Journal of Nuclear and Particle Physics 2020, 10(2): 23-30
DOI: 10.5923/j.jnpp.20201002.01

Nonrelativistic Open String Model – Neutrino Mass and Lifetime Values

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Abstract Candidate neutrino string parameters are investigated using a nonrelativistic open string model with fixed endpoints. String parameters and lifetime values are derived as a function of the neutrino mass. Large variations in string parameter and lifetime values are predicted for the various neutrino mass values utilized in this paper. The neutrino lifetime values exceed 10^{55} y for the 0.0001 – 1.0 eV/c^2 neutrino mass range considered in this paper.

Keywords String theory, Neutrino string parameter values, Open string model, Neutrino mass, and neutrino lifetime

1. Introduction

String theory is an elegant mathematical formulation [1-7] that has yet to be experimentally verified. Specific particle parameter values and associated decay modes are uncertain and have been qualitatively discussed [8-28]. These uncertainties are exemplified by estimates of the neutrino mass and lifetime values [25,29,30]. This paper applies the nonrelativistic open string model proposed by the author [28] to calculate a range of neutrino string parameter and lifetime values as a function of assumed neutrino mass values.

Within the Standard Model of Particle Physics, there are three generations each with a specific neutrino. These generations include the electron, muon, and tau neutrinos that are presumed to have different mass values. The nonrelativistic open string model [28] will be applied to the determination of the neutrino string parameters and lifetime values. However, the model is not sufficiently accurate to distinguish between the various neutrino types. Accordingly, this paper does not distinguish between the three generations of neutrinos, and calculates generic neutrino string characteristic and lifetime values as a function of the neutrino mass.

Neutrino mass values are uncertain [25,29,30], but the KATRIN Collaboration [29] recently reported an upper limit neutrino mass of < 1.1 eV/c^2 (90% C.L.). The KATRIN Collaboration results [29] significantly improved previous lifetime estimates by almost a factor of two.

The magnitude of the neutrino mass and associated lifetime values have implications for both particle physics and cosmology. For particle physics, it narrows the allowed range of quasi-degenerate neutrino mass models. With respect to cosmology, a model-independent limit can be used as input for determination of the structural evolution in cosmological models.

This paper defines a model to calculate the neutrino lifetime and associated string parameters as a function of neutrino mass using the nonrelativistic open string model with fixed endpoints [28]. By constraining the model to reproduce a selected neutrino mass, a set of parameters that provide an initial representation for the neutrino string and associated lifetime are derived.

Determination of these string parameters and lifetime values is fraught with obvious uncertainty. The present approach provides string parameters that establish an initial, but not definitive, set as the basis to explore in future work. Subsequent work will include a string model incorporating charge, electric and magnetic fields, multiple interacting strings including loops, various boundary conditions, interaction types, gauge theories, and symmetry conditions. The deviation in string parameters from the base case values established in this paper will illuminate the dependence of the various parameters on specific string properties.

2. Nonrelativistic Open String Model Overview

The model proposed in this paper assumes the production of cosmic strings following the big bang or during a big bang/crunch cycle of cosmic events. In this paper, it is assumed that particles result from the emission of the vibrational energy of the string. The fields associated with these particles can be derived from a number of symmetry classes. A simple example would be an Abelian-Higgs

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Published online at http://journal.sapub.org/jnpp

theory with a complex scalar field and a U(1) gauge field [27]. This class of fields is shown by Matsunami et al. [27] to produce a string with a lifetime, defined in Section 6.0 that is proportional to the square of the string length.

Following the Abelian-Higgs field theory with a U(1) gauge approach, the decay of strings into requisite particles occurs episodically with an associated energy loss. This energy loss is associated with the neutrino mass.

In previous work [28], a representative sample of string parameters for a set of baryons, leptons, and mesons were determined. This determination was based on specific mass and lifetime values for the set of selected particles that included the proton, neutron, and lambda baryons; electron, muon, and tau leptons; and charged pions and charged B mesons [28].

Since the neutrino mass and lifetime values are uncertain [25,29,30], these circumstances require a somewhat different approach than utilized in [28]. Given these uncertainties, neutrino masses are assumed to vary between 0.0001 eV/c^2 and 1.0 eV/c^2 where 1.1eV/c^2 was the upper bound noted by the KATRIN Collaboration [29]. For each assumed mass, string parameter and lifetime values are derived from the best three fits to the particle mass value. These parameter values and lifetimes are summarized in Table 1 – 5.

3. Model Parameter Specification

The string utilized in this paper is limited to nonrelativistic velocities. The energy of the string available for neutrino decay is based on its total vibrational energy (kinetic plus potential energy). In this paper, assumed neutrino mass values are utilized to calculate the associated neutrino lifetime and string parameter values.

