Journal of Civil Engineering Research 2012, 2(2): 1-5
DOI: 10.5923/j.jce.20120202.01

Stress Concentration Near a Half-Plane Weakening Hole

Kuliyev S. A.

Azerbaijan Engineering , Architecture University, 5, Ayna Sultanova, Baku, Azerbaijan

Abstract In the paper, stress state of an elastic, isotropic and homogeneous weighty half-plane weakened by an elliptic hole with two rectilinear cracks is considered. The domain S occupied by a body consists of a "lower" half-plane bounded by the straightline L_2 and a contour of the hole L_1. The elastic medium, in addition to its specific weight at the infinity is subjected to the action of uniformly-distributed pressure of intensity q on the segment $-a \leq t \leq a$ of a rectilinear boundary L_1 of the half-plane. The hole's contour is free from external forces. Many aspects of such problems have been considered in the papers[1; 3; 6; 7; 8; 10]. But the case that we investigate is considered for the first time (this is connected with definition of the mapping function and also complexity of acting force factors).

Keywords Weighty Half-Plane, Specific Weight, Elastic Medium, Compatibility Conditions, Additional Stress Uniformly Distributed Pressure, Distance Piece Coefficient

1. Introduction

The problem solution is reduced to definition of two analytic functions $\phi(z)$ and $\psi(z)$ satisfying the boundary conditions. After some mathematical transformations and reasonings, we get two systems of infinite linear algebraic equations with respect to the expansion coefficients α_k and β_k of the functions $\phi(z)$ and $\psi(z)$.

In order to illustrate the obtained solution, numerical examples are considered.

The centre of the hole coincides with the origin of coordinates oxy, the linear cracks are arranged symmetrically along the axis OX, the coordinates of the end points of the cracks are denoted by $\pm e$ (fig.1). The domain S occupied by a body consists of a "lower" half-plane bounded by a straight line L_2 and contour of the hole L_1 (an ellipse with two linear cracks).

The problem on definition of stresses in domain S occupied by an elastic medium has a great value in such a statement in the practice of mining, tunnel building (metro-building) and also in many problems of hydro engineering.

2. The Problem for a Weighty Half-plane

On the base of the known principle of mechanics – independence of actions of applied forces, the problem may be divided into two separate cases, the problem for a weighty half-plane, and a problem for a weightless half-plane under the action of uniformly distributed pressure P.

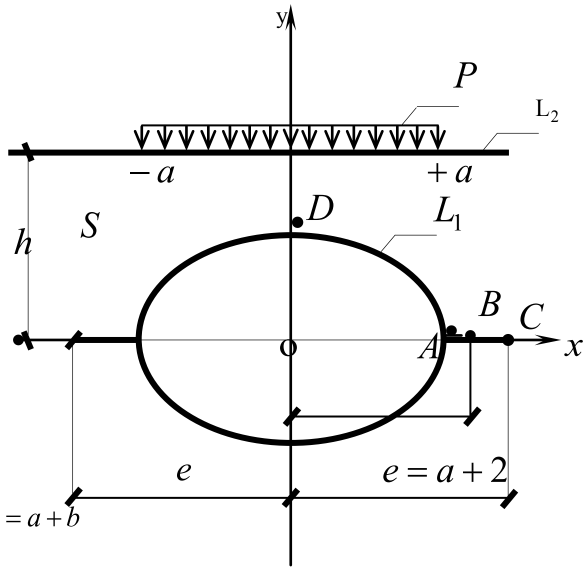

Figure 1. Cross-section of a half-plane with the elliptic holes and two cracks.

In the domain S of a half plane bounded by the straight line L_2 and contour of holes L_1 (ellipse with two linear cracks) the stress components σ_{1x}, σ_{1y} and τ_{1xy} satisfy the differential equilibrium equations and compatibility conditions (for the first case of the weighty half-plane).

$$\frac{\partial \sigma_{1x}}{\partial x} + \frac{\partial \tau_{1xy}}{\partial y} = 0 \quad ; \quad \frac{\partial \tau_{1xy}}{\partial x} + \frac{\partial \sigma_{1y}}{\partial y} - \rho \cdot g = 0 ;$$

$$\Delta\left(\sigma_{1x} + \sigma_{1y}\right) = 0 \qquad (2.1)$$

where ρ is density of elastic medium, g is free fall acceleration.

