

# On the Photonuclear Rates Calculation for Nuclear Transmutation of Beta Decay Nuclides by Application of the Ginzburg-landau Theory

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**Abstract** The main aim of this paper is to find which are the conditions the photonuclear reactions in order to accelerate the beta decay of radionuclides. Therefore, it was developed a new model where the photonuclear reaction at above the giant resonance cross section (GR) is viewed as an incident photon creating superconducting hot spot (hot belt) across nucleons from a neutron-proton[n-p] pair (a *quasi-deuteron*) of the valence bosons of a unstable nuclide, accordingly with IBAF model and, followed by a thermally induced vortex crossing, which turns superconducting hot belt into the normal state (vacuum) resulting in a vortex assisted photon beta decay with the fermion ( $e^+, e^-$ ) release (capture), and eventually the pair break-up by a reaction  $(\gamma, n)$  or  $(\gamma, p)$ .

Since, a such model requires data on energies of vortex and on currents from inside of the nucleon, an analogue of a superconductor model was developed for the nucleon, and tested on the data of natural beta decay viewed as dark count. The results consist in obtaining a precise value of the threshold energy of the photons when the beta decay of the radionuclides is accelerated. Such efficient installations could be, for example, the laser ELI-NP in construction in Romania for the higher energy photons source.

**Keywords** Beta Decay, Nuclear Transmutation, Photonuclear Reactions, High Energy Lasers, W,Z, Higgs Bosons, G-L Theory

## 1. Introduction

Photonuclear reactions are proven to be of potential interest for nuclear transmutation[1] in the handling of radioactive waste, which is a big problem in nuclear energy.

Photonuclear cross sections are quite large at the giant resonance (GR) region of  $E_\gamma = 15 \div 30 \text{ MeV}$ . Major  $(\gamma, 2n)$  and  $(\gamma, n)$  reactions on long-lived nuclei may transmute them to short-lived of beta decay type or stable nuclei.

Coherent photonuclear isotope transmutation (CPIT) produces exclusively radioactive isotopes (RIs) by coherent photonuclear  $(\gamma, n)$  and  $(2\gamma, n)$  reactions via E1 giant resonances[1]. Photons to be used are medium energy  $E(\gamma) = 12 - 25 [\text{MeV}]$  photons produced by laser photons backscattered off  $\text{GeV}$  electrons.

The photon intensity of around  $10^{8-12}/\text{s}$  is realistic by using intense lasers and intense electrons. In future high intensity photon sources with the intensity of the order of  $10^{14-15}/\text{s}$  will be possible. Therefore, are two conditions for a

photonuclear reaction the intensity, and the threshold energy. For example, intense electron accelerators provide bremsstrahlung photons for photonuclear isotope transmutation but most of photons are below the threshold energy of the photonuclear reaction.

Otherwise, in [2a] is argued that photon counts in a superconducting nanowire single-photon detector (SNSPD) are caused by the transition from a current-biased metastable superconducting ( $s$ ) state to the normal ( $n$ ) state. Such a transition is triggered by vortices crossing the thin film superconducting strip from one edge to another due to the Lorentz force. Hence, the general picture is that the vortex crossing may trigger the  $s \rightarrow n$  transition. A photon makes this process much more probable by creating a spot with suppressed order parameter and thus with lower energy barrier for vortex crossing. The model developed in [2a] is based entirely on the Ginsburg-Landau theory of superconductivity.

In the present paper-section 3, based on the above models[1],[2a], a new model is introduced, respectively, a photonuclear reaction at above the (GR) viewed as an incident photon creating superconducting hot spot (hot belt) across a nucleon from a neutron-proton[n-p] pair (a quasi-deuteron as was baptized by Levinger[36]) of the valence bosons of a unstable nuclide, accordingly with IBA

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model[25],[31] and, followed by a thermally induced vortex crossing, which turns superconducting hot belt into the normal state (vacuum) resulting in a vortex assisted photon beta decay with the fermion ( $e^+, e^-$ ) release (capture)[25], and eventually the pair break-up by a reaction  $(\gamma, n) \text{ or } (\gamma, p)$ .

The first mention in literature of the connection between the photonuclear reaction and superconducting it was when the characteristic feature of thermalized non-equilibrated matter is that, in spite of energy relaxation–equilibration, a phase memory of the way the many-body system was excited remains[37]. There, in order to demonstrate that, it is analysed the proton yield of the  $Bi(\gamma, p)$  photonuclear reaction produced by 24 MeV bremsstrahlung, when the properly scaled angle-integrated spectrum for the outgoing proton energy  $\varepsilon \leq 8 \text{ MeV}$  has an exponential shape with a slope of 0.55 MeV. This is characteristic for the decay of thermalized compound nucleus with a “temperature”  $T = 0.55 \text{ MeV}$  (or  $6.3109 \text{ K}$ ) of the residual nucleus. The average excitation energy of the compound nucleus was evaluated at  $E^* = 14 \text{ MeV}$ , i.e. slightly above the center of the dipole giant resonance peak at 13.5 MeV. The characteristic time for the formation of such thermalized many-body states is given by the inverse spreading width,  $\tau_{th} = \hbar/\Gamma_{spr}$ , where

$\Gamma_{spr}$  is the width of a dipole giant resonance, which is about 4.5 MeV for Bi. Notice that  $\hbar/\Gamma_{spr} = 1 \times 10^{-22} \text{ s}$  is the energy relaxation time and  $\hbar/\beta = 6 \times 10^{-14} \text{ s}$  is phase memory time or phase relaxation time, and phase relaxation width  $\beta \cong 0.01 \text{ eV}$ . Therefore, the phase relaxation is at least 8 orders of magnitude slower than energy relaxation. This argues that thermalized non-equilibrated matter resembles a high temperature superconducting state in quantum many-body systems.

Since, this model requires data on energies of vortex and on currents from inside of the nucleon, an analogue of a superconductor model was developed for the nucleon[37], and described in section 2.

In section 4 is applied this model to the mechanism of natural beta decay viewed as dark count.

The results consist in obtaining a precise value of the threshold energy of the photons when the beta decay of the radionuclides is accelerated.

If these kind of predictions continue to be verified by appropriate experiments, one may have a cheap solution to remove the radioactive waste of used-up nuclear fuel rods of fission reactors in a time period of a few years.

## 2. The Description of the Analogue Model of Nucleon to a Superconductor

Usually, the masses of  $W, Z$ , are calculated by taking into account an a-priori a Higgs field, and the default using the Higgs mechanism and Higgs boson.

Soon after the advent of QCD, 't Hooft and Mandelstam[3] proposed the dual superconductor scenario of confinement; the QCD vacuum is thought to behave analogously to an electrodynamic superconductor but with the roles of electric and magnetic fields being interchanged: a condensate of magnetic monopoles expels electric fields from the vacuum. If one now puts electric charge and anti-charge into this medium, the electric flux that forms between them will be squeezed into a thin, eventually string-like, Abrikosov-Nielsen-Olesen (ANO) vortex which results in linear confinement.

The dual superconductor mechanism[3] is an alternative that does not require the ad hoc introduction of a Higgs field but instead uses dynamically generated topological excitations to provide the screening supercurrents. For example, U(1) lattice gauge theory contains Dirac magnetic monopoles in addition to photons. The dual superconductor hypothesis postulates that these monopoles provide the circulating color magnetic currents that constrain the color electric flux lines into narrow flux tubes. 't Hooft has shown that objects similar to the Dirac monopoles in U(1) gauge theory can also be found in non-Abelian SU(N) models.

The results are consistent with a dual version of the Ginzburg-Landau model of superconductivity. Important in understanding field (magnetic) dependence was Abrikosov's field theoretical approaches based on Ginzburg-Landau theory[4] for type I superconductors ( $\kappa < 1/\sqrt{2}$ ,  $\kappa = \lambda/\xi$ ,  $\lambda$  is the penetration depth,  $\xi$  the coherence length) and type ones ( $\kappa > 1/\sqrt{2}$ ) II superconductors, which allows magnetic flux  $\Phi$  to penetrate the superconductor in a regular array quantized in units of elementary flux quantum  $\Phi = \pi\hbar c/e$ . Important was the quantization in a ring, flux  $\Phi = \left(n + \frac{1}{2}\right) \frac{\hbar c}{e}$ , ( $n = 0, \pm 1, \dots$ )

Accordingly, we revisited G-L model[4-13] in order to calculate the values of the Lorentz force, the current, and the energies of the Abrikosov vortex lines inside of the nucleon, in natural units, firstly, in view to search for its relevance; for the time being, it was found that this would correspond to energies for subatomic particles, such as that of  $W, Z$ , bosons, and of pion  $\pi^+$ , and secondarily to be used in the model of vortex-assisted single photon count. In this model to a superconductor analogue, we do not use any a-priori field (Higgs) and hence a Higgs boson, we use only a comparison with the electrical field generated by the pair (dipole)  $q\bar{q}$ .

It is known, that the normal cores that exist in type-II superconductors in the mixed state are not sharply delineated. The value of number density of superelectrons  $n_s$  is zero at the centers of the cores and rises over a characteristic distance  $\xi$ , the coherence length. The magnetic field associated with each normal core is spread over a region with a diameter of  $2\lambda$ , and each normal core is surrounded by a vortex of circulating current.

The QCD vacuum can be viewed as a dual superconductor characterized by a monopole condensate[3,9,11], when

embedding a static  $q\bar{q}$  pair into the vacuum. The core of the flux tube is just a normal conducting vortex which is stabilized by solenoidal magnetic supercurrents,  $j_s$ , in the surrounding vacuum.

In order to calculate distinctly the energy states (masses) in natural units, firstly we re-derive the field equations of magnetic monopoles current, of the electric flux and energy states.

