

# Squeezing Interferometry for Optical Phase Extraction in Digital Speckle Pattern Interferometry

Vamara Dembele, Khalid Assid\*, Faïçal Alaoui, Abdel Karim Nassim

Laboratoire Instrumentation de Mesure et de Contrôle, Equipe Metrologie Optique et Traitement, Image, Université Chouaib Doukkali-FSJ, BP 20, El Jadida 24000, Morocco)

**Abstract** The purpose of the present work is the use of squeezing interferometry Technique to provide an optical phase mapping from DSPI fringes. The main advantage of this technique is to demodulate by means of a quadrature Gabor filter, a single carrier-frequency fringe pattern formed from intermixing M shifted fringe patterns that are synthesised numerically by the combination of a primary interferogram and its quadrature, which leads directly to the phase without any speckle denoising algorithm, and this after unwrapping the phase with a standard phase unwrapping algorithm and using a simple median filter to smooth the result. This approach was tested on simulated interferograms for different speckle sizes and fringes densities; it was found that the procedure was able to give the result with a good accuracy for all these cases. An application of the proposed procedure to retrieve the phase for experimental fringes recorded from the thermomechanical study of the MOS power transistor is also presented.

**Keywords** Speckle Interferometry, Squeezing Interferometry, Phase Shifting, Phase Unwrapping

## 1. Introduction

Digital speckle pattern interferometry (DSPI) is a whole field optical method for non- contact and nondestructive surface analysis. It's now considered as a powerful tool for industrial measurements. It enables full-field measurement of optical phase changes via the acquisition of speckle patterns[1-3]. After acquisition, a simple subtraction is usually performed to obtain a correlation fringe pattern. The greatest challenges in speckle interferometry focus on relating fringe patterns to phase mapping, permitting the direct determination of surface deformation. However, as DSPI fringes are characterized by a strong speckle noise background, a denoising method[4-6] must be used before the phase evaluation. The development of more sophisticated phase evaluation algorithms such as Phase Shifting methods, are continuously needed[7-9].

The phase shifting technique is the most used in many areas of precise metrology. It ensures a maximum accuracy of the calculated phase, providing quantitative information from interferograms. The phase-shifting technique requires a sequence of interferograms shifted in phase. Usually a minimum of three phase-shifted images is needed. There are many techniques available in the literature for phase shifting interferometry. Malacara, *et. al.*, [10] has an exhaustive amount of phase shifting algorithms.

The aim of this paper is to use the Squeezing interferometry Technique[11] to retrieve a phase mapping from only two  $\pi/2$  shifted speckle fringe patterns without any speckle denoising filtering. The principle of squeezing interferometry is based on transforming N phase shifted fringe patterns by a blending technique into a single carrier fringe pattern and then demodulating this carrier interferogram with a Gabor filter[12]. After unwrapping the result by a special phase unwrapping algorithm named PUMA[13], then we applied a median filter to smooth the unwrapped phase.

The paper is organized as follows. In section 2, we present a brief description of Digital Speckle Pattern Interferometry (DSPI) technique, the Squeezing interferometry Technique and our procedure to provide the phase mapping. In section 3, we present different results on simulated speckle fringe patterns and experimental interferograms acquired from the real fringes of an MOS power transistor to perform the method. Finally, a conclusion is given in section 4.

## 2. Squeezing Interferometry Technique In Digital Speckle Pattern Interferometry

### 2.1. Speckle Fringe Correlation

The speckle fringe patterns are obtained by subtraction of a reference speckled image from the image of a displaced surface. The intensity distribution of a reference speckled image (before displacement) is

$$I_1(x, y) = a(x, y) + b(x, y) \cos \phi_r(x, y) \quad (1)$$

\* Corresponding author:

khalidassid@gmail.com (Khalid Assid)

Published online at <http://journal.sapub.org/optics>

Copyright © 2012 Scientific & Academic Publishing. All Rights Reserved

where  $a(x,y)$  is the bias intensity,  $b(x,y)$  the modulation intensity and  $\phi_s(x,y)$  is the original phase from the speckle that appears as the high frequency and apparently random pixel-by-pixel intensity variation.