The boundary conditions:

* Corresponding author:
nigar_sabir@yahoo.com (Kuliyev S. A.)
Published online at http://journal.sapub.org/jce

$$\tau_{1xy}=0;\quad \sigma_{1y}=0$$

on the linear boundary L_2 (2.2)

$$\left.\begin{aligned}\sigma_{1x}\cdot\cos(n,x)+\tau_{1xy}\cos(n,y)=0\\ \tau_{1x}\cdot\cos(n,x)+\sigma_{1xy}\cos(n,y)=0\end{aligned}\right\}$$

on the contour of the hole L_1, (2.3)

where n is normal to L_1 and L_2 directed from the interior of the domain S to the outside.

Each of the stresses σ_{1x},σ_{1y} and τ_{1xy} may be represented in the form of the sum [3:9]:

$$\sigma_x(I)=\sigma_{1x}=\sigma_{1x}^{(0)}+\sigma_{1x}^{(1)};\ \sigma_y(I)=\sigma_{1y}=\sigma_{1y}^{(0)}+\sigma_{1y}^{(1)};$$

$$\tau_{xy}(I)=\tau_{1xy}=\tau_{1xy}^{(0)}+\tau_{1xy}^{(1)}; \qquad (2.4)$$

where: $\sigma_{1x}^{(0)},\sigma_{1y}^{(0)}$ and $\tau_{1xy}^{(0)}$ are stresses in solid (without holes) weighty half-plane with the indicated above acting forces (uniformly distributed pressure on the segment $-\alpha\le t\le\alpha$ of the linear boundary L_2) and are some special solutions of differential equations (2.1).

The constituents $\sigma_{1x}^{(1)},\sigma_{1y}^{(1)}$ and $\tau_{1xy}^{(1)}$ are additional stresses stipulated by the availability of the hole L_1 in medium that weakens it.

Since the components $\sigma_{1x}^{(0)},\sigma_{1y}^{(0)}$ and $\tau_{1xy}^{(0)}$ satisfy equations (2.1), (as special solution of these systems of equations), we can take them in the form[3;9]:

$$\sigma_{1x}^{(0)}=\gamma\cdot\rho\cdot g\left(y+c_1^{(0)}\right);$$

$$\sigma_{1y}^{(0)}=\rho\cdot g\left(y+c_2^{(0)}\right)\ ;\ \tau_{1xy}^{(0)}=0 \qquad (2.5)$$

Here, γ is a constant dependent on medium's property, so called a distance piece factor; for a considerable depth, γ is very close to a unit.

The integration constants $c_1^{(0)}$ and $c_2^{(0)}$ should be defined from the condition of inversion of the components $\sigma_{1x}^{(0)}$ and $\sigma_{1y}^{(0)}$ in $\sigma_{1x}^{(0)}=0$ and $\sigma_{1y}^{(0)}=0$, respectively on the linear boundary L_2. In (2.5), by accepting $y=h$, we get

$$c_1^{(0)}=-\gamma\cdot\rho\cdot g\cdot h;\ c_2^{(0)}=-\rho\cdot g\cdot h \qquad (2.6)$$

Thus, the components $\sigma_{1x}^{(0)}$ and $\sigma_{1y}^{(0)}$ will take the form:

$$\sigma_{1x}^{(0)}=\gamma\cdot\rho\cdot g(y-h);$$

$$\sigma_{1y}^{(0)}=\rho\cdot g(y-h); \qquad (2.7)$$

The stress components $\sigma_{1x}^{(1)},\sigma_{1y}^{(1)}$ and $\tau_{1xy}^{(1)}$ satisfy the homogeneous system of equilibrium equations and compatibility equations (2.1) assuming that they vanish at infinity and on the linear boundary L_2.