Therefore, here is adopted a basic dual ( $B \Leftrightarrow E$ ) form of Ginzburg-Landau (G-L) theory[4-8], which generalizes the London theory to allow the magnitude of the condensate density to vary in space. As before, the superconducting order parameter is a complex function  $\psi(\vec{x})$ , where  $|\psi(\vec{x})|^2$  is the condensate density  $n_s$ . Also is defined the wave function  $\psi(\vec{x}) = \sqrt{n_s} \exp(i\phi(\vec{x}))$ , where  $n_s$  is the London (bulk) condensate density, and  $\phi$  are real functions describing the spatial variation of the condensate.

The characteristic scale over which the condensate density varies is  $\xi$ , the G-L coherence length or the vortex core dimension. The  $x$  denote the radial distance of points from the  $z$ -axis, the superconductor occupying the half space  $x > 0$ . Outside of the superconductor in the half space  $x < 0$ , one has  $B = E = H = H_0$ , where, “the external” vector  $H_0$  is parallel to the surface. The  $\Psi$  theory of superconductivity[3,4] is an application of the Landau theory of phase transitions to superconductivity. In this case, some scalar complex  $\psi$  function fulfils the role of the order parameter.

The final expression for the free energy then takes the form

$$f = f_n + \int \left\{ \frac{\hbar^2}{2m} \left| \left( \nabla - i \frac{2e}{\hbar c} A \right) \psi \right|^2 + a |\psi|^2 + \frac{b}{2} |\psi|^4 + \frac{B^2}{8\pi} \right\} dV \quad (1)$$

One can obtain the basic equations of Ginzburg-Landau theory by varying this functional with respect to  $A$  and  $\psi^*$ . Carrying first variation with respect to  $A$ , we find after a simple calculation:

$$\delta f = \int \left[ c \frac{ie\hbar}{2m} (\psi^* \nabla \phi - \psi \nabla \psi^*) + \frac{2e^2}{m} |\psi|^2 A + \frac{curl B}{4\pi} \right] \delta A dV + \int div(\delta A \times B) \frac{dV}{4\pi} = 0 \quad (2)$$

The second integral can be transformed into an integral over remote surface and disappears. To minimize the free energy, the expression in the brackets must be equal to zero. This results in the Maxwell equation

$$curl B = \frac{4\pi}{c} j_s = \frac{1}{c^2 \epsilon_0} j_s \text{ (in SI)} \quad (3)$$

or

$$\nabla \times \nabla \times A = \frac{4\pi}{c} j_s = \frac{1}{c^2 \epsilon_0} j_s \quad (4)$$

provided that the current density is given by

$$j_s = \frac{ie\hbar}{2m} (\psi^* \nabla \psi - \psi \nabla \psi^*) + \frac{2e^2}{m.c} |\psi|^2 A \quad (5)$$

The following equations (6) and (7) form the complete system of the Ginzburg-Landau(G-L) theory.

$$\nabla \times \nabla \times A = \frac{4\pi}{c} j_s = \frac{1}{c^2 \epsilon_0} j_s \quad (6)$$

According to the definition of  $n_s$  we use  $\psi = \sqrt{n_s} \exp(i\phi)$ .

The second integral is over the surface of the sample. The volume integral vanishes when

$$-\frac{\hbar^2}{4m} \left( \nabla - i \frac{2e}{\hbar c} A \right)^2 \psi + a\psi + b|\psi|^2 \psi = 0 \quad (7)$$

Where :  $\lambda = \left( \frac{\epsilon_0 m c^2}{n_s e^2} \right)^{1/2} = 1.17e-16[m]$ , and the quantized

flux:  $\Phi_0 = \frac{\pi \hbar c}{e}$ , and  $|\Psi|^2 = n_s$ ;  $n_s = 3_{\text{monopoles}}/V * 1.e-45 m^3$ ,  $V = 4/3 \pi r^3 = 2$ ,  $r = 0.48[fm]$   $\epsilon_0 = 8.8e-12[C^2.N^{-1}.m^2]$ .

Since, the magnetic charge of monopole being[17]

$g_d = 4\pi\epsilon_0 \frac{\hbar c}{2e} = \frac{4\pi\epsilon_0 \hbar c}{e^2} e = \frac{137}{2} e = 68.5e$ , and assuming that the classical electron radius be equal to “the classical monopole radius” from which one has the monopole mass  $m_M = g_d^2 m_e / e^2 = 4700 m_e$ , the value of  $\lambda$  remains unmodified.

The superconductors of second kind are those with  $\kappa > 1/\sqrt{2}$ , and  $\lambda > \xi$ .

We now consider the phase transition in superconductors of the second kind.

For this we can omit the non-linear  $(|\psi|^2 \psi)$  term in (7), we have

$$\frac{1}{4m} \left( -i\hbar - \frac{2e}{c} A \right)^2 \psi = |a| \psi \quad (8)$$

This equation coincides with the Schrodinger equation for a particle of mass  $2m$  and charge  $2e$  (in the case of dual, the factor 2 for the charge, which is specific to the “pairs”, it is actually 1) in a magnetic field  $H_0$  (in our case the chromo-electrical flux  $E(0)$ ). The quantity  $|a|$  plays the role of energy ( $E\psi$ ) of that equation. The minimum energy for a such particle in a uniform electro-magnetic field is

$$\epsilon_{(0)} = \frac{1}{2} \hbar \omega_B = \frac{1}{2} \hbar \frac{2eH_0}{8mc} \Rightarrow [J], \quad H_0 \text{ -an “external”}$$

electro-magnetic field of a dipole created by the pair  $u\bar{u}$  (the chromoelectrical colors field)

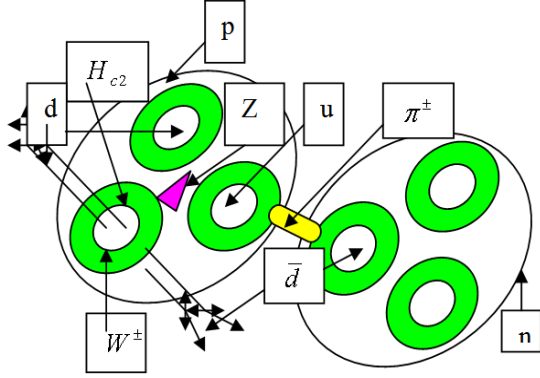
$$H_0 = E_0 = \frac{de}{4\pi\epsilon_0 r^3} \cong 8.33e24 \left[ \frac{N}{C} \right] \quad (9)$$

where  $r \cong 0.05[fm]$  -is the electrical flux tube radius,  $d = 0.48[fm]$  -the distance between the two quarks charges,

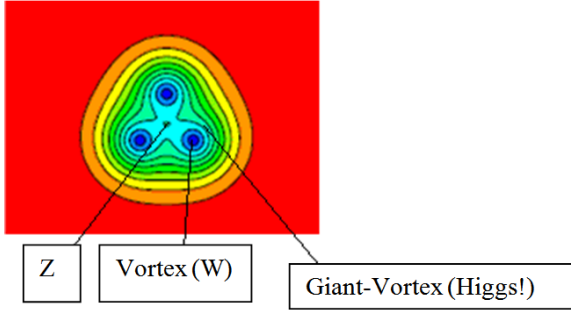
usually  $H[A/m]$ , but here is used as  $B = \mu_0 H \left[ \frac{J}{Am^2} \right]$

Hence, equation (8) has a solution only if  $|a| > 2\hbar * eH_0/8mc$ , when following power-law conformal map is applied for complex number of the r.h.s of (8), or equivalently if the electro-magnetic field is less than an

upper critical field, see fig.1a



**Figure 1a.** Abrikosov's triangular lattice for a nucleon (proposal)



**Figure 1b.** The Giant-Vortex (after Ref.[24]) type arrangement for the nucleon (only illustration)

$$H_0 \leq \frac{4mc|a|}{\hbar e} \leq H_{c2} = \frac{\Phi_0}{2\pi\xi^2} = \frac{\pi\hbar c}{2\pi e\xi^2} = 8.33e24 \left[ \frac{N}{C} \right] \quad (10)$$

and in terms of

$$B_{c2} = H_{c2} = \frac{\pi\hbar c}{2\pi e\xi^2 \cdot c} = 2.7e16 \left[ \frac{J}{Am^2} \right]$$

The particle energy is

$$\varepsilon_{(0)} = \frac{1}{2} \frac{\hbar^2}{8m\xi^2} = 6.28e-08[J] \Rightarrow 392[GeV]$$

with  $\lambda \gg \xi$ ;  $\xi = \frac{\lambda}{\kappa} = \frac{0.117}{1.05} = 0.1114[fm]$ , or  $\kappa \gg 1/\sqrt{2} \gg 1 = 1.05$  (of type II-superconductor).

One of the characteristic lengths for the description of superconductors is called the coherence length. For example, a transition from the superconducting state to a normal state will have a transition layer of finite thickness which is related to the coherence length.

