

On the Structure of $A=3$ Nuclei

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Abstract $A=3$ is the lightest genuine “nucleus” with an “inside” with a centre and an outside/surface. As $A=2$ nucleus does not have an “inside” what transpires in $A=3$ to make this happen? The hole in the charge distribution of ${}^3\text{He}$ is a major problem in the $A=3$ nuclei. The canonical wave function of $A=3$ nuclei which does well for the electromagnetic properties of $A=3$ nuclei fails to produce the hole in $A=3$ nuclei. We show that this wave function does give the surface while the “inside” is completely missed by it. The reproduction of the inside of the $A=3$ nuclei demands that a different wave function be used. This new wave function gives these $A=3$ nuclei a mathematical centre which arises fundamentally and which also reproduces the hole in ${}^3\text{He}$. Thus this hybrid wave function, given in terms of the nucleonic degrees of freedom, provides us with a better and a consistent understanding of the structure of the $A=3$ nuclei.

Keywords Nuclear Structure, Helion/Triton, Charge Distribution, Clustering

1. Introduction

Clusters are playing an important role in the current nuclear physics. Of these α -clusters are known to play a prominent role. These are known to be spontaneously emitted from heavy nuclei. Other clustering in particular $A=3$ clustering is also possible and this was recently studied in detail by us [1]. Strong experimental evidences of clustering of $A=3$ nuclei- ${}^3\text{He}$ (Helion) and ${}^3\text{H}$ (Triton) were pointed out and a consistent theoretical understanding was attempted therein. One would have thought that nuclei as simple as $A=3$ should have been well understood by now. But in keeping with the complexity in the structure of another three body system that of baryons made up of three quarks things are not all that easy.

There are deficiencies as to the agreement between the experimental and the conventional nuclear models in explaining the ground state properties of light nuclei. For example, calculations of the binding energies for the few particle systems such as ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$, using as inputs various two-body potentials only, show an under binding relative to their corresponding experimental values. Several mechanisms have been proposed to minimize the deficiencies between the experiments and the theoretical models. One of the most important of these is the effect of the three-body interactions, which cannot be deduced from the two-body data. This may give the necessary components to the ground state properties of $A=3$ nuclei. Within the nonrelativistic framework of nuclear physics a

three-body force is expected to have an appreciable contribution relative to other mechanisms, especially for systems with cluster structures. The importance of three-body forces in nuclear physics is well known, and quantum Monte Carlo (QMC) calculations of light nuclei have provided evidence in its favour. There the $A=3$ systems can be described as a three-body quantum cluster entity where the dominant three body forces acts between two neutrons and a proton or between two protons and a neutron. Thus there has been growing evidence in nuclear physics that three-body forces should exist among the nucleons inside nuclei, for a large number of isotopes [2-5].

A major goal of nuclear physics is to understand the correct nature of the many-body interactions. Being the fundamental theory of strong interaction, Quantum Chromodynamics is still unable to quantitatively describe the two-nucleon interactions from the first principle, specially, in the non perturbative regime of low energy nuclear physics. Further, the two-body potentials constructed, by using the large amount of experimental data on nucleon-nucleon scattering are inadequate to explain the ground state properties as well as many of two-body scattering data. The composite nature of the nucleons, indicates that many-nucleon forces, and at the least the three-nucleon forces are necessary to explain many bound state properties. A lot of effort has been devoted to understand the correct nature of nuclear forces and currents and this has resulted in developments of accurate many-body techniques for evaluating them [2-19].

There are other features of the nuclear matter which point to the need of real three-body force. For finite nuclei one needs a real three-body force in order to reproduce the spin-orbit splitting between the single particle orbits [14]. There are indeed pointers from the experiments and the