So, we have:

$$\sigma_{1x}^{(1)}=0;\ \sigma_{1y}^{(1)}=0\ \text{ on } L_2 \qquad (2.8)$$

$$\sigma_{1x}^{(1)}\cdot\cos(n,x)+\tau_{1xy}^{(1)}\cdot\cos(n,y)=-\left[\gamma\rho g(y-h)\right]\cos(n,x)$$

$$\tau_{1xy}^{(1)}\cdot\cos(n,x)+\sigma_{1y}^{(1)}\cdot\cos(n,y)=-\left[\rho g(y-h)\right]\cos(n,y)$$

on L_1. (2.9)

Since the contour L_1 is free from external forces, then for releasing the stresses $\sigma_{1x}^{(0)},\sigma_{1y}^{(0)}$ and $\tau_{1xy}^{(0)}$ from this contour we should apply the forces with opposite sign $\left(-X_n^{(0)}\right)$ and $\left(-Y_n^{(0)}\right)$.

In the present case, we are interested mainly in the stress state in the vicinity of the contour L_1 (ellipse with two linear cracks) including at the end points of the cracks $\pm e$. Therewith, taking into account the conditions $h>>l>>b$ (for considerable depth of medium), not making any series error, we can reject exact satisfaction of boundary condition (2.8) and in expression (2.9) neglect the quantity y contained in parenthesis of the right hand side. Thus, condition (2.9) is reduced to the form:

$$\begin{aligned}\sigma_{1x}^{(1)}\cdot\cos(n,x)+\tau_{1xy}^{(1)}\cdot\cos(n,y)=\left[h\cdot\gamma\rho g\right]\cdot\cos(n,x)\\ \tau_{1xy}^{(1)}\cdot\cos(n,x)+\sigma_{1y}^{(1)}\cdot\cos(n,y)=\left[h\cdot\rho g\right]\cdot\cos(n,y)\end{aligned} \qquad (2.10)$$

We can introduce the functions $\phi(z)$ and $\Psi(z)$ of the complex variable $z=x+iy$ that are regular in domain S and connected with stress components $\sigma_{1x}^{(1)},\sigma_{1y}^{(1)}$ and $\tau_{1xy}^{(1)}$ by Kolosov – Muskheleshvili – Sherman formulae:

$$\sigma_{1x}^{(1)}+\sigma_{1y}^{(1)}=4R_e\phi^1(z);$$

$$\sigma_{1y}^{(1)}-\sigma_{1x}^{(1)}+2i\tau_{1xy}^{(1)}=2\left[(\bar{z}-z)\cdot\phi''(z)+\psi_1'(z)\right] \qquad (2.11)$$

Taking into account these expressions, boundary conditions (2.10) are transformed to the following form:

$$\phi(t)+(t-\bar{t})\cdot\overline{\phi'(t)}+\overline{\psi(t)}=f_1(t,\bar{t})$$

on L_1 (2.12)

3. Problem for a Weightless Half-Plane under the Action of Uniformly Distributed Pressure P

In the second case, when a uniformly – distributed load of intensity P acts on the linear boundary L_2 on the segment $-a\le t\le a$, the stresses $\sigma_{2x}^{(0)},\sigma_{2y}^{(0)}$ and $\tau_{2xy}^{(0)}$ for an entire weightless half-plane are determined by the expressions [1;6]:

$$\sigma_{2x}^{(0)}=a_1+b_1\cdot(y-h);$$

$$\sigma_{2y}^{(0)}=a_1+b_1\cdot(y-h);$$

$$\tau_{2xy}^{(0)}=b_2\cdot(y-h); \qquad (3.1)$$

where $a_1=-\dfrac{P}{\pi}\cdot(\theta_1-\theta_2)$;

$$\mathrm{b}_1=2\mathrm{P}\cdot\mathrm{a}\cdot\frac{\cos(\theta_1+\theta_2)}{\rho_1\cdot\rho_2};\ b_2=-\frac{2P\cdot a}{\rho_1\cdot\rho_2}\cdot\sin(\theta_1+\theta_2);$$

θ_1 and θ_2 are the angles under which the loaded segment $-a\le t\le a$ from the point Z of a half-plane is seen,

ρ_1 and ρ_2 are the distances from the point Z to the end of the segment $-a \le t \le a$ (i.e. to the point $-a$ and a on the linear boundary L_2).