More exactly, this quantity is called the correlation or healing length[4], and is defined as

$$\xi(T) \cong \xi_0 \left( \frac{T_c}{T_c - T} \right)^{1/2} \gg \xi_0$$

where

$$\xi_0 = a \hbar v_F / E_g \quad (11)$$

is for  $T \rightarrow 0$ ,  $a = 2/\pi$  from[19],  $E_g$  -gap energy,  $k_B$

Boltzmann constant; at confinement  $T_c = 175[MeV] \rightarrow 2e12[K]$ , and the Fermi velocity of electrons (monopoles) is

$$v_F = \frac{\sqrt{2 \cdot 4700 m_e E_F}}{4700 m_e} \quad (12)$$

where as the Fermi energy we have for monopoles condensate viewed as bosons condensate

$$E_g = E_{bosonCond} = 3.31 \frac{\hbar^2 n_s^{2/3}}{4700 m_e} \cong T_c k_B = 0.7 E_F$$

where

$$E_F = \frac{\hbar^2}{2 \cdot 4700 m_e} (3\pi^2 n_s)^{2/3} \quad (13)$$

numerically, we have:

$$E_F = \frac{1.06e-34^2}{2 \cdot 4700 \cdot 9.e-31} (3\pi^2 \cdot 3/(V \cdot 1.e-45))^{2/3} = 9.32e-12[J] \rightarrow 55[MeV]$$

where  $V = 1.5[fm]^3$ , and the velocity of monopole is

$$v_F = 0.55e8 < c = 2.997e8[m/s]$$

and

$$\xi_0 = 0.6 \cdot 0.7 \frac{1.06e-34 \cdot 0.55e8}{1.38e-23 \cdot 2.e12} = 1.02e-16[m] \text{ at } T \rightarrow 0 \quad (14)$$

In the following we will consider the structure of the mixed state. The main problem is to understand how the electric field penetrate in the bulk of the superconductor. Let us again consider a superconducting cylinder in the electric field. It is natural to expect that the normal regions, with their accompanying electric field, are cylindrical tubes parallel to the field. The electrical flux inside such tube must be integral multiple  $n$  of the flux quantum

$$\Phi_0 = \pi\hbar c/e \rightarrow \text{usually } \frac{\pi\hbar}{e} = 2.07e-15[Tm^2] \quad (15)$$

The electrical field is concentrated inside the tube. At large distances from the tube it is shielded by annular superconducting flowing around the tube. This current is analog of the superfluid velocity field surrounding the vortex lines in the superfluid liquid. We can then picture the mixed state as an array of quantized vortex lines. Such vortex lines were predicted by A.A. Abrikosov in 1957. Their existence is crucial for explaining the proprieties of type II superconductors (dual in our case).

The presence of a vortex line in the center of the tube increases the free energy of the superconducting media. The G-L equations are solved analytically only for  $\lambda \gg \xi$  (near  $T_c$  this means  $\kappa \gg 1$ ). Thus, when the electrical flux is applied parallel to the superconducting cylinder, the first flux penetrating should be located along the axis of the cylinder.

In the range

$$\lambda \gg x \gg \xi \quad (16)$$

The induction  $B$  is

$$B(x) = \frac{2\Phi_0}{2\pi\lambda^2 c} \log\left(\frac{\lambda}{x}\right) \quad (17)$$

This equation is valid in the interval (16) with logarithmic accuracy.

This equation is valid only at all distances

$$x \gg \xi \quad (18)$$

The solution at  $x \rightarrow \infty$  is  $B(r) = \text{const} \cdot K_0(x/\lambda)$ , where  $K_0$  is the Hankel function of imaginary argument. The coefficient must be defined by matching with the solution of (17). Using the asymptotic formula  $K_0(x) \approx \log(2/\gamma x)$  for  $x \ll 1$ ,

where  $\gamma = e^C \approx 1.78$  ( $C$  is Euler's constant), we finally have

$$B(x) = \frac{2\Phi_0}{2\pi\lambda^2 c} K_0(x/\lambda) \quad (19)$$

Using equation (19) we can rewrite (17) as:

$$B(x) = \frac{2\Phi_0}{2\pi\lambda^2 c} \log \frac{2\lambda}{\gamma x}, \quad x \ll \lambda \quad (19.1)$$

In opposite limit of large distances one can use the asymptotic expression  $K_0(x) \approx (\pi/2x)^{1/2} e^{-x}$  for  $x \gg 1$ . Thus, at large distances from the axis of the vortex line the field decreases according to

$$B(x) = \frac{2\Phi_0}{c(8\pi x \lambda^3)^{1/2}} e^{-x/\lambda}, \quad x \gg \lambda \quad (20)$$

Accordingly the superconductive current density decreases (in SI):

$$j_\phi = -\frac{c}{4\pi} \frac{dB}{dx} (4\pi c \varepsilon_0) = \frac{2c^2 \varepsilon_0 \Phi_0}{8(2\pi^3 x \lambda^5)^{1/2} c} e^{-x/\lambda} \quad (21)$$

We can now calculate the energy  $\mathcal{E}$  of the vortex line, we obtain for the energy per unit length of vortex line.

$$\mathcal{E} = c^2 \varepsilon_0 \left( \frac{2\Phi_0}{4\pi\lambda c} \right)^2 \log \left( \frac{\lambda}{\xi} \right) \quad (22)$$

Here, the factor  $4\pi c \varepsilon_0$  is used to convert from (cgs)  $\rightarrow$  (SI).

Because the magnetic induction of the monopoles current which is powered by electric field given by a pair of quarks ( $H_0$ ), it has the raw flow consequences squeezing this cromoelectrical flux into a vortex line, followed by forcing an organization into a triangular Abrikosov lattice, see figure 1.

The core of every vortex can be considered to contain a vortex line, and every particle in the vortex can be considered to be circulating around the vortex line. Vortex lines can start and end at the boundary of the fluid or form closed loops.

The presence of vortex line which increases the free energy of the superconducting media with  $\varepsilon L$ , it is thermodynamically favorable if the contribution is negative;

i.e. if  $\varepsilon L - \Phi_0/c \cdot H_0 L/4\pi\mu < 0$ , and  $B_0 = \frac{H_0}{\mu_0}$ ,  $\mu_0 = 1/c^2 \varepsilon_0$ , or

$$H_0 > H_{cl} = \frac{4\pi\varepsilon\mu_0 c}{\Phi_0} \quad (23)$$

Substituting (22) in (23), we find the lower critical field

$$B_0 = H_{cl} = \frac{2\Phi_0}{2\pi\lambda^2} \log \left( \frac{\lambda}{\xi} \right) = \quad (24)$$

$$\frac{\pi\hbar c}{\pi\lambda^2 c} \log(\kappa) = 1.15 \left[ \frac{J}{Am^2} \right]$$

$\xi = 0.1114$ , and when near the axis, for  $x \approx 0.116 \approx \xi$  from (20) when the induction is

$$B(\xi) \approx 2.15 \approx 2H_{cl} \quad (25)$$

Let us the results obtained to the calculation of the energy of interaction of vortex lines. Let us consider two vortex lines separated by a distance  $d$  from each other.

$$\varepsilon_{\text{int}}(d) = \frac{2\Phi_0}{4\pi} B_d(x) = \frac{4\Phi_0^2}{8\pi^2 \lambda^2} K_0 \left( \frac{d}{\lambda} \right) \quad (26)$$

One can use the asymptotic expression for  $\varepsilon_{\text{int}}$  (see eq. (20))

$$\varepsilon_{\text{int}} = \frac{4\Phi_0^2}{2^{7/2} \pi^{3/2} \lambda^2} \left( \frac{\lambda}{d} \right)^{1/2} e^{-x/\lambda}, \quad x \gg \lambda \quad (27)$$

When the distance  $d \approx \lambda \gg \xi$  the cores of vortex lines overlap[4].

Let us consider a closed contour near the surface of the cylinder. The change of wave function on passing round the contour is  $2\pi\nu S$ , where  $S$  is the cross-section area of the cylinder and  $\nu$  - the number of vortex lines. The electric flux is

$$\Phi = 2\Phi_0 \nu S - \frac{2m}{e\hbar} \oint \frac{j_s}{n_s} \cdot dl \quad (28)$$

Let us recall that a similarly relationship[4],[5], it was introduced for the first time by London, called fluxoid equation.

Each fluxoid, or vortex, is associated with a single quantum of flux represented as  $\Phi_0$ , and is surrounded by a circulating supercurrent  $j_s$ , of spatial extent,  $\lambda$ . As the applied field increases, the fluxoids begin to interact and as the consequence ensembles themselves into a lattice. A simple geometrical argument for the spacing,  $d$  of a triangular lattice then gives the flux quantization condition[13],

$$Bd^2 = \frac{2}{\sqrt{3}} \Phi_0 \quad (29)$$

where  $B$ , is the induction.

The solution of Ginzburg-Landau phenomenological free energy (13) is useful for understanding the Abrikosov flux lattice. The coordinate-dependent order parameter  $\phi$  describes the flux vortices of periodicity of a triangular lattice. Fluctuations from  $\phi$  change the state to  $\psi$ , the minimization of free energy with respect to  $\psi$ , gives the ground state  $\phi(r/0)$ .

The free energy is given by,

$$f = f_n + \int \left\{ \frac{\hbar^2}{2m} \left| \nabla - i \frac{2e}{\hbar c} A \right| \psi \right|^2 + a |\psi|^2 + \frac{b}{2} |\psi|^4 + \frac{(B - H_0)^2}{8\pi} \right\} dV \quad (30)$$

the average magnetic induction is  $\bar{B}(-y, 0, 0)$ . The free energy has solutions of vortices of triangular form. The

coordinates of the three vertices of a triangular vortex are given by  $(0,0)$ ,  $(l,0)$ , and  $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)l$ . The fluctuation from ground state corresponding to that of triangular lattice is that for small fluctuations. The deviation of the free energy from the mean-field value  $F - F_{FM}$  with respect to the thermal energy,  $k_B T$ , can be used to obtain the physical properties of the fluctuations which are useful for understanding the melted vortex lattices. The deviation from the triangular Abrikosov lattice is defined as

$$D = \langle |\psi - a_1 \phi(r/0)|^2 \rangle / a_1^2 \quad (31)$$

which uses the spatial and thermal averages calculated with the probability  $\exp(-F/k_B T)$ . Classically,

$$D = \frac{k_B T}{F - F_{MF}} \quad (32)$$

measures the fluctuations from the triangular vortex state. The fluctuation in the distance between vortices becomes:

-case 1,

$$(1 - T_{FM}/T_c) \cong 10^{-5} c^{-4/3} B^{2/3} \quad (33)$$

-case 2,

$$1 - T_{FM}/T_c \cong c^{-1} B^{5/4} \quad (34)$$

-case 3, a vortex transition below the transition temperature see[13]

where,  $T_{FM}$  -the flux-lattice melting temperature, and  $c = 0.1$  from Lindemann criterion of lattice melting when  $d^2 = c^2 l^2$ , the flux quantization condition  $l^2 = \Phi_0/B$ ,

$B = 2\pi m/\kappa$ , where  $\kappa = \frac{e\sqrt{2}}{\hbar c} \lambda^2 H_c(T)$ , and  $H_c = \kappa/\ln \kappa H_{c1}$ .

