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Free Vibration Analysis of Ring Shaped Plate of Polygonal Cross-Sections Immersed in Fluid

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Abstract The free vibration analysis of ring shaped plate of polygonal cross-sections immersed in fluid is studied using the Fourier expansion collocation method. The equations of motion based on two-dimensional theory of elasticity is applied under the plane stress-strain assumptions of elastic plate of polygonal cross-sections namely, triangle, square, pentagon and hexagon is made of isotropic material. The frequency equations are obtained for longitudinal and flexural anti symmetric modes of vibrations. The computed non-dimensional frequencies are plotted in the form of dispersion curves for the material copper. Comparison is made between the frequency responses of plate in space and plate immersed in fluid. The general theory can be used to study any kinds of ring shaped plate using the proper geometrical relations.

Keywords Solid-Fluid Interface, Wave Propagation in Plate, Vibration of Thermal Plate, Piezoelectric Plate, Plate Immersed in Fluid, Generalized Thermo Elastic Plate, Cylinder Immersed In Fluid

1. Introduction

The plates of circular and plates of polygonalcross-sections are often used as structural components and their vibration characteristics are important for practical design. The frequency responses of rotating arbitrary / polygonal cross-sectional plates has many applications in various fields of science and technology, namely, submarine structures, pressure vessel, bore wells, ship building industries and have many other engineering applications. Nagaya[1-3] studied the simplified method for solving problems of vibrating plates of doubly connected arbitrary shape using the Fourier expansion collocation method, he has detained the frequency equations for a doubly connected polygonal cross-sectional plate and numerical calculation were carried for the above said cross-sections. Following Nagaya, Ponnusamy[4,5] discussed the frequency responses of thermo-elastic plate of arbitrary cross-sections and generalized thermo-elastic plate of polygonal cross-sections by using the Fourier expansion collocation method developed by Nagaya[1,-3]. Venkatesan and Ponnusamy[6,7] studied the wave propagation in solid and generalized solid cylinder of arbitrary cross-sections immersed in fluid using the Fourier expansion collocation method. Sinha et al[8] have studied the axisymmetric wave propagation in circular cylindrical shell immersed in a fluid, in two parts. In Part I, the theoretical analysis of the propagation modes is discussed and in Part II, the axisymmetric modes excluding tensional modes are obtained theoretically and experimentally and are compared. Berliner and Solecki[9] have studied the wave propagation in a fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results. Easwaran and Munjal[10] investigated the effect of wall compliance on lowest order mode propagation in a fluid filled or submerged impedance tubes. Based on the closed form analytical solution of the coupled wave equations and applying the boundary conditions at the fluid-solid interface, an eigen equation was obtained and then the dispersion behavior of wave motion was analyzed. Also, they investigated axial attenuation characteristics of plane waves along water filled tubes submerged in water or air.

This paper demonstrates the in-plane vibration of rings of polygonal cross section composed of homogeneous isotropic material immersed in a fluid. In the first part, solutions to the equations of motion for an isotropic medium using the two dimensional theory of elasticity and for the fluid in and around the solid medium are described. It is assumed that, there is no vibration and displacement along the z-axis, that is the displacement along the z-axis, w is zero. To satisfy the boundary conditions, the Fourier expansion collocation method is performed to the equations of the boundary conditions along the boundary of the inner and outer surface of the ring. Also, the frequency equations are obtained for the symmetric and antisymmetric cases. To illustrate the validity of the model, some numerical examples are provided for the frequency equation. The equation is solved by omitting the fluid medium and the results are compared with the

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frequency responses of plate in space and plate immersed in fluid. Also, the results are obtained for polygonal (triangle, square, pentagon and hexagon) rings for the longitudinal and flexural antisymmetric modes of vibrations.

2. Formulation of the Problem

We consider a homogeneous, isotropic, elastic ring shaped plate of polygonal cross-sections immersed in fluid. The system displacements and stresses are defined by the polar coordinates r and θ in an arbitrary point inside the plate and denote the displacements u_r in the direction of r and u_θ in the tangential direction θ. The in-plane vibration and displacements of ring shaped plate of polygonal cross-sections immersed in fluid is obtained by assuming that there is no vibration and displacement along the z-directions in the cylindrical coordinate system(r,θ,z).

The two dimensional stress equations of motion, strain displacement relations in the absence of body forces for a linearly elastic medium are considered from Ponnusamy[4] as

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho\ddot{u}_r$$
$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + 2r^{-1}\sigma_{r\theta} = \rho\ddot{u}_\theta \tag{1}$$

and

$$\sigma_{rr} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{rr}$$
$$\sigma_{\theta\theta} = \lambda\left(e_{rr} + e_{\theta\theta}\right) + 2\mu e_{\theta\theta}$$
$$\sigma_{r\theta} = 2\mu e_{r\theta} \tag{2}$$

Where $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}$ are the stress components, $e_{rr}, e_{\theta\theta}, e_{r\theta}$ are the strain components, ρ is the mass density, λ and μ are Lame's constants.

The strain e_{ij} related to the displacements are given by

$$e_{rr} = u_{r,r},\ e_{\theta\theta} = r^{-1}\left(u_r + u_{\theta,\theta}\right),\ e_{r\theta} = u_{\theta,r} - r^{-1}\left(u_\theta - u_{r,\theta}\right) \tag{3}$$

in which u_r and u_θ are the displacements components along radial and circumferential directions respectively. The comma in the subscripts denotes the partial differentiation or derivative with respect to the variables.

Substituting the Eqs.(2) and (3) in Eqs.(1), the displacement equations of motions are obtained as

$$\left(\lambda+2\mu\right)\left(u_{r,rr} + r^{-1}u_{r,r} - r^{-2}u_r\right) + \mu r^{-2}u_{r,\theta\theta} + \left(\lambda+\mu\right)r^{-1}u_{\theta,r\theta} - \left(\lambda+3\mu\right)r^{-2}u_{\theta,\theta} = \rho u_{r,tt}$$
$$\mu\left(u_{\theta,rr} + r^{-1}u_{\theta,r} - r^{-2}u_\theta\right) + \left(\lambda+2\mu\right)r^{-2}u_{\theta,\theta\theta} + \left(\lambda+3\mu\right)r^{-2}u_{r,\theta} + \left(\lambda+\mu\right)r^{-1}u_{r,r\theta} = \rho u_{\theta,tt}. \tag{4}$$

3. Solutions of Ring Shaped Plate

To obtain the free vibration of ring shaped plate of polygonal cross-sections, we seek the solution of Eq.(4) in the form

$$u_r(r,\theta,t) = \sum_{n=0}^{\infty}\varepsilon_n\left[\left(\phi_{n,r} + r^{-1}\psi_{n,\theta}\right) + \left(\bar{\phi}_{n,r} + r^{-1}\bar{\psi}_{n,\theta}\right)\right]e^{i\omega t}$$
$$u_\theta(r,\theta,t) = \sum_{n=0}^{\infty}\varepsilon_n\left[\left(r^{-1}\phi_{n,\theta} - \psi_{n,r}\right) + \left(r^{-1}\bar{\phi}_{n,\theta} - \bar{\psi}_{n,r}\right)\right]e^{i\omega t} \tag{5}$$

where $\varepsilon_n = \frac{1}{2}$ for $n=0$, $\varepsilon_n = 1$ for $n \geq 1$, $i=\sqrt{-1}$, ω is the frequency, $\phi_n(r,\theta)$, $\psi_n(r,\theta)$, $\bar{\phi}_n(r,\theta)$ and $\bar{\psi}_n(r,\theta)$ are the displacement potentials.