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theory which hint that a real three-body force may be needed in shell model studies as well. For example, the inclusion of a real three-body force gives the correct spin assignment in case of ^{10}B [15]. The differential cross sections for the $\gamma + {}^4\text{He} \rightarrow p + t$ reaction were measured as a function of photon-beam energy and proton-scattering angle and are compared with the latest model calculations. At 0.6-1.2 GeV, the data is in good agreement only with the calculations that include three-body mechanisms, thus confirming their importance. These results reinforce the conclusion of some previous study of the three-body break-up of ${}^3\text{He}$ that demonstrated the great importance of the three-body mechanisms in the energy region 0.5-0.8 GeV[11-13]. DAPHNE Collaboration have measured the differential cross sections for the ${}^3\text{He}(\gamma, \text{pd})$ reaction. The results confirm the rapid decrease of forward-angle cross sections with increasing photon energy, whilst at backward angles, aided by the high precision of the measurement, which show the clear enhancement in the angular distribution and is a possible signature of three-body mechanisms[17]. Also a schematic three-body forces in addition to a general two-body forces leads to a strongly improved description of the energies of light and medium mass nuclei[18], and there are effects of three-body force on the longitudinal response function of ${}^4\text{He}$ [19]. Further, while trying to fit the nuclear matter binding energy and density simultaneously the results of the best two body interactions, all fall on the so called Coester band. No matter what we do, if one confines oneself to two body interaction then there is no way one can do any better? This clearly hints at including at least a three body force[3BF].

There are also several strong reasons as to why it seems that it is not enough to confine oneself to only two body interactions. For example, there are basic phenomenological reasons[20] why a three body force is needed and also more fundamental reasons[21] for the same. Note that these 3BF can not be reduced to any two body force and thus have their own independent existence.

The realization that in nonrelativistic framework 3BF is indispensable in a complete and consistent description of nuclei has grown in recent years. One knows that $A=3$ bound states ${}^3\text{H}$ and ${}^3\text{He}$ are theoretically underbound using only NN (two body) forces. So also is ${}^4\text{He}$ underbound and this is represented by the so called Tjon lines[22]. Proper inclusion of 3BF in different models is essential to fit the binding energies of all ${}^3\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$ [22]. The same paper also indicated that the presence of four-body forces (4BF) is ruled out for ${}^4\text{He}$.

This point is very succinctly made by Wiringa and Peiper[23] "Modern nucleon nucleon (NN) potentials, such as the Argonne v_{18} , CD Bonn, Reid93, NijmI, and NijmII, fit over 4300 elastic NN scattering data with a $\chi^2=1$. These potentials are very complicated, including spin, isospin, tensor, spin-orbit, quadratic momentum-dependent, and charge-dependent tensor, with ~ 40 parameters adjusted to fit the data. Despite this sophistication, these potentials can not reproduce the binding energy of few body nuclei such as ${}^3\text{H}$

and ${}^3\text{He}$ without the assistance of a three-nucleon potential".

Hence we see that the 3BF manifest itself right away in the structure of $A=3$ nuclei, so much so that it is not possible to fit triton's and helion's binding energy, rms radii etc. without involving a 3BF[22-23]. Note that at the same time they also found that 4BF in α -nucleus is negligible[22].

Experiments confirm our prediction, based on multi quarks in QCD, of a tendency for triton clustering in light neutron rich nuclei[24]. As such the neutron halo phenomena are naturally explained[25]. Prediction of exotic molecular states rich in triatomic clusters is made through new Ikeda-like diagram configurations for the nuclei ${}^Z_Z A_{2Z}$. Recent experiments clearly demonstrate the importance of the excited state 2^+ at 1.8 MeV in the substructure ${}^6\text{He}$ for the ground state of the nucleus ${}^8\text{He}$. This means that the ground state structure of ${}^8\text{He}$ rather than being $\alpha + 4n$ is actually ${}^3\text{H} + {}^3\text{H} + 2n$.

Is there some new symmetry which supports this tritonic clustering? We know that n and p combine to give a bound triplet state ($S=1, T=0$). It has no excited states. The singlet state ($S=0, T=1$) is unbound by 64 keV. This being isospin partner of (n-n) ($S=0, T=1$) and (p-p) ($S=0, T=1$), all these are unbound. Here we proposed a new symmetry called the "mucosin" with h and t belonging to the fundamental representation of this new and distinct SU(2) group[25]. The binding energies of ${}^3\text{He}$ and ${}^3\text{H}$ are 7.72 MeV and 8.48 MeV, respectively and also these two have no excited states. Hence for low-energy excitations, we may consider these as elementary. That is we treat h and t as elementary particles with a new "isospin". Hence, we would expect for (h-t) the triplet ($S=1, T=0$) to be bound and singlet ($S=0, T=1$) to be unbound. Also, its isospin partners (h-h) ($S=0, T=1$) and (t-t) ($S=0, T=1$) would be unbound too. This puts triton clustering in nuclei on a stronger pedestal.