In the present case, the additional stresses $\sigma_{2x}^{(1)}, \sigma_{2y}^{(1)}$ and $\tau_{2xy}^{(1)}$, stipulated by availability of a hole satisfy the following boundary conditions (similar to the first case, i.e. formulae 2.4). Each of the stresses $\sigma(II) = \sigma_{2x}$, $\sigma_y(II) = \sigma_{2y}$ and $\tau_{xy}(II) = \tau_{2xy}$ are represented in the form of the sum

$$\begin{vmatrix} \sigma_{2x}^{(1)} \cdot \cos(n,x) + \tau_{2xy}^{(1)} \cdot \cos(n,y) \\ = -\left[a_1 + (y-h) \cdot b_1\right] \cdot \cos(n,x) + \left[b_2 (y+h) \cos(n,y)\right] \\ \tau_{2xy}^{(1)} \cdot \cos(n,x) + \sigma_{2y}^{(1)} \cdot \cos(n,y) \\ = -\left[b_2 + (y-h)\right] \cdot \cos(n,x) - \left[a_1 - b_1 (y-h) \cos(n,y)\right] \end{vmatrix} \quad (3.2)$$

Similar to the first problem, the stress components $\sigma_{2x}^{(1)}, \sigma_{2y}^{(1)}$ and $\tau_{2xy}^{(1)}$ are defined by analytic functions by the known formulae (see. 2.11). Then, boundary conditions (3.2) are transformed also to the following form

$$\phi(t) + (t - \bar{t}) \cdot \overline{\phi'(t)} + \overline{\psi(t)} = f_2(t, \bar{t}) \quad (3.3)$$

In equations (2.12) and (3.3), the variable t is an affix of the points of the contour L_1 as for a weighty half-plane (the first problem):

$$f_1(t, \bar{t}) = k_1 \cdot t + k_2 \cdot \bar{t};$$
$$k_1 = \frac{1}{2}(1+\gamma)\rho g \cdot h;$$
$$k_2 = \frac{1}{2}(1-\gamma)\rho g \cdot h; \quad (3.4)$$

At a considerable depth $\gamma = 1$, then we get $k_1 = \rho g h$; $k_2 = 0$

For a weightless half-plane, under the action the uniformly-distributed load of intensity P, on the segment $-a \le t \le a$, of the boundary L_2, the right side of equation (3.3) has the form [1;6;7]:

$$f_2(t, \bar{t}) = a_1 \cdot (y - h);$$
$$a_1 = -\frac{p}{\pi}(\theta_1 - \theta_2) \quad (3.5)$$

It is seen that boundary conditions for determining additional stresses at both variants of active external forces are reduced to determining two analytic functions $\phi(z)$ and $\psi(z)$ (by comparing 2.12 and 3.3 it is seen that the left side are same in both cases, the right side has the form of 3.4 and 3.5, respectively).

For both variants, the desired functions $\phi(z)$ and $\psi(z)$ are representable in the form [3-10].

$$\phi(z) = \phi_0(z) + \phi_1(z);$$
$$\psi(z) = \psi_0(z) + \psi_1(z). \quad (3.6)$$

Here, $\phi_0(z)$ and $\psi_0(z)$ are analytic functions regular everywhere in the lower half-plane, with a rectilinear boundary L_2, $\phi_1(z)$ and $\psi_1(z)$ are holomorphic functions in domain outside of the hole L_1 (ellipse with two rectilinear cracks).

The functions $\phi_1(z)$ and $\psi_1(z)$ regular outside of the contour L_1 (outside of the hole) are representable in the form [1;6;9;10]

$$\phi_1(z) = \sum_{k=1}^{\infty} \alpha_k \cdot \xi_1^{-k};$$
$$\psi_1(z) = \sum_{k=1}^{\infty} \beta_k \cdot \xi_1^{-k} \quad (3.7)$$