After displacement, the intensity distribution becomes

$$I_2(x, y) = a(x, y) + b(x, y) \cos(\phi_s(x, y) + \varphi(x, y)) \quad (2)$$

where  $\varphi$  is the phase change in the light resulting from the displacement. We assume that the displacements are sufficiently small that speckle decorrelation effects can be ignored.

The intensity distribution for the speckle interferogram in the subtraction mode is expressed by

$$I(x, y) = I_2(x, y) - I_1(x, y) \quad (3)$$

$$I(x, y) = 2b(x, y) \sin(\varphi(x, y) / 2) \sin(\phi_s(x, y) + \varphi(x, y) / 2) \quad (4)$$

In DSPI, the fringes obtained are affected by speckle noise and usually; a noise reduction method must be used before the phase distribution is evaluated. The desired information in the  $\sin(\varphi / 2)$  fringe term (shape of envelope modulating the random speckle term) may be rectified and filtered by appropriate computer image processing in order to remove the high frequency  $\sin(\phi_s + \varphi / 2)$  noise.

Squaring Eq. (4) leads to

$$I^2(x, y) = 4b^2(x, y) \sin^2(\varphi(x, y) / 2) \sin^2(\phi_s(x, y) + \varphi(x, y) / 2) \quad (5)$$

Since  $\phi_s(x, y)$  changes rapidly across the speckle pattern, the ensemble average of the second sine squared term in Eq. (5) across the whole measurement area leads to

$$\begin{aligned} \langle \sin^2(\phi_s(x, y) + \varphi(x, y) / 2) \rangle &\approx \\ \frac{1}{2n\pi} \int_0^{2\pi} \sin^2(\phi_s(x, y) + \varphi(x, y) / 2) d(\phi_s(x, y) + \varphi(x, y) / 2) &\quad (6) \\ &= \frac{1}{2} \end{aligned}$$

Substituting Eq. (6) into Eq. (5) yields

$$\begin{aligned} I^2(x, y) &= 2b^2(x, y) \sin^2(\varphi(x, y) / 2) \\ &= b^2(x, y) [1 - \cos(\varphi(x, y))] \end{aligned} \quad (7)$$

Eq. (7) yields a cosine profile and should be more suitable to generate correlation fringe images as an input for phase shifting methods. The new form of the speckle correlation fringe pattern is thus rendered in the classical form of a cosine fringe pattern:

$$I^2(x, y) = J(x, y) = A(x, y) + B(x, y) \cos(\varphi(x, y)) \quad (8)$$

## 2.2. Synthetic Shifted Fringe Patterns

The major phase-shifting techniques require a sequence of shifted interferograms by adding progressively a known phase step. Usually a minimum of three phase-shifted interferograms are needed. In our work, we will generate numerically multiple interferograms with arbitrary phase-shift amount between them from only two interferograms shifted by  $\pi/2$ .

The cosine fringe pattern is

$$J(x, y) = A(x, y) + B(x, y) \cos(\varphi(x, y)) \quad (9)$$

The  $\pi/2$  shifted fringe pattern is

$$Q(x, y) = A(x, y) + B(x, y) \sin(\varphi(x, y)) \quad (10)$$

After removing the background  $A(x, y)$  by a highpass filter, the resultant fringe patterns are:

$$J_a(x, y) = B(x, y) \cos(\varphi(x, y)) \quad (11)$$

$$Q_a(x, y) = B(x, y) \sin(\varphi(x, y)) \quad (12)$$

We propose to combine numerically the interferogram  $J_a(x, y)$  and its quadrature  $Q_a(x, y)$  with  $\cos(\beta)$  and  $\sin(\beta)$  respectively which leads to the  $\beta$  shifted fringe pattern.

$$J_\beta(x, y) = J_a(x, y) \cdot \cos\beta - Q_a(x, y) \cdot \sin\beta \quad (13)$$

$$J_\beta(x, y) = B(x, y) \cdot \cos(\varphi(x, y) + \beta) \quad (14)$$

By this way, numerical generation of multiple shifted interferograms with arbitrary phase steps, from only two primary interferograms, will enable us to extract the phase with an appropriate phase shifting algorithm.

