

Masses of the Neutrinos and Two Flavor Mixing

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Abstract The absolute mass eigen-states of the neutrinos and the absolute mass eigen states of the corresponding charged leptons are assumed to obtain their effective mass through their interaction with the Higgs field. However Electron and its neutrino of the Standard model have non-diagonal mass matrix. Similarly the muon and its neutrino have non-diagonal mass matrix. The Tau-lepton and its neutrino have a non-diagonal mass matrix in the Standard model. By diagonalizing these mass matrices, we can arrange a two flavor neutrino mixing. This mixing allows the estimation of the deviation of neutrino flux ratio from one flavor to the other flavor.

Keywords Neutrino mass eigenstates, Mass of charged lepton, Flavor mixing

1. Introduction

The deviation of muon-neutrino flux to electron- neutrino flux is what is known as the Atmospheric neutrino anomaly. This observation of flavor violation is also observed by man made neutrinos after propagating sizable distances. To explain this, each neutrino flavor is assumed to be mixture of different mass eigenstates. As neutrinos propagate, each component mass eigen state acquires a different phase, so neutrino of a definite flavor will convert to a mixture different flavors.

To obtain mixing It is the mass that causes mixing. we should consider three Neutrino mass and mixing scheme. Here we follow Two flavor mixing much like the Cabibbo-mixing in the quark sector.

2. Absolute Mass Eigenstates of Neutrinos and the Charged Leptons

The absolute value of the electron-neutrino mass comes from the Tritium beta decay [1] and appears to be about 2 eV. The muon neutrino also has mass. In the Standard Model [2] each lepton starts out with no intrinsic mass. The charged leptons obtain an effective mass through their interaction with the Higgs field. The neutrino is automatically massless because it is left-handed in the Standard model. Right-handed neutrinos have no interaction with other particles and so are not a functional part of the Standard Model. But neutrino oscillations confirm that Neutrinos have non-zero mass.

Does the neutrino obtain its mass through interaction with the same Higgs field like all other particles?

For this to take place we should start with a mass eigen state for the neutrino.

Let there be a mass eigenstate for the electron neutrino and its effective mass be generated through the Lagrangian,

$$L = - h \bar{\nu} \nu \phi - h \bar{e} e \phi - i \alpha_1 \bar{e} \gamma_5 e \phi, \quad (1)$$

where, the Higgs field is ϕ with the VEV V_0 , and h in Eq. (1), is very small and we are not assuming that the neutrino is left-handed. It is a full mass eigenstate. The electron is coupled to the Higgs field through a γ_5 coupling also. We recover the standard model scenario when h is set zero and $i \gamma_5$ is replaced by one. Given a Dirac field say, ψ , the Hermitian Scalar $\bar{\psi}\psi$ and $i\bar{\psi}\gamma_5\psi$ have opposite CP and T transformation properties (In this respect they are unlike the vector and axial vector.) The CP violation is now caused by the exchange of ϕ' fields. Since the coupling of Higgs Field is usually rather small, it is possible to arrange for the CP violation to be of roughly milliweak magnitude [Mohapatra 1980, Bilenkey, 3&4].

After spontaneous symmetry breaking from (1) we note that,

$$L = - h V_0 \bar{\nu} \nu - h \bar{\nu} \nu \phi' - h V_0 \bar{e} e - h \bar{e} e \phi' - i \alpha_1 \bar{e} \gamma_5 e V_0 - i \alpha_1 \bar{e} \gamma_5 e \phi'.$$

And with, $m = h V_0$, which is now the electron-neutrino mass,

$$L = - m \bar{\nu} \nu - h \bar{\nu} \nu \phi' - m \bar{e} e - h \bar{e} e \phi' - i \alpha_1 \bar{e} \gamma_5 e V_0 - i \alpha_1 \bar{e} \gamma_5 e \phi' \quad (2)$$

In the above m , and also h are presumably very small because m is neutrino mass.