Assuming that the coefficients α_k and β_k in expansions (3.7) are known, by the Muskheleshvili method [1](the method based on the use of a Cauchy type integral) on the linear boundary L_2 from the boundary condition 2.12 or from 3.3 for $f_1(t, \bar{t}) = f_2(t, \bar{t}) = 0$ we find the functions $\phi_0(z)$ and $\psi_0(z)$

$$\phi_0(z) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\beta_k}{(z-2ih)^k} + (\bar{z} + 2ih) \cdot \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k \cdot \alpha_k}{(z-2ih)^k} \cdot \frac{1}{\omega'(z-2ih)}$$

$$\psi_0(z) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\alpha_k}{(z-2ih)^k} + (\bar{z} + 2ih) \cdot \phi_0^1(\bar{z} + 2ih) \quad (3.8)$$

We should recall that the following expressions hold on the linear boundary L_0 : $t = -\bar{t}$; $t = x + ih$; $x + ih$; $\bar{t} = x - ih$; $t - \bar{t} = x + ih - x + ih = 2ih$; $t = \bar{t} + 2ih$; $\bar{t} = t - 2ih$.

Now, pass to boundary conditions on the contour of the hole L_1.

The exterior of the contour L_1 (ellipse with two rectilinear cracks) is mapped onto the exterior of a unit circle by means of the function [4;5].

$$z = A \cdot \xi_1 \sum_{n-0}^{\infty} \Pi_n \cdot \xi_1^{-n}; \quad A = \frac{a_1 + b_1}{2};$$
$$m = \frac{a-b}{a+b}, \quad (3.9)$$

where $\Pi_n = \sum_{n-0}^{\infty} \gamma_{n-1} \cdot T_{n-k}$;

$$T_{(\nu_1)} = \sum_{n_1 = \nu_1 - 2E\left(\frac{\nu_1}{2}\right)}^{\infty} {}^{*} m^{\frac{\nu_1 - n_1}{2}} \cdot \gamma_{-1}^{-(\nu - n_1)} \cdot L_n .$$

The quantities L_n are determined according to[4;5] from the following condition

---

[1] The method elaborated by N. I. Muskheleshvili [6] is based on the use of a Cauchy type integral. For example:

$$\frac{1}{2\pi i} \int_{\gamma} \frac{\phi(\tau) d\tau}{\tau - \xi} = \phi(\xi); \quad \frac{1}{2\pi i} \int_{\gamma} \frac{\overline{\psi(\tau)} d\tau}{\tau - \xi} = 0; \quad \frac{1}{2\pi i} \int_{\gamma} \frac{\overline{\phi(\tau)} d\tau}{\tau - \xi} = 0;$$

$$\frac{1}{2\pi i} \int_{\gamma} \frac{\omega(\tau)}{\omega'(\tau)} \frac{\overline{\phi'(\tau)}}{\tau - \xi} d\tau = 0 \text{; when } |\xi| \le 1 \quad \frac{1}{2\pi i} \int_{\gamma} \frac{d\tau}{\tau} = -1;$$

$$\frac{1}{2\pi i} \int_{\gamma} \frac{\omega(\tau)}{\omega'(\tau)} \frac{\overline{\phi'(\tau)}}{\tau - \xi} d\tau = \frac{\omega(\xi)}{\omega'(\xi)} \overline{\phi'(\xi)} \text{ when } |\xi| \ge 1 \text{ and etc.}$$

$$L_n + \sum_{n_1=1}^{n} L_{n-n_1} \cdot g_{n_1}^{(n)} = 0 \tag{3.10}$$

where $g_n^{(k)} + \sum_{n_1=0}^{n} g_{n_1}^{(1)} \cdot g_{n-n_1}^{(k-1)}$; $g_{n_1}^{(1)} = \frac{\gamma_{n-1}}{\gamma_{-1}}$

The function $\xi = f(z)$ inverse to the mapping function (3.9) is found in the form [4;5]:

$$\xi_1 = \frac{z}{A} \cdot \sum_{n_1=0}^{n} V_n \left(\frac{A}{z}\right)^n \tag{3.11}$$

Where $V_n = \sum_{\nu=n-2E\left(\frac{n}{2}\right)}^{n} {}^{*}\alpha^{(1)}_{\frac{n-\nu}{2}} \cdot H_\nu$;

$H_\nu = \sum_{k=\nu-2E\left(\frac{\nu}{2}\right)}^{\nu} \delta_{k-1} \cdot \frac{\chi_{\nu-k}^{(n)}}{2}$;

The quantities $\chi_n^{(n)}$ are determined similar to the quantities L_n replacing in (3.10) $g_n^{(k)}$ by $\alpha_n^{(k)}$, moreover $\alpha_n^{(n)}$ will be defined from the recurrent formula (3.11) having taken $\alpha_n^{(k)}$ instead of $g_n^{(k)}$.