By introducing the dimensionless quantities $\bar{\lambda} = (\lambda/\mu)$, $\Omega^2 = \rho\omega^2a^2/\mu$, $T = t\sqrt{\mu/\rho}/a$, $x = r/a$ and substituting Eq.(5) in Eq.(4), we obtain

$$\left[\left(2+\bar{\lambda}\right)\nabla^2 + \Omega^2\right]\phi_n = 0 \tag{6}$$

and

$$\left[\nabla^2 + \Omega^2\right]\psi_n = 0 \tag{7}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\,\partial/\partial x + x^{-2}\,\partial^2/\partial\theta^2$

The solution of Eq. (6) for symmetric mode is

$$\phi_n = \left[A_{1n}J_n\left(\alpha_1 ax\right) + B_{1n}Y_n\left(\alpha_1 ax\right)\right]\cos n\theta \tag{8}$$

and the solution for the anti symmetric mode $\bar{\phi}_n$ is obtained by replacing $\cos n\theta$ by $\sin n\theta$ in Eq. (8), we get

$$\bar{\phi}_n = \left[\bar{A}_{1n} J_n\left(\alpha_1 ax\right) + \bar{B}_{1n} Y_n\left(\alpha_1 ax\right)\right]\sin n\theta \tag{9}$$

where J_n is the Bessel function of first kind of order n and Y_n is the Bessel function of second kind of order n and $\left(\alpha_1 a\right)^2 = \Omega^2/\left(2+\bar{\lambda}\right)$. Solving Eq. (7), we obtain

$$\psi_n = \left[A_{2n} J_n\left(\alpha_2 ax\right) + B_{2n} Y_n\left(\alpha_2 ax\right)\right]\sin n\theta \tag{10}$$

for symmetric mode. For the antisymmetric mode $\bar{\psi}_n$ is obtained from Eq. (10) by replacing $\sin n\theta$ by $\cos n\theta$ in Eq. (10), we obtain

$$\bar{\psi}_n = \left[\bar{A}_{2n} J_n\left(\alpha_2 ax\right) + \bar{B}_{2n} Y_n\left(\alpha_2 ax\right)\right]\cos n\theta \tag{11}$$

where $\left(\alpha_2 a\right)^2 = \Omega^2$. If $\left(\alpha_1 a\right)^2 < 0$ and $\left(\alpha_2 a\right)^2 < 0$, then the Bessel functions J_n and Y_n are replaced by the modified Bessel functions I_n and K_n respectively.

4. Solutions of Fluid Medium

In cylindrical polar coordinates r and θ the acoustic pressure and radial displacement equation of motion for an inviscid fluid are of the form Achenbach[11] ,

$$p^f = -B^f\left(u^f_{r,r} + r^{-1}(u^f_r + u^f_{\theta,\theta})\right) \tag{12}$$

and

$$c_f^{-2} u^f_{r,tt} = \Delta,_r \tag{13}$$

respectively, where B^f is the adiabatic bulk modulus, ρ^f is the density, $c_f = \sqrt{B^f/\rho^f}$ is the acoustic phase velocity in the fluid, $\left(u^f_r, u^f_\theta\right)$ is the displacement vector and

$$\Delta = \left(u^f_{r,r} + r^{-1}(u^f_r + u^f_{\theta,\theta})\right) \tag{14}$$

Substituting

$$u^f_r = \phi^f_{,r}, u^f_\theta = r^{-1}\phi^f_{,\theta} \tag{15}$$

and seeking the solution of Eq. (13) in the form

$$\phi^f(r,\theta,t) = \sum_{n=0}^{\infty} \varepsilon_n\left[\phi^f_n(r)\cos n\theta + \bar{\phi}^f_n(r)\sin n\theta\right]e^{i\omega t}, \tag{16}$$

the oscillating waves propagating in the inner fluid located in the annulus is given by

$$\phi^f_n = A_{3n} J_n(\alpha_3 ax) \tag{17}$$

where $\left(\alpha_3 a\right)^2 = \Omega^2/\bar{\rho}^f_1 \bar{B}^f_1$, in which $\bar{\rho}^f_1 = \rho_1/\rho^f$, $\bar{B}^f_1 = B^f_1/\mu$. If $\left(\alpha_3 a\right) < 0$, the Bessel function J_n in the Eq. (17) is to be replaced by modified Bessel function I_n. Similarly, for the outer fluid that represents the oscillatory waves propagating away is given as

$$\phi^f_n = B_{3n} H^{(2)}_n(\alpha_4 ax) \tag{18}$$

where $\left(\alpha_4 a\right)^2 = \Omega^2/\bar{\rho}^f_2 \bar{B}^f_2$, in which $\bar{\rho}^f_2 = \rho_2/\rho^f$, $\bar{B}^f_2 = B^f_2/\mu$, $H^{(2)}_n$ is the Hankel function of the second kind of order n. If $(\alpha_4 a)^2 < 0$, then the Hankel function of second kind is to be replaced by K_n, where K_n is the modified Bessel function of the second kind. By substituting Eq. (16) in Eq. (12) along with the Eqs. (17) and (18), the acoustic pressure for the inner fluid can be expressed as

$$p^f_1 = A_{3n}\Omega^2 \bar{\rho}_1 J_n(\alpha_3 ax)\cos n\theta e^{i\Omega T_a} \tag{19}$$

while for the outer fluid, it is given by

$$p^f_2 = B_{3n}\Omega^2 \bar{\rho}_2 H^{(2)}_n(\alpha_4 ax)\cos n\theta e^{i\Omega T_a} \tag{20}$$

In the antisymmetric case, the solutions for the inner and outer fluid are obtained as

$$\bar{\phi}^f_n = A_{3n} J_n(\alpha_3 ax) \tag{21}$$

and

$$\phi_n^f = B_{3n} H_n^{(2)}(\alpha_4 ax) \tag{22}$$

Similarly, the acoustic pressure for the antisymmetric mode are obtained as

$$\overline{p}_1^f = \overline{A}_{3n} \Omega^2 \overline{\rho}_1 J_n(\alpha_3 ax) \sin n\theta e^{i\Omega T_a} \tag{23}$$

for the inner fluid and

$$\overline{p}_2^f = \overline{B}_{3n} \Omega^2 \overline{\rho}_2 H_n^{(2)}(\alpha_4 ax) \sin n\theta e^{i\Omega T_a} \tag{24}$$

for the outer fluid.

5. Boundary Conditions and Frequency Equations

The boundary conditions along the curved surfaces of the ring are

$$\left(\sigma_{xx} + p_1^f\right)_l = \left(\sigma_{xy}\right)_l = \left(u_r - u_r^f\right)_l = 0 \tag{25}$$

for the inner boundary and

$$\left(\sigma_{xx} + p_2^f\right)_l = \left(\sigma_{xy}\right)_l = \left(u_r - u_r^f\right)_l = 0 \tag{26}$$

for the outer boundary, where x is the coordinate normal to the boundary and y is the coordinate in the tangential direction, σ_{xx} is the normal stress, σ_{xy} are the shearing stresses and $(\ )_l$ is the value at the $l-th$ segment of the boundary. The first and last conditions in Eqs. (25) and (26) are due to the continuity of the stresses and displacements of the ring shaped plate and fluid on the curved surfaces. Since the boundary of the cross section is irregular in shape, it is difficult to satisfy the boundary conditions along both inner and outer surfaces of the ring shaped polygonal cross sectional plate immersed in fluid directly. If γ_l is the angle between normal to the segment and the reference axis and if it is assumed to be constant, then the transformed expressions for the stresses are Nagaya[3] as

$$\begin{aligned}\sigma_{xx} &= \lambda\left(u_{r,r} + r^{-1}\left(u_r + u_{\theta,\theta}\right)\right) + 2\mu[u_{r,r}\cos^2(\theta-\gamma_l) + r^{-1}\left(u_r + u_{\theta,\theta}\right)\sin^2(\theta-\gamma_l) \\ &\quad + 0.5\left(r^{-1}\left(u_\theta - u_{r,\theta}\right) - u_{\theta,r}\right)\sin 2(\theta-\gamma_l)] \\ \sigma_{xy} &= \mu[u_{r,r} - r^{-1}\left(u_{\theta,\theta} + u_r\right)\sin 2(\theta-\gamma_l) + \left(r^{-1}\left(u_{r,\theta} - u_\theta\right) + u_{\theta,r}\right)\sin 2(\theta-\gamma_l)]\end{aligned} \tag{27}$$