Key model parameters include the string density, which is related to the tension, and the length, amplitude, and velocity. Bounds on the string tension (S), derived from pulsar timing measurements [22-24,27], are based on the gravitational wave background produced by decaying cosmic string loops. This bound, $\mathbf{GS \le 10^{-11}}$, is based on Newton's gravitational constant (G) and is derived from simulations that ignore the field composition of the string. This would correspond to a string mass density of about 1.4×10^{17} kg/m. As a matter of comparison, a density of 1.4×10^{27} kg/m is derived from the Planck energy divided by the Planck length. Davis and Kibble [20] suggest that a string density of 10^{21} kg/m is an appropriate string density. These results imply that a range of density values are possible. Accordingly, the string density is permitted to vary over a range of values.

Matsunami et al. [27] suggest that particle radiation is associated with a string length that is < 10^{-19} m. Longer lived particles that do not decay or that have extended lifetimes (e.g., protons and electrons) would be expected to have significantly longer string lengths. This assertion was also noted in [28]. In addition, cosmological strings are expected to be mildly relativistic [27]. Matsunami et al. [27] utilize values of 0.33 c and 0.6 c in their calculations. The model proposed in this paper [28] uses a nonrelativistic approach and limits the string velocity to values less than used in [27] (i.e., $\beta \le 0.05$).

These parameter values will be used as a guide and not a specific limitation in this paper. Reasonable variations will be considered in subsequent discussion. In particular, the density is permitted to vary between 10^7 and 1.4×10^{27} kg/m. The string length is permitted to vary within the 10^{-21} to 10^{46} m. String velocity is assumed to be nonrelativistic and is limited to $\beta = v/c \le 0.05$. Amplitude values are restricted to be less than the string length. The lifetime value of Chacko et al. [30] was used as a lower limit for the calculated neutrino values.

4. Base Case String Model

Cosmic strings have extremely large masses that greatly exceed the particle masses considered in this paper. The particle masses are assumed to be generated by the kinetic and potential energies of the vibrating string. The resulting particle mass does not depend on the total inclusive string mass. In this paper, the inherent string mass is treated as a renormalized vacuum or zero point energy with particles associated with the vibrational energy of the string.

As a base case, a one-dimensional string of finite length and fixed endpoints is assumed to vibrate with peak amplitude A

$$\mathbf{\Phi(x,t) = A\cos(kx - \omega t)} \quad (1)$$

where $\mathbf{\Phi(x,t)}$ is the displacement of the string from its linear, nonoscillating state, k is the wave number, x is the position as measured from one end of the string, ω is the angular frequency, and t is the time. The string is assumed to be under tension and has mass per unit length (μ), and is fixed at x=0 and x=L where L is the nonvibrating string length. For small displacements, summing the kinetic and potential energies determines the amount of energy contained within a segment of length dx.

The kinetic energy (dT) is the product of the string segment's mass and its velocity

$$\mathbf{dT = \frac{1}{2}\mu dx\left(\frac{\partial \Phi}{\partial t}\right)^2} \quad (2)$$

Using Eq. 1, this yields

$$\mathbf{dT = \frac{1}{2}\mu dx A^2 \omega^2 \sin^2(kx - \omega t)} \quad (3)$$

The potential energy (dU) arises from the string tension. When a segment associated with an interval dx is displaced from equilibrium, it has a length

$$\mathbf{dL = \left[(dx)^2 + (d\Phi)^2\right]^{1/2} = dx\left[1 + \left(\frac{\partial \Phi}{\partial x}\right)^2\right]^{1/2} \approx dx\left[1 + \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2\right]} \quad (4)$$

The net result is an expansion of the string length by an amount δL due to the tension

$$\delta L = dL - dx = \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2 dx \tag{5}$$

The expansion creates a potential energy change

$$dU = S\delta L = \frac{1}{2} S\left(\frac{\partial \Phi}{\partial x}\right)^2 dx \tag{6}$$

Using Eq. 1 and the relationships $k = \omega / v$ and $v = (S/\mu)^{1/2}$ yields

$$Sk^2 = S\left(\frac{\omega}{v}\right)^2 = S\omega^2 \frac{\mu}{S} = \mu\omega^2 \tag{7}$$

The solution of Eq. 1 and Eq. 7 permits dU to be written as

$$\begin{aligned} dU &= \frac{1}{2} Sk^2 A^2 Sin^2 (kx - \omega t) dx \\ &= \frac{1}{2} \mu A^2 \omega^2 Sin^2 (kx - \omega t) dx = dT \end{aligned} \tag{8}$$