Hence we see that three body-force is essential for the consistent understanding of the nuclear phenomena. Clearly the 3BF does not even exist in the $A=2$ system, i.e. in N-N scattering or in the $A=2$ deuteron ground state. But the 3BF raises his head right away in the $A=3$ nuclei themselves. So, clearly in the $A=3$ nuclei both the 2BF and the 3BF are acting. What basic and fundamental difference is introduced in the $A=3$ nuclei? It is clear that the $A=2$ system is only a "surface" with no "inside" component. On the other hand the $A=3$ is the lightest nucleus with an additional "inside". This hints that the "inside" in this nucleus must be a result of the 3BF. We wish to study this effect in this paper. Does the "inside" in the $A=3$ nucleus is anything to do with the 3BF? If so, how does it manifest itself physically? We show below that indeed the "hole" in the $A=3$ nuclei as well as the creation of the unique "centre" of the $A=3$ nucleus is a manifestation of the 3BF. We shall show that this physical effect is ensured by the mathematical property of the wave function at the centre of the $A=3$ nucleus.

2. Objectives

To understand the clustering of A=3 nuclei we have to see what aspects are the ones which contribute to its clustering property. Hence a better and consistent understanding of the structure of A=3 is required.

What is the structure of A=3 nucleus vis a viz A=2 nucleus and even A=1 nucleon? Although in general one considers A=2 deuteron as the first nucleus, in structure it resembles A=1 nucleon rather than A=3 and heavier nuclei. This is clearly manifested in the similarity of the form factors of proton and that of deuteron which both fall off smoothly (with no undulation)[26, 27]. So these both are pure “surface” nuclei with nothing like an “inside” in it. But a typical nucleus as we know has Fermi charge distribution with a distinct inside and a distinct surface. The form factor of ^3He (in contrast to A=1 & A=2 form factors) has an oscillatory pattern[26] thereby including an “inside” and an “outside/surface”. A=3 is the lightest nucleus showing this characteristics and hence should be treated as the lightest nucleus.

What changes as we go from the A=2 to the A=3 that an additional “inside” to an already existing “outside” is created in the A=3. An outside also means that, somehow this light system is able to create a “centre” of the A=3 nucleus. For heavier nuclei we justify a self bound nucleus as arising from a Hartree-Fock self consistent calculations which produces self consistently a central potential for large number of nucleons which also justifies shell structure etc. But the A=3 is much too small and this logic cannot hold here. Not to forget that the A=2 deuteron does not have an “inside” and a centre. The A=1 & A=2 are bound due to proper symmetrisation of all degrees of freedom relative to nucleon and deuteron, respectively. So for the A=3 one cannot or should not invoke a potential concept and so we are forced to seek an explanation of this dual characterisation, an “inside” with a centre and an “outside/surface” in terms of proper wave functions. What wave function of the A=3 will give this dual characterisation of the A=3 nucleus? Also we want to know how the 3BF arises phenomenologically. After all the 3BF acts in the A=3 nucleus and not in the A=2 system!

3. Our Model

To understand as to what makes A=3 so special, let us study the two properties of the ground state magnetic moment of ^3He and ^3H , and next the ^3He charge density.

The degrees of freedom relevant for ^3He and ^3H are spin, isospin and orbital space. The total wave function should be anti-symmetric. For three nucleons the spin wave function in standard notation are of mixed symmetry χ_ρ and χ_λ where ρ and λ correspond to mixed symmetric state with the first two spins in anti-symmetric and symmetric states respectively. The corresponding mixed symmetric states or the three nucleons are ϕ_ρ and ϕ_λ . If we ensure full anti-symmetry in spin-isospin space

$$\Psi_A = \frac{1}{\sqrt{2}}(\chi_\lambda \phi_\rho - \chi_\rho \phi_\lambda) \quad (1)$$

then the orbital part of the wave function for A=3 should be completely symmetric which we take as Fadeev wave function[28]

$$\Psi_{A=3} = \frac{\alpha^3}{\pi^{3/2}} e^{-\alpha^2(\rho^2 + \lambda^2)/2} \quad (2)$$

Where,

$$\begin{aligned} \vec{\rho} &= (\vec{r}_1 - \vec{r}_2)/\sqrt{2} \\ \vec{\lambda} &= (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6} \end{aligned} \quad (3)$$

So the three nucleons in the ground are in an orbital symmetric S-state. Pauli Principle require that in A=3, the two like nucleons would be in spin zero state. Then the magnetic dipole moment of ^3H would be entirely due to the odd proton and thus would arise from the odd neutron in ^3He .