For numerical values  $T_c = 175[MeV]$ , in case of symmetry breaking, in case 1, results  $T_{FM} \cong T_c$ , and in case 2, results  $T_{FM} \cong 100[keV]$  by using (29) in place of  $\Phi_0/B \cong d^2$  with  $d = 0.3982[fm]$  (a very precisely value), and  $\kappa \cong 1$ , which is the temperature of fusion (melting!) of two protons.

This triangular lattice corresponds to the arrangement of the quarks pairs  $u\bar{u}, u\bar{u}, d\bar{d}$  in the frame of a nucleon, see fig.1a; fig.1b.

A direct numerical analysis allows to obtain the following values for the current, force and energy. Thus, from (21) the current is given by:

$$j_\phi \cong 1.15e7[A/fm^2] \quad (35)$$

where  $x \cong \lambda$ .

For  $x = 2 \cdot \lambda$ , the current density decreases at  $j_F \cong 3.e5[A/fm^2]$

Note that velocity  $v_F$ , moreover, if one considers the monopole current given by equation as  $j_\phi = n_s v_F g_D$

where the magnetic charge is:

$$g_d = 4\pi\epsilon_0 \frac{\hbar c}{2e} = \frac{4\pi\epsilon_0 \hbar c}{e^2} e = \frac{137}{2} e = 68.5e \quad (36)$$

If we use the range  $x \cong 0.1 \leq \lambda$ , then the current is obtained by derivation of (19.1):

$$j_\phi = \frac{1}{4} \frac{1}{x} \frac{1}{\lambda^2} \frac{4\pi\hbar c}{e} \frac{c^2 \epsilon_0}{c} = \frac{1}{4} \frac{1}{x} \frac{1}{\lambda^2} g_D c = \frac{n_s}{V} v_{Fi} g_D \rightarrow \quad (37)$$

$$\frac{v_{Fi}}{c} = \frac{1}{9.89} = 0.302e8[m/s]$$

The Lorentz' force to squeeze the flux tube, is:

$$F_L = qv_{Fi} B \cong 1.6e-19 * 0.1 * 2.992e8 * 4.7e15 = 2.25.e4[N] \quad (38)$$

when  $B$  is given by (19.1) and  $x \cong \lambda$ , for the upper limit:

$$B(x) \cong 2.35e15[J/Am^2] \quad (39)$$

With  $B$  from (20) and for  $x \gg \lambda$ , we have

$$B(x) = 3.8e-16 \left[ \frac{J}{Am^2} \right] \quad (40)$$

where  $x = 72\lambda = 8.4[fm]$

then, the force becomes  $F_L \cong 1.8e-26[N]$ , or in terms of energy

$$\mathcal{E}_{barrier} = F_L * x = 945[MeV] \quad (41)$$

or the nucleon overall.

In case of  $x \cong \xi \rightarrow (0)$ , in (17)

$$B(0) = 1.03e15 \left[ \frac{J}{Am^2} \right] \quad (42)$$

which respect (25).

The magnetic energy results from (22), and for  $(\lambda \gg x \gg \xi)$  from (16):

$$\mathcal{E} = 1.09e-10[J/fm] \Rightarrow 0.66[GeV/fm] \quad (43)$$

the force on the flux tube (string tension).

Now, from (27) and  $d \approx (4 \div 6)\lambda \gg \xi$ , we have

$$\mathcal{E}_{int} = 2.3e-11[J/fm] \Rightarrow 144[MeV] \quad (44)$$

What would be the value of the mass of the pion  $\pi^+$ , composed of a pair of quarks  $u\bar{d}$  interacting at a distance  $d \approx 2/3 \cong 5.65 * \lambda \cong 0.66[fm]$  of the radius of the nucleus.

Now, others important values of energy:

$$\mathcal{E}_0(0) = \mathcal{E}_{int}(d = x - \lambda; x = 0.14) * 0.117[fm] \cong 1.e-09[J] \quad (45)$$

and from (38) with

$$x \cong \xi = 0.107[fm]; \quad (46)$$

$$\mathcal{E}_{0h} = Vc^2 \epsilon_0 (2H_{c1})^2 / 8\pi = 5.e-11[J] \quad (47)$$

Now, the vortex energy is:

$$\mathcal{E}_{vortex} = Vc^2 \epsilon_0 H_{c2}^2 / 8\pi = 1.16e-08[J] \quad (47.1)$$

where  $V$  -is the volume, see fig.1a, fig.1b, accordingly, the corresponding equivalently masses are  $M = \mathcal{E}_{vortex}/c^2 \Rightarrow 73[GeV]$ , which seems to be equal to the mass of  $W$  boson.

The energy of the neutral boson  $Z$  is assimilated with the vortex-vortex three pairs interaction energy[24],  $\mathcal{E}_Z = 3 * \mathcal{E}_{int-pair} = 91[GeV]$ , when from (27)

$$\begin{aligned} \mathcal{E}_{int-pair} &= \mathcal{E}_{int}(d = 0.117 - 0.116; \\ x &= 1.4\lambda) = 4.85e-09[J] \rightarrow 30.33[GeV] \end{aligned} \quad (48)$$

is the energy of each of three pairs of vortex outermost ( $d \cong 0$ ) vortices lines which interacting (repel) at the center of the triangle situated at  $x = 1.4\lambda$ , thus, being generated a neutral current in the zone of  $Z$  during the triangular arrangement of the lattice, see fig.1a, or fig.1b.

Now, is possible that the vortices start to coalesce into a giant vortex (GV)[26], see, fig. 1b. ,

Thus, from (27), results an another energy state-maximum possible ( $d \neq 0$ ), probable that of Higgs boson (H):

$$\begin{aligned}\mathcal{E}_H &= 3 * \mathcal{E}_{\text{int}}(d = 0.117 - 0.116; x = \lambda) \\ &= 2.17e - 08[J] \rightarrow 135[GeV]\end{aligned}$$

Notice that this is not in fact a particle, since contain others subparticles ( $W, Z, u, d, \text{monopoles}$ ), so during high energy protons collision (CERN) can not be obtained as itself.

Here, a factor of 2 was introduced to correct on  $2e$  for “pairs” in the G-L model.

### 3. The Photonuclear Reaction Mechanism in G-L theory

Like in[2a], we use only the processes when a single incident photon with insufficient energy to create a normal-state belt but initiating a subsequent single-vortex crossing, which provides the rest of the energy needed to create the normal-state belt (vortex-assisted single photon count). We derive the current dependence of the rate of vortex assisted photon counts.

Hence, our general picture is that the vortex crossing may trigger the  $s \rightarrow n$  transition. A photon makes this process much more probable by creating a spot with suppressed order parameter and thus with lower energy barrier for vortex crossing.

As in[2a], we derive the energy barrier for vortex crossing and crossing rate from the standard Ginzburg-Landau (GL) functional with respect to the superconducting order parameter  $\psi(r)$  (normalized to its zero-field value in the absence of current) and the vector potential  $A$  in the presence of monopoles current induced magnetic field  $H$ . The Gibbs energy is given by (30).

Like in[2], we assume that in sufficiently narrow strips,  $w \cong r_{\text{nucleon}}$ , (a) the cloud covers the entire width of strip when it reaches its maximum size, (b) the quasiparticle density in the cloud (hot belt) is close to uniform, and (c) quasiparticles suppress the superconducting order parameter inside the hot belt, but their density is not sufficient to convert the hot belt to the normal state. Thus the superconducting condensation energy density  $H_{ch}^2/8\pi$  in the hot belt satisfies the inequalities  $0 < H_{ch}^2/8\pi < H_{c(0)}^2/8\pi$ .

It follows that the vortex crossing rate via hot belt (denoted by subscript  $h$ ),  $R_{vh}(I)$ , is enhanced in comparison with that for dark counts, because the parameter  $v_h = \varepsilon_{0h}/k_B T \approx H_{ch}^2/8\pi k_B T$  is reduced in comparison with  $v_0 = \varepsilon_0/T$ , and  $\varepsilon_0 = \frac{\Phi_0 I_0}{\pi c} = \varepsilon_{\text{int}} \cong 1.e - 09[J]$ , from (45)

There are three superconducting current-biased regions to be considered, which are separated by three characteristic currents  $I_h^* < I_{ch} < I_{c0}$ . For currents below  $I_h^*$  counts are absent, because vortex crossings do not result in the formation of normal-state belt. The “hot” current  $I_h$  is determined through the energy balance for destroying the

superconducting condensate in the hot belt similar to that for dark counts, however, with the thermodynamic field  $H_{ch} < H_c(0) = H_{c1}$ , from (24)

For lower currents in the interval  $I_{ch} > I > I_h^*$ , the rate of photon counts shows a power-law current dependence,

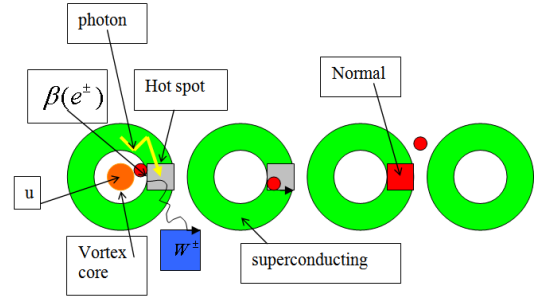
$$R_{pc}(I) \approx I^{v_h+1}$$

Here, we model the temperature dependent parameters  $I_{ch}(T) = I_{ch}(0)\varepsilon_T^3$ ,  $v_i(T) = v_i(0)\varepsilon_T^2/T$ , with  $\varepsilon_T = (1 - T/T_c)^{1/2}$ .

