

Study of the Stress Intensity Factor in Artificially Damaged Structures

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Abstract The appearance of cracks in macrostructures creates heterogeneity which manifests itself by a change of the state of stress and strain in the structure. Subjected to stresses, the structures are the seat of a crack growth; this propagation follows a life cycle starting from the initiation phase to the phase where unstable fracture may occur at any moment. In order to understand the mechanism of structural damage by cracking, a study based on the evaluation of the stress distribution in artificially damaged structures is conducted and an assessment of the level of damage is defined by calculating the static damage. This work was also interested to the evaluation of the stress intensity factor theoretically and numerically by energy approach. In the light of the results obtained in this study we were able to assess the effect of the single and the double notch on the mechanical behavior of structures subjected to tension. We also pointed out the criticality of the simple notch compared to the double notch.

Keywords Crack, Life cycle, Damage, Energy approach, Traction, Stress intensity factor

1. Introduction

A component is said damaged when a defect is primed. The defects are classified into two types, the volume defects and planar defects. These are often presented as a notch with different radius notch [1]. Generally, the presence of these notches or defects induces a perturbation in the structures. These disturbances are manifested by high stress concentrations in the vicinity of the notch front. Therefore the analysis of the stress distribution in the vicinities of defects is a key driver to understand the crack behavior and then to the mechanical behavior of the damaged structure.

One knows that there are three failure modes that depend on the load type [2]:

- Mode I: the opening mode where displacements of the crack lips are perpendicular to the direction of propagation.
- Mode II: shear mode in the plane where the displacements of crack lips are parallel to the direction of propagation.
- Mode III: anti-plane shear mode, corresponds to a transverse sliding of crack

Several researchers have presented methods for evaluation of stress intensity factors for different types of

specimens. For many configurations a analytical and numerical methods was used [3-7].

P. Livieri proposes a simplified procedure for the evaluation of the stress intensity factors of cracks by means of an equivalent hole. The procedure is performed to give an engineering response both for numerical or experimental investigations [8].

In S. Reddy study, a combination of crack-stop hole and carbon fiber reinforced polymer (CFRP) overlays under static loads are studied numerically using finite element analysis [9].

Three-dimensional effects near crack front were subject of many analytical, numerical and experimental studies. Different experimental techniques for determining stress intensity factor fully rely on the measurements taken outside the region affected by the three-dimensional effects [10].

This study aims to present a practical method to calculate stress intensity factors using different theories of the mechanics of brittle fracture to determine the mode I brittle fracture risk

2. Experiment

2.1. Study Models

To study the effect of defects on the mechanical behavior of structures, Two types of ABS specimen; simply notched beam and double notched beam were used.

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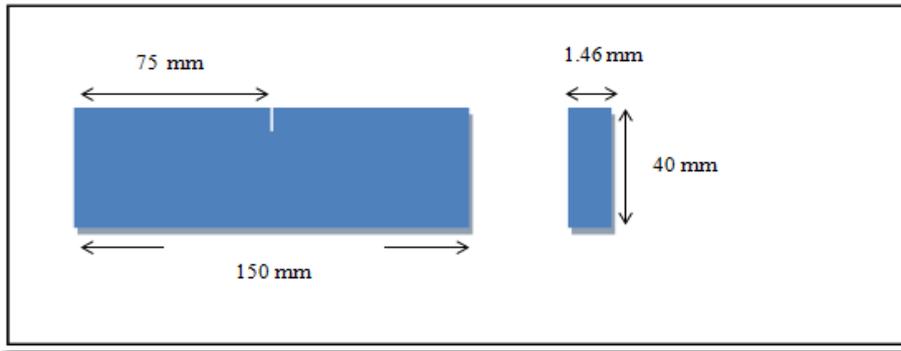