Therefore this picture predicts

$$\begin{aligned} \mu(^3\text{H}) &= \mu(p) = 2.79 \text{ nm} \\ \mu(^3\text{He}) &= \mu(n) = -1.91 \text{ nm} \end{aligned} \quad (4)$$

This is in good agreement with the experimental value of 2.97 nm and -2.12 nm, respectively. Thus clearly a symmetric space wave function in S-state does reasonably well for the magnetic moment of A=3 nuclei.

What will such a wave function predict for the charge distribution for ^3He for example? Using the symmetric space wave function (2) the proton nuclear charge distribution are given in Fig. 1.

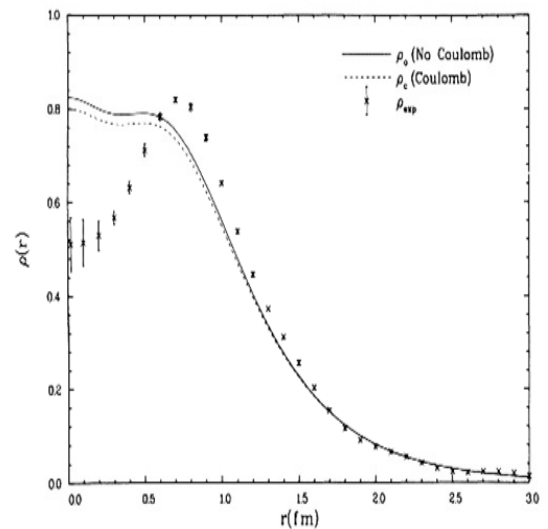


Figure 1. Point nucleus ^3He theoretical charge density and the corresponding experimental density indicated as data points[28]

This is plotted against experimental results[28]. Such a wave function does reasonably well for $r \geq 1$ but fails miserably for small r . Experimentally; there is a “hole” in the centre of ^3He charge distribution. So, the symmetric space part of the wave function which does so well for the magnetic dipole moment of A=3 nuclei is failing badly at small distances for the charge distribution. How can we accommodate this?

One technique is to split the wave function in two parts. For some r_0 for $r > r_0$ the wave function is assumed to look like that of (1) and (2) while for $r < r_0$ it is assumed that quark degree of freedom may play a role [21, 29]. At small relative distances the nucleons are likely to overlap strongly and as such probability of existence of 6 quark [29] clusters or 9 quark [21] clusters towards the centre of ${}^3\text{He}$ would be non-negligible and thus the hole may be explained as a consequence of these quark degrees of freedom in A=3 nuclei (note ${}^3\text{H}$ also has a similar distribution with a hole at the centre [21]).

In this way the short range part of the wave function is arising due to the explicit quark degrees of freedom. In this paper we ask the question- Is it possible to understand this hole in A=3 nuclei within only the nucleonic degrees of freedom without invoking the quark degrees of freedom. Below we show that indeed this is possible!

We accept that there are two parts of the A=3 wave function. For some r_0 , for $r > r_0$ let the wave function be described in the canonical way through equations (1) and (2). This is good as it explains the A=3 magnetic dipole moment satisfactorily.

For nucleons in A=3 the degree of freedom are spin-isospin and orbital part. In equation (1) the spin-isospin part was anti-symmetric and thus the space part was symmetric (2).

Note that it is possible to have an entirely symmetric spin-isospin wave function for the A=3 system

$$\Psi_S = \frac{1}{\sqrt{2}} (\chi_\lambda \phi_\lambda + \chi_\rho \phi_\rho) \quad (5)$$

If this holds then what we require is anti-symmetric space wave function. So the question is-is it possible to have an anti-symmetric part of space wave function for the ground state of A=3 nuclei with three nucleons?