Substituting Eqs. (8)-(11), (19), (20), (23), (24) in the Eqs. (25) and (26), the boundary conditions are transformed as follows:

$$\left[\left(S_{xx}^1\right)_l + \left(\overline{S}_{xx}^1\right)_l\right]e^{i\Omega T_a} = 0, \quad \left[\left(S_{xy}^1\right)_l + \left(\overline{S}_{xy}^1\right)_l\right]e^{i\Omega T_a} = 0$$
$$\left[\left(S_r^1\right)_l + \left(\overline{S}_r^1\right)_l\right]e^{i\Omega T_a} = 0 \tag{28}$$

for the inner surface and

$$\left[\left(S_{xx}\right)_l + \left(\overline{S}_{xx}\right)_l\right]e^{i\Omega T_a} = 0, \quad \left[\left(S_{xy}\right)_l + \left(\overline{S}_{xy}\right)_l\right]e^{i\Omega T_a} = 0$$
$$\left[\left(S_r\right)_l + \left(\overline{S}_r\right)_l\right]e^{i\Omega T_a} = 0 \tag{29}$$

for the outer surface, where

$$S_{xx}^1 = 0.5\left(p_0^1 A_{10} + p_0^2 B_{10} + p_0^5 A_{50}\right) + \sum_{n=1}^{\infty}\left(p_n^1 A_{1n} + p_n^2 B_{1n} + p_n^3 A_{2n} + p_n^4 B_{2n} + p_n^5 A_{5n}\right)$$

$$S_{xy}^1 = 0.5\left(q_0^1 A_{10} + q_0^2 B_{10}\right) + \sum_{n=1}^{\infty}\left(q_n^1 A_{1n} + q_n^2 B_{1n} + q_n^3 A_{2n} + q_n^4 B_{2n}\right)$$

$$S_r^1 = 0.5\left(r_0^1 A_{10} + r_0^2 B_{10} + r_0^5 A_{50}\right) + \sum_{n=1}^{\infty}\left(r_n^1 A_{1n} + r_n^2 B_{1n} + r_n^3 A_{2n} + r_n^4 B_{2n} + r_n^5 A_{5n}\right)$$

$$S_{xx} = 0.5\left(p_0^1 A_{10} + p_0^2 B_{10} + p_0^6 B_{50}\right) + \sum_{n=1}^{\infty}\left(p_n^1 A_{1n} + p_n^2 B_{1n} + p_n^3 A_{2n} + p_n^4 B_{2n} + p_n^6 B_{5n}\right)$$

$$S_{xy} = 0.5\left(q_0^1 A_{10} + q_0^2 B_{10}\right) + \sum_{n=1}^{\infty}\left(q_n^1 A_{1n} + q_n^2 B_{1n} + q_n^3 A_{2n} + q_n^4 B_{2n}\right)$$

$$S_r = 0.5\left(r_0^1 A_{10} + r_0^2 B_{10} + r_0^6 B_{50}\right) + \sum_{n=1}^{\infty}\left(r_n^1 A_{1n} + r_n^2 B_{1n} + r_n^3 A_{2n} + r_n^4 B_{2n} + r_n^6 B_{5n}\right) \tag{30}$$

$$\overline{S}^1_{xx} = 0.5\left(\overline{p}^3_0 \overline{A}_{20} + \overline{p}^4_0 \overline{B}_{20}\right) + \sum_{n=1}^{\infty}\left(\overline{p}^1_n \overline{A}_{1n} + \overline{p}^2_n \overline{B}_{1n} + \overline{p}^3_n \overline{A}_{2n} + \overline{p}^4_n \overline{B}_{2n} + \overline{p}^5_n \overline{A}_{5n}\right)$$

$$\overline{S}^1_{xy} = 0.5\left(\overline{q}^3_0 \overline{A}_{20} + \overline{q}^4_0 \overline{B}_{20}\right) + \sum_{n=1}^{\infty}\left(\overline{q}^1_n \overline{A}_{1n} + \overline{q}^2_n \overline{B}_{1n} + \overline{q}^3_n \overline{A}_{2n} + \overline{q}^4_n \overline{B}_{2n}\right)$$

$$\overline{S}^1_{r} = 0.5\left(\overline{r}^3_0 \overline{A}_{20} + \overline{r}^4_0 \overline{B}_{20}\right) + \sum_{n=1}^{\infty}\left(\overline{r}^1_n \overline{A}_{1n} + \overline{r}^2_n \overline{B}_{1n} + \overline{r}^3_n \overline{A}_{2n} + \overline{r}^4_n \overline{B}_{2n} + \overline{r}^5_n \overline{A}_{5n}\right)$$

$$\overline{S}_{xx} = 0.5\left(\overline{p}^3_0 \overline{A}_{20} + \overline{p}^4_0 \overline{B}_{20}\right) + \sum_{n=1}^{\infty}\left(\overline{p}^1_n \overline{A}_{1n} + \overline{p}^2_n \overline{B}_{1n} + \overline{p}^3_n \overline{A}_{2n} + \overline{p}^4_n \overline{B}_{2n} + \overline{p}^6_n \overline{B}_{5n}\right)$$

$$\overline{S}_{xy} = 0.5\left(\overline{q}^3_0 \overline{A}_{20} + \overline{q}^4_0 \overline{B}_{20}\right) + \sum_{n=1}^{\infty}\left(\overline{q}^1_n \overline{A}_{1n} + \overline{q}^2_n \overline{B}_{1n} + \overline{q}^3_n \overline{A}_{2n} + \overline{q}^4_n \overline{B}_{2n}\right)$$

$$\overline{S}_{r} = 0.5\left(\overline{r}^3_0 \overline{A}_{20} + \overline{r}^4_0 \overline{B}_{20}\right) + \sum_{n=1}^{\infty}\left(\overline{r}^1_n \overline{A}_{1n} + \overline{r}^2_n \overline{B}_{1n} + \overline{r}^3_n \overline{A}_{2n} + \overline{r}^4_n \overline{B}_{2n} + \overline{r}^6_n \overline{B}_{5n}\right) \tag{31}$$

The terms p_n^i to $\overline{r}_n^i$ are given in the Appendix A.

Performing the Fourier series expansion to Eqs. (25) and (26) along the boundary, the boundary conditions along the inner and outer surfaces are expanded in the form of double Fourier series. When a plate is symmetric about more than one axis, the boundary conditions, in the case of symmetric mode can be written in the form of a matrix as given below:

$$\begin{bmatrix}
\hat{P}^1_{00} & \hat{P}^2_{00} & \hat{P}^5_{00} & 0 & \hat{P}^1_{01} & \cdots & \hat{P}^1_{0N} & \hat{P}^2_{01} & \cdots & \hat{P}^2_{0N} & \hat{P}^3_{01} & \cdots & \hat{P}^3_{0N} & \hat{P}^4_{01} & \cdots & \hat{P}^4_{0N} & \hat{P}^5_{01} & \cdots & \hat{P}^5_{0N} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\hat{P}^1_{N0} & \hat{P}^2_{N0} & \hat{P}^5_{N0} & 0 & \hat{P}^1_{N1} & \cdots & \hat{P}^1_{NN} & \hat{P}^2_{N1} & \cdots & \hat{P}^2_{NN} & \hat{P}^3_{N1} & \cdots & \hat{P}^3_{NN} & \hat{P}^4_{N1} & \cdots & \hat{P}^4_{NN} & \hat{P}^5_{N1} & \cdots & \hat{P}^5_{NN} & 0 & \cdots & 0 \\
\hat{Q}^1_{10} & \hat{Q}^2_{10} & 0 & 0 & \hat{Q}^1_{11} & \cdots & \hat{Q}^1_{1N} & \hat{Q}^2_{11} & \cdots & \hat{Q}^2_{1N} & \hat{Q}^3_{11} & \cdots & \hat{Q}^3_{1N} & \hat{Q}^4_{11} & \cdots & \hat{Q}^4_{1N} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\hat{Q}^1_{N0} & \hat{Q}^2_{N0} & 0 & 0 & \hat{Q}^1_{N1} & \cdots & \hat{Q}^1_{NN} & \hat{Q}^2_{N1} & \cdots & \hat{Q}^2_{NN} & \hat{Q}^3_{N1} & \cdots & \hat{Q}^3_{NN} & \hat{Q}^4_{N1} & \cdots & \hat{Q}^4_{NN} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\hat{R}^1_{00} & \hat{R}^2_{00} & \hat{R}^5_{00} & 0 & \hat{R}^1_{01} & \cdots & \hat{R}^1_{0N} & \hat{R}^2_{01} & \cdots & \hat{R}^2_{0N} & \hat{R}^3_{01} & \cdots & \hat{R}^3_{0N} & \hat{R}^4_{01} & \cdots & \hat{R}^4_{0N} & \hat{R}^5_{01} & \cdots & \hat{R}^5_{0N} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\hat{R}^1_{N0} & \hat{R}^2_{N0} & \hat{R}^5_{N0} & 0 & \hat{R}^1_{N1} & \cdots & \hat{R}^1_{NN} & \hat{R}^2_{N1} & \cdots & \hat{R}^2_{NN} & \hat{R}^3_{N1} & \cdots & \hat{R}^3_{NN} & \hat{R}^4_{N1} & \cdots & \hat{R}^4_{NN} & \hat{R}^5_{N1} & \cdots & \hat{R}^5_{NN} & 0 & \cdots & 0 \\
P^1_{00} & P^2_{00} & 0 & P^6_{00} & P^1_{10} & \cdots & P^1_{0N} & P^2_{10} & \cdots & P^2_{0N} & P^3_{10} & \cdots & P^3_{0N} & P^4_{10} & \cdots & P^4_{0N} & 0 & \cdots & 0 & P^6_{10} & \cdots & P^6_{0N} \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
P^1_{N0} & P^2_{N0} & 0 & P^6_{N0} & P^1_{N1} & \cdots & P^1_{NN} & P^2_{N1} & \cdots & P^2_{NN} & P^3_{N1} & \cdots & P^3_{NN} & P^4_{N1} & \cdots & P^4_{NN} & 0 & \cdots & 0 & P^6_{N1} & \cdots & P^6_{NN} \\
Q^1_{10} & Q^2_{10} & 0 & 0 & Q^1_{11} & \cdots & Q^1_{1N} & Q^2_{11} & \cdots & Q^2_{1N} & Q^3_{11} & \cdots & Q^3_{1N} & Q^4_{11} & \cdots & Q^4_{1N} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
Q^1_{N0} & Q^2_{N0} & 0 & 0 & Q^1_{N1} & \cdots & Q^1_{NN} & Q^2_{N1} & \cdots & Q^2_{NN} & Q^3_{N1} & \cdots & Q^3_{NN} & Q^4_{N1} & \cdots & Q^4_{NN} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
R^1_{00} & R^2_{00} & 0 & R^6_{00} & R^1_{00} & \cdots & R^1_{0N} & R^2_{00} & \cdots & R^2_{0N} & R^3_{00} & \cdots & R^3_{0N} & R^4_{00} & \cdots & R^4_{0N} & 0 & \cdots & 0 & R^6_{00} & \cdots & R^6_{0N} \\
\vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
R^1_{N0} & R^2_{N0} & 0 & R^6_{N0} & R^1_{N1} & \cdots & R^1_{NN} & R^2_{N1} & \cdots & R^2_{NN} & R^3_{N1} & \cdots & R^3_{NN} & R^4_{N1} & \cdots & R^4_{NN} & 0 & \cdots & 0 & R^6_{N1} & \cdots & R^6_{NN}
\end{bmatrix}
\begin{bmatrix} A_{10} \\ B_{10} \\ A_{50} \\ B_{50} \\ A_{11} \\ \vdots \\ A_{1N} \\ B_{11} \\ \vdots \\ B_{1N} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ A_{51} \\ \vdots \\ A_{5N} \\ B_{51} \\ \vdots \\ B_{5N} \end{bmatrix} = 0 \tag{32}$$

where

$$\hat{P}^j_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} p_n^j\left(\hat{R}_l,\theta\right)\cos m\theta d\theta, \quad \hat{Q}^j_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} q_n^j\left(\hat{R}_l,\theta\right)\sin m\theta d\theta$$

$$\hat{R}^j_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} r_n^j\left(\hat{R}_l,\theta\right)\cos m\theta d\theta, \quad P^j_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} p_n^j\left(R_l,\theta\right)\cos m\theta d\theta$$

$$Q^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} q^{j}_{n}\left(R_l,\theta\right)\sin m\theta d\theta\,,\quad R^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} r^{j}_{n}\left(R_l,\theta\right)\cos m\theta d\theta \tag{33}$$