The total energy (dE) is just the sum of dT and dU

$$\begin{aligned} dE &= dT + dU = 2dT \\ &= \mu dx A^2 \omega^2 \sin^2 (kx - \omega t) \end{aligned} \tag{9}$$

The energy per unit length or energy density (ρ) is

$$\rho(x,t) = \mu A^2 \omega^2 \sin^2 (kx - \omega t) \tag{10}$$

The average energy per unit length or energy density over a full wavelength or a full period is

$$\bar{\rho} = \frac{1}{2} \mu A^2 \omega^2 = \frac{dE}{dx} \tag{11}$$

Assuming a uniform energy density over the string length, the energy of a particle corresponding to the string vibrational energy density defined by Eq.11 with total length L is

$$E = \frac{1}{2} \mu A^2 \omega^2 L \tag{12}$$

5. Neutrino Mass

In order to estimate the characteristics of a string corresponding to a neutrino's mass, a simplifying assumption is made (i.e., the string is vibrating in its fundamental frequency). This is a reasonable first approximation because the neutrino is not a resonance of a lighter particle.

An application of Eq. 12 permits an estimate of neutrino's rest mass energy (ε). This can be accomplished using the relationship between angular frequency and frequency (ν)

$$\omega = 2\pi\nu \tag{13}$$

and string velocity and wavelength

$$v = \nu\lambda \tag{14}$$

Eqs.13 and 14 permits Eq. 12 to be written as

$$E = 2\pi^2 \mu A^2 \left(\frac{v}{\lambda}\right)^2 L = \frac{\pi^2}{2} \mu A^2 \frac{v^2}{L} \approx \varepsilon \tag{15}$$

where λ = 2L based on the first harmonic assumption.

As noted previously, the neutrino lifetime and mass are uncertain. To facilitate the calculation of neutrino string parameters and lifetime values, the upper bound on neutrino mass of 1.1 eV/c^2 is utilized [29]. Lifetime values are calculated as a function of particle mass for a range of neutrino masses between 0.0001 to 1.0 eV/c^2.

6. Neutrino Lifetime

Matsunami et al. [27] provide a relationship for the string lifetime (τ)

$$\tau \approx \frac{SL^2}{\xi\varepsilon c} = \frac{v^2 \mu L^2}{\xi\varepsilon c} \tag{16}$$

where ξ is the number of episodes per period, and ε is the average energy lost per unit time which the model assumes to be the neutrino rest mass energy. The string described in Section 4 is used as the basis for estimating the neutrino lifetime.

7. Model Assumptions and Limitations

The neutrino lifetime and associated string parameters are derived by assuming the following:

1. The model, defined in Sections 2 – 4, specifies the string parameters that characterize the neutrino.
2. One episode per period is assumed which is consistent with the fundamental mode assumption of Section 5.
3. The average energy lost per unit time (e.g., over a period) is the string kinetic plus potential energy. Since the string is nonrelativistic, this is assumed to be the neutrino's rest mass. The neutrino lifetime is derived from the rest mass energy of the particle (ε) and is defined by Eqs. 15 and 16.
4. Only the string kinetic plus potential energy contributes to the neutrino mass. The inherent string mass ($\bar{\rho}L$) is essentially a constant (i.e., it is the vacuum or zero point energy), because the neutrino energy is much smaller than this inherent mass.
5. No distinction is made for the three neutrino generations that occur in the Standard Model. The lifetime and string parameter values should be considered as a global average over the three generations of electron, muon, and tau neutrinos.
6. The mass hierarchy of neutrinos is not considered or specifically evaluated.
7. Neutrino oscillations are not included in the model.
8. Additional neutrino generations or the specific nature

of neutrinos (i.e., Dirac or Majorana) are not considered.

9. The specific neutrino decay modes and their associated decay products are not specified or considered.

8. Results and Discussion

The model results provide specific neutrino string parameter values and neutrino mean lifetime values as a function of neutrino mass. Model results suggest that long-lived neutrino lifetime values are obtained for a wide range of string parameters. The string parameters (i.e., density, length, amplitude, and velocity) supporting these lifetime values are addressed, and their variation with neutrino mass are discussed in subsequent commentary. Tables 1, 2, 3, 4, and 5 summarize, as a function of neutrino mass, the neutrino string density, length, amplitude, beta value, and lifetime values, respectively. The three best fits to the assumed neutrino mass are provided in these tables.

Given the nature of the proposed calculations and associated uncertainties, a preliminary goal of fitting the particle masses and lifetimes to within 1% of their experimental values was set. This appears to be a reasonable criterion for the initial calculations.