$$\text{Let, } e = \exp\left(-\frac{1}{2} i \alpha_1 \gamma_5\right) e', \quad (3)$$

where α_1 is a real parameter. Vector and axial vector interactions are not affected by this transformation. We

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choose α_1 in such a way that the constant coefficient of $\bar{e}' \gamma_5 e'$ is zero. This gives,

$$\begin{aligned} & -[m \cos \alpha_1 + V_0 a_1 \sin \alpha_1] \bar{e}' e' \\ & -[-i m \sin \alpha_1 + i a_1 V_0 \cos \alpha_1] \bar{e}' \gamma_5 e' \\ & -a_1 \bar{e}' [\sin \alpha_1 + i \gamma_5 \cos \alpha_1] e' \phi', \end{aligned} \quad (4)$$

and set the coefficient of the second term equal to zero to yield,

$$\tan \alpha_1 = \frac{a_1 V_0}{m}. \quad (5)$$

The mass of the electron is now given by,

$$m_e^2 = m^2 \sec^2 \alpha_1 = m^2 \left[1 + \frac{V_0^2 a_1^2}{m^2} \right] = m V_0 \left[\frac{V_0 a_1^2}{m} + \frac{m}{V_0} \right]. \quad (6)$$

The very last term within the bracket is independent of the VEV. It is the sum of $\left[\frac{a_1^2}{h} + h \right]$, which are the interaction constants of the Higgs field with the electron and its neutrino, [Eq.(2)]. Let,

$$q_0 = \left[\frac{a_1^2}{h} + h \right]. \quad (7)$$

A similar Lagrangian can be chosen to obtain the effective mass for the muon and its neutrino through their interaction with the same Higgs field

$$L = -h_1 \bar{\nu}_\mu \nu_\mu \phi - h_1 \bar{\mu} \mu \phi - i a_2 \bar{\mu} \gamma_5 \mu \phi. \quad (8)$$

In Eq. (8), h_1 and a_2 are real positive numbers and after symmetry breaking, the muon neutrino obtains the following mass:

$$m_1 = h_1 V_0. \quad (9)$$

Following the same steps as in Eq. (2, 3, 4, and 5) for the Lagrangian (8) we readily observe that,

$$m_\mu^2 = m_1^2 \sec^2 \alpha_2 = m_1^2 \left[1 + \frac{V_0^2 a_2^2}{m_1^2} \right] = m_1 V_0 \left[\frac{V_0 a_2^2}{m_1} + \frac{m_1}{V_0} \right]. \quad (10)$$

Again, we note that in Eq. (10) the parameter at the end in the brackets is independent of the VEV and is equal to $\left[\frac{a_2^2}{h_1} + h_1 \right]$. Let,

$$q_1 = \left[\frac{a_2^2}{h_1} + h_1 \right]. \quad (11)$$

After symmetry breaking, the muon neutrino obtains the mass $m_1 = h_1 V_0$, and with the same VEV,

$$V_0 = 246.22 \text{ GeV}, \quad (12)$$

There is another massive charged lepton, the τ lepton. Its mass is,

$$m_\tau = 1.777 \text{ GeV}. \quad (13)$$

This lepton also has a neutrino. Like the other leptons, they start out with no intrinsic mass and obtain effective mass with their interaction to the same Higgs field with the VEV = V_0 . The Lagrangian in this case is

$$L = -h_2 \bar{\nu}_\tau \nu_\tau \phi - h_2 \bar{\tau} \tau \phi - i a_3 \bar{\tau} \gamma_5 \tau \phi. \quad (14)$$

Following by now the familiar steps, after spontaneous symmetry breaking, we have,

$$m_{\nu\tau} = m_2 = h_2 V_0, \quad (15)$$

where m_2 is the mass of the τ -neutrino and the mass of the charged τ lepton is now given by,

$$m_\tau^2 = m_2^2 \sec^2 \alpha_3 = m_2^2 \left[1 + \frac{V_0^2 a_3^2}{m_2^2} \right] = m_2 V_0 \left[\frac{V_0 a_3^2}{m_2} + \frac{m_2}{V_0} \right]. \quad (16)$$

Again, the very last factor in the brackets is independent of the VEV.