For currents  $I \geq I_c/3$ , the energy released by phase slip is sufficient to destroy superconductivity.

In the region  $I_h^* < I < I^* \cong I_{c0}/3$ , vortex crossings via the hot belt destroy the superconducting state inside the belt during the time  $\tau$  of the hot spot's lifetime. During this time, the belt is in the normal state, which causes the “vortex-assisted” photon count.

Hence, our general picture is that the vortex crossing may trigger the  $s \rightarrow n$  transition. A photon makes this process much more probable by creating a spot with suppressed order parameter and thus with lower energy barrier for vortex crossing. A sketch of the strip and of the belt across are shown in Fig. 2.



**Figure 2.** The photonuclear mechanism. From left to right, illustration of incident photon creating superconducting hot spot (hot belt) across nucleon, followed by a thermally induced vortex crossing together with an electron (bias current), which turns superconducting hot belt into the normal state resulting in a vortex assisted photon beta decay

If  $I_{c0} > I > I_{ch}$ , the rate of single photon counts,  $R_{pc}$ , is the same as the rate for hot spot formation  $R_h$  because the barrier for vortex crossing is now absent. For hot counts  $R_h$ , we have  $R_h = \eta_h R_p$ , where  $R_p$  is the photon rate and  $\eta_h$  is the quantum efficiency of hot spot formation caused by a single photon.

The vortex assisted photon count rate[2a] is:

$$R_{pc} = R_h [1 - \exp(-\mathfrak{R})] \quad (49)$$

where:

$$R_v(I, v_h) = \frac{4k_B T c^2 R}{\pi \Phi_0^2} \frac{\xi}{w} \left( \frac{v_h}{2\pi} \right)^{1/2} \left( \frac{I}{I_{ch}} \right)^{v_h+1} \quad (50)$$

$$\mathfrak{R} \approx \frac{\tau_{GL} R_v}{v_0 - v_h} \frac{1 - (I_{ch}/I)^{v_h - v_1 - 1}}{\ln(I_{ch}/I)} \quad (51)$$

$$I_{ch} = I_{c0} (v_h/v_0)^{3/2} \quad (52)$$

In the case of beta disintegration  $n \rightarrow p + e^- + \bar{\nu}_e$ , or



$$udd \rightarrow uud + e^- + \nu_e + W^- (80 \text{ GeV})$$

$$\text{or } d(-1/3e) + 2 \cdot 3/3e = u(2/3e) + e$$

or,

$$e + u(2/3e) - 2 \cdot 3/3e = d(-1/3e) \quad (53)$$

when the vortex equilibrium is disturbed by the transformation of one quark ( $d \rightarrow u$ ), then this is accompanied by a release of a  $W$  boson or the crossing vortex in this model.

We can suppose than along the hot belt induced by the incident photon, the charge  $2e + W$  creates a bias current ( $I > 2/3e[(1/(\nu_h/\nu_0)^{3/2}) \approx 3] \Rightarrow 2e$  who circulates due of the potential difference between the vortex and the rest of isotope.

At the first sight, the ohmic resistance of this ad-hoc electrical circuit created by the bias current is given as:

$$R = \frac{U_\beta}{\tau_{GL}} \frac{1}{V_{vortex}^2} \quad (54)$$

where the vortex potential is  $V_{vortex} = H_0 \xi$ ,  $H_0$  from (30.1), and the power is  $U_\beta/\tau \approx \varepsilon_{vortex(W^\pm)}/\tau$ , where  $U_\beta$  is given by (47.1), and  $\tau_{GL} = \pi\hbar/(8k_B T_c) = 1.5e-24[s]$  -the Ginzburg-Landau life time of  $W^\pm$  bosons.

Numerically, with  $T_c = 175 \text{ MeV}$ ,  $E_{prag} = k_B T$ , result  $\nu_0 = \varepsilon_W/k_B T_c = 1.e-09/(1.38e-23 \cdot 2.e12) = 36.2$ , where  $\varepsilon_W$  from (43);  $\nu_h = \varepsilon_{int>2\lambda}/E_{ph} = 5.e-11/E_{prag}$ , where  $\varepsilon_{int}$  from (46).

The value of  $E_{prag}$  is determined by trials in order to have  $R_{pc}/R_h \approx 1$ .

So, based on the above models[1,2], a photonuclear reaction is viewed as an incident photon creating superconducting hot spot (hot belt) across nucleon from the composition of a unstable nuclide (radioisotope), followed by a thermally induced vortex crossing, which turns superconducting hot belt into the normal state (vacuum) resulting in a vortex assisted photon beta decay.

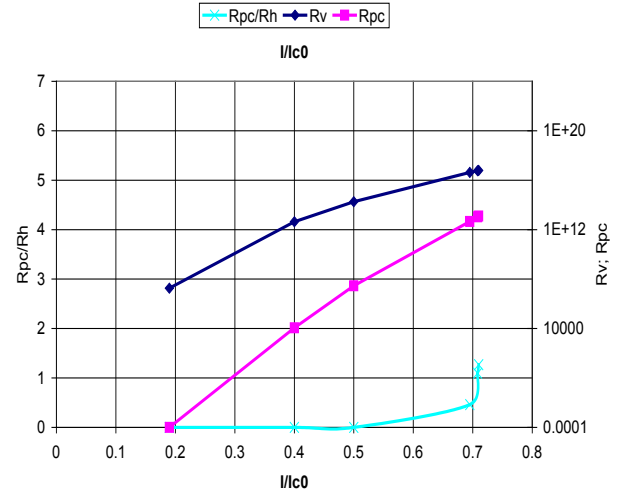
So, with the above data related on energies of vortex and on currents from inside of the nucleon, are calculated the beta decay rates.

Thus, let us discuss the  $^{99}\text{Mo} \rightarrow ^{99m}\text{Tc}$  isotopes, which are widely used as SPECT isotopes[1]. The  $^{99}\text{Mo}$  ( $\beta^-, T_{1/2} = 65h, Q = 1.356 \text{ MeV}$ ) isotopes are produced by  $^{100}\text{Mo}(\gamma, n)$  reactions and also  $^{100}\text{Mo}(\gamma, p)$  reactions followed by  $\beta^-$  decays. CPIT with  $N(\gamma) \approx 10^{13-15}/s$  and enriched  $^{100}\text{Mo}$  isotopes provides the RIs of  $^{99}\text{Mo}$  and  $^{99m}\text{Tc}$  with the production rate of  $10^{11-13}/s$ , and the RI density of  $(5-10)10^{8-10} \text{ GBq/mg}$  but after sufficiently long irradiation (240h).

Since the product  $^{99}\text{Mo}$  have the same decay rate as the reaction rate, we can not say if the beta decay itself is accelerated or no. In order to say that, it needs to have data on longer lived beta decay products. Thus, from our model

results the same beta decay rate of  $R_{pc} = 1.e13 \text{ counts/s}$ , with the incident photon rate of  $R_h = 1.e13 \text{ photons/s}$ ,  $R_{pc}/R_h \approx 1$ , or the same happens, i.e. much faster (instantly), if the photons energy will be at threshold value  $E_{prag}$ , see fig.3.

The same type of instant rates it happens for all beta-decay isotopes, like:  $^{85}\text{Kr}$ ,  $Q_\beta = 0.687 \text{ MeV}$  and  $T_{1/2} = 48h$ ,  $\gamma\text{-ray} = 514 \text{ KeV} (0.46\%) \rightarrow T_{1/2} = 10.756 \text{ yr}$ ;  $\gamma\text{-ray} = 151 \text{ KeV} (95\%) \rightarrow T_{1/2} = 4.48h$   $^{137}_{55}\text{Cs} \rightarrow ^{137}_{56}\text{Ba} + e^- + \bar{\nu}$ ,  $Q_\beta = 1.175 \text{ MeV}$ ,  $\gamma\text{-ray} = 661 \text{ KeV} (85\%) \rightarrow T_{1/2} = 30.8 \text{ yr}$  etc., i.e. these rates are not dependent of the nuclide type, are dependent only on  $E_{prag}$ .



**Figure 3.** The vortex-assisted photon count rate  $R_{pc}/R_h$  vs. bias current given in Eqs. (49 ÷ 53)

As it was mentioned in[1], CPIT with large efficiency is quite attractive from ecological view points. Since the  $\text{GeV}$  electrons stored in a storage ring lose little their energy via interactions with laser photons, they remain in the ring. The laser electron photons are efficiently used for production of desired  $RIs$ . Then the overall efficiency of the  $RI$  production is many orders of magnitude larger than that of the charged particle accelerators and nuclear reactors. Fast neutrons are used for nuclear transmutation but only a small fraction of the charged particles are used to produce the fast neutrons. Intense electron accelerators provide bremsstrahlung photons for photonuclear isotope transmutation but most of photons are below the threshold energy of the photonuclear reaction. There are several programs of intense photon sources under progress MAGa-ray is the project at LLNL for high intensity photons with  $10^{12}$  photons/sec in the MeV region and ELI-NP is the one at Romania[35] for higher energy photons with  $10^{13}$  photons/sec in the GR energy region of  $E(\gamma) \geq 19.5 [\text{MeV}]$ . They plan to achieve the intensity of around  $10^{15}/s$ . In Tokai Japan, the ERL(Energy Recovery Linia) project is under progress to provide intense photons for resonance fluorescence



#### 4. Beta Decay Halftime Calculation by G-L Theory

Below we will demonstrate that the mechanism for beta decay of radioisotopes is the same as the dark counts in the case of superconductors[2b].

