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A Game-Theoretical Approach to Current Nuclear Proliferation Policies between the U.S. and the IRI

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Abstract The purpose of this paper is to illuminate the emergent effects of conflict resolution in the international nuclear diplomatic arena. A pertinent examination is the skirmish currently underway between the United States and its allies with respect to the Islamic Republic of Iran (IRI) and its tenacity towards nuclear capabilities. A comparison between an emphasis on diplomatic versus military U.S.-IRI policies is conducted and compared to illustrate how policy differences are potentially required based on differences on utility functions of other players.

Keywords Rational choice, Nash equilibrium, Nuclear proliferation

1. Introduction

The word "competition" is typical vocabulary when dealing with private industry, athletic events, or with games. However it need not be constrained to simply those arenas. One more subtle area, yet with larger implications of competition is within international diplomacy. Nations or groups of nations vie for their interests over those with dissimilar ones. Here the competition is to establish you and your allies policy over the alternative policies. A particular branch of mathematics focuses on exactly the dynamics of how this interplay of competing interests play out - Game Theory. Although some might not think policy setting should be referred to as a "game" the dynamics are very much the same.

2. Origins of Game Theory

The origins of game theory date as early back as the 18th century but had been established as a respectable branch of mathematics in the 1920s through the work of Emile Borel [1] and John von Neumann. The true framework for game theory as we know it today was established in the 1944s when John von Neumann and Oskar Morgenstern published their book Theory of Games and Economic Behaviour [2] [6]. In 1950 John F. Nash reasoned that a player must determine their action profile while considering what all other players will decide to do. Until that point, decisions were based solely on maximizing one's own utility. Depending on the situation, this would lead to constant conflict between each player. A better approach would be to pick an action profile for each player so that there would be no incentive of any player to deviate from that action profile. This was Nash's contribution as for this sake the action profile that enables this steady-state solution among players is called a Nash Equilibrium. [2]

Game theory is a study of strategic decision under competition or more formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" [3]. The term "rational" implies that players are making choices to satisfy their best selfinterests. Each situation and player has a portfolio of actions (called the action profile) that they can perform and the associated payoffs (or utility) for those actions. In the Theory of Rational Choice a person would evaluate their action profiles and then pick that profile that would maximize their utility. Formally we define the function $u(\bullet)$ as the utility function for a player and a and b as two different action profiles. If a player acts rationally, then they would pick action profile a if

$$u(a) > u(b) \tag{1}$$

Notice that we need not know the actual function $u(\bullet)$ in order to determine which action profile will be used given that the player acts rationally. It suffices for a player to know which action profile he prefers over another. This does make intuitive sense. I might prefer product A over product B because it has a certain feature but I have not evaluated with mathematical precision exactly by how much I prefer product A over product B.

3. Essentials of Nash Equilibrium

The Nash equilibrium is the steady state of the game. If a game has a Nash equilibrium then each player has an incentive to retain their decision over changing it to a

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different one. Unlike a decision that is simply based on personal (think greedy decision making) incentives, a Nash equilibrium takes into account each players potential decisions. Thus for a player to determine the optimal - and steady state - decision strategy a player should ask himself "Given my opponent's decision X, what should my decision be?". If we let $A_{r,s,i} = D_i\,(P_r|P_s = d_{s,i})$ represent the optimal action profile P_r can make, given P_s's action profile is $d_{s,i}$, where *r* represent Player 1, and *s* represents Player 2, then the set of Nash equilibria (SNE) can be represented as:

$$S_{NE} = \left(\bigcup_{i=1}^{n} \left\{ A_{r,s,i}, d_{2,i} \right\} \right) \bigcap \left(\bigcup_{i=1}^{n} \left\{ A_{r,s,i}, d_{1,i} \right\} \right) \quad (2)$$

If $S_{NE} \neq \emptyset$; then there exists an action profile for each player such that if both players choose their respective action profile, there is no incentive to deviate from this action profile. Let $|S_{NE}|$ be the cardinality (the number of elements) of the set S_{NE}. If $|S_{NE}| > 1$ then multiple Nash equilibria exist however Nash equilibria need not be equally preferable. Depending on the player utility function or payoff values an action profile that is a Nash equilibrium could be better than another action profile that is a Nash equilibrium too. Further analysis would need to be made after S_{NE} has been obtained to decide which action profile to focus on.