And it is given by, q_2 where,

$$q_2 = \left[\frac{a_3^2}{h_2} + h_2 \right]. \quad (17)$$

Even though we do not know what is the value of q , it is a real positive number. The masses of W and Z bosons are given in terms of the gauge constants g_L and g' of SU(2)_L XU(1) through Higgs Mechanism. The mass generating interaction constants of the charged leptons with the Higgs boson are put by hand in the Standard model as there is no theory which will require a particular choice for these constants. In Eq. (2) the electron is coupled to the ϕ [$\bar{e}e\phi$ and $\bar{e}\gamma_5 e\phi$] in two ways, and this sort of coupling is also there with the standard Z-boson. Using this clue, we tried to relate the masses of the electron and muon to the interaction constants g_V and g_A of the Z boson in Ref. [5].

First let us note how a simple choice of q_2 enables the prediction of Tau -neutrino mass.

$$\text{If } q_2 = 1, \text{ then, } m_2 = 12.825 \text{ MeV}, \quad (18)$$

$$\text{If } q_2 = 0.5 \text{ then } m_2 = 25.65 \text{ MeV}, \text{ and} \quad (19)$$

$$\text{If } q_2 = 21.33239 \times 10^3 \text{ then, } m_2 = 601.19 \text{ eV}, \quad (20)$$

The choice of these values will be explained shortly. But it is clear that Neutrinos have mass. In the above, three possible values for the mass of the tau -neutrino are given. If $h = h_1 = h_2$ then all the three neutrinos will have the same mass. The electron neutrino mass can be computed from the following in which q_0 is chosen as a function of gauge constants [5].

$$m_e^2 = m V_0 q_0 = m V_0 \frac{(g_V/g_A)_{\nu e}^4}{(g_V/g_A)_{e\mu}^4} \left[1 - \left\{ 1 - (g_V/g_A)_{e\mu}^4 \right\}^{1/2} \right], \quad (21)$$

The factor $q_0 = \left(\frac{a^2}{h} + h \right)$ is given by the expression that is after $m V_0$ in Eq. (21), where g_V and g_A are the vector and axial vector coupling constants of the particles indicated by the subscripts with the Z-boson of the Standard model. The ratio $g_V/g_A = 1$ for all the neutrinos and it is introduced for future generalization. On the other hand,

$$(g_V/g_A)_{e\mu}^2 = [-1 + 4 \sin^2 \vartheta_W]^2. \quad (22)$$

Where $\sin^2 \vartheta_W$ is the Weinberg mixing parameter. Eq.(21) is approximately $\frac{1}{2} m V_0$ irrespective of the value of the mixing parameter. The mass of the electron neutrino is therefore, $m \approx 2 m_e^2 / V_0 = 2.12 \text{ eV}$. We took 0.5 for q_2 in Eq. (19) assuming a possibility that $q_2 \approx q_0$.

Exact mass of the electron neutrino is, (with $\sin^2 \vartheta_W = 0.2254$),

$$m = 2.12098 \text{ eV}. \quad (23)$$

The above value appears correct. In Eq. (10) also the very

last factor involving the constants $\left[\frac{a_2^2}{h_1} + h_1\right]$ must as well be a function of gauge constants and from Ref. [5], it can be noted that,

$$m_\mu^2 = m_1 V_0 q_1 = m_1 V_0 \frac{(g_V/g_A)_{\nu\mu}^4}{(g_V/g_A)_{e\mu}^4} \left[1 + \left\{1 - (g_V/g_A)_{e\mu}^4\right\}^{1/2}\right], \quad (24)$$

From the above the muon neutrino mass m_1 is given by,

$$m_1 = 2.1254 eV. \quad (25)$$

While computing the mass of the Tau-neutrino, we used the possibility in Eq. (20), that $q_2 \approx q_1 = 21.33239 \times 10^3$, which is q_1 is from Eq. (24). The electron-neutrino and the muon -neutrino masses are nearly equal.