## 2.3. Squeezing Interferometry Technique

The principle of squeezing interferometry[11] is based on transforming  $M$  temporal-spatial phase shifted fringe patterns by a blending technique into a single carrier fringe pattern and then demodulating this carrier interferogram to retrieve the phase.

Squeezing interferometry rearranges the data of  $M$  frames of phase shifted interferograms with size  $L \times L$  to a spatial linear carrier interferogram with extended size  $ML \times L$ , which converts the temporal phase shift to the spatial carrier.

A sequence of  $M$  phase shifted interferograms can be written as

$$J_m(x, y) = B(x, y) \cdot \cos(\varphi(x, y) + 2\pi f m) \quad (15)$$

with  $m=0, 1, \dots, M-1$ . The normalized phase shift frequency  $f=1/M$  and the phase shift step  $\beta$  is

$$\beta = 2\pi f = 2\pi / M$$

Squeezing interferometry rearranges the data of the  $M$  interferograms[11] to obtain the frequency carrier interferogram by

$$J_{sq}(Mx + m, y) = J_m(x, y) \quad (16)$$

Intermixing the  $M$  fringe patterns by this blending technique, we obtained a single spatial carrier frequency fringe pattern  $J_{sq}(x', y)$ . The fringe pattern representation of this re-arrangement is defined by

$$J_{sq}(x', y) = B(x', y) \cdot \cos[\varphi(x', y) + \beta x'] \quad (17)$$

with  $x'=1, 2, \dots, ML$  and  $\varphi(x', y)$  is the unknown phase.

To demodulate the phase from this kind of fringe pattern, one may use a wider range of interferometric techniques such as wavelet or Fourier techniques. In our work, we adopted the Gabor quadrature filter[12] for its higher signal to noise ratio of the estimated phase.

## 2.4. Gabor Filter Demodulation

The carrier frequency interferogram can be demodulated using the Gabor filter[12] described as

$$G_{\beta}(x, y) = [\cos(\beta x) + j \sin(\beta x)] \exp[-((x^2 / N_{\sigma})^2) + (y^2 / \sigma^2)] \quad (18)$$

where  $\beta$  is the phase shift among the  $N$  interferograms and  $\sigma$  is a spreading parameter. By convolution of the spatial carrier frequency interferograms and the Gabor filter, the phase value can be retrieved as

$$\phi(x, y) = \tan^{-1} \left[ \frac{\text{Im} [J_{\text{sq}}(x', y) \otimes G_{\beta}(x, y)]}{\text{Re} [J_{\text{sq}}(x', y) \otimes G_{\beta}(x, y)]} \right] \quad (19)$$

Where  $\otimes$  denotes convolution;  $\phi(x, y)$  is the retrieved phase, and the operators  $\text{Im}(\cdot)$  and  $\text{Re}(\cdot)$  are the imaginary and real parts of the two-dimensional convolution product  $J_{\text{sq}}(x', y) \otimes G_{\beta}(x, y)$ .

### 3. Numerical Results

#### 3.1. DSPI Phase Retrieval Without Speckle Interferogram Denoising

The presentation given in the previous section is a generalization for  $M$  shifted interferograms with a phase step of  $2\pi/M$ . As our method enables generation of multiple interferograms with arbitrary phase shifts, we will limit our study to 3 frames shifted with  $2\pi/3$ .