In Tables 1 – 5, the notation H (high), M (medium), and L (low) is used to label the columns of the three best parameter fits to the assumed neutrino mass value. The parameter set yielding the largest lifetime for each string mass is listed under the H column. The L (M) columns record the lowest (middle) lifetime for each of the assumed neutrino mass values.

8.1. Neutrino Masses

The neutrino masses summarized in Tables 1 – 5 are limited to values between 0.0001 and 1.0 eV/c^2 [29]. The 1.1 eV/c^2 value represents the current best estimate for an upper bound to the neutrino mass [29]. The string parameters and lifetime values are calculated as a function of these assumed neutrino mass values. The neutrino mass values were fit to within 0.1% for all masses considered in Tables 1 – 5.

Given the simplistic nonrelativistic, uncharged, fixed endpoint open string model, the mass results are encouraging. However, the model parameter assumptions and associated parameter ranges are still lacking in experimental verification.

8.2. String Density

As noted in Table 1, there is significant variation in the string density as a function of neutrino mass for the L, M, and H Cases. In particular, the string density values reside within the range of $10^{11} – 10^{26}$ kg/m. In view of this variation, definitive conclusions regarding the string density are not possible. Therefore, a more global analysis must be utilized.

To facilitate a global analysis, an averaged logarithmic string parameter (ALSP) Ω(E) is defined by the relationship:

$$\log_{10}\Omega(E) = \frac{\log_{10}\Omega_L(E) + \log_{10}\Omega_M(E) + \log_{10}\Omega_H(E)}{3} \tag{17}$$

where the averaged logarithmic string parameters are ALSμ for the string density, ALSL for the string length, ALSA for the string amplitude, and ALAτ for the string lifetime. The averaged string velocity (ASβ) is addressed in subsequent discussion.

The ALSμ for the string density is plotted as a function of neutrino mass in Fig. 1. As expected, the ALSμ (Fig. 1 dashed curve derived from the Table 1 data) still exhibits considerable variation, but it is less severe than the individual Case L, M, and H variations.

Table 1. Neutrino String Density (kg/m)[a]

Neutrino Mass (eV/c^2)	Case L	Case M	Case H
0.0001	3.88x10^{20}	8.98x10^{20}	8.14x10^{13}
0.0002	1.01x10^{15}	4.81x10^{21}	1.95x10^{22}
0.0005	7.49x10^{10}	1.17x10^{17}	3.19x10^{23}
0.001	1.62x10^{12}	7.87x10^{22}	1.65x10^{16}
0.002	6.15x10^{13}	1.17x10^{17}	3.19x10^{23}
0.005	4.81x10^{21}	1.11x10^{22}	2.26x10^{24}
0.01	3.82x10^{16}	5.48x10^{19}	5.13x10^{20}
0.02	5.67x10^{10}	4.81x10^{21}	2.93x10^{20}
0.05	1.31x10^{11}	5.22x10^{24}	1.38x10^{23}
0.1	6.25x10^{17}	4.08x10^{15}	1.04x10^{23}
0.2	1.38x10^{23}	2.62x10^{26}	6.90x10^{24}
0.5	1.02x10^{19}	5.67x10^{10}	6.46x10^{25}
1.0	5.48x10^{19}	1.88x10^{14}	1.91x10^{18}

[a]Cases L(low), M(Medium), and H(high) are based on the relative mean lifetime values of Table 5.

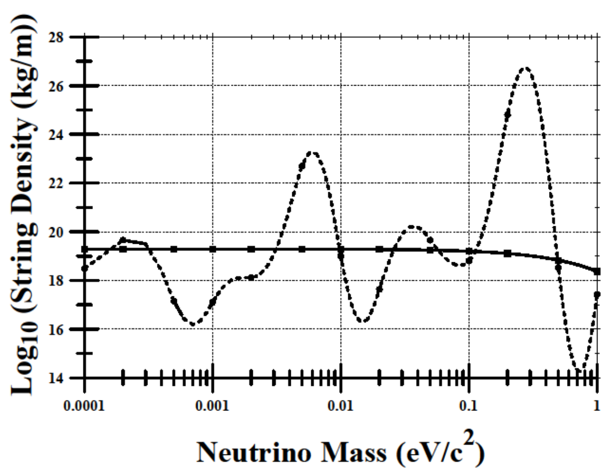

Figure 1. Neutrino string density as a function of neutrino mass

The solid curve in Table 1 represents a linear fit to the ALSμ values defined by the relationship:

$$\mu(E) = a\log_{10}\mu_{ALS\mu}(E) + b \tag{18}$$

where a = -0.92201118 kg/m and b = 19.29387042 kg/m.