Similarly, the matrix for the antisymmetric mode is obtained as

$$\begin{bmatrix}
\overline{\widehat{P}}^{3}_{10} & \overline{\widehat{P}}^{4}_{10} & \overline{\widehat{P}}^{1}_{11} & \cdots & \overline{\widehat{P}}^{1}_{1N} & \overline{\widehat{P}}^{2}_{11} & \cdots & \overline{\widehat{P}}^{2}_{1N} & \overline{\widehat{P}}^{3}_{11} & \cdots & \overline{\widehat{P}}^{3}_{1N} & \overline{\widehat{P}}^{4}_{11} & \cdots & \overline{\widehat{P}}^{4}_{1N} & \overline{\widehat{P}}^{5}_{11} & \cdots & \overline{\widehat{P}}^{5}_{1N} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{\widehat{P}}^{3}_{N0} & \overline{\widehat{P}}^{4}_{N0} & \overline{\widehat{P}}^{1}_{N1} & \cdots & \overline{\widehat{P}}^{1}_{NN} & \overline{\widehat{P}}^{2}_{N1} & \cdots & \overline{\widehat{P}}^{2}_{NN} & \overline{\widehat{P}}^{3}_{N1} & \cdots & \overline{\widehat{P}}^{3}_{NN} & \overline{\widehat{P}}^{4}_{N1} & \cdots & \overline{\widehat{P}}^{4}_{NN} & \overline{\widehat{P}}^{5}_{N1} & \cdots & \overline{\widehat{P}}^{5}_{NN} & 0 & \cdots & 0 \\
\overline{\widehat{Q}}^{3}_{00} & \overline{\widehat{Q}}^{4}_{00} & \overline{\widehat{Q}}^{1}_{01} & \cdots & \overline{\widehat{Q}}^{1}_{0N} & \overline{\widehat{Q}}^{2}_{01} & \cdots & \overline{\widehat{Q}}^{2}_{0N} & \overline{\widehat{Q}}^{3}_{01} & \cdots & \overline{\widehat{Q}}^{3}_{0N} & \overline{\widehat{Q}}^{4}_{01} & \cdots & \overline{\widehat{Q}}^{4}_{0N} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{\widehat{Q}}^{3}_{N0} & \overline{\widehat{Q}}^{3}_{N0} & \overline{\widehat{Q}}^{1}_{N1} & \cdots & \overline{\widehat{Q}}^{1}_{NN} & \overline{\widehat{Q}}^{2}_{N1} & \cdots & \overline{\widehat{Q}}^{2}_{NN} & \overline{\widehat{Q}}^{3}_{N1} & \cdots & \overline{\widehat{Q}}^{3}_{NN} & \overline{\widehat{Q}}^{4}_{N1} & \cdots & \overline{\widehat{Q}}^{4}_{NN} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\overline{\widehat{R}}^{3}_{10} & \overline{\widehat{R}}^{4}_{10} & \overline{\widehat{R}}^{1}_{11} & \cdots & \overline{\widehat{R}}^{1}_{1N} & \overline{\widehat{R}}^{2}_{11} & \cdots & \overline{\widehat{R}}^{2}_{1N} & \overline{\widehat{R}}^{3}_{11} & \cdots & \overline{\widehat{R}}^{3}_{1N} & \overline{\widehat{R}}^{4}_{11} & \cdots & \overline{\widehat{R}}^{4}_{1N} & \overline{\widehat{R}}^{5}_{11} & \cdots & \overline{\widehat{R}}^{5}_{1N} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{\widehat{R}}^{3}_{N0} & \overline{\widehat{R}}^{4}_{N0} & \overline{\widehat{R}}^{1}_{N1} & \cdots & \overline{\widehat{R}}^{1}_{NN} & \overline{\widehat{R}}^{2}_{N1} & \cdots & \overline{\widehat{R}}^{2}_{NN} & \overline{\widehat{R}}^{3}_{N1} & \cdots & \overline{\widehat{R}}^{3}_{NN} & \overline{\widehat{R}}^{4}_{N1} & \cdots & \overline{\widehat{R}}^{4}_{NN} & \overline{\widehat{R}}^{5}_{N1} & \cdots & \overline{\widehat{R}}^{5}_{NN} & 0 & \cdots & 0 \\
\overline{P}^{3}_{10} & \overline{P}^{4}_{10} & \overline{P}^{1}_{11} & \cdots & \overline{P}^{1}_{1N} & \overline{P}^{2}_{11} & \cdots & \overline{P}^{2}_{1N} & \overline{P}^{3}_{11} & \cdots & \overline{P}^{3}_{1N} & \overline{P}^{4}_{11} & \cdots & \overline{P}^{4}_{1N} & 0 & \cdots & 0 & \overline{P}^{6}_{11} & & \overline{P}^{6}_{1N} \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & & & \\
\overline{P}^{3}_{N0} & \overline{P}^{4}_{N0} & \overline{P}^{1}_{N1} & \cdots & \overline{P}^{1}_{NN} & \overline{P}^{2}_{N1} & \cdots & \overline{P}^{2}_{NN} & \overline{P}^{3}_{N1} & \cdots & \overline{P}^{3}_{NN} & \overline{P}^{4}_{N1} & \cdots & \overline{P}^{4}_{NN} & 0 & \cdots & 0 & \overline{P}^{6}_{N1} & & \overline{P}^{6}_{NN} \\
\overline{Q}^{3}_{00} & \overline{Q}^{4}_{00} & \overline{Q}^{1}_{01} & \cdots & \overline{Q}^{1}_{0N} & \overline{Q}^{2}_{01} & \cdots & \overline{Q}^{2}_{0N} & \overline{Q}^{3}_{01} & \cdots & \overline{Q}^{3}_{0N} & \overline{Q}^{4}_{01} & \cdots & \overline{Q}^{4}_{0N} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{Q}^{3}_{N0} & \overline{Q}^{4}_{10} & \overline{Q}^{1}_{N1} & \cdots & \overline{Q}^{1}_{NN} & \overline{Q}^{2}_{N1} & \cdots & \overline{Q}^{2}_{NN} & \overline{Q}^{3}_{N1} & \cdots & \overline{Q}^{3}_{NN} & \overline{Q}^{4}_{N1} & \cdots & \overline{Q}^{4}_{NN} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\overline{R}^{3}_{10} & \overline{R}^{4}_{10} & \overline{R}^{1}_{11} & \cdots & \overline{R}^{1}_{1N} & \overline{R}^{2}_{11} & \cdots & \overline{R}^{2}_{1N} & \overline{R}^{3}_{11} & \cdots & \overline{R}^{3}_{1N} & \overline{R}^{4}_{11} & \cdots & \overline{R}^{4}_{1N} & 0 & \cdots & 0 & \overline{R}^{6}_{11} & \cdots & \overline{R}^{6}_{11} \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{R}^{3}_{N0} & \overline{R}^{4}_{N0} & \overline{R}^{1}_{N1} & \cdots & \overline{R}^{1}_{NN} & \overline{R}^{2}_{N1} & \cdots & \overline{R}^{2}_{NN} & \overline{R}^{3}_{N1} & \cdots & \overline{R}^{3}_{NN} & \overline{R}^{4}_{N1} & \cdots & \overline{R}^{4}_{NN} & 0 & \cdots & 0 & \overline{R}^{6}_{N1} & \cdots & \overline{R}^{6}_{NN}
\end{bmatrix}
\begin{bmatrix} A_{20} \\ B_{20} \\ A_{11} \\ \vdots \\ A_{1N} \\ B_{11} \\ \vdots \\ B_{1N} \\ A_{21} \\ \vdots \\ A_{2N} \\ B_{21} \\ \vdots \\ B_{2N} \\ A_{51} \\ \vdots \\ A_{5N} \\ B_{51} \\ \vdots \\ B_{5N} \end{bmatrix} = 0 \tag{34}$$

where

$$\overline{\widehat{P}}^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} \bar{p}^{j}_{n}\left(\hat{R}_l,\theta\right)\sin m\theta d\theta\,,\ \overline{\widehat{Q}}^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} \bar{q}^{j}_{n}\left(\hat{R}_l,\theta\right)\cos m\theta d\theta$$

$$\overline{\widehat{R}}^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} \bar{r}^{j}_{n}\left(\hat{R}_l,\theta\right)\sin m\theta d\theta\,,\ \overline{P}^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} \bar{p}^{j}_{n}\left(R_l,\theta\right)\sin m\theta d\theta$$

$$\overline{Q}^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} \bar{q}^{j}_{n}\left(R_l,\theta\right)\cos m\theta d\theta\,,\ \overline{R}^{j}_{mn} = \left(2\varepsilon_n/\pi\right)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} \bar{r}^{j}_{n}\left(R_l,\theta\right)\sin m\theta d\theta \tag{35}$$

and where $j = 1,2,3,4,5$ and 6, L is the number of segments, $\hat{R}_l$ is the coordinate r at the inner boundary, R_l is the coordinate r at the outer boundary and N is the number of truncation of the Fourier series. For the nontrivial solution of the systems of equations given in Eqs. (32) and (34), the determinant of the coefficient matrix must vanish and these determinants give the frequencies of symmetric and antisymmetric modes respectively.

6. Particular Case

The frequency equations for a ring shaped plate of polygonal cross-sections is obtained by omitting the fluid medium in the corresponding expressions and solutions of the above sections. This frequency equations are further reduced to obtain the frequency equations of a solid polygonal cross-sectional plate by considering the outer surface of the plate Y_n and Y_{n+1} are equal to zero in the Eqs. (25) and (26), this frequency equations are matches well with the frequency equations of Ponnusamy[5] by considering the thermal fields are equal to zero. Using this frequency equation, the numerical results of the frequency for a plate in space is obtained, this results is used to compare with the frequency obtained for a plate immersed in fluid.

7. Numerical Results and Discussions

In order to illustrate the nature and general behavior of the solution, some numerical examples are considered in this section. The resulting frequency equations of the symmetric and antisymmetric cases of the ring shaped plate of polygonal cross sections immersed in a fluid given in Eqs. (33) and (35) are transcendental in nature with respect to the dimensionless frequency Ω. The computation of Fourier coefficients given in Eqs. (34) and (36) is carried out using the five point Gaussian quadrature. To obtain the roots of the frequency equation, the secant method applicable for the complex roots (Antia[12]) is employed. The material properties used for the computations are as follows: for solid, the Poisson ratio $\nu = 0.3$, density $\rho = 7849 kg/m^3$ and Young's modulus $E = 2.139 \times 10^{11} N/m^2$ and for fluid the density $\rho^f = 1000 kg/m^3$ and phase velocity $c = 1500 m/\sec$.