From Eq. (13), we generated the three fringe patterns with the phase step  $2\pi/3$  according to the following formulas

$$J_0(x, y) = B(x, y) \cdot \cos(\phi(x, y))$$

$$J_{2\pi/3}(x, y) = B(x, y) \cdot \cos(\phi(x, y) + 2\pi/3) \quad (20)$$

$$J_{4\pi/3}(x, y) = B(x, y) \cdot \cos(\phi(x, y) + 4\pi/3)$$

The computer simulated DSPI fringes were generated by the method reported in ref[5]; the DSPI fringes were generated with a resolution of 256x256 pixels and an average speckle size of 2 pixels.

To evaluate the performance of the squeezing interferometry Technique, the following phase distribution was simulated

$$\phi(x, y) = \pi \left[ 1 + 16x_1 + 8x_1^2 + \frac{1}{x_1 + y_1 + 0.06} + \frac{1}{x_1 + y_2 + 0.06} \right] \quad (21)$$

$$\text{with } x_1 = (x - 64)/256, \quad y_1 = (y - 32)/256, \quad y_2 = (y - 160)/256.$$

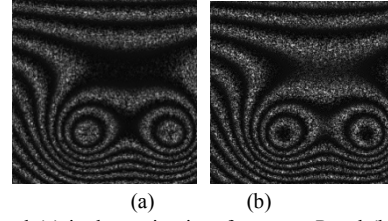
To evaluate the accuracy of this technique, we adopted the fidelity measure  $f$ [13] which is defined by

$$f = 1 - \left( \frac{\sum_{x=1}^L \sum_{y=1}^L |\phi_s(x, y) - \phi(x, y)|}{\sum_{x=1}^L \sum_{y=1}^L \phi_s(x, y)} \right) \quad (22)$$

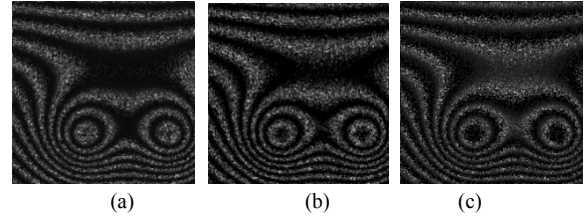
Here  $\phi_s$  represents the synthesised phase,  $\phi$  is the retrieved phase. The fidelity quantifies how well the details are preserved in the retrieved image. Desirable values for  $f$  are close to 1.

The interferogram and its quadrature are shown in Fig.1. In Fig.2, we show the three synthesized  $2\pi/3$  shifted inter-

ferograms.

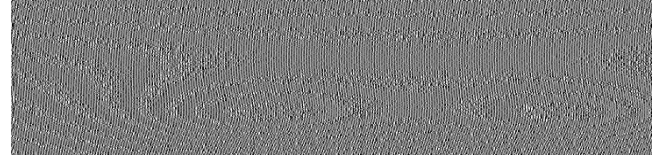


**Figure 1.** Panel (a) is the cosine interferogram. Panel (b) is the sine interferogram



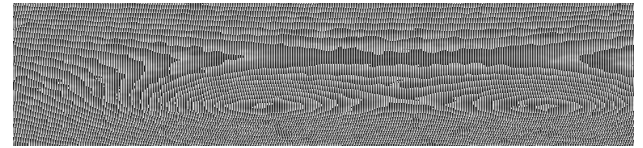
**Figure 2.** Three synthesized interferograms shifted by  $2\pi/3$

In Fig.3, the spatial-carrier frequency interferogram blended from the three synthesized fringe patterns is shown.



**Figure 3.** Single image spatial-carrier frequency interferogram

The phase distribution obtained from Gabor filter before being re-arranged is presented in Fig.4.