The linear fit suggests an averaged neutrino string density on the order of $10^{18} – 10^{19}$ kg/m for neutrino masses in the range of 0.0001 – 1.0 eV/c^2.

The neutrino sting density is higher than noted by Bevelacqua [28] for unstable baryons (neutrons and lambdas), leptons (muons and taus), and mesons (charged pions and charged B mesons). However, the electron and proton string density range of values is similar to the neutrino values of $10^{11} – 10^{26}$ kg/m summarized in Table 1.

Baryon densities, derived in a previous paper [28], are $10^{12} – 10^{18}$ kg/m for neutrons, $10^{10} – 10^{27}$ for protons, and about 10^{12} kg/m for lambdas. Lepton string densities are also tend to be lower than the corresponding neutrino values summarized in this paper with values of $10^{11} – 10^{21}$ kg/m, $10^{12} – 10^{16}$ kg/m, and $10^{11} – 10^{12}$ kg/m for electron, muon, and tau leptons, respectively. Meson string densities for charged pions ($10^{11} – 10^{14}$ kg/m) and charged B mesons ($\approx 10^{11}$ kg/m) also exhibit a lower value than the neutrino string density.

These results suggest that higher string densities are exhibited for long-lived particles. Although the neutrino lifetime is not well established [25,29,30], the string density results suggest the neutrino is also a long-lived particle with a lifetime as long or longer than the proton and electron.

8.3. String Length

Following [27], the string length associated with decay of unstable particles should be $<10^{-19}$ m. As noted in previous discussion, this value provides an indication of an expected unstable particle string length and the results of the open string nonrelativistic model may differ.

The neutrino string length values summarized in Table 2 varies over a range of $10^6 – 10^{17}$ m. These string length values are much larger than noted for unstable particles [27,28].

For baryons, the neutron and lambda string lengths are in the range of 10^{-15} to 10^{-12} m and $\approx 10^{-19}$ m, respectively. A similar range of string values is found for leptons. The muon and tau string lengths are in the range of 10^{-19} to 10^{-17} m and $\approx 10^{-19}$ m, respectively. The meson values are 10^{-19} to 10^{-17} m and $\approx 10^{-19}$ m for the charged pion and charged B meson, respectively.

For long-lived particles, string lengths have an increased value. Proton and electron string lengths are in the range of $10^6 – 10^{11}$ m and $10^4 – 10^{14}$ m, respectively [28]. Eq. 16 suggests that the increased proton and electron lifetime values should correspond with string lengths that are much longer than those values encountered in unstable baryons, leptons, and mesons [28]. The results summarized in Table 2 further suggest a long-lived neutrino with a lifetime as long or longer than the proton and electron.

The neutrino string length results are further summarized in Fig. 2. In Fig. 2, the dashed curve represents the ALSL values derived from Table 2. The solid curve in Fig. 2 represents a linear fit to the ALSL values:

$$\mathbf{L(E) = a \log_{10} L_{ALSL}(E) + b} \quad (19)$$

where a = -1.94415416 m and b = 12.25890117 m. The linear fit of Eq. 19 suggests an averaged string length of about 10^{10} - 10^{12} m for neutrino masses between 0.0001 and 1.0 eV/c^2.

Table 2. Neutrino String Length (m)[a]

Neutrino Mass (eV/c^2)	Case L	Case M	Case H
0.0001	2.99×10^{7}	2.03×10^{11}	5.29×10^{14}
0.0002	1.59×10^{15}	6.68×10^{13}	1.82×10^{13}
0.0005	1.25×10^{7}	4.30×10^{15}	1.66×10^{16}
0.001	3.52×10^{6}	7.71×10^{10}	4.98×10^{16}
0.002	8.46×10^{12}	4.30×10^{15}	1.66×10^{16}
0.005	6.68×10^{13}	1.06×10^{13}	5.78×10^{12}
0.01	2.66×10^{7}	3.37×10^{7}	5.66×10^{8}
0.02	9.72×10^{11}	6.68×10^{13}	1.68×10^{16}
0.05	2.91×10^{11}	1.35×10^{10}	1.50×10^{13}
0.1	2.21×10^{6}	7.80×10^{13}	1.45×10^{12}
0.2	1.50×10^{13}	4.61×10^{11}	8.63×10^{12}
0.5	1.06×10^{6}	9.72×10^{11}	2.76×10^{12}
1.0	1.03×10^{7}	7.14×10^{10}	6.40×10^{14}

[a]Cases L(low), M(Medium), and H(high) are based on the relative mean lifetime values of Table 5.