We take the cue from quark model of hadrons. As is well known [30] nucleons are made up of three constituent quarks. It was found that in space-isospin-orbital space a symmetric wave function worked well. Normally one would have a symmetric L=0 state for the ground state and in which case it was a puzzle as to how the three spin-1/2 quarks requires symmetric spin-isospin-orbital wave function. One may fix for this by involving an additional color degree of freedom to carry the anti-symmetry. With hindsight, this was the correct way to solve the problem. But in the early days of quark model, without color degree of freedom, one possibility was to seek for an L=0 orbital anti-symmetric wave function. And indeed it is possible to construct L=0 wave function for baryons which is totally anti-symmetric [30]. Though in keeping with Pauli principle this was possible, a difficulty was pointed out by Mitra and Majumdar [31]. What they showed was that with such an anti-symmetric orbital wave function for 3 quarks in L=0 state there would be zeros in the form factors of proton, and such a situation does not arise experimentally. So this ruled out the possibility of absorbing anti-symmetry of 3 quarks in the orbital space in quark model [30-32]. And as we know in quark model- this problem

was solved through the correct invocation of a new three color degrees of freedom.

Here for us the L=0 anti-symmetric wave function of three fermions is significant. If for A=3 the spin-isospin wave function is symmetric as in Eq. (5) then the orbital part should be anti-symmetric here. What failed for 3 quarks in baryons works wonderfully for 3 nucleons in A=3 nuclei. We see this below.

Let the completely anti-symmetric wave function in orbital space be given as $f(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ [30] [p.31] (the exact form of f is not important for our discussion here). Then the charge density is

$$\rho(\vec{r}) = \int d^3\vec{r}_2 |f(\vec{r}_1, \vec{r}_2, -(\vec{r}_1 + \vec{r}_2))|^2 \quad (6)$$

We choose the coordinate of the three nucleons in the centre of mass system in which

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0 \quad (7)$$

As f is anti-symmetric the integrand in Eq. (6) vanishes and thus the charge density at the origin is zero. This becomes the centre of the A=3 nucleus. It is a unique mathematical property of three fermionic orbital anti-symmetric wave functions that defines generically an "origin" with charge zero at this point. Thus an inside is created as well as a unique origin as a centre of the A=3 nucleus.

Thus the existence of anti-symmetric orbital wave function for A=3 demands that the charge density vanishes at $r = 0$ for these nuclei. So the wave function predicts a hole at the centre of A=3 nuclei exactly as found for ${}^3\text{He}$ (Fig. 1). One may expect that the finite size of nucleons would smear out the hole somewhat but should still leave a prominent depression in the density near the centre which would be seen as a hole (see Fig. 1).

Thus we suggest that for $r < r_0$ (where $r_0 \sim 1\text{fm}$) the wave function is

$$\Psi_{\text{Symmetric}}(\text{spin-isospin}) \times \Psi_{\text{Anti-symmetric}}(\text{orbital})$$

and thus the hole is naturally predicted by such a wave function.

Note the $\langle \Psi_A | \Psi_S \rangle = 0$ (from Esq. (1) and (5)) and thus the two parts of the wave function, one for $r > r_0$ (for some suitable values say $r_0 \sim 1\text{fm}$ for ${}^3\text{He}$) and the other $r < r_0$ with Eq. (5) would work well for both the magnetic moment etc. and for reproducing the charge/matter densities of A=3 nuclei, and thus is entirely within the nucleonic degree of freedom only.

Hence both anti-symmetric and symmetric orbital parts play a complementary role in predicting the magnetic moment as well as the hole in A=3 nuclei. It is like two different phases (with orbital part anti-symmetric in one and completely different, that is symmetric, in the other one) coexisting for the same nucleus A=3.

4. Conclusions

Summarising, note that the light nuclei A=1 and A=2 are all pure 'surface' nuclei. A genuine nucleus should have a

distinct 'interior' plus a clear 'surface' outside. $A=3$ is hence, the first genuine nucleus. It has managed to build an "interior" while retaining the "surface". And the new structure suggested here plays a fundamental role in creating a first clear cut "interior" along with a centre (which appears as a hole) while still retaining the "surface" outside. Thus what is needed is a hybrid model with one part describing the outside and the surface while another orthogonal term giving the interior. Now producing the interior is highly non-trivial job. It is so as it has to produce a point inside which should become a centre. This has to be ensured mathematically and this is what our function does. Note that all this is done entirely within nucleonic degrees of freedom itself.

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