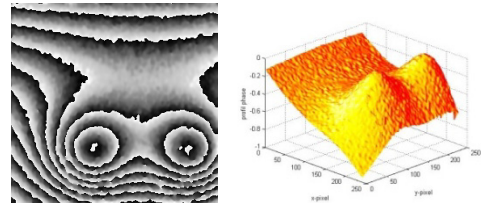
**Figure 4.** Wrapped Phase distribution obtained from Gabor filter

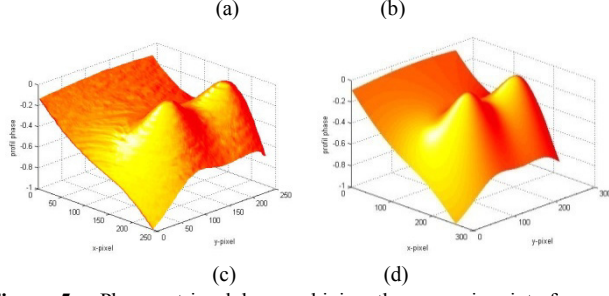
The phase obtained is constant over three consecutive pixels; it is stretched out three times along the  $x$  coordinate. We take the central pixel as the estimate of the wanted phase.

Once having the unwrapped phase, we propose to use a specified phase unwrapping algorithm named PUMA (for phase unwrapping maximum flow)[13] to unwrap the retrieved phase. This algorithm is based on network programming.

Based on the peak signal to noise ratio (PSNR), the effect of noise for demodulated data, the view of unwrapping result and the elapsed time, PUMA presents the best result compared to other phase unwrapping algorithms[14].

Finally, the results are illustrated in (fig.5) where we can see clearly the success of our method in retrieving a complex phase with a fidelity value of 0.996.





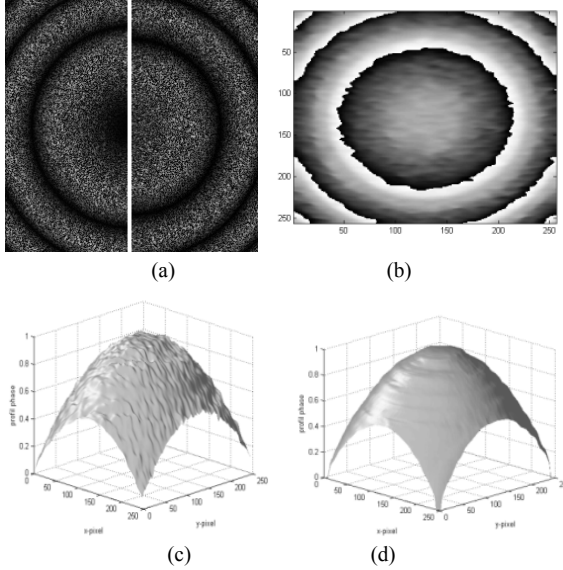
**Figure 5.** Phase retrieval by combining the squeezing interferometry technique, the PUMA algorithm and a median filter. The fidelity value is 0.996. (a). the wrapped phase, (b). the unwrapped phase by PUMA, (c). the smoothed phase by median filter, (d) the noiseless simulated phase

### 3.2. Effect of the Speckle Size

To emphasise the speckle size effect on this technique, we conducted a series of DSPI simulations for different average speckle sizes. The following phase distribution was simulated

$$\phi(x, y) = 0.0006 \left[ (x - 127)^2 - (y - 127)^2 \right] \quad 1 \leq x, y \leq 256 \quad (23)$$

The results are presented in table 1. These results show that combining squeezing interferometry with the PUMA algorithm and a median filter continues to give good results even in the case of large average speckle sizes.