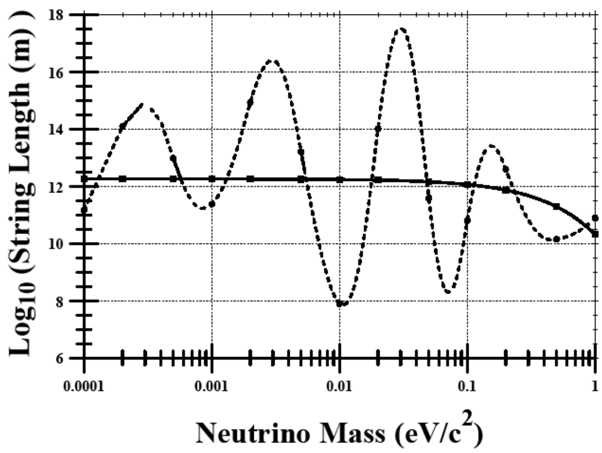

Figure 2. Neutrino string length as a function of neutrino mass

8.4. String Amplitude

The neutrino string amplitude summarized in Table 3 has a range between 10^{-26} m and 10^{-15} m. As noted with the other string parameters, there is considerable variability in the amplitude values. This variability is reduced using the ALSA values.

Using Eq. 17, an ALSA value is calculated and is represented by the dashed curve in Fig. 3. The solid curve in Fig. 3 represents the linear fit to the ALSA values

$$\mathbf{A(E) = a \log_{10} A_{ALSA}(E) + b} \quad (20)$$

where a = 0.70396182 m and b = -21.18500958 m. Over the neutrino mass range of 0.0001 – 1.0 eV/c^2, the string amplitude is about 10^{-21} m. This amplitude is significantly larger than the values for short-lived baryons, leptons, and mesons [28].

Table 3. Neutrino String Amplitude (m)[a]

Neutrino Mass (eV/c^2)	Case L	Case M	Case H
0.0001	$4.04x10^{-26}$	$7.85x10^{-24}$	$3.80x10^{-19}$
0.0002	$2.28x10^{-19}$	$5.72x10^{-22}$	$5.86x10^{-24}$
0.0005	$5.99x10^{-21}$	$2.57x10^{-19}$	$1.42x10^{-22}$
0.001	$5.59x10^{-22}$	$3.58x10^{-24}$	$4.40x10^{-18}$
0.002	$2.34x10^{-19}$	$2.57x10^{-19}$	$1.42x10^{-22}$
0.005	$5.72x10^{-22}$	$3.56x10^{-23}$	$2.74x10^{-24}$
0.01	$3.37x10^{-23}$	$1.10x10^{-24}$	$3.82x10^{-24}$
0.02	$1.28x10^{-16}$	$5.72x10^{-22}$	$2.32x10^{-20}$
0.05	$7.99x10^{-16}$	$1.55x10^{-25}$	$1.47x10^{-22}$
0.1	$4.19x10^{-23}$	$3.18x10^{-18}$	$2.30x10^{-23}$
0.2	$1.47x10^{-22}$	$2.87x10^{-25}$	$1.17x10^{-23}$
0.5	$3.69x10^{-24}$	$1.28x10^{-16}$	$5.75x10^{-24}$
1.0	$6.43x10^{-24}$	$2.45x10^{-19}$	$2.48x10^{-19}$

[a]Cases L(low), M(Medium), and H(high) are based on the relative mean lifetime values of Table 5.

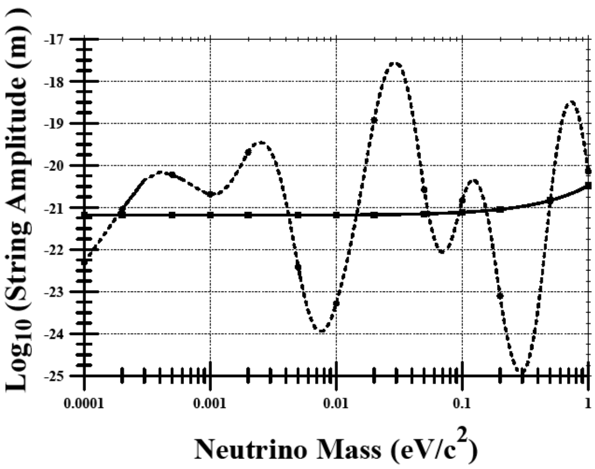

Figure 3. Neutrino string amplitude as a function of neutrino mass

The neutron amplitude is in the range of 10^{-29} to 10^{-25} m, and the heavier lambda amplitude is $\approx 10^{-28}$ m. For short-lived leptons and mesons, larger amplitude values suggest a larger mass and shorter lifetime. The muon amplitude is in the range of 10^{-30} to 10^{-27} m, and the heavier tau has an amplitude of $\approx 10^{-27}$ m. Meson amplitudes follow a similar pattern, but the differences are not as large. The charged pion amplitude is in the range of 10^{-29} to 10^{-26} m, and the heavier charged B meson has a value of $\approx 10^{-27}$ m.