In[2], are discussed three types of possible fluctuations in superconducting strip (assimilated in our case with the nucleon) which result in dissipation.

(a) Spontaneous nucleation of a normal-state belt across the strip with  $2\pi - \phi$  phase slip as in thin wires (a every phase slip meaning  $\Phi_0$  energy released).

(b) Spontaneous nucleation of a single vortex near the edge of the strip and its motion across to the opposite edge accompanied by a voltage pulse.

(c) Spontaneous nucleation of vortex-antivortex pairs and their unbinding as they move across the strip to opposite edges due to the Lorentz force, as well as the opposite process of nucleation of vortices and antivortices at the opposite edges and their annihilation in the strip middle.

In[2b] are derived the energy barriers for three dissipative processes mentioned within the  $G-L$  theory. Consider a thin-film strip (one of three vortexes of the nucleus) of width  $w = r^* \lambda$ , see fig 1a, fig.1b. We choose the coordinates so that  $0 \leq x \leq w$ . Since we are interested in bias currents which may approach depairing values, the suppression of the superconducting order parameter ( $\psi$ ) must be taken into account. Also in[2b], is used the standard GL functional, given above in (1).

We will use the case (a), a vortex crossing from one strip edge to the opposite one induces a phase slip without creating a normal region across the strip (one of three vortexes of nucleus) width. When, is treating the vortex as a particle moving in the energy potential formed by the superconducting currents around vortex center inside the strip and by the Lorentz force induced by the bias current. In[2b], it was derived the energy potential and is found the vortex crossings rate (phase slips and corresponding voltage pulses) in the framework of Langevin equation for viscous vortex motion by invoking the known solution of the corresponding Fokker-Planck equation.

Finally, from[2] the asymptotic estimate for the dark counts rate, results as:

$$R_v(I, \nu_h) = \frac{4k_B T c^2 R_\Omega}{\pi \Phi_0^2} \left( \frac{\pi \nu_h^3}{2} \right)^{1/2} \left( \frac{\pi \xi}{w} \right)^{\nu_h+1} Y \left( \frac{I}{\mu^2 I_0} \right) \quad (55)$$

and where the bias current is :

$$I = \frac{2w}{\pi \xi} I_0 \kappa (1 - \kappa^2) \quad (56)$$

$$I_0 = \varepsilon_0 \frac{c \Phi_0}{8 \pi \Lambda}; \Lambda = \frac{2 \lambda^2}{h_z} \quad (57)$$

Here,  $h_z \equiv \lambda$  -the axial ( $z$ ) height of the monopole condensate.

Here, the critical current at which the energy barrier vanishes for a single vortex crossing:

$$I_c = \frac{2\mu^2 w I_0}{2.72 \pi \xi} \quad (58)$$

And the thermodynamic critical field is :

$$H_c = \frac{\Phi_0}{2\sqrt{2} \pi \xi \lambda c} \quad (59)$$

where  $\mu^2 = 1 - \kappa^2$ , and

$$Y(z) = \left(1 + z^2\right)^{(\nu+1)/2} \exp \left[ \nu z \tan^{-1} (1/z) \right] \quad (60)$$

where

$v_h = \tau_{GL} (\varepsilon_{vortex} - Q_{bind}) / \hbar$  -is the energy of the vortex during crossing the barrier of height  $\varepsilon_{vortex} - Q_{bind}$  by quantum tunneling in place of the thermal activation used in[2b], and overpassing an ohmic resistance along a transverse path way of the nuclide:

$$R_\Omega = \frac{R_q R_{nuclide}}{R_q + 2\pi (\xi/w)^2 R_{nuclide}} \quad (61)$$

Here,  $\varepsilon_{vortex} = M_W$  from (47.1), and  $Q_{bind}$  is the beta decay energy as obtained from the data of each radionuclide of beta decay type (Nuclides chart 2010).

In the case of beta disintegration  $n \rightarrow p + e^- + \nu_e$ , or

$$udd \rightarrow uud + e^- + \nu_e + W^- (80 GeV),$$

or and the bias current is:  $d(-1/3e) + 2*3/3e = u(2/3e) + e$

In  $\beta^+$  decay, energy is used to convert a proton into a neutron, while emitting a positron ( $e^+$ ) and an electron neutrino ( $\nu_e$ ):

$$Energy + p \rightarrow n + e^+ + \nu_e$$

In all the cases where  $\beta^+$  decay is allowed energetically (and the proton is a part of a nucleus with electron shells) it is accompanied by the electron capture (EC) process, when an atomic electron is captured by a nucleus with the emission of a neutrino:  $Energy + p + e^- \rightarrow n + \nu_e$

Therefore, the ad-hoc bias current created during vortex crossing through the energy barrier is:  $e + u(2/3e) - 2*3/3e = d(-1/3e)$

At the first sight, the ohmic resistance of this ad-hoc electrical circuit created by the bias current  $I(e^\mp)$  due of quarks transformation ( $d \rightarrow u$ ), or ( $EC(u \rightarrow d)$ ), is given as:

$$R_{nuclide} = \frac{Q_{bind}}{\tau_{GL}} \frac{1}{V_{vortex}^2} \quad (62)$$

and the superconducting quantum resistance is:  $R_q = \hbar / (2e)^2 = 6.5 k\Omega$ , where the vortex potential is  $V_{vortex} = H_0 \xi$ ,  $H_0$  from (9)

Giordano[21] has suggested that phase slips due to macroscopic quantum tunneling may be the cause of the low temperature resistance tail in the 1D wires he studied in zero field. One possible mechanism for our low temperature resistivity tail could be quantum tunneling of vortices through the energy barrier[22]. One expects a crossover from thermal activation to quantum tunneling to occur when[23], in (55) in place of thermal activation we use the quantum tunneling:  $k_B T = \hbar / \tau_{GL}$

A vortex moving from  $x = 0 \rightarrow w$ , during the time  $\tau_{GL}$ .

We estimate the total interaction energy interaction with the neighborhood vortexes or with the one giant-vortexes, fig.1b, of others nucleons from the nuclide nucleus, during the time  $\tau_{GL}$  along the vortex path by matching (56), (57) and (59) as:

$$Q_{bind} \cong (\Phi_0 I / c) = \frac{\Phi_0}{c} \frac{2w}{\pi \xi} \kappa (1 - \kappa^2) \frac{\varepsilon_0 c \Phi_0 h_z}{8\pi 2\lambda^2} = \frac{c^2 \varepsilon_0 \pi \xi^2 h_z}{2} H_c^2 \frac{I}{I_0} \quad (63)$$

where the ratio  $I/I_0 \mu^2 = z$

was chosen as a variable in (55), through (60);

This is, in fact, the work done by the Lorentz force on the vortex path of the length  $w$ .

Now, we proceed to application to some radionuclides which decay beta, and beginning with the neutron.

Thus, the lifetime of the free neutron is a basic physical quantity, which is relevant in a variety of different fields of particle and astrophysics. Being directly related to the weak interaction characteristics it plays a vital role in the determination of the basic parameters like coupling constants or quark mixing angles as well as for all cross sections related to weak  $p-n$  interaction. From the most precise measurement within this class of experiments, results  $\tau_n = 886.3[s]$ .

From Nuclide chart-2010, result:  $^{99}Mo$  ( $\beta^-$ ,  $T_{1/2} = 65h$ ,  $Q = 1.356MeV$ );

$^{85}Kr$ ,  $Q_\beta = 0.687MeV$  and  $T_{1/2} = 48h$ ,  $\gamma$ -ray =  $514KeV$  (0.46%)  $\rightarrow T_{1/2} = 10.756yr$ ;  $\gamma$ -ray =  $151KeV$  (95%)  $\rightarrow T_{1/2} = 4.48h$ ;  $^{137}Cs \rightarrow ^{137}Ba + e^- + \bar{\nu}$ ,  $Q_\beta = 1.175MeV$ ,  $\gamma$ -ray =  $661KeV$  (85%)  $\rightarrow T_{1/2} = 30.8yr$  etc

Numerically, with these data result:  $\tau_{GL} = 1.5e-24[s]$ ,  $Y = 2560$ ,  $\nu_h \cong 157$ , and with  $w = r\lambda$  as variable, the evolution of dark count rate  $R_v$ , and of  $R_\Omega$ , for different isotopes are given in fig. 3.

Here, the fraction of the bias current to critical current  $I/I_0$  as used in  $Y(z)$  from (60): was deduced separately for  $^{137}Cs$  as that of the plateau zone in fig.4, respectively of  $I/\mu^2 I_0 \cong 0.005$ , by using the condition  $R_v/T^{1/2} \cong 1$ . We can observe that this value corresponds with the expected value from quarks transformation of  $\cong l(e^\pm) * \nu_{vortex} * 1.6e-19 r_0 [Am] \rightarrow 124[A]$ , where the vortex crossing velocity is  $\nu_{vortex} = \lambda/\tau_{GL}$ , and  $r_0$ -K shell radius. Therefore, the bias current, which is perpendicularly on the monopoles current, is  $I = 0.005 \mu^2 I_0 = 150[A]$ ; where,  $\mu^2 = 0.111$ , and the monopoles current is:  $I_0 \cong 3.e5[A]$  as given by (57), and  $I_c = 1.6e4[A]$  from (58). From (62), results  $Q_{bind} \cong 2.92e-13[J]$  or near equally with  $Q_\beta$  of the almost of nuclides, for example, for  $^{100}Mo$ ,  $Q_\beta = 1.356MeV \rightarrow 2.17e-13[J]$ .