**Figure 6.** The procedure for retrieving the phase with simulated DSPI fringes with speckle size of 1 pixel: (a) computer generated cosine and sine DSPI fringes, (b) the wrapped phase retrieved by squeezing interferometry, (c) the unwrapped phase retrieved by PUMA, (d) the final phase after smoothing with a median filter

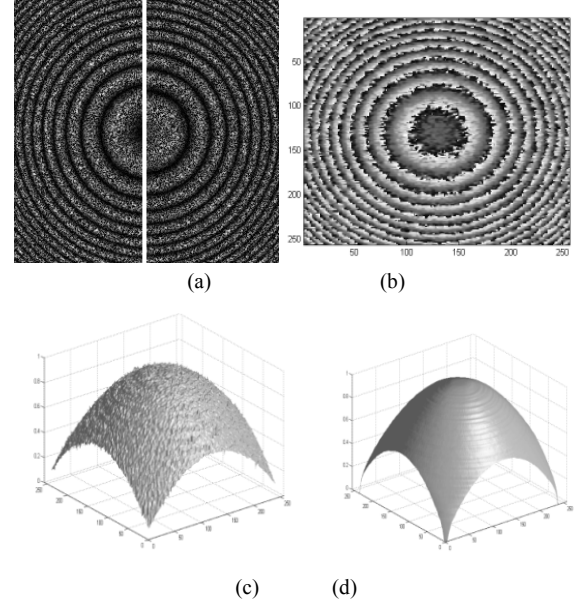
**Table 1.** the fidelity values for different average speckle sizes

Speckle size (pixels)	F fidelity
1	0.9993
2	0.9987
3	0.9985
4	0.9981

### 3.3. Effect of the fringe density

To emphasise the fringe density effect on this technique, we conducted a series of DSPI simulations for different fringe densities and an average speckle size of 1 pixels.

The results are presented in table 2. These results show that our procedure continues to give very good results even in the case of high density fringes.



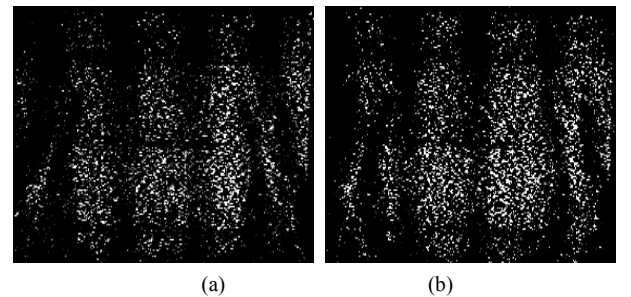
**Figure 7.** The procedure for retrieving the phase with simulated DSPI fringes having 15 fringes: (a) computer generated cosine and sine DSPI fringes, (b) the wrapped phase retrieved by squeezing interferometry, (c) the unwrapped phase retrieved by PUMA, (d) the final phase after smoothing with a median filter

**Table 2.** The fidelity values for different fringe number

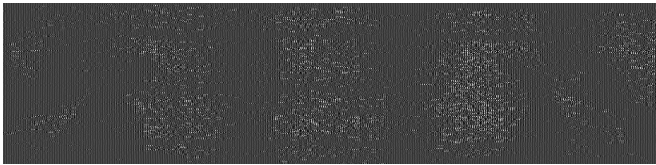
Fringe number	F fidelity
1	0.9985
5	0.9985
10	0.9985
15	0.999

### 3.4. Experimental results

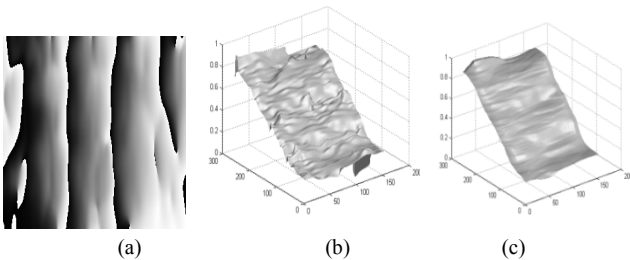
The reliability of power devices depends strongly on their working temperature and the related thermal dilatation. To perform the method, we have used two  $\pi/2$  shifted fringe patterns obtained from the MOS transistor thermomechanical deformation experiment. This study allows determining displacement components of points of the surface of an active MOS transistor in thermal equilibrium and the transient deformation of the out-of-plane deformation.