The neutrino amplitude is smaller in magnitude than the proton and electron values [28]. As noted in previous work [28], the proton and electron amplitude values are in the range of 10^{-20} – 10^{-13} m and 10^{-19} – 10^{-17} m, respectively. These results suggest that long-lived particles have longer amplitudes than short-lived particles [28]. This result continues to suggest that the neutrino has a long lifetime consistent with Chacko et al. [30].

8.5. String Velocity

The string velocity is restricted to $\beta \leq 0.05$. In a previous paper [28], the baryon, lepton, and meson results suggested that there was no general velocity relationship between values of β and the particle mass or mean lifetime and associated string parameters. There is also considerable scatter in the neutrino string velocity values summarized in Table 4.

The L, M, and H Case values were averaged to obtain the ASβ value:

$$\beta_{AS\beta}(E) = \frac{\beta_L(E) + \beta_M(E) + \beta_H(E)}{3} \quad (21)$$

where the $\beta_{AS\beta}(E)$ values were fit to the linear relationship

$$\beta(E) = a\beta_{AS\beta}(E) + b \quad (22)$$

with a = 0.0142220174 and b = 0.0241823682.

Table 4. Neutrino String Beta[a]

Neutrino Mass (eV/c^2)	Case L	Case M	Case H
0.0001	0.0413	0.0115	0.0403
0.0002	0.0468	0.00175	0.0443
0.0005	0.0290	0.0100	0.0215
0.001	0.0500	0.00525	0.00750
0.002	0.0425	0.0200	0.0430
0.005	0.00875	0.0368	0.0248
0.01	0.0470	0.0430	0.0165
0.02	0.00275	0.0175	0.0278
0.05	0.000250	0.0440	0.00950
0.1	0.00850	0.00825	0.0308
0.2	0.0190	0.0393	0.0258
0.5	0.0370	0.0138	0.0153
1.0	0.0405	0.0478	0.0443

[a]Cases L(low), M(Medium), and H(high) are based on the relative mean lifetime values of Table 5.

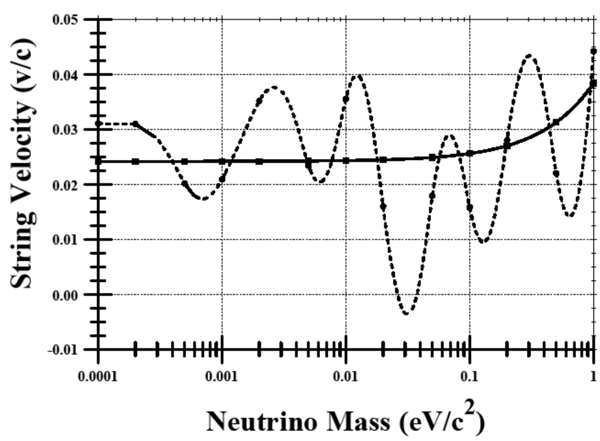

Figure 4. Neutrino string velocity as a function of neutrino mass

In Fig. 4, the dashed curve represents the $\beta_{AS\beta}(E)$ values, and the solid curve illustrates the linear fit values of Eq. 22. The averaged $\beta_{AS\beta}(E)$ values still exhibit considerable scatter, but the linear fit suggests the neutrino velocity values lie in the range of about 0.025 c – 0.040 c.

These Table 4 and Fig. 4 values are not clustered near the maximum β value (i.e., 0.05) that suggests that the model is favoring a nonrelativistic solution. This conclusion is model dependent and must be verified with a more refined approach including electromagnetic fields and other symmetry assumptions that were noted previously.

8.6. Particle Lifetime

Following Eq. 16 and the associated discussion, the particle lifetime values are strongly dependent on the string length, tension, and particle mass. The particle mass (Eq. 15) involves multiple parameters, but the lifetime (Eq. 16) only depends on a subset of these parameters.