We will illustrate how to obtain the set of Nash equilibria with a celebrated game called the Prisoner's Dilemma. The Prisoner's Dilemma deals with two suspects, P_1 and P_2. After their arrest they are kept separate so they cannot corroborate their stories. Both have the option to confess or remain quiet and both would like to minimize the time they would have to serve. If both stay quiet, the police can only get them for a lesser charge of 1 year. However if only one of them confess then he who confesses will receive no sentence while the other will get the full sentence of 3 years. Finally, if both confess then both sill receive a higher sentence of 2 years. This is summarized in the following payoff matrix:

		P_2	
		Confess (C)	Quiet (Q)
P_1	Confess (C)	[2, 2]	0, 3
	Quiet (Q)	3, 0	1,1

If each player simply acted in their own self-interest (that is trying to maximize their utility by minimizing their time) each would confess trying to obtain no sentence at all. However in acting in their own self-interest only they receive a 2 year sentence each. Now if each player tried to maximize their utility by taking the action profile of their opponent into account, we obtain the following decisions:

- Given that P1 chooses Quiet, P2 chooses Quiet.
- Given that P1 chooses Confess, P2 chooses Quiet.
- Given that P2 chooses Quiet, P1 chooses Quiet.
- Given that P2 chooses Confess, P1 chooses Quiet.

Utilizing (2) our viable set of Nash equilibria becomes:

$$\left(\{Q,Q\} \cup \{C,Q\}\right) \cap \left(\{Q,Q\} \cup \{Q,C\}\right) = \{(Q,Q)\}$$

Therefore the Nash equilibrium is for both to choose **Quiet** with cardinality 1. Therefore the optimal action profile of **Quiet** is the only Nash equilibrium and will result in only a 1 year sentence for both suspects.

		P_2	
		Confess (C)	Quiet (Q)
P_1	Confess (C)	2, 2	0, 3
	Quiet (Q)	3, 0	[1, 1]

4. Diplomacy between the U.S. and the IRI in Nuclear Proliferation

At the same time that Game Theory was becoming an established field of mathematics, the United States was assisting Iran launch their nuclear program as part of the Atoms for Peace Program [4]. This participation continued until the 1979 Iranian Revolution. [5] All further research was halted due to the new regime under Ayatollah Ruhollah Khomeini who regarded nuclear, biological and chemical weapons forbidden under Muslim ethics and jurisprudence. [7]. As recent as 2003 however signs of a reinvigorated effort to resurrect its nuclear program have surfaced. [8]

The general argument is that a nuclear Iran would destabilize the region and cause a nuclear arms race in the Middle East. In order to prevent further turmoil in the region, the United States has invoked harsh and effective sanctions against the IRI. This has caused, among other things, the Iranian regime to work in overdrive to try to shield its public from the affects in order to keep them subdued. Due to these sanctions, the Iranians have lost a major source of income, namely their oil exports to Western European nations. These stressors enable the U.S. to force Iran to negotiate.

To try to understand current policy dynamics we will focus on the most likely subset of possible policy options. We define the action profile for the U.S. as: Strengthen Embargo (**E**$^{+}$), Sustain Embargo (**E**), Relax Embargo (**E**$^{-}$), Remove Embargo (**E**^{0}), and Military Action (**M**). We also define the IRI's action profile as: Develop Nuclear Bomb (**DNB**), Enrich for Nuclear Bomb (**ENB**), Enrich for Nuclear Power Only (**ENPO**), and Disband Centrifuge Capability (**DCC**). Furthermore, define u(•) and v(•) be the respective utility functions for the U.S. and the IRI. We also consider two possible preferences of each side: Diplomatically-minded (D) and Militaristically-minded (M). Therefore we obtain four combinations of utility functions preferences given these action profiles. They are a diplomatically-minded U.S. (3), a militaristically-minded U.S. (4), a diplomatically-minded IRI (5), and lastly a militaristically-minded IRI (6). Furthermore, let S_1 and S_2 denote the sets of action profiles available to P_1 and P_2 respectively. It is important to note that deviations might arise since the author's subjective assessment played into determining these relationships. None of these relationships seem to violate reasonable assumptions about the order.