**Figure 8.** Thermomechanical study of transistor, (a) the primary interferogram, (b) the quadrature interferogram



**Figure 9.** Frequency carrier fringe pattern obtained from the  $2\pi/3$  shifted fringe patterns



**Figure 10.** Retrieved phase by squeezing interferometry technique for the CMOS transistor. (a) wrapped phase, (b) unwrapped phase, (c) smoothed phase

## 5. Conclusions

In this paper, we have presented and tested a squeezing interferometry technique for phase extraction from two  $\pi/2$  shifted interferograms. We have introduced digitally the three  $2\pi/3$  shifted fringes patterns, and applied the blending technique; we generate a single spatial-carrier frequency interferogram using a two dimensional quadrature Gabor filter. After unwrapping the result by the PUMA algorithm, we applied a simple median filter to smooth the phase. We have obtained the phase distribution with a good accuracy avoiding the complex step of speckle denoising. We performed our results on a thermomechanical MOS transistor deformation experiment, where the phase map was easily obtained.

## ACKNOWLEDGEMENTS

We want to thank Dr Joseph W. Goodman for his constructive and helpful comments.

## REFERENCES

- [1] K. Creath, "Phase-shifting speckle interferometry", *Appl. Opt.*, vol 24, pp. 3053–3058, 1985.
- [2] S. Nakadate, and H. Saito, "Fringe scanning speckle-pattern interferometry", *Appl. Opt.*, vol 24, pp. 2172–2180, 1985.
- [3] A. Federico, and G. H. Kaufmann, "Denoising in digital speckle pattern interferometry using wave atoms", *Opt. Lett.*, 32, pp. 1232-1234, 2007.
- [4] D. L. Donoho, "De-noising by soft-thresholding", in *proceedings of IEEE transactions on Information. Theory*, 41, pp. 613–627, 1995.
- [5] A. Federico, and G.H. Kaufmann, "Comparative study of wavelet thresholding methods for denoising electronic speckle pattern interferometry fringes", *Opt. Eng.*, 40, pp. 2598–2604, 2001.
- [6] E. M. Barj, M. Afifi, A. A. Idrissi, A. Nassim, and S. Rachafi, "Speckle correlation fringes using stationary wavelet transform. Application in the wavelet phase evaluation technique", *Optics and Laser Technology*, 38, pp. 506-511, 2006.
- [7] K. Creath, "Phase measurement interferometry techniques", *Progress in optics*, 26, pp. 349–393, 1988.
- [8] B.V. Dorrio, and J. L. Fernandez, "Phase evaluation methods in whole-field optical measurement", *Meas. Sci. Technol.*, 10, pp. 33-55, 1999.
- [9] E. M. Barj, M. Afifi, A. A. Idrissi, S. Rachafi and K. Nassim, "A digital spatial carrier for wavelet phase extraction", *Optik - International Journal for Light and Electron Optics*. 116, pp. 507-510, 2005.
- [10] D. Malacara, M. Servin, and Z. Malacara, *Interferogram Analysis for Optical Testing*, Taylor & Francis, 2005.
- [11] M. Servin, M. Cywiak, D. Malacara-Hernandez, J. C. Estrada, and J. A. Quiroga, "Spatial carrier interferometry from M temporal phase shifted interferograms: Squeezing Interferometry", *Opt. Express*, 16, pp. 9276–9283, 2008.
- [12] Javier R. Movellan, "Tutorials on Gabor Filters, pp.1-20, GNU Free documentation License 1.1, Kolmogorov Project, 2002.
- [13] J. Bioucas-Dias, and G. Valadao, "phase unwrapping via graph cuts", *IEEE Transactions on Image processing*. 16, pp. 698-709, 2007.
- [14] syakrani, N. Mengko, T. L. R. Suksmono, and A. B. Baskoro, "comparison of PUMA and CUNWRAP to 2-D phase unwrapping", in *proceedings of IEEE conference on electrical engeneering and informatics*, pp. 1-6, 2011.