Because of this the electron and muon mass ratio is given by,

$$\frac{m_e}{m_\mu} \approx \frac{1}{2} \left(\frac{g_V}{g_A}\right)_{e\mu}^2 = 0.00484128, \text{ if } \sin^2 \vartheta_W = 0.2254. \quad (26)$$

If $h = h_1$, On the other hand, both these neutrinos will have the same mass. This mass can be arranged to be a Majorana mass, and these two Neutrinos together are equivalent to a single Dirac neutrino with the same mass, Ref. [5],

$$m = \frac{m_e m_\mu}{V_0} \left(\frac{g_V}{g_A}\right)_{e\mu}^2 = 2.131996 eV. \quad (27)$$

With this there is no necessity of any right handed component for these Neutrinos. But then the electron and muon are subject to an electroweak Model, SU(2) \times SU(2) \times U(1) gauge model with the mixing parameters 0.2254 and 0.2746. Identical mass for these two neutrinos is not supported by the observed neutrino oscillation. Hence the electron and Muon neutrinos have different masses and these are given by Eqs. (23) and (25).

In place of $(g_V/g_A)_{e\mu}^2$ we take $(g_V/g_A)_\tau^2$ in both Eqs. (21) & (24) and find that, Ref. [6],

$$m_\tau^2 = m_2 V_0 \frac{(g_V/g_A)_{\nu\tau}^4}{(g_V/g_A)_\tau^4} \left[1 \pm \left\{1 - (g_V/g_A)_\tau^4\right\}^{1/2}\right] \\ = m_2 V_0 q_2. \quad (28)$$

In Eq.(28), the expression after $m_2 V_0$ is similar to Eq.(24) with the plus Sign and with Eq.(21) with the negative sign.

$$\text{If, } (g_V/g_A)_\tau^2 = (-1 + 4\sin^2 \vartheta_\tau)^2 = 1, \quad (29)$$

the expression after $m_2 V_0$ in the middle is just one and so $q_2 = 1$, that means that the mixing parameter for the electroweak model of the tau lepton is 0.5. It is SU(2) \times U(1) gauge model with a different mixing parameter. The $e-\mu-\tau$ universality is no longer valid. Thus, the experimental determination of Tau-neutrino mass is very crucial [7,8,9]

$$m_2 = 12.825 MeV. \quad (30)$$

There is no evidence as of now for the non-universality of $e-\mu-\tau$, and Hence the mass of tau-neutrino may not be given by Eq.(30). But Eq.(27) may well be recast for the

mass of the Tau neutrino,

$$m_2 = \frac{m_\tau m_e}{V_0} \frac{g_V^2}{g_A^2} = 0.124177 MeV. \quad (31)$$

Where we replaced $m_\mu m_e$ by $m_\tau m_e$ assuming some sort of generality. We believe that the mass of the Tau neutrino is given by Eq.(31). While Estimating the above value the Weinberg mixing parameter is taken to be 0.2254. If we take it to be 0.23 these numbers will be substantially quite different. In order to obtain Eq.(31), we note that Eq.(28) must be,

$$m_\tau^2 = m_2 V_0 \frac{g_A^2}{g_V^2}. \quad (32)$$

Experiments suggest that the theoretical estimations here of all the neutrino masses are right. The assumption that the neutrino mass eigen state exists and obtains its mass through its interaction with the same Higgs field through which the corresponding charged lepton obtains its mass is a departure from the Standard Model. But

$$m_e^2 = V_0^2 h \left[\frac{a_1^2}{h} + h\right] = V_0^2 [a_1^2 + h^2]. \quad (33)$$