Figure 1. ABS beam simply notched

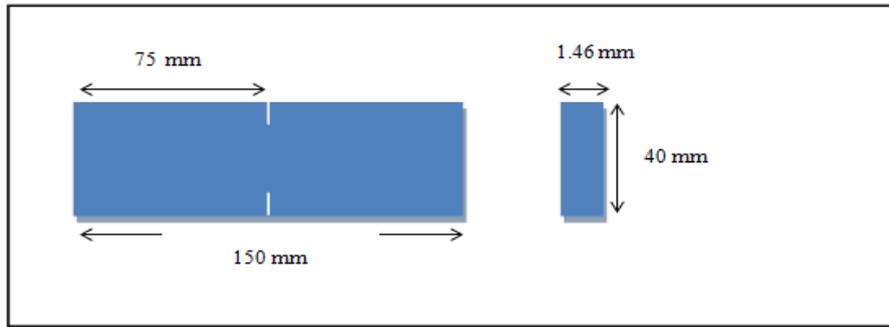


Figure 2. ABS beam doubly notched

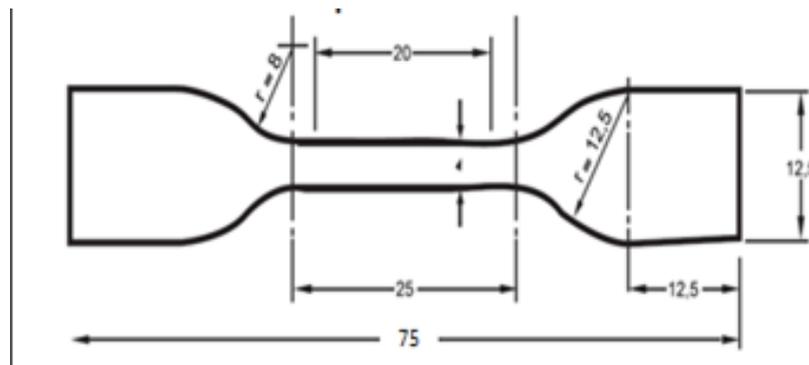


Figure 3. Dumbbell specimen Dimensions

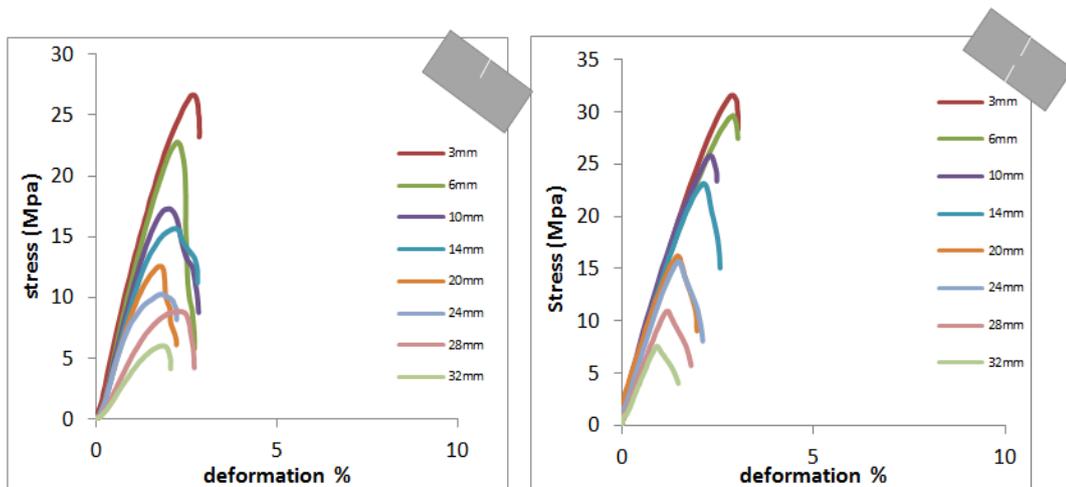


Figure 4. Tensile curves of the applied stress according to deformation for beam simple and double notch

Tests on rectangular beams with sizes of notch from 3 to 32 mm were conducted. Then, a tensile testing was applied on three specimens for each notch size with speed 1 mm/min

2.2. Mechanical Characteristics

For a good characterization of the studied material, the dumbbell shape specimen according to IEC6800-1-1 norms was used (Figure 3).