Table 5. Neutrino String Mean Lifetime (y)[a]

Neutrino Mass (eV/c^2)	Case L	Case M	Case H
0.0001	$3.50 x 10^{56}$	$2.91 x 10^{63}$	$2.19 x 10^{64}$
0.0002	$1.64 x 10^{66}$	$1.95 x 10^{67}$	$3.73 x 10^{69}$
0.0005	$1.18 x 10^{45}$	$2.57 x 10^{67}$	$4.82 x 10^{75}$
0.001	$3.00 x 10^{45}$	$7.66 x 10^{62}$	$1.37 x 10^{68}$
0.002	$2.36 x 10^{59}$	$2.57 x 10^{67}$	$4.82 x 10^{75}$
0.005	$1.95 x 10^{67}$	$1.99 x 10^{67}$	$5.48 x 10^{68}$
0.01	$3.55 x 10^{50}$	$6.86 x 10^{53}$	$2.66 x 10^{56}$
0.02	$1.20 x 10^{51}$	$1.95 x 10^{67}$	$1.90 x 10^{71}$
0.05	$8.23 x 10^{47}$	$2.18 x 10^{63}$	$3.32 x 10^{66}$
0.1	$1.31 x 10^{47}$	$1.00 x 10^{60}$	$1.22 x 10^{65}$
0.2	$3.32 x 10^{66}$	$2.55 x 10^{67}$	$1.01 x 10^{68}$
0.5	$1.88 x 10^{48}$	$1.20 x 10^{51}$	$1.36 x 10^{67}$
1.0	$5.70 x 10^{50}$	$1.30 x 10^{53}$	$9.12 x 10^{64}$

[a]Cases L(low), M(Medium), and H(high) are based on the relative mean lifetime values.

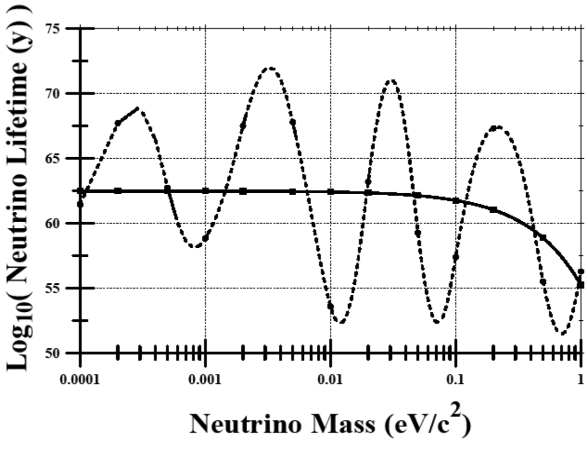

Figure 5. Neutrino mean lifetime as a function of neutrino mass

The variation in lifetime values as a function of neutrino mass is illustrated by an examination of Table 5. As summarized in Table 5, the neutrino lifetime values vary significantly and range between 10^{45} and 10^{75} y. In the spirit of the model assumptions and limitations, the results of Table 5 were fit to the functional form of Eq. 17.

The ALSτ values are plotted in Fig. 5 (dashed curve) and still exhibit considerable variation. In Fig. 5, the solid curve represents the linear fit to the ALSτ values

$$\tau(E) = a \log_{10} \tau_{ALS\tau}(E) + b \qquad (23)$$

where the parameters a = -7.23562858 y and b = 62.47429718 y. The linear fit provides a more stable set of lifetime values.

The linear fit lifetime is relatively constant between 0.0001 and 0.1 eV/c^2, and is > 10^{55} y for all assumed mass values. The calculations also provide a range of model predictions for the neutrino lifetime as a function of mass. For example, lifetimes of about 10^{55} – 10^{62} y would be predicted for neutrino masses in the range of 0.0001 – 1.0 eV/c^2.

The predicted neutrino lifetime range is similar to that for the string model lifetimes for the proton and electron [28]. Nonrelativistic string model predictions for the proton (electron) lifetime are 10^{37} - 10^{58} y (10^{29} – 10^{59} y), respectively. The relative consistency of the string density, length, and amplitude values for the proton, electron, and neutrino further support a long-lived value for the neutrino lifetime.

9. Generalization to Closed String Models

Bagchi et al. [26] note that there is a natural emergence of an open string from a closed string given selected parameter limits. There is also a condensation of perturbative closed string modes to an open string. Bagchi et al. [26] provide an important calculation that has the potential to generalize the open string model of this paper to closed string models.

10. Conclusions

The proposed nonrelativistic open string model with fixed endpoints provides an initial set of neutrino string parameters that yield mean lifetime values in excess of 10^{55} y for masses in the range of 0.0001 – 1.0 eV/c^2. The derived neutrino string parameters and lifetime values are based on a simplistic open string model, and will likely change as the model becomes more complex through the inclusion of charge, electric and magnetic fields, multiple strings with loops, additional boundary conditions, and specific symmetries and gauge theories. The validity of the proposed and subsequent models will be determined by experimental verification. Experimental verification is ultimately the requirement that will determine the validity of all string theories. However, this initial set of neutrino parameters provides a base case for future investigation, development, and determination of observable string characteristics.

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