$$u(E) > u(E^+) > u(E^-) > u(E^0) > u(M), \\ u(DCC) > u(ENPO) > u(ENB) > u(DNB) \tag{3}$$

$$u(E) > u(M) > u(E^+) > u(E^-) > u(E^0), \\ u(DCC) > u(ENPO) > u(ENB) > u(DNB) \tag{4}$$

$$v(E^0) > v(E^-) > v(E) > v(E^+) > v(M), \\ v(ENPO) > v(ENB) > v(DNB) > v(DCC) \tag{5}$$

$$v(E^0) > v(M) > v(E^-) > v(E) > v(E^+), \\ v(DNB) > v(ENB) > v(ENPO) > v(DCC) \tag{6}$$

The next step is to assign a utility value for both the U.S. and the IRI for each combination of policies. Since the U.S. possesses 5 action profiles its values range from 1 through 5. Likewise for the IRI, it possesses 4 action profiles and therefore we assign values ranging from 1 through 4. Overall preference is determined by the summation of the U.S. and IRI preference values. Furthermore, we also force each action profile to be strictly less than or strictly greater than any other action profile. Therefore when a tie between multiple action profiles occur, then that action profile is chosen that drives the IRI to the U.S.'s ultimate goal. As an example, we utilize the US(D) table below. The action profile $(u + v)(E^-,ENB) = (u + v)(E^0,ENPO) = 8$. This tie is broken providing $(u + v)(E^0,ENPO) = 9$ since this moves the IRI closer to DCC - the U.S.'s stationary solution for the IRI unless another summation is 8 as well then the next possible value that can be assigned is 10. We obtain the following tables using these ideas.

Table 1. Diplomatically-minded U.S. preference values

US (D) (Strict Order)					
		DNB	ENB	ENPO	DCC
		1	2	3	4
E^+	4	7	12	16	19
E	5	11	15	18	20
E^-	3	4	8	13	17
E^0	2	2	5	9	14
M	1	1	3	6	10

Table 2. Militaristically-minded U.S. preference values

UD (M) (Strict Order)					
		DNB	ENB	ENPO	DCC
		1	2	3	4
E^+	3	4	8	13	17
E	5	11	15	18	20
E^-	2	2	5	9	14
E^0	1	1	3	6	10
M	4	7	12	16	19

Table 3. Diplomatically-minded IRI preference values

IRI (D) (Strict Order)					
		DNB	ENB	ENPO	DCC
		2	3	4	1
E^+	2	5	8	11	3
E	3	9	12	15	6
E^-	4	13	16	18	10
E^0	5	17	19	20	14
M	1	2	4	7	1

Table 4. Militaristically-minded IRI preference values

US (M) (Strict Order)					
		DNB	ENB	ENPO	DCC
		4	3	2	1
E^+	1	10	6	3	1
E	2	14	9	5	2
E^-	3	17	13	8	4
E^0	5	20	18	15	11
M	4	19	16	12	7

The following are the four combinations of the application action profiles for the U.S. and the IRI with the Nash equilibria highlighted in bold. Following each table is the actions of the one player given the actions of the other player.

		IRI (D)			
		DNB	ENB	ENPO	DCC
US (D)	E^+	7,5	12,8	16,11	19,3
	E	11,9	15,12	**18,15**	20,6
	E^-	4,13	8,16	13,18	17,10
	E^0	2,17	5,19	9,20	14,14
	M	1,2	3,4	6,7	10,1

- Given that the U.S. chooses E^+, IRI chooses ENPO.
- Given that the U.S. chooses E, IRI chooses ENPO.
- Given that the U.S. chooses E^-, IRI chooses ENPO.
- Given that the U.S. chooses E^0, IRI chooses ENPO.
- Given that the U.S. chooses M, IRI chooses ENPO.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses ENPO, U.S. chooses E.
- Given that the IRI chooses DCC, U.S. chooses E.

IRI (M)

		DNB	ENB	ENPO	DCC
US (D)	E^+	7,10	12,6	16,13	19,1
	E	**11,14**	15,9	18,5	20,2
	E^-	4,17	8,13	13,8	17,4
	E^0	2,20	5,18	9,15	14,11
	M	1,19	3,16	6,12	10,7

- Given that the U.S. chooses E^+, IRI chooses DNB.
- Given that the U.S. chooses E, IRI chooses DNB.
- Given that the U.S. chooses E^-, IRI chooses DNB.
- Given that the U.S. chooses E^0, IRI chooses DNB.
- Given that the U.S. chooses M, IRI chooses DNB.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses ENPO, U.S. chooses E.
- Given that the IRI chooses DCC, U.S. chooses E.