The mechanical characteristics obtained are shown in the table below:

Table 1. Mechanical characteristics of the studied material

Young's Modulus (GPa)	σ_e (MPa)	σ_{ur} (MPa)	Poisson coefficient
E=2	29,779	34,39	$\nu=0.43$

2.3. Tensile Tests

Tensile tests were conducted on samples defined in Figures 1 and 2, in Public Studies and Trials laboratory (LPEE). The results of these static tests are shown in the figures below

3. Results and Analysis

3.1. Evaluation of the Damage in Simply and Doubly Notched Beam

3.1.1. Static Damage Calculation

The structure state is related to the damage degree. The latter is directly related to the physical properties hence the importance of adopting a model of the damage depending on the stress state in the studied structure. And therefore the expression of the static damage model formulated by [6]:

$$D_s = \frac{1 - \frac{\sigma_{ur}}{\sigma_u}}{1 - \frac{\sigma_a}{\sigma_u}} \tag{1}$$

And the life fraction is defined as:

$$\beta = \frac{a}{w} \tag{2}$$

With:

a: crack length;

w: beam width ;

σ_{ur} : residual ultimate stress;

σ_u : ultimate stress of virgin material;

σ_a : strain just before fracture.

The figure 5 illustrates the behavior of the damage between the beam not damaged in its virgin state that corresponding to no damage and the totally damaged beam whose damage is egal to 1.

It is found from this result that the damage is even higher than the fraction of life is high and approaches to 1 indicating a total failure of the material or otherwise its break and this for two cases simple and double notch.

It follows that the damage reflects the fragility of the material more so than increasing of damage level is associated with the degradation of the physical capacity of the material to break.

3.1.2. Reliability Study of Beam with Single and Double Notch

It is noted that the static reliability curves maintain the same pace for both types of default. At the beginning of the life cycle reliability is 100%, which corresponds to zero damage. Gradually as the damage increases the reliability decreases.

The intersection between reliability curve and damage curve informs us the initiation of life fraction that announces the start of the seed stage.

Table 2. Summary of different stages of damage of beam simply and doubly notched

	Stage I	Stage II	Stage III
Baem simply notched	$\beta \in [0 ; 0,22]$	$\beta \in [0,22 ; 0,65]$	$\beta \in [0,65 ; 1]$
Baem doubly notched	$\beta \in [0 ; 0,4]$	$\beta \in [0,4 ; 0,78]$	$\beta \in [0,78 ; 1]$

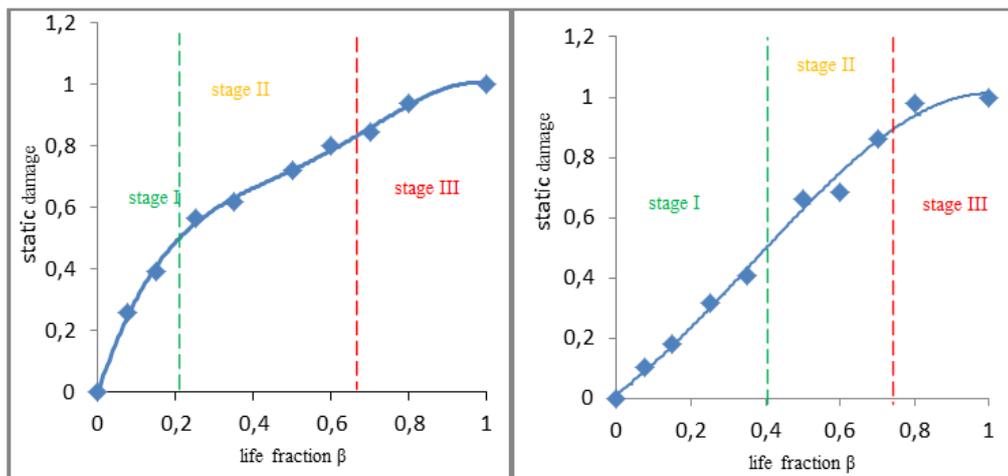


Figure 5. Evolution of static damage depending on the life fraction for simple and double notch

