small values compare with single-waveguide. However bandwidth of double-waveguide is smaller than single one.

## 3. Simulation Results and Discussions

In this work, the effects of variation of parameters are analysed and simulated. Due to the coupled mode theory, the coupling coefficient depends on several design and structural parameters such as distance gap between ring and waveguide.

Ring resonators can produce slow and fast light. Their applications are in optical delay lines and optical buffers, so on. Several structures have been proposed until now such as double-knot resonator structure [23]. The main purpose is improvement of structural characteristics in order to produce adjustable slow and fast light and to increase group delay and operation bandwidth. Adjustable slow and fast light can be achieved by using mutual mode coupling [24]. Here, we are going to increase group delay. This considerable improvement is obtained by optimization parameters which is existed in transmission relation.

The SOI platform provides highly compact photonic devices. The accumulated phase shift  $\Phi$  is constant for a certain value of  $L$  at operating wavelength. According to the equation in the previous section, the transfer function is only affected by coupling and loss coefficient. In these systems, the amount of loss is expressed by dB/cm, therefore for a fix structure, the amount of loss is nearly constant. By varying the distance gap between ring and waveguide, coupling coefficient is altered. In such a way the coupling coefficient is decreased with increasing the gap distance. Optimized coupling coefficient can be obtained by mathematical analysis.

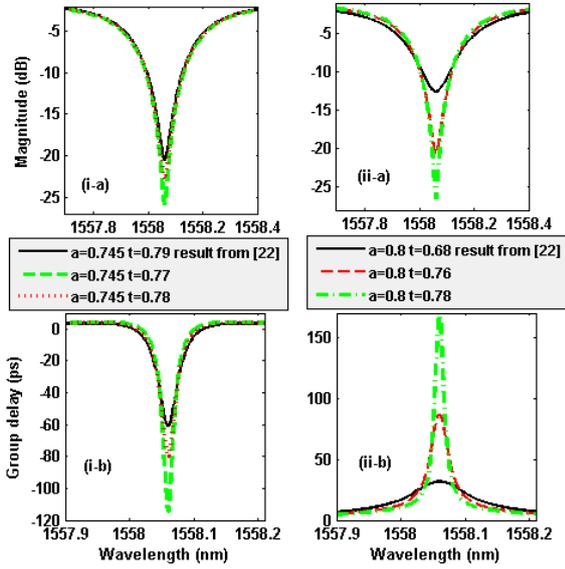
### 3.1. Single Waveguide MRR

Consider a single waveguide coupled to ring with  $340\mu\text{m}$  circumference of the ring in SOI structure. According to Table 1 slow and fast light can be achieved in under and over coupled, respectively. Our simulations agree with experimental and measured results in [22], under same coupling and loss conditions. The optimized coupling coefficient is extracted and simulated in Fig.5 and final results are collected in Table 2.

As can be seen, there is a noticeable increase in group delay for single waveguide in both under and over coupled condition. Group delay enhanced by the factor of more than two.

Table 2. Pulse Delay and Advance of Single Waveguide Ring Resonator for under and over Coupled Condition

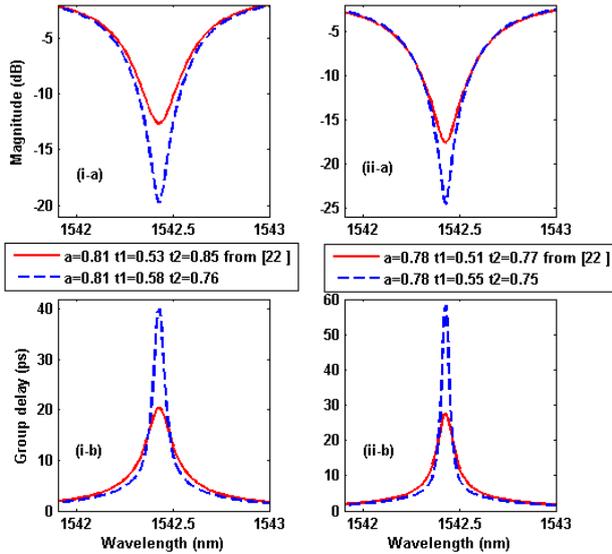
| Coupling                   | Conditions | Delay (ps)              |     |
|----------------------------|------------|-------------------------|-----|
| Under Coupled (fast light) | a=0.745    | t=0.79 (data from [22]) | 60  |
|                            |            | t=0.78                  | 80  |
|                            |            | t=0.77                  | 114 |
| Over Coupled (slow light)  | a=0.8      | t=0.68 (data from [22]) | 32  |
|                            |            | t=0.76                  | 88  |
|                            |            | t=0.78                  | 166 |



**Figure 5.** Simulation results. (a) Magnitude, and (b) group delay for under coupled (i) and over coupled (ii) condition

### 3.2. Double Waveguide MRR

Based on calculations for double waveguide coupled resonator, the coupling coefficient  $t_1$  and  $t_2$  are set, respectively 0.55 and 0.75. Simulation results are compared with experiments in [22] with the same condition for loss coefficient. Group delay is increased which means, this proposed structure has more buffering capacity. Fig.6 shown magnitude and group delay for different coupling coefficients of  $t_1$  and  $t_2$  with equal loss factor. Group delay enhanced from  $\sim 20$  ps to  $\sim 40$  ps.