Now, in case of quantum tunneling the transmission coefficient[20] is  $T = e^{-\frac{\varepsilon_0 \tau_{GL}}{h}} \cong 1.e-66$ , which is too small, so,

a vortex crossing from one strip edge to the opposite one induces a phase slip, case a), is the only viable mechanism for dark counts (beta decay).

Thus, it was established a logarithmic equation of the  $\beta$  decay rate which resulting a straight line as a function of the barrier width ( $w = r\lambda$ ) for every nuclide, fig.4, it decreasing in case of long lived nuclides, like  $^{60}Fe$ . Therefore, this evolution is a decisive validation test of entirely model. The factor  $r$  describe the interactions of the nucleons inside of the nucleus, being a complex function of  $Z, N, A, Q_{bind}$ .

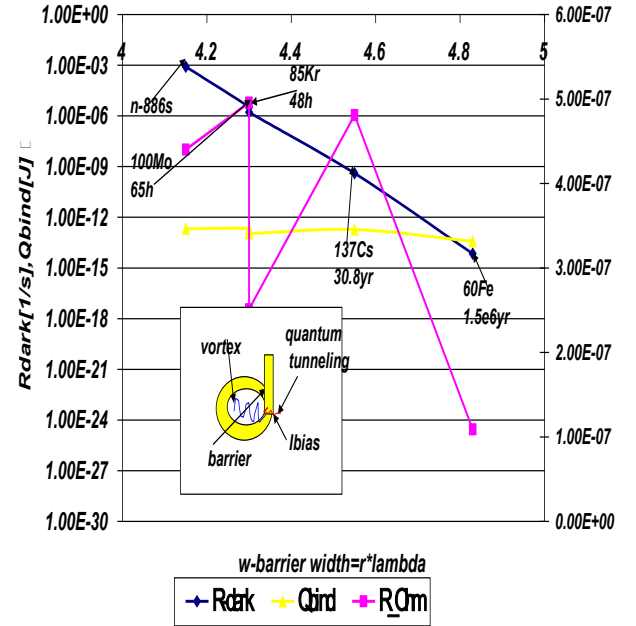


Figure 4. The evolution of dark count ( $\beta$  decay) rate as function of barrier width

What other signification have the width  $w = r\lambda$ ?

In order to answer to this question, in the following we will look to the vortexes interactions[24] vis-à-vis of the nucleus substructure models [25÷33].

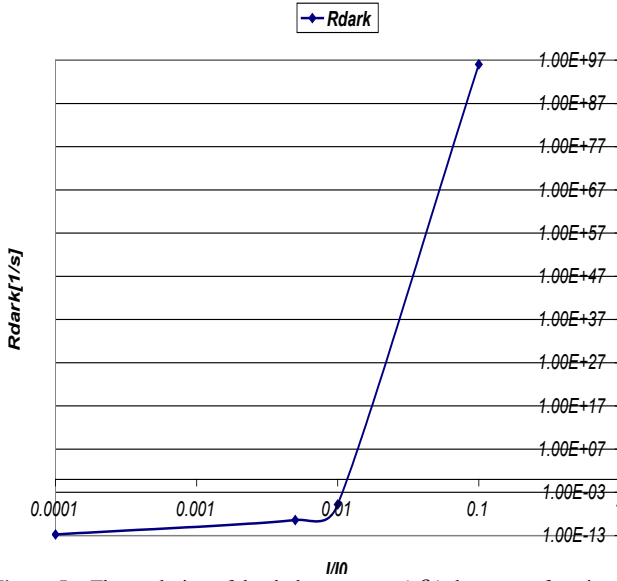
Thus, firstly, from[24], for vortex-vortex interaction (V-V) at large separation, using the asymptotic form of the modified Bessel functions, is:

$$\Omega(d \rightarrow \infty) = \gamma d^{-1/2} (\delta e^{-d} - \sqrt{\mu} e^{-\mu d}) \quad (64)$$

where:  $d(\lambda) = r\lambda$  -the separation distance;  $\mu = \sqrt{2}\kappa$ , and  $\gamma = -12.503$ ;  $\delta = 0.3127$  are fitting parameters of the analytical fitting function for the numerically obtained curves by using the finite difference technique and a relaxation method for the Euler-Lagrange equations for the present problem setting.

Furthermore, in[24], is show that the interaction between an antivortex and a giant vortex is always attractive as well.

In the absence of lateral confinement, a giant vortex is a stable (unstable) state in type-I (type-II) systems and can interact as such with other vortexes, and this motivated us to investigate the interaction force between a vortex and a giant vortex.



**Figure 5.** The evolution of the dark count rate ( $\beta$ ) decay as a function of the bias current

Also in [24], is suggested that the fitting function given by the following equation which can not only be used for the  $V-V$ , but also for the  $V-GV$  interaction force.

$$\Omega_{fit}(d) = \eta_1 \frac{d^{\eta_3}}{1 + \eta_2 d^{\eta_3 + \frac{1}{2}}} \left( \eta_4 e^{-d} - \sqrt{\mu} e^{-\mu d} \right) \quad (65)$$

where  $\eta_i (i=1 \div 4)$  are four fitting parameters.

The values of the four fitting parameters are given in Table II of [24], for  $\mu$  from 0.2 to 2.5

In our case  $\mu = \sqrt{2}\kappa \cong 1.4$ ; result:  $\eta_1 = 1.237e-02$ ;  $\eta_2 = 1.55e-03$ ;  $\eta_3 = 4.248$ ;  $\eta_4 = 1.183$

In the previous subsections, we showed that when two vortices or a vortex and a giant vortex are brought close to each other, they merge forming a single giant vortex state with vorticity  $n = n_1 + n_2$ , and in the limit of small separation the  $V-V$  or  $V-GV$  forces are very weak. Conversely, a vortex and an antivortex attract and annihilate, both in type-I and type-II superconductors. In what follows, the behavior of the force for the vortex-antivortex ( $V-AV$ ) interaction as a function of the  $V-AV$  separation is also studied. As follows from theory [24], the V-AV interaction is always attractive.

However, at some critical  $V-AV$  separation  $d_E(\lambda)$ , the solution with well defined supercurrents around each vortex and antivortex ceases to be the lowest energy state of the system. A solution with lower energy exhibits a strong suppression of the amplitude of the order parameter and super-current in the region between the vortex and the antivortex, and represents the ground state for small  $V-AV$  distances. A hysteresis is observed in the vicinity of the critical separation  $d_E$ . The dependence of the numerically obtained critical separation  $d_E$  for the  $V-AV$  interaction on the GL parameter  $\mu$  is illustrated as the squares in Fig. 11(c) from Ref. [1], and can be fitted to a function similar to

the one used for the critical separations in the  $V-V$  and  $V-GV$ , given by

$$d_E = 0.337 + 31.249(1 + 10.264\mu)^{-0.6855}, \text{ or } d_E = 4.96\lambda$$

and becomes unstable as the  $V-AV$  separation is reduced at  $d < d_A$ , while the solution with suppressed current and order parameter between the vortex and antivortex, shown by the gray curves, becomes the lowest energy state for  $d < d_E$  and the only stable solution for  $d < d_A$ .

The suppressed order parameter in the region between vortices observed in the only stable solution for  $d < d_A$  suggests that a vortex and an antivortex cannot coexist at these distances, unless somehow pinned, in which case this string solution is formed. This is reasonable, since at  $\mu$  and can be fitted by :

$$d_A = 0.337 + 12.222(1 + 2.461\mu)^{-0.79}; \text{ or } d_A = 3.95\lambda$$

When the  $V-AV$  separation is large, the currents around the vortex and short distances the fields of the vortex and the antivortex compensate each other, and the flux quantization as an essential property of a(n) (anti)vortex is lost. Notice that this is different from the case of two merging vortices, which can coexist at short distances, deform and interact as described in previous sections, since the flux quantization of the  $V-V$  pair is preserved even at small  $V-V$  separations. For molecular dynamics studies of the  $V-AV$  motion, one should consider the critical separation  $d_A$  as the separation where the  $V-AV$  pair annihilates.

The fit of the numerically obtained  $V-AV$  interaction force for  $d > d_E$ , yields a single expression:

$$\Omega = -(2.879 + .415\mu^{-3.166})K_1(d) + (0.2258 - 1.044e^{1.866\mu})\mu K_1(\mu d) \quad (66)$$

where the approximation of the Bessel function

$$K_1(d) = \frac{\pi}{\sqrt{2\pi d}} \exp(-d)$$

which is expected to provide an accurate description of the V-AV interaction force, at separations  $d > d_E$ , for any value of  $\mu$ .

Thus, in [25] is mentioned that the results obtained the study of "Effect of a fermion on quantum phase transitions in bosonic systems" are of interest not only for applications to nuclei, but also for applications to other systems in which a fermion is immersed in a bath of bosons, for example, the simple case of a spin 1/2 particle in a bath of harmonic oscillator bosons [34] page 217.

In the literature [25] of the Interacting Boson Fermion Model (IBFM) the bosonic part has a cubic term leading to the possibility of a first order transition. The analogy with bosonic systems in an external field also suggests that these results apply to the study of phase transition in superconductors in the presence of magnetic fields (which is our case of phase slip, as discussed above).