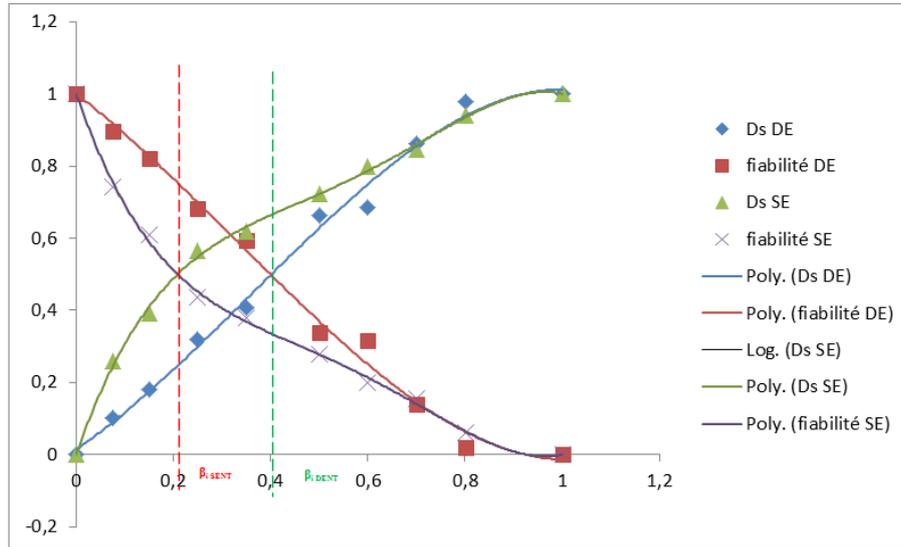


Figure 6. Curves of static damage –Reliability of beam simply and doubly notched

3.1.3. Comparative Study of Damage Level of Beam Simply and Doubly Notched

From the results shown in the figure above (fig.6) and collected in Table 2, one notes that:

- The damage curve of beam simply notched is located above the doubly notched during the first two stages, that it reflect the speed evolution of damage level of simply notched compared to doubly notched. From the critical life fraction β_c that announces the beginning of stage III the two curves merge, It is the unstable phase. The damage becomes uncontrollable; the specimen may at any time express a fatal rupture.
- The initial life fraction is $\beta_{i_SENT}=0,22$ for beam simply notched while for beam doubly notched is $\beta_{i_DENT}=0,4$;
- the stage I which represents the elastic damage area and that is the safe area where one can control the default; is reduced for simple notch , so the stage considered the most stable phase is the most important in the case of the double notch

Facing these findings, one can say that the beam simply notched is most fragile and that the default which has the highest criticality is simple notch which it comes mainly to the stress distribution in test pieces where there is a high concentration of stresses at the simple notch whereas for the double notch this concentration is distributed. Therefore, and in our case the most tolerable defect is the double notch because it presents a more interesting life.

3.2. Calculation of the Stress Concentration Coefficient

Indeed, in the absence of defaults, the stress distribution is uniform in structures. The perturbation of the part geometry is at the origin of stress concentration phenomenon. To illustrate this problem we will expose the case of our studied beam with discontinuities in the form of single and double

notch

To assess the density of this phenomenon, a theoretical calculation of the stress concentration factor for both cases was conducted:

• Beam simply notched

For tension case and for simple geometry having a default in the form of single notch the mathematical formula of the stress concentration factor is given by Peterson formula [7]:

$$Kt = 1 + \sqrt{\frac{a}{\rho}} \tag{3}$$

From equation (3) and every calculation made (Tab.3) it was possible to trace evolution curve of the stress concentration coefficient cf. fig.7:

Table 3. Stress concentration factor value of beam simply notched

A	Kt
0	1
3	4,464
6	5,899
10	7,324
14	8,483
20	9,944
24	10,798
28	11,583
32	12,314

It is noted that the stress concentration factor progressively changed with size notch since it is a value of 4,4 of 3 mm crack size and increases up to 12 for 32 mm crack size. This stress concentration factor values reflect the stress concentration degree at the discontinuity and therefore assesses the degree of criticality of the simple notch.