For an ellipse, the first six quantities $\alpha_n^{(k)}$, $g_n^{(k)}$ are found and arranged in the table (see. [4;5]).

Allowing for expansions (3.6), (3.7), (3.8) and (3.9), boundary condition (2.12) or (3.3) on the contour of the hole L_1 is reduced to the form (after some mathematical transformations and reasonings, passing to a new variable τ, and taking into account that $\tau \cdot \overline{\tau} = 1$ holds on a unit circle):

$$\begin{aligned} &\sum_{k=1}^{\infty} \alpha_k \cdot \tau^{-k} + \tau^2 \sum_{k=0}^{\infty} \tau^{-k} \cdot T_7(k) + \tau^2 \cdot \sum_{k=1}^{\infty} \tau^k \cdot T_8(k) \\ &+ \tau^2 \cdot \sum_{k=1}^{\infty} \tau^k \cdot T_6(k) - \sum_{k=1}^{\infty} \tau^k \cdot T_5(k) + \sum_{k=1}^{\infty} \beta_k \cdot \tau^k \\ &= \delta_1 \cdot t + \delta_2 \cdot \overline{t} = \delta_1 \cdot A \cdot \tau \cdot \sum_{n=0}^{\infty} \Pi_n \cdot \tau^{-n} \\ &+ \delta_2 \cdot A \cdot \tau^{-1} \cdot \sum_{n=0}^{\infty} \Pi_n \cdot \tau^n \quad on \quad L_1 \end{aligned} \tag{3.12}$$

where $\delta_1 = k_1 = \frac{1}{2}(\gamma - 1) \cdot \rho \cdot g \cdot h$; $\delta_2 = k_2 = \frac{1}{2}(\gamma + 1) \cdot \rho \cdot g \cdot h$ for a weighty half-plane.

$\delta_1 = -a = \frac{P}{2\pi} \cdot (\theta_1 - \theta_2)$, $\delta_2 = 0$ for a weightless half-plane under the action of a uniformly distributed load of intensity P on the segment $-a \le t \le a$ of the rectilinear boundary L_0.

In equation (3.12) having equated the coefficients under the same degrees of τ, we get the following two systems of infinite algebraic equations for determining the coefficients α_k and β_k:

$$\alpha_k + T_7(k-2) \cdot \varepsilon_1 = \delta_1 A \cdot \Pi_{k+1} + \delta_2 A \cdot \Pi_0 \cdot \varepsilon_2 \tag{3.13}$$

$$\begin{aligned} &T_7(2-k) \cdot \varepsilon_3 + T_6(k-2) \cdot \varepsilon_1 - T_5(k) + \beta_k + T_8(k-2) \cdot \varepsilon_1 = \\ &= \delta_1 \cdot A \cdot \Pi_0 \cdot \varepsilon_2 + \delta_2 \cdot A \cdot \Pi_{k+1}, \end{aligned} \tag{3.14}$$

where:

$$\varepsilon_1 = \begin{vmatrix} 0 & for\ k \le 2 \\ 1 & for\ k > 2 \end{vmatrix};\ \varepsilon_2 = \begin{vmatrix} 0 & for\ k \ne 2 \\ 1 & for\ k = 2 \end{vmatrix};\ \varepsilon_3 = \begin{vmatrix} 0 & for\ k > 2 \\ 1 & for\ k \le 2 \end{vmatrix};$$

The values of the quantities $T_1(k) \div T_7(k)$ are not cited here because of their bulky form.

Solving jointly systems of equations (3.13) and (3.14) (having taken some first terms from each system), the coefficients α_k and β_k are determined. Analytic functions $\phi_1(z)$ and $\psi_1(z)$ are determined for each of indicated problems respectively, from formula (3.7).