7.1. Geometrical Relations of Ring Shaped Polygonal Cross-Sections

The geometrical relations for the ring shaped plate of polygonal cross-section given by Nagaya[3] as follows.

$$R_i/a = \left[\cos\left(\theta - \gamma_l\right)\right]^{-1}$$
$$R_i/b = \left[\cos\left(\theta - \widehat{\gamma_l}\right)\right]^{-1}$$
$$\gamma_l = \widehat{\gamma_l} \tag{36}$$

where $a(=b+h)$ and b is the apothems, h is the thickness of the plate. Here the apothem b is taken as the reference length which is used to obtain the dimensionless expressions, and γ_l is the angle between the reference axis and the normal to the segment.

7.2. Longitudinal Mode

In longitudinal mode of square and hexagonal cross-section, the cross-section vibrates along the axis of the cylinder, so that the vibration displacements in the cross-sections are symmetrical about both the major and the minor axes. Hence the frequency equations are obtained by choosing both the terms of m and n as $0, 2, 4, 6, \ldots$ in the Eq. (32) for the numerical calculations. In the case of triangle and pentagonal cross-sectional plate, the vibration and displacements are symmetrical about the major axis alone, hence the frequency equations are obtained from the Eq. (32) by choosing m and n as $0, 1, 2, 3, \ldots$. Since the boundary of the cross-sections namely, triangle, square, pentagon and hexagon are irregular, it is difficult to satisfy the boundary conditions along the curved surface, and hence Fourier expansion collocation method is applied. That is the curved surface, in the range $\theta = 0$ and $\theta = \pi$ is divided into 20 segments, such that the distance between any two segments is negligible and the integrations is performed for each segment numerically by using the Gauss five point formula .The non-dimensional frequencies are computed for $0 < \Omega \le 1.0$, using the secant method (applicable for the complex roots, (Anita[12])).

7.3. Flexural Mode

In flexural mode of square and hexagonal cross-section, the vibration and displacements are antisymmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equation may be obtained from Eq. (34) by choosing $n, m = 1, 3, 5, 7\ldots..$ In the case of triangle and pentagonal cross-sections, the vibration and displacements are antisymmetrical about the minor axis, hence the frequency equations may be obtained from Eq. (34)by choosing $n, m = 1, 2, 3\ldots.$

7.4. Comparison between the Frequency Responses of Plate in Space and Plate Immersed in Fluid

Table 1. Comparison between non-dimensional frequencies of Plate in Space (PS) and Plate Immersed in Fluid (PIF) for longitudinal modes of the Polygonal (triangle, square, pentagon and hexagon) cross-sectional plates

Cross-sections	Mode	PS	PIF
Triangle	S1	2.3472	2.7667
	S2	3.9541	4.1810
	S3	5.4340	5.5957
Square	S1	1.4157	1.4158
	S2	2.8305	2.8426
	S3	7.0773	7.0781
Pentagon	S1	1.4156	1.4270
	S2	2.3341	2.7232
	S3	5.4287	5.5569
Hexagon	S1	2.8314	2.8420
	S2	5.6614	5.6668
	S3	7.0728	7.0761

A comparison is made between the non-dimensional frequencies of Plate in Space (PS) and Plate Immersed in Fluid (PIF) for the longitudinal and flexural anti symmetric modes of vibrations and are shown in Tables 1 and 2 respectively. From the Tables 1 and 2, it is observed that, the non-dimensional frequencies increases for increasing modes of vibrations. Comparing the frequency responses of PS and PIF, the frequencies are higher for a plate immersed in fluid than the plate in space, this is the proper physical behavior for solid-fluid interface problems. The notations namely S1, S2, S3 and A1, A2, A3 used in the Tables respectively represents the symmetric and anti symmetric modes of vibration, and the number 1, 2, 3 represents the first, second and third modes of vibrations.

Table 2. Comparison between non-dimensional frequencies of Plate in Space (PS) and Plate Immersed in Fluid (PIF) for flexural anti symmetric modes of the Polygonal (triangle, square, pentagon and hexagon) cross-sectional plates

Cross-sections	Mode	PS	PIF
Triangle	A1	1.2996	1.3763
	A2	2.7079	2.7955
	A3	4.1218	4.2101
Square	A1	1.2934	1.3616
	A2	2.6543	2.7779
	A3	4.0691	4.1971
Pentagon	A1	1.2896	1.3627
	A2	2.6701	2.7746
	A3	4.0660	4.1896
Hexagon	A1	1.3005	1.3567
	A2	2.6282	2.7683
	A3	4.0549	4.1840

7.5 Dispersion Curves

The frequency responses of longitudinal and flexural antisymmetric modes of vibration are plotted in the form of dispersion curves. The notations used in the figures, namely, PS, PF respectively denotes the plate in space and plate immersed in fluid. The notations 1, 2 and 3 respectively represcents the first, second and third modes of vibrations. The cross over points in various dispersion curves of different modes indicate that for a particular frequency of vibrations, the mechanical energy is communicated between the directions of frequency in the respective modes. Another aspect, which can be seen from the graph is that the frequencies are increases with respect to its modes of vibration. It is also observed that the displacement of particles takes irregular path when a plate in space (vacuum).

A dispersion curve is drawn to compare the frequency responses of longitudinal modes of triangular cross-sectional plate immersed in fluid and in space and is shown in Fig.1. From the Fig.1, it is observed that, the non-dimensional frequencies are increased with respect to its modes of vibrations and also the frequencies are linearly increased for a plate immersed in fluid. Similarly, a comparison is made between the frequency responses of flexural antisymmetric modes of triangular cross-sectional plate immersed in fluid and in space and is shown in Fig.2. From Fig.2, it is observed that the displacement of particles for a plate immersed in fluid and in space receive a similar pattern. The displacement of energy for a plate in space is lesser than the plate immersed in fluid.

Figs.3 and 4 respectively shows the comparison between the frequency responses of longitudinal and flexural antisymmetric modes of square cross-sectional plate immersed in fluid and in space. From the Figs.3 and 4, it is observed that, the non-dimensional frequencie increases with respect to its modes of vibrations. From the Fig.3, for the first and third modes of vibrations, the frequencies will be the same for a particular period, after that both the plate immersed in fluid and in space have dispersion in different patterns. A similar type of behavior is observed for the flexural antisymmetric modes of vibrations also.

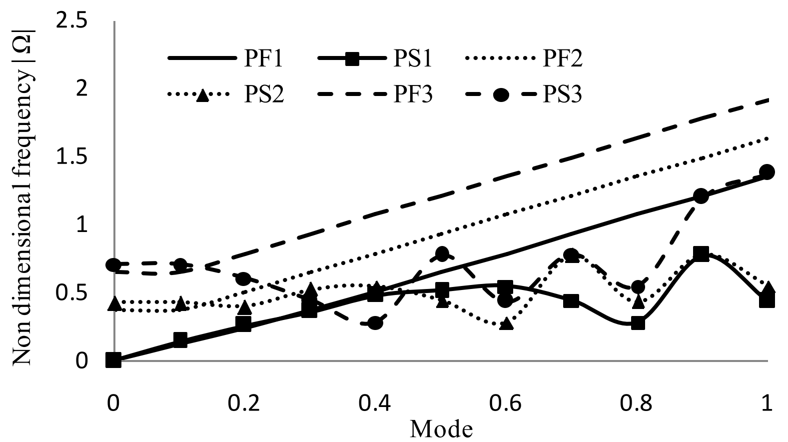

Figure 1. Comparison between the non-dimensional frequencies of longitudinal modes of triangular cross-sectional plate immersed in fluid and in space