In which we shifted the neutrino mass constant. Suppose h^2 is very small compared to a_1^2 , then we can assume that the neutrino has no interaction at all and our neutrino is massless and left-handed, but the electron mass is given by its interaction with the Higgs field. In a similar way the muon mass can be arranged and the muon neutrino has no mass apparently and it is left-handed like the electron neutrino. Note that the constant h is about 8.6×10^{-12} , from the electron neutrino mass. The square of this constant is definitely very small compared to the other number in Eq. (33)

$$\frac{M_i^2}{V_0 m_i} = q_i, \quad (34)$$

where M_i is the mass of the corresponding charged lepton and V_0 is the VEV which is known. From m_i the parameter h can be found and then the corresponding a_i can as well be calculated and it fixes the CP transformation.

3. Two State Mixing of Neutrinos

As in the case of quarks the lepton mass matrix may not be diagonal.

It will lead to a mixed state of the two neutrinos much like Cabibbo Mixing of quarks. For obtaining the electron-neutrino & muon-neutrino Mixing we proceed in the following, Ref. [10,11], way.

Let the electron and its neutrino mass matrix be given by, M_e , where,

$$M_e = \begin{pmatrix} 0 & \sqrt{m_e m} \\ \sqrt{m_e m} & m_e + m \end{pmatrix}. \quad (35)$$

This mass matrix is diagonalized by an orthogonal matrix, O_e , where,

$$O_e = \begin{pmatrix} \cos \phi_1 & -\sin \phi_1 \\ \sin \phi_1 & \cos \phi_1 \end{pmatrix}, \quad (36)$$

Where,
$$\tan\phi_1 = \sqrt{\frac{m}{m_e}} = 0.0020373, \quad (37)$$

From the above we note that,

$$\phi_1 = 0.116729 \text{ degree}. \quad (38)$$

Let the mass matrix for the muon and its neutrino be given by the matrix, M_μ where,

$$M_\mu = \begin{pmatrix} 0 & \sqrt{m_1 m_\mu} \\ \sqrt{m_1 m_\mu} & m_\mu + m_1 \end{pmatrix}. \quad (39)$$

The above mass matrix is diagonalized by an orthogonal matrix $O_\mu(\phi_2)$,

Where,
$$\tan\phi_2 = \sqrt{\frac{m_1}{m_\mu}} = 0.00014278. \quad (40)$$

The angle $\phi_2 = 0.008181$ degree. (41)

The absolute mass eigen-states ν_e of the electron-neutrino and the absolute mass eigen state ν_μ of the muon-neutrino are not the eigen States that take part in the electroweak model. Instead they mix and appear as a mixed states much like the Cabibbo-mixed states.

$$\begin{aligned} \nu_e' &= \nu_e \cos\vartheta_1 - \nu_\mu \sin\vartheta_1 \\ \nu_\mu' &= \nu_e \sin\vartheta_1 + \nu_\mu \cos\vartheta_1, \end{aligned} \quad (42)$$

Where $\vartheta_1 = 0.108549 \text{ degree} = \phi_1 - \phi_2$ (43)

This is much like the Cabibbo mixing in the quark sector. To obtain this mixing angle we used the non-diagonal mass matrices, M_e and M_μ . [10]. In view of the mixing of ν_e and ν_μ with the mixing angle ϑ_1 the relative Phase of ν_e and ν_μ changes because of the mass difference so that a neutrino originating as ν_e' has a non-zero probability of being detected as ν_μ . If an electron-type of neutrino is propagating with momentum P_e at time $t=0$, it will have a probability of oscillation $P_1 = P_{\nu_e \rightarrow \nu_\mu}$, where,

$$P_1 = \sin^2 2\vartheta_1 \sin^2 \left[\frac{1.27 \Delta m^2 L}{E_e} \right]. \quad (44)$$

Here, ϑ_1 is given by Eq. (43), and,

$$\Delta m^2 = m_1^2 - m^2 = (2.154 \text{ eV})^2 - (2.12098 \text{ eV})^2. \quad (45)$$

Moreover E_e is the initial energy of the electron-neutrino in GeV and L is in km. Ref. [9].