Having known the values of the functions $\phi(z)$ and $\psi(z)$, the additional stresses $\sigma_x^{(1)}, \sigma_y^{(1)}$ and $\tau_{xy}^{(1)}$ are determined by formula (2.11).

The found stress components $\sigma_{ix} = \sigma_{ix}^{(0)} + \sigma_{ix}^{(1)}$, $\sigma_{iy} = \sigma_{iy}^{(0)} + \sigma_{iy}^{(1)}$ and $\tau_{ixy} = \tau_{ixy}^{(0)} + \tau_{ixy}^{(1)}$; $i = 1;2$ for each of the mentioned variants of active loads are given below (calculated at typical points of the holes $z = \pm ib$; $z = a$ for $l = a$, where l is the length of the aperture and also at the points $z = e$, for $l = e - a$).

On the base of force action independence principle, by the superposition method we can determine stress state under simultaneous action of loads indicated in both cases:

$$\begin{aligned} \sigma_x &= \sigma_x(I) + \sigma_x(II); \\ \sigma_y &= \sigma_y(I) + \sigma_y(II); \\ \tau_{xy} &= \tau_{xy}(I) + \tau_{xy}(II), \end{aligned} \tag{3.15}$$

where $\sigma_x(I)$, $\sigma_y(I)$ and $\tau_{xy}(I)$ belong to the first, $\sigma_x(II)$, $\sigma_y(II)$ and $\tau_{xy}(II)$ to the second problem.

4. Numerical Results

4.1. Uniformly Distributed Pressure of Intensity P Acts on a Rectilinear Boundary L_2 of a Weightless Half-Plane on the Segment $-a^* \le t \le a^*$

For an entire (without hole) half-plane, this problem has been solved in [1;6;7;8]. Using the obtained results for an entire half-plane by the method indicated above, i.e. solving the system of equations (3.13 and 3.14) for the right side in the form (3.12), by the known formulae (2.11) we find additional stresses stipulated by the availability of a hole. The value of complete – total stresses for different variants for dimensions of a hole and depth h are the followings: (for typical points of the hole)

Variant I: $h = a$; $a = 2b$;

$e = a$; $l = 0$; $z = a$; the point « A » in the draft

$\sigma_x = 4{,}62 \cdot \frac{P}{2\pi}$; $\sigma_y = 1{,}18 \cdot \frac{P}{2\pi}$

$z = ib$ at the point «D».

$\sigma_x = 6,94\frac{P}{2\pi}$; $\sigma_y = 10,74\cdot\frac{P}{2\pi}$

for $e = a + l;\ l = b;$ at the point $z = a + b$; $\sigma_x = 3,93\cdot\frac{P}{2\pi}$; $\sigma_y = 1,26\cdot\frac{P}{2\pi}$ for $z = ib$; $\sigma_x = 5,65\frac{P}{2\pi}$; $\sigma_y = 1,82\cdot\frac{P}{2\pi}$.

for $e = a + l;\ l = 2b;$ at the point $z = a + 2b$; $\sigma_x = 3,01\frac{P}{2\pi}$; $\sigma_y = 1,42\cdot\frac{p}{2\pi}$.

Variant II: $h = 2a$; $a = 2b$; $a^* = a$; $l = 0;\ e = a;$ at the point $z = a$; $\sigma_x = 3,213\cdot\frac{P}{2\pi}$; $\sigma_y = 0,35\cdot\frac{P}{2\pi}$;

For $z = ia$; $\sigma_x = 2,77\cdot\frac{P}{2\pi}$; $\sigma_y = -0,68\cdot\frac{P}{2\pi}$; $l = b;\ e = a + l;$ at the point $z = a + b$; $\sigma_x = 2,57\cdot\frac{P}{2\pi}$; $\sigma_y = 0,57\cdot\frac{P}{2\pi}$;

Variant III: $h = 10a$; $a = 2b$; $a^* = a$ $l = 0;$ at the points $z = a;$ $z = ib;$ $\sigma_x^{(1)} = \sigma_y^{(1)} = 0$

i.e. at considerable depths the additional stresses equal to zero (influence of a hole and load on a rectilinear boundary L_2 almost doesn't influence the stress distribution).