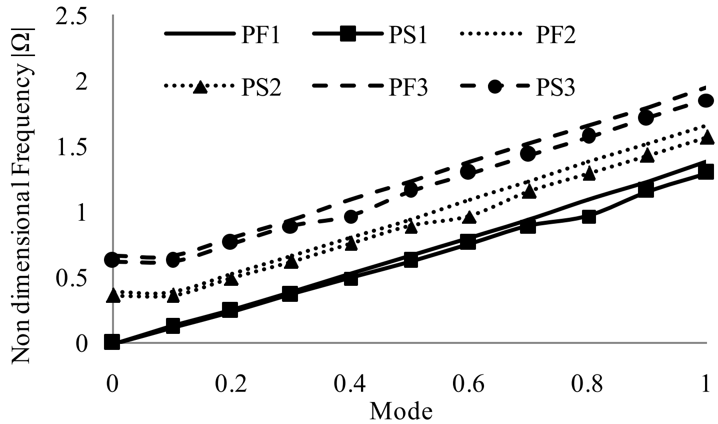

Figure 2. Comparison between the non-dimensional frequencies of triangular cross-sectional plate immersed in fluid and in space for different flexural anti symmetric modes

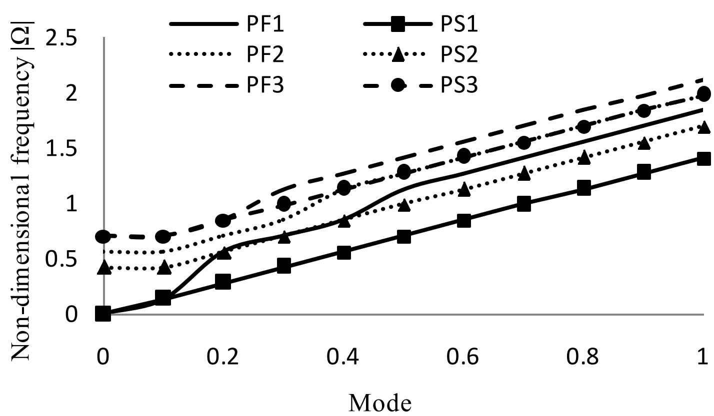

Figure 3. Comparison between the frequencies of flexural anti symmetric modes of square cross sectional plate immersed in fluid and in space

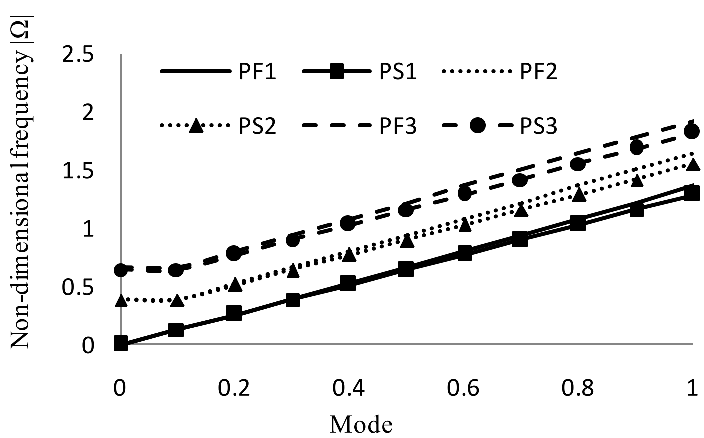

Figure 4. Comparison between the frequencies of longitudinal modes of square cross sectional plate immersed in fluid and in space

Graph is drawn to compare the frequencies of longitudinal and flexural antisymmetric modes of vibration for pentagonal cross-sectional plate immersed in fluid and in space is respectively and is shown in the Figs.5 and 6.

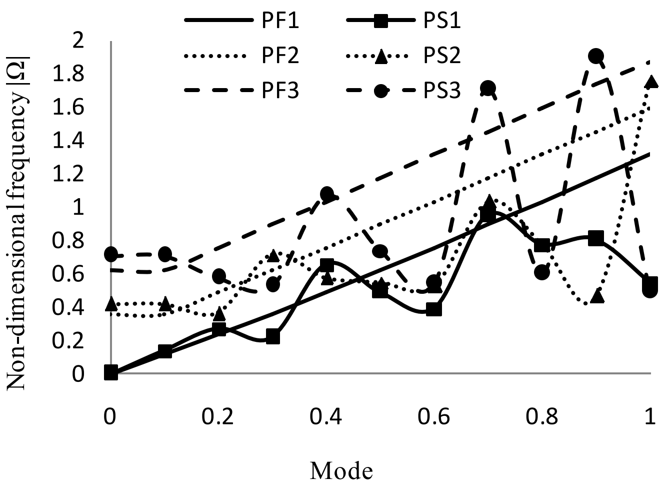

Figure 5. Comparison between the frequencies of longitudinal modes of pentagonal cross sectional plate immersed in fluid and in space

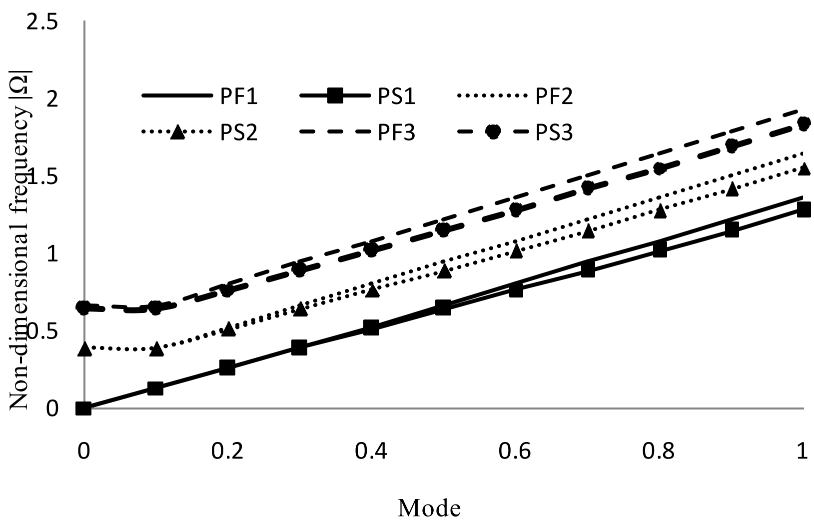

Figure 6. Comparison between the frequencies of flexural anti symmetric modes of pentagonal cross-sectional plate immersed in fluid and in space

The dispersion of a plate immersed in fluid linearly increases, whereas the dispersion pattern is irregular when the plate is in space. The cross-over points between the modes of vibration indicates that, there is a transfer of energy between the modes of vibrations. The plate is highly dispersive when it is placed in vacuum. A comparison graph is drawn between the non-dimensional frequencies of longitudinal and flexural antisymmetric modes of hexagonal plates and are shown in the Figs. 7 and 8 respectively. From the Figs. 7 and 8, it is observed that the non-dimensional frequencies are higher for plate immersed in fluid than the plate is space. This is the proper physical behavior of solid-fluid interaction problems.

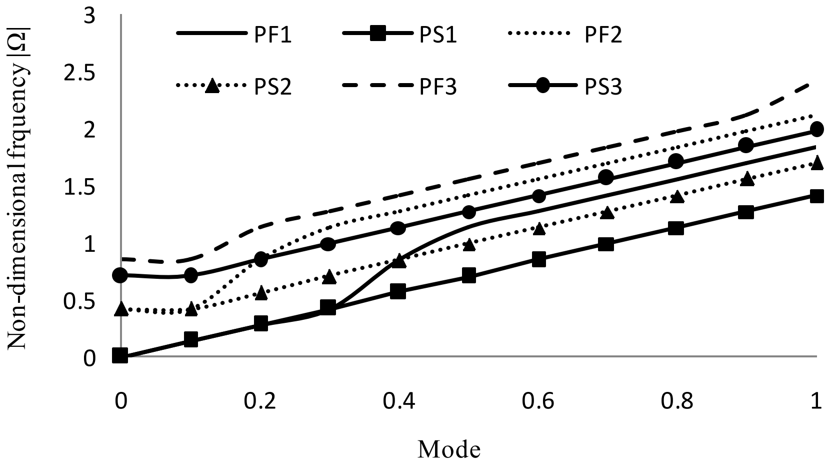

Figure 7. Comparison between the non-dimensional frequencies of longitudinal modes of hexagonal cross sectional plate immersed in fluid and in space