IRI (D)

		DNB	ENB	ENPO	DCC
US (M)	E^+	4,5	8,8	13,11	17,3
	E	11,9	15,12	**18,15**	20,6
	E^-	2,13	5,16	9,18	14,10
	E^0	1,17	3,19	6,20	10,14
	M	7,2	12,4	16,7	19,1

- Given that the U.S. chooses E^+, IRI chooses ENPO.
- Given that the U.S. chooses E, IRI chooses ENPO.
- Given that the U.S. chooses E^-, IRI chooses ENPO.
- Given that the U.S. chooses E^0, IRI chooses ENPO.
- Given that the U.S. chooses M, IRI chooses ENPO.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses ENPO, U.S. chooses E.
- Given that the IRI chooses DCC, U.S. chooses E.

IRI (M)

		DNB	ENB	ENPO	DCC
US (M)	E^+	4,10	8,6	13,3	17,1
	E	**11,14**	15,9	18, 5	20,2
	E^-	2,17	5,13	9,8	14,4
	E^0	1,20	3,18	6,15	10,11
	M	7,19	12,16	16,12	19,7

- Given that the U.S. chooses E^+, IRI chooses DNB.
- Given that the U.S. chooses E, IRI chooses DNB.
- Given that the U.S. chooses E^-, IRI chooses DNB.
- Given that the U.S. chooses E^0, IRI chooses DNB.
- Given that the U.S. chooses M, IRI chooses DNB.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses DNB, U.S. chooses E.
- Given that the IRI chooses ENPO, U.S. chooses E.
- Given that the IRI chooses DCC, U.S. chooses E.

After evaluating S_{NE} from (2), we obtain the Nash equilibrium of (*E,ENPO*) when both players act diplomatically. Using this approach we can calculate the Nash Equilibrium action profiles for the other preferences. Observe that since all preferences are strictly greater than one another our scenario will always have $|S_{NE}| = 1$.

These results are intriguing. For all combinations the U.S. remains undeterred from maintaining current economic sanctions regardless of whether it prefers a diplomatic or militaristic solution. The only distinction arises from the IRI. Here their decision seems to only depend on their preference and not on the U.S.'s preference. If the IRI prefers a diplomatic solution then it will aim for nuclear power only and if it prefers a more militaristic solution then it will prefer developing a nuclear bomb. Naturally this analysis depends on the current game. The nature of foreign policy is to shape the game in such a way as to influence the rational behavior of the other player or players. Given the current game as shown, the U.S. has no influence on how to influence the IRI. The U.S. decisions is strictly contingent on the IRI's preference. Thus to retain an active role the U.S. must change the game. This entails fundamentally influencing, through political means, the utility function the IRI currently exhibits with their action profile.

5. Conclusions

Game-theoretical applications are pervasive in many aspects of life. Since the likelihood of any two parties fully agreement is low it behoves us to analyse the situation in such a framework to establish a best course of action. Establishing viable international policy is no exception.

Nations as the IRI still serve their objectives in order to maximize their utility. Although their motivation may seem irrational to an outsider we must still attempt to understand their incentive structure if we want to author a viable and stable policy. The models seem to suggest that the IRI would be willing to deescalate their nuclear ambitions given the proper incentives. However these assessments are contingent on how they view their economic health with respect to the nuclear capabilities. Given the current policy environment it seems to suggest that the IRI is more diplomatic than militaristic. No similar assessment can be made for the U.S given the current action profile.

With regard to the IRI it can be especially difficult to assess their incentive structure with a Western mindset. This paper makes an attempt to emulate their incentive structure and to predict what their possible action profile might be. All scenarios require perfect information about the players' intentions which is never fully realizable and especially difficult given current tensions between the U.S. and the IRI. An avenue for further discussion would be to eliminate the assumption of perfect information and to observe how this relaxation affects strategies of players for both scenarios using a mixed-strategy approach.

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