4.2. A weighty Half-Plane Weakened by a Hole L_1 (Ellipse with Two Rectilinear Cracks) is Subjected Only to the Action of Gravity

The contours L_1 and L_2 are free from external forces. Solving the system of equations for the right side in the form (3.12), we define the desired coefficients α_k and β_k for different relative dimensions. Then, by formula (2.11) we find additional (as the result, total stresses) stresses: $\sigma_{1x}^{(1)}, \sigma_{1y}^{(1)}$ and $\tau_{1xy}^{(1)}$ at the typical points of the hole L_1 (since we are interested only in stress concentration at the points of hole's contour). Below we give the found values of these stresses for different variants:

Variant I: $h = 2a$; $l = a$; $b = 0$ for $z = a$; $\sigma_x^{(1)} = 1,81H$; $H = \rho gh$; $\sigma_x^{(total)} = 0,81H$; for $z = ib = 0$; $\sigma_x^{(1)} = 2,3H$; $\sigma_x^{(total)} = 1,3H$.

Variant II: $h = 4a$; $l = a$; $b = 0$ for $z = a$; $\sigma_x^{(1)} = 1,17H$; $\sigma_x^{(total)} = 0,17H$ при $z = a$; $\sigma_x^{(1)} = 1,27H$; $\sigma_x^{(total)} = 0,27H$.

Variant III: $h = \infty$; $\gamma = 1$ (distance piece factor equals to one), then at all the points of the contour L_1 we have

$\sigma_x^{(1)} = \rho gh$; $\sigma_x^{(total)} = 0$.

5. Conclusions

The quantities of additional $\sigma_x^{(1)}$ and total stresses on the $\sigma_x^{(sides)}$ of a rectilinear aperture (i.e. at the end points of the crack and in the middle of this crack when $x = \frac{l}{a};\ y = 0$) considerably increase when the crack of L_1 approaches to the linear contour L_2.

As calculations show, such type stresses have a particular local character and as a rule, they rapidly drop (weaken) when they are removed from the vertex of the crack, where they have maximal value. When $l = 0$, $a/b = 2$; $e = a$ the internal contour L_1 will become an ellipse, and the problem on definition of additional stresses will be similar to that in the paper[9]. Therefore, here we don't cite the results.

REFERENCES

[1] Amanzadeh Yu. A. Theory of elasticity. M. Vysshaya shkola 1976. 272 p. (Russian)

[2] Berezhnitskiy L. T., Delyavskiy M. V., Panasyuk V. V. Bending of elastic plates with crack type defects. Kiev, Naukova Dumka, 1979. 400 p. (Russian)

[3] Kosmodomianskiy A. S. Plane problem of theory of elasticity for plates with holes, cuts and bulgs. Kiev. Vysshaya Shkola. 1975. 227 p. (Russian)

[4] Kuliyev S. A. Two-dimensional problems of theory of elasticity. M. Stroyizdat 1991, 351 p. (Russian)

[5] Kuliyev S. A. Conformally-mapping functions of complex domains. Baku, Azerneshr. 2004, 372 p. (Russian)

[6] Muskheleshvili N. I. Some basic problems of mathematical theory of elasticity. M. Nauka, 1966, 707 p. (Russian)

[7] Savin T. N. Stress distribution near holes. Kiev, Naukova dumka, 1968, 887 p. (Russian)

[8] Timoshenko S. R., Goodyear Dt. Theory of elasticity M. Nauka, 1975, 575 p. (Russian)

[9] Sherman D. I. Elastic weighty half-plane weakened by an elliptic from hole arranged close to its boundary. Collection: Problemy mechaniki sploshnoy sredy: M. Izd-vo ANSSSR, 1961. pp. 527-563. (Russian)

[10] Sherman D. I. On stresses of weighty half-plane weakened by two annular holes. PMM, 1952, vol. XV, issue 3. pp. 297-316. (Russian)

[11] Sie s., Paris P., Erdogan F. Coefficients of stress concentration at the vertex of the crack under plane tension and bending of plates. // Tr. Amer. Obsh. inzh. mech. Ser. E. Prikl. Mekh. 1962. №2, pp.102-108.