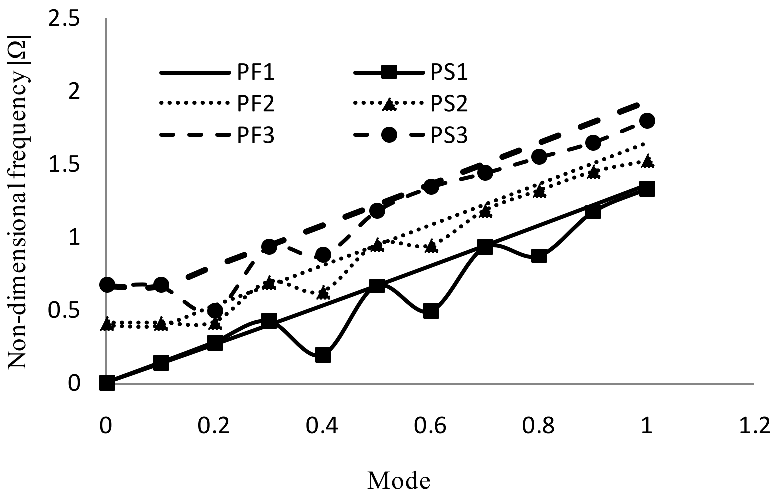

Figure 8. Comparison between the non-dimensional frequenciesof flexural anti symmetric modes hexagonal cross sectional plate immersed in fluid and in space

8. Conclusions

The in-plane vibration of rings of polygonal cross section composed of homogeneous isotropic material immersed in a fluid is analyzed using the Fourier expansion collocation method. The frequency equations are obtained for longitudinal and flexural antisymmetric modes of vibrations. The computed non-dimensional frequencies for PS and PIF are compared by plotting the dispersion curves. Comparison is made between non-dimensional frequencies of Plate in Space (PS) and Plate Immersed in Fluid (PIF) for longitudinal modes of the Polygonal (triangle, square, pentagon and hexagon) cross-sectional plates.

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Appendix A

$$\begin{aligned} p_n^1 &= 2\left\{n(n-1)J_n(\alpha_1 ax)+(\alpha_1 ax)J_{n+1}(\alpha_1 ax)\right\}\cos 2(\theta-\gamma_l)\cos n\theta \\ &-x^2\left\{(\alpha_1 a)^2\left(\overline{\lambda}+2\cos^2(\theta-\gamma_l)\right)\right\}J_n(\alpha_1 ax)\cos n\theta \end{aligned} \tag{A1}$$

$$\begin{aligned} p_n^2 &= 2\left\{n(n-1)Y_n(\alpha_1 ax)+(\alpha_1 ax)Y_{n+1}(\alpha_1 ax)\right\}\cos 2(\theta-\gamma_l)\cos n\theta \\ &-x^2\left\{(\alpha_1 a)^2\left(\overline{\lambda}+2\cos^2(\theta-\gamma_l)\right)\right\}Y_n(\alpha_1 ax)\cos n\theta \end{aligned} \tag{A2}$$

$$\begin{aligned} p_n^3 &= 2n\left\{(n-1)J_n(\alpha_2 ax)+(\alpha_2 ax)J_{n+1}(\alpha_2 ax)\right\}\cos 2(\theta-\gamma_l)\cos n\theta \\ &+2\left\{\left(n(n-1)-(\alpha_2 ax)^2\right)J_n(\alpha_2 ax)+(\alpha_2 ax)J_{n+1}(\alpha_2 ax)\right\}\sin 2(\theta-\gamma_l)\sin n\theta \end{aligned} \tag{A3}$$

$$\begin{aligned} p_n^4 &= 2n\left\{(n-1)Y_n(\alpha_2 ax)-(\alpha_2 ax)Y_{n+1}(\alpha_2 ax)\right\}\cos 2(\theta-\gamma_l)\cos n\theta \\ &+2\left\{\left(n(n-1)-(\alpha_2 ax)^2\right)Y_n(\alpha_2 ax)+(\alpha_2 ax)Y_{n+1}(\alpha_2 ax)\right\}\sin 2(\theta-\gamma_l)\sin n\theta \end{aligned} \tag{A4}$$

$$p_n^5 = \Omega^2\overline{\rho}_1 J_n(\alpha_3 ax)\cos n\theta \tag{A5}$$

$$p_n^6 = \Omega^2\overline{\rho}_2 H_n^{(2)}(\alpha_4 ax)\cos n\theta \tag{A6}$$

$$\begin{aligned} q_n^1 &= 2\left\{n(n-1)-(\alpha_1 ax)^2 J_n(\alpha_1 ax)+(\alpha_1 ax)J_{n+1}(\alpha_1 ax)\right\}\sin 2(\theta-\gamma_l)\cos n\theta \\ &+2n\left\{(\alpha_1 ax)J_{n+1}(\alpha_1 ax)-(n-1)J_n(\alpha_1 ax)\right\}\cos 2(\theta-\gamma_l)\sin n\theta \end{aligned} \tag{A7}$$

$$\begin{aligned} q_n^2 &= 2\left\{n(n-1)-(\alpha_1 ax)^2 Y_n(\alpha_1 ax)+(\alpha_1 ax)Y_{n+1}(\alpha_1 ax)\right\}\sin 2(\theta-\gamma_l)\cos n\theta \\ &+2n\left\{(\alpha_1 ax)Y_{n+1}(\alpha_1 ax)-(n-1)Y_n(\alpha_1 ax)\right\}\cos 2(\theta-\gamma_l)\sin n\theta \end{aligned} \tag{A8}$$

$$\begin{aligned} q_n^3 &= 2n\left\{(n-1)J_n(\alpha_2 ax)-(\alpha_2 ax)J_{n+1}(\alpha_2 ax)\right\}\sin 2(\theta-\gamma_l)\cos n\theta \\ &+2\left\{(\alpha_2 ax)J_{n+1}(\alpha_2 ax)-\left(n(n-1)-(\alpha_2 ax)^2\right)J_n(\alpha_2 ax)\right\}\cos 2(\theta-\gamma_l)\sin n\theta \end{aligned} \tag{A9}$$

$$\begin{aligned} q_n^4 &= 2n\left\{(n-1)Y_n(\alpha_2 ax)-(\alpha_2 ax)Y_{n+1}(\alpha_2 ax)\right\}\sin 2(\theta-\gamma_l)\cos n\theta \\ &+2\left\{(\alpha_2 ax)Y_{n+1}(\alpha_2 ax)-\left(n(n-1)-(\alpha_2 ax)^2\right)Y_n(\alpha_2 ax)\right\}\cos 2(\theta-\gamma_l)\sin n\theta \end{aligned} \tag{A10}$$

$$r_n^1 = \left\{nJ_n(\alpha_1 ax)-(\alpha_1 ax)J_{n+1}(\alpha_1 ax)\right\}\cos n\theta \tag{A11}$$

$$r_n^2 = \left\{nY_n(\alpha_1 ax)-(\alpha_1 ax)Y_{n+1}(\alpha_1 ax)\right\}\cos n\theta \tag{A12}$$

$$r_n^3 = nJ_n(\alpha_2 ax)\cos n\theta \tag{A13}$$

$$r_n^4 = nY_n(\alpha_2 ax)\cos n\theta \tag{A14}$$

$$r_n^5 = -\left[nJ_n(\alpha_3 ax)-(\alpha_3 ax)J_{n+1}(\alpha_3 ax)\right]\cos n\theta \tag{A15}$$

$$r_n^6 = -\left[nH_n^{(2)}(\alpha_4 ax)-(\alpha_4 ax)H_{n+1}^{(2)}(\alpha_4 ax)\right]\cos n\theta \tag{A16}$$

The barred expressions for the anti symmetric case are obtained by replacing $\cos n\theta$ by $\sin n\theta$ and $\sin n\theta$ by $\cos n\theta$ in the Appendix A.

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