4. Electron and Tau Neutrino Mixing

We follow an exact mass mixing procedure to obtain the electron and Tau-neutrino mixing, again we assume two flavor mixing. In order to find the mixing angle of the Tau-neutrino with the electron-neutrino, we consider the following mass matrix for the Tau and its neutrino again assuming as though there are two generations.

$$M_\tau = \begin{pmatrix} 0 & \sqrt{m_2 m_\tau} \\ \sqrt{m_2 m_\tau} & m_\tau + m_2 \end{pmatrix}. \quad (46)$$

This matrix is diagonalized by the orthogonal matrix $O_\tau(\phi_3)$, where

$$\tan\phi_3 = \sqrt{\frac{m_2}{m_\tau}} = \sqrt{\frac{0.124177}{1.777 \times 10^3}} = 0.00835944 \quad (47)$$

From the above, the angle ϕ_3 is given by,

$$\phi_3 = 0.47895 \text{ degrees}. \quad (48)$$

The electron-neutrino and the Tau-neutrino mix where the mixing angle

$$\text{Is, } \vartheta_2 = \phi_3 - \phi_1 = 0.36222 \text{ degrees}, \quad (49)$$

The mixed neutrino states are given by,

$$\begin{aligned} \nu_e' &= \nu_e \cos\vartheta_2 - \nu_\tau \sin\vartheta_2, \\ \nu_\tau' &= \nu_e \sin\vartheta_2 + \nu_\tau \cos\vartheta_2. \end{aligned} \quad (50)$$

Because of this mixing, a neutrino ν_e' which starts with energy E_e will Oscillate into a neutrino ν_τ' with a probability, $P_2 = P_{\nu_e \rightarrow \nu_\tau}$ given by

$$P_2 = \sin^2 2\vartheta_2 \sin^2 \left[1.27 \frac{\Delta m^2 L}{E_e} \right], \quad (51)$$

Where ϑ_2 is given by Eq.(49), and, [see Ref. [9]].

$$\begin{aligned} \Delta m^2 &= m_2^2 - m^2 \\ &= (0.124177 \times 10^6 \text{ eV})^2 - (2.12098 \text{ eV})^2. \end{aligned} \quad (52)$$

5. Muon-Tau Neutrino Mixing

The muon neutrino can also mix with the tau neutrino exactly like the electron neutrino. The Cabibbo type of mixing angle in this case is given by, ϑ_3 , where,

$$\vartheta_3 = \phi_3 - \phi_2 = 0.470768 \text{ degrees}. \quad (53)$$

The mixed neutrino states are given by,

$$\begin{aligned} \nu_\mu' &= \nu_\mu \cos\vartheta_3 - \nu_\tau \sin\vartheta_3 \\ \nu_\tau' &= \nu_\mu \sin\vartheta_3 + \nu_\tau \cos\vartheta_3. \end{aligned} \quad (54)$$

Because of the above mixing a ν_μ' neutrino with initial Energy E_μ oscillates into a ν_τ' with a probability,

$$\begin{aligned} P_3 &= P_{\nu_\mu \rightarrow \nu_\tau}, \text{ where,} \\ P_3 &= \sin^2 2\vartheta_3 \sin^2 \left[\frac{1.27 \Delta m^2 L}{E_\mu} \right]. \end{aligned} \quad (55)$$

Here, $\Delta m^2 = (0.124177 \times 10^6 \text{ eV})^2 - (2.1254 \text{ eV})^2. \quad (56)$

6. Conclusions

From the mass matrices we obtained the necessary theoretical data to Verify the flavor violation. Instead of two flavor mixing we should consider three by three mass mixings. But the present analysis should Yield at least approximate results. The masses of the neutrinos will be Slightly different if we take the Weinberg mixing parameter = 0.23.

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