**Figure 6.** Simulation results. (a) Magnitude, and (b) group delay for two different loss factor

Also, port 4 as an output is monitored. Transfer function of this port can be calculated with transfer matrix method [18] when there is no signal at the port 3 as an input. The transfer function is obtained as:

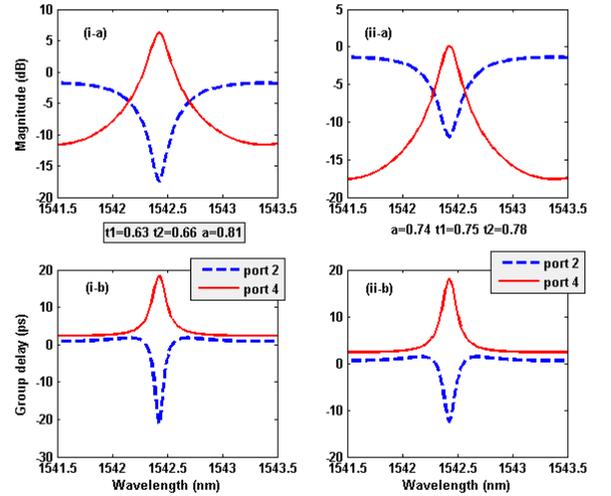
$$\frac{E_4}{E_1} = \exp\left(\frac{a}{(1-t_1^2)(1-t_2^2)}\right) \times \frac{t_1^2 + t_2^2 - 1 - t_1^2 t_2^2}{t_2 - t_1 a e^{i\phi}} \times e^{i\phi} \quad (9)$$

Similar to previous section, group delay can be calculated as:

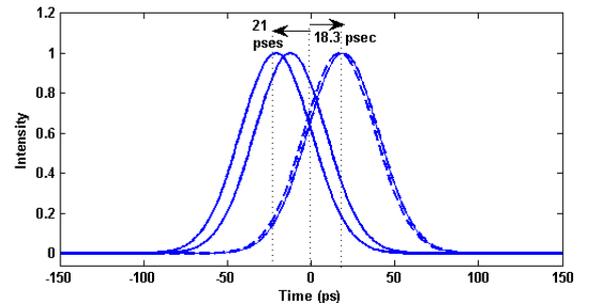
$$\tau_d = \frac{d}{d\omega} \left( \arg \left( \frac{E_4}{E_1} \right) \right) \quad (10)$$

At this time, different values for  $t_1$ ,  $t_2$  and loss are used. Outputs of port 2 and port 4 are drawn in Fig.7. As can be seen, both slow and fast light can be achieved in this device at single wavelength.

We assume that a Gaussian pulse as an input, propagates over the structure. The output of port 2 and 4 are shown in Fig.8 with respect to off resonance propagation. So, simultaneous slow and fast light are possible in this structure under certain conditions.



**Figure 7.** Simulation results. (a) Magnitude, and (b) group delay of double waveguide. Light pulse propagate as slow from port 4 and as fast light from port 2



**Figure 8.** Output of port 2 and port 4 for two different coupling setting. Simultaneous slow and fast light achieve

## 4. Conclusions

In summary, the transmission function is used to analyse group delay of single- and double-waveguide coupled resonator in SOI technology. This structure can produce slow and fast light by controlling the coupling coefficient. Transmission relation can be obtained from coupled mode theory or transfer matrix method. We study pulse propagation through single and double waveguide with different coupling setting. Simulation results with initial values for coupling coefficient agree with previous experimental ones. Coupling coefficient is optimized by variation of distance gap between ring and waveguide and group delay is enhanced and doubled for double waveguide coupled resonator respect to experimental results in the same structure and loss coefficient. Similarly for single waveguide coupled resonator group delay is enhanced from 32 ps to 166 ps for slow light and increased from 60 ps to 114 ps for the fast light. Also, simultaneous slow and fast light is investigated in double waveguide coupled resonator. Two ports are used as outputs to produce time delay and advance at single frequency. This structure is particularly important in applications.

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