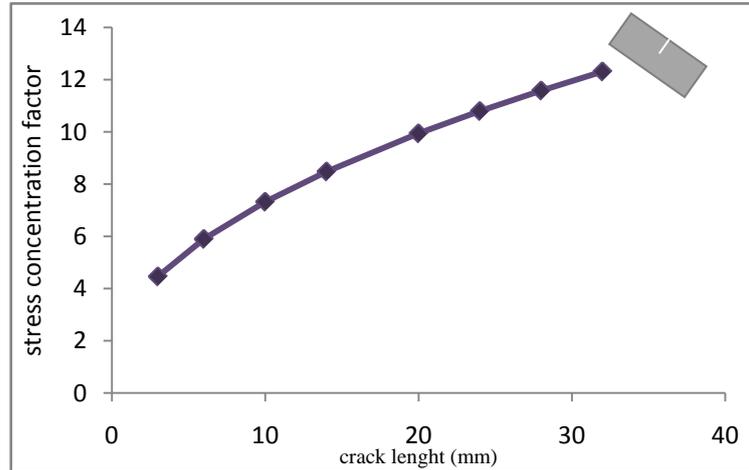


Figure 7. Evolution of stress concentration factor of beam simply notched

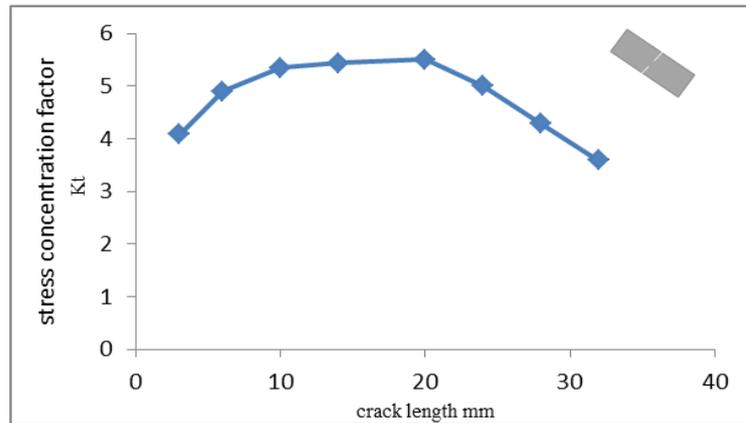


Figure 8. Evolution curve of stress concentration factor of double notch

This allows to say that default « single notch » present a level of significant damage this justifies the results of damage study.

• **Beam doubly notched**

For double notched the theoretical formula of stress concentration factor according to Neuberse provides follows [8]:

$$Kt = 1 + \frac{(Scs-1)(Scd-1)}{\sqrt{(Scs-1)^2+(Scd-1)^2}} \quad (4)$$

With:

$$Scs = 1 + 2\sqrt{\frac{a}{r}} \quad (5)$$

$$Scd = \frac{2(\frac{b}{r}+1)\sqrt{\frac{b}{r}}}{\sqrt{\frac{b}{r}+(\frac{b}{r}+1)}\arctan\sqrt{\frac{b}{r}}} \quad (6)$$

After all calculation made one find the results in the table below:

It is noted that the evolution of the stress concentration factor increases with the crack size to a size of 24 mm where there is a decreasing of the curve Figure 8.

This drop coincides with the unstable area of the damage where it can no longer control the structure behavior and

therefore the formulas are no longer valid.

Table 4. Stress concentration factor summary of double notch

a	b	Scd	Scs	Kt
0	40	8,066	1	1
3	37	7,759	4,464	4,083
6	34	7,439	5,899	4,899
10	30	6,991	7,324	5,349
14	26	6,512	8,483	5,438
20	20	5,719	9,944	5,542
24	16	5,124	10,797	4,801
28	12	4,452	11,583	4,282
32	8	3,661	12,314	3,590

Comparing stress concentration factor results of the both studied cases: the simple and double notch, it is clearly that the simple notch present important stress concentration factor which reflecting the existence of high stress concentration area at the simple notch which justifies its criticality degree relative to double notch.

3.3. Stress Intensity Factor

The stress intensity factor characterizes the structure state in the vicinity of the notch defining the intensity of the stress field at the tip of the notch. It depends on the stress distribution and the notch geometry. In this part the SIF simply notched and doubly notched structure was calculated using analytical formulas [9], [10].