Atomic nuclei are known to exhibit changes of their energy levels and electromagnetic transition rates among them when the number of protons and/or neutrons is

modified, resulting in shape phase transitions from one kind of collective behaviour to another. These transitions are not phase transitions of the usual thermodynamic type. They are quantum phase transitions *QPT* [25] (initially called ground state phase transitions).

However, since the proton-neutron quadrupole interaction dominates over the proton-proton and neutron-neutron ones for medium-heavy and heavy deformed nuclei, the axial deformation parameters  $\beta$  are related by a constant of proportionality determined by equating the corresponding intrinsic quadrupole moments. The geometrical variable  $\beta$  is obtained by multiplying the boson.

The nuclear  $\beta$ -decay provides a severe test of the nuclear model because the decay rates are very sensitive to the wave functions of both the initial and the final nuclei.

The description of  $\beta$  decay of odd-mass nuclei in the interacting boson-fermion model (IBFM) was formulated by [25].

Thus, in [25], are presented the results of an investigation of the effect of a fermion (in our case the electron  $e^\pm$ ) on QPTs in bosonic systems. That is done in atomic nuclei by making use of the Interacting Boson Fermion Model (IBFM), a model of odd-even nuclei in terms of correlated pairs with angular momentum  $J = 0, 2(s, d)$  and unpaired particles with angular momentum  $J = j$  ( $j$  fermions). To note, however, that the method of analysis from [25] can also be used for systems with other values of the fermion,  $j$ , and boson,  $J$ , angular momenta, for example the spin-boson systems, the simplest case of which is a fermion with  $j = 1/2$  (i.e., a single spin) in a bath of harmonic oscillator one-dimensional bosons of interest in dissipation and light phenomena. Here the focus is on the effect of a fermionic impurity on QPTs in bosonic systems. The main results are that, (a) the presence of a single fermion greatly influences the location and nature of the phase transition, the fermion acting either as a catalyst or a retarder of the QPT, and (b) there is experimental evidence for quantum phase transitions in odd-even nuclei (bosonic systems plus a single fermion).

A main conclusion from [25] is that the effect of the fermionic impurity is to wash out the phase transition for states with quantum numbers:  $K = 1/2, 3/2, 5/2$  and to enhance it for states with  $K = 7/2, 9/2, 11/2$ . In other words, the fermion acts as a catalyst for some states and as a retarder for others. Also, when the coupling strength becomes very large, the minima for some large  $K$  like  $K = 11/2$ , shift to negative values (oblate deformation).

An important property of atomic nuclei is that they provide experimental evidence for shape QPTs, in particular, of the spherical to axially-deformed transition ( $U(5)$ - $SU(3)$ ). One of three signatures have been used to experimentally verify the occurrence of shape phase transitions in nuclei, namely: the behavior of the gap between the ground state and the first excited  $0^+$  state.

The nucleus  $^{152}\text{Sm}$ , with  $N = 90$  and  $Z = 62$ , lies intermediate between nuclei of known spherical shape and well-deformed axially-symmetric rotor structure. A sudden change in deformation occurs at  $N \approx 88-90$  for the *Sm* and

neighboring isotopic chains, see Figure 6.

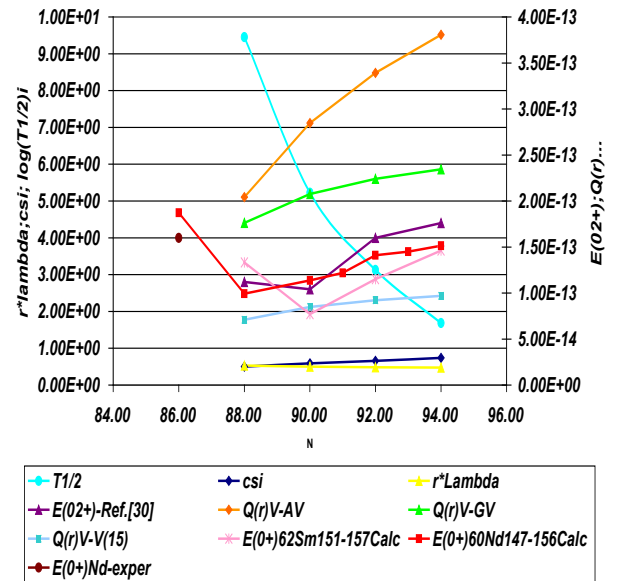
New data obtained [31] constrain the description of this nucleus within the *IBM* to parameter values near the critical point of the transition from oscillator to rotor structure.

The performed IBA calculations in [31] for the entire region  $N \approx 90$  with the simple IBA Hamiltonian which involves only one parameter,  $\xi$ , which depends only on the neutron number  $N$ , for all the isotopic chains.

Figure 6 shows that the evolution of some basic observables in *Nd-Sm* is very well reproduced by these calculations. The energy of the intrinsic excitation  $0^+$  has a minimum at the phase transition point. The phase/shape transition is mirrored in the calculations and, as noted, the IBA parameter  $\xi$  plays the role of a control parameter.

The total energy surface [31] corresponding to the *Sm* isotopes obtained from the IBA with increasing neutron number changes the location of the deformation minimum, from  $\beta = 0$  to finite  $\beta$  when the neutron number increases from 88 to 92.

Notice that the bosons ( $Z(62-50)/2 = 6; N(90-82)/2 = 4$ ) pairing appears for  $^{152}\text{Sm}^{90}(\text{stable}) \leftarrow EC \leftarrow ^{152}\text{Eu}^{89}$ , and  $^{150}\text{Sm}^{88}(\text{stable}) \leftarrow \beta^- \leftarrow ^{150}\text{Pm}^{89}$ , until a free proton appears for  $^{151}\text{Sm}^{89} + \beta^- \rightarrow ^{151}\text{Eu}^{88}(\text{stable})$ , and  $^{153}\text{Sm}^{91} \rightarrow \beta^- \rightarrow ^{153}\text{Eu}^{92}(\text{stable})$ ,  $^{155}\text{Sm}^{93} \rightarrow \beta^- \rightarrow ^{155}\text{Eu}^{92}$ , the same situation for  $^{147}\text{Nd}^{87} \rightarrow \beta^- \rightarrow ^{147}\text{Pm}^{86} \rightarrow \beta^- \rightarrow ^{147}\text{Sm}^{85}(\text{stable})$ , etc.



**Figure 6.** The evolution of empirical values of  $E(0_2^+)$  in the *Nd-Sm* isotopic chains with  $N > 82$  compared with the IBA results as extracted from [31], and of  $E(r)$  as calculated by the present model

In terms of our model defined interactions, it means that a proton is composed of 2 vortexes (*V*) and 1 antivortex (*AV*) of different spin, a neutron is a Giant-vortex (*GV*). Therefore, for  $N \leq 90$ , we have an interaction ( $V-V$ )-( $V-AV$ ), and above of ( $V-AV$ )-( $V-GV$ ) type,

respectively. Based on these rationales we have obtained the values of  $E(0^+)$ , as shown in figure 6, where, we show, also, the values of interactions  $(V - GV); (V - V); (AV - V)$  calculated with equations (64÷66).

In this model, the phase/shape changes are functions of a control parameter  $\xi$  and a quadrupole deformation  $\beta$  as an order parameter, and in our case the equivalently phase slip which conducts to a dark count accompanied by a  $W$  boson energy, in place of  $\xi, \beta$  parameter appears to be necessarily only one:  $w = r \cdot \lambda$ , see fig. 6.

Our model have the advantage to explain the cause of phase change and its probability associated, as expressed by the inherently dark counts rate or the decay half-time  $T_{1/2}$ .

## 4. Conclusions

A model was developed in which a photonuclear reaction is viewed as an incident photon creating superconducting hot spot (hot belt) across nucleon from the composition of a unstable nuclide (radioisotope), followed by a thermally induced vortex crossing, which turns superconducting hot belt into the normal state (vacuum) resulting in a vortex assisted photon beta decay. Because this model, firstly developed for superconducting nanowire single-photon detector (SNSPD), requires data on energies of vortex and on currents from inside of the nucleon, an analog of a superconductor model was developed for the nucleon.

Thus, it was re-visited a model G-L, in order to calculate the Lorenz force, the current, and the energies of the Abrikosov vortex lines inside of the nucleon. Thus, it was found that these energies correspond of subatomic particles,  $W$ ,  $Z$ , Higgs bosons, and of meson  $\pi$ . So, the nucleon can be seen as a triangular lattice with three pairs of quarks-antiquarks in the tips of the triangle. These axial vortexes (filaments) interacting in the lateral plane.

All of the keys of this model, as being analogous to a superconductor clinging very well, starting with the use of Maxwell's equation with monopoles, further mass, charge, and the number of monopoles (density), which define the penetration depth and the coherence length which could be considered as new fundamentals constants for nucleons. Also, we can say that, because no free quarks were detected, the same is true for monopoles, both of which are confined together and in full in nucleons, when the temperature of the universe has reached  $2 \cdot 10^{12} K$ .

Finally the model provides counts (decay) rates ( $R_{pc}$ ) comparable with the experiments, and allows the estimation of intensity  $R_h$  (photons/s), and of threshold value of photons energy ( $MeV$ ), in order to achieve  $R_{pc}/R_h \cong 1$ , or (100%) efficiency of beta decay type radionuclides in the frame of the nuclear transmutation in the reducing of radioactive waste, together with  $\alpha$  decay enhancement method[20]. As a validation test is the application of this model to the mechanism of natural beta decay viewed as dark count rate, which it was discussed in comparison with the results of one recently model on the effect of a fermion on quantum phase

transitions in bosonic systems.

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