• **Beam simply notched**

The stress intensity factor of beam simply notched is calculated with following formula:

$$K_I = \sigma \cdot \sqrt{\pi a} \cdot f(\bar{a}) \tag{7}$$

With:

$$\bar{a} = \frac{a}{w};$$

$$f(\bar{a}) = 1,12 - 0,231\bar{a} + 10,55\bar{a}^2 - 21,72\bar{a}^3 + 30,39\bar{a}^4$$

After all calculation made one find the following results:

The figure 9 above shows the evolution of the stress intensity factor in mode I according to crack size, it is observed that the stress intensity factor increases exponentially with the advancing of default, it marks a sharp increase when the life fraction exceeds 0.6, after this life

fraction there is the damage unstable region, failure can occur at any time and consequently the formulas are no longer applicable in this area where high values of the SIF.

To validate this theoretical calculation a numerical finite element of SIF by J-integral approach was used. The results are presented in graph figure 10.

It is clear that the curve that represents the numeric SIF develops exponentially with crack size and follows the theoretical curve to a crack size of 30 mm, which coincides with the unstable region of the damage.

Table 5. SIF calculation of beam simply notched

W (m)	σ (MPa)	a (m)	a/w	$KI(Mpa\sqrt{m})$
0,04	11,908	0,001	0,025	0,748
0,04	11,908	0,002	0,05	1,068
0,04	11,908	0,004	0,1	1,580
0,04	11,908	0,008	0,2	2,588
0,04	11,908	0,012	0,3	3,838
0,04	11,908	0,016	0,4	5,617
0,04	11,908	0,02	0,5	8,438
0,04	11,908	0,024	0,6	13,168

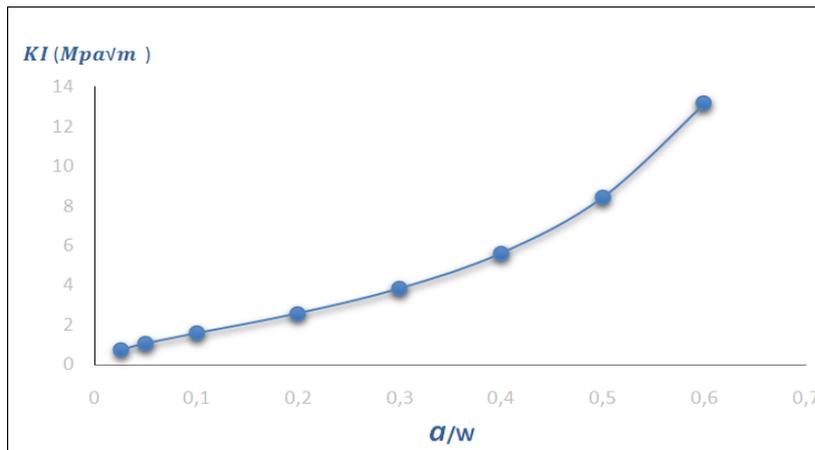


Figure 9. Stress intensity factor evolution of beam simply notched

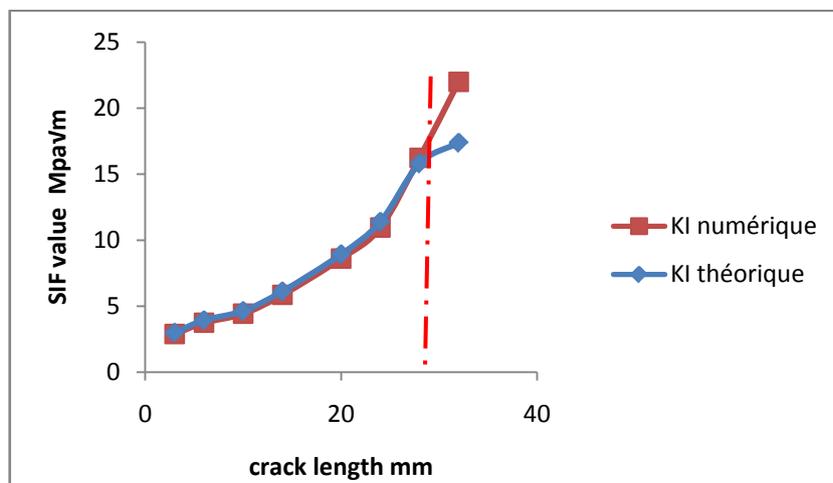


Figure 10. Numerical and theoretical curves of the SIF

• **Beam doubly notched:**

The approach adopted in the calculation of the stress intensity factor at the beam having a double notch is given by the following expression:

$$K_I = \sigma \cdot \sqrt{\pi a} \cdot f(\bar{a}) \tag{8}$$

With:

$$\bar{a} = \frac{a}{w}$$

$$f(\bar{a}) = \frac{1,122 - 0,561\bar{a} - 0,205\bar{a}^2 + 0,471\bar{a}^3 - 0,190\bar{a}^4}{\sqrt{1 - \bar{a}}}$$

After all calculation made we find the results in table 6 below

The figure 11 shows the SIF variation in mode I according life fraction, it is observed that the SIF increases exponentially and it becomes important when life fraction exceeds 0.8 which coincides with the unstable area where one cannot judgmental because the formulas are no longer applicable in this area and the values are no longer reliable.

From the results obtained by the two graphs of the stress intensity factor, one observes that it increases exponentially with notch size advancing.

Is also noticed that the SIF of simple notched case is larger than double notch case this is mainly due to stress distribution where one have previously noted that greater stress concentration at the simple notch.

4. Conclusions

This manuscript provides a spotlight on the mechanical behavior in mode I artificially damaged structures by means of the simple and the double notch solicited by uniaxial tension.

The study is presented in three parts; the first was devoted to the study of static damage of both cases studied. This work led to:

- ❖ Distinguish three stages of damage of beam simply and doubly notched; they are the initiation stages, the controllable propagation and then the unstable stage where the structure failure can occur at any time.
- ❖ Identify initial and critical life fraction of both studied cases

Thus the results obtained in this part allowed judging on the criticality of default type, in our case simple notch is more harmful than double notch.

The second part analyzes the beam stress distribution by theoretical calculation of stress intensity factor for every default size which allowed deducing that the simply notched structure present important concentration factor and consequently a high concentration reflecting a high level of criticality.

Table 6. SIF calculation of beam doubly notched

W (m)	σ (MPa)	a (m)	2a/w	$KI(Mpa\sqrt{m})$
0,04	11,908	0,001	0,05	0,74895585
0,04	11,908	0,002	0,1	1,0591556
0,04	11,908	0,004	0,2	1,50030607
0,04	11,908	0,008	0,4	2,16969934
0,04	11,908	0,012	0,6	2,88390017
0,04	11,908	0,016	0,8	4,21150446
0,04	11,908	0,018	0,9	5,99860301
0,04	11,908	0,019	0,95	8,49915817

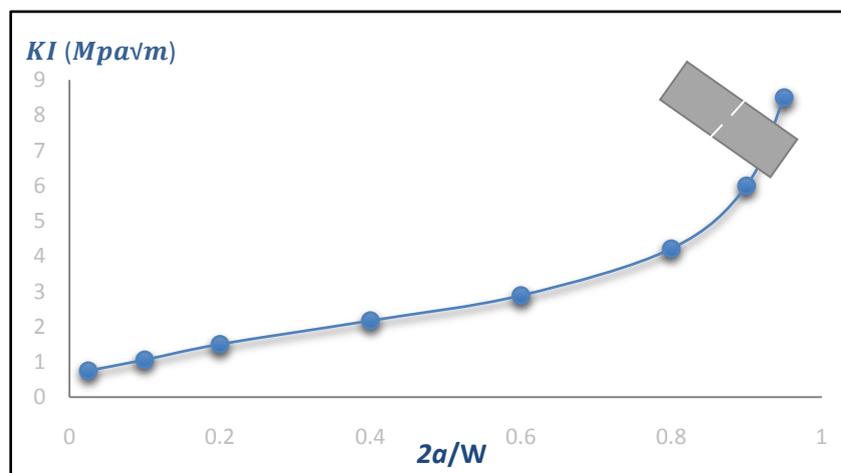


Figure 11. Stress intensity factor evolution according to 2a/w

The third part evaluates the stress intensity factor of both cases by analytical approach which was validated by a numerical finite element study. The results of this part come consolidate those other parties that one scored a bigger SIF at the single notch.

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