The Prevailing Time Dilation Formula $t = \dfrac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ of Einstein's SRT is Proven Invalid and Untenable Based on Accurate Analysis

Liu Ke Dian

China

Abstract The prevailing formula of time dilation assertion of Einstein's SRT is physically and mathematically proven invalid and untenable based on accurate analysis from decisive points of view. This article theoretically and practically falsifies the time dilation formula $t = \dfrac{t'}{\sqrt{1-\frac{v^2}{c^2}}}$ through analyzing and clarifying the exact actual physical situations. Concretely, the article falsifies the formula through two decisive points of view: Point 1: the derivation of the prevailing time dilation formula is based solely on a particular situation; hence the formula is incompatible with and contradicting to any other formulas that are deduced based on universal situations. Point 2: when considering wave property and particle property of the light pulse, the geometrical relationship that meet the Pythagorean Theorem between the two measured light speeds by stationary observer and moving observer which being based to derive the time dilation formula is not existed.

Keywords Time dilation, Special relativity theory, Einstein, Frame of reference, Length contraction

1. Briefing the Derivation of Time Dilation Formula

1.1. Extraction of Book *Fundamental of Physics*, 8th Edition

"A vehicle moving to the right with a speed v is shown in Figure 1.1a. A mirror is fixed to the ceiling of the vehicle, and observer O' at rest in this system holds a laser a distance d below the mirror. At some instant, the laser emits a pulse of light directed toward the mirror (event 1), and at some later time after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer O' carries a clock C' and uses it to measure the time interval $\triangle t_p$ between these two events.

The Pythagorean Theorem gives Solving for $\triangle t$ gives

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Solving for Δt gives $\Delta t = \dfrac{2d}{\sqrt{c^2-v^2}} = \dfrac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$

Because $\triangle t_p = 2d/c$, we can express this result as

$$\Delta t = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\Delta t_p$$

Where $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

The final report briefly rewritten as $\Delta t = \gamma\Delta t_p$ is, in fact, called 'dilation of time'."

1.2. Extraction of Book *Concepts of Modern Physics*, Sixth Edition

"To see how time dilation comes about, let us consider two clocks shown in **Fig.1.2**. In each clock a pulse of light is reflected back and forth between two mirrors L_0 apart.

One clock is at rest on the ground and the other is in a spacecraft that moves at the speed v. An observer in the laboratory watches both clocks:

The time interval between ticks is the proper time t_0 and the time needed for the light pulse to travel between the mirrors at the speed of light c is $t_0/2$. Hence $t_0/2 = L_0/c$.

Since L_0 is the vertical distance between the mirrors, then proper time is

$$t_0 = \frac{2L_0}{c} \Longrightarrow L^2 = \left(\frac{ct}{2}\right)^2 = L_0{}^2 + \left(\frac{vt}{2}\right)^2 \Longrightarrow t^2$$

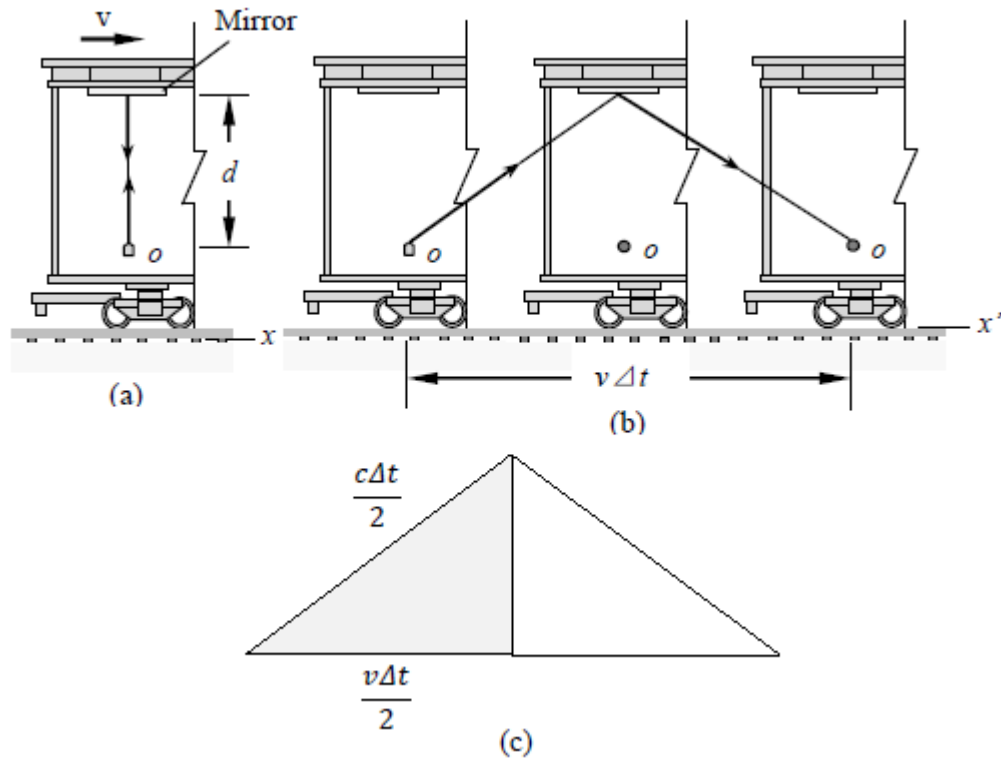$$= \frac{(2L_0)^2}{c^2(1-\frac{v^2}{c^2})} \Longrightarrow t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Figure 1.1. (a) A mirror is fixed to a moving vehicle, and a light pulse is sent out by observer O' at rest in the vehicle. (b) Relative to a stationary observer O standing alongside the vehicle, the mirror and O' move with a speed v. Note that what observer O measures for the distance the pulse travels is greater than 2d. (c) The right triangle for calculating the relationship between $\triangle t$ and $\triangle t_p$
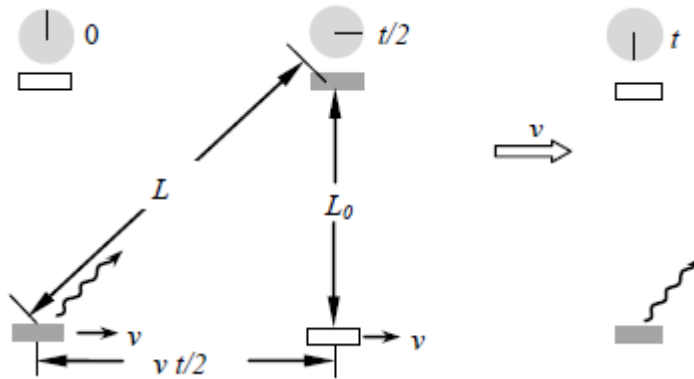

Figure 1.2

2. (Point of View 1): Derives Based on Universal Situation Manifest Time Dilation Formulas are Self-Confliction and Untenable

No matter whether it is right or wrong actually, the prevailing time dilation formula $t = \dfrac{t'}{\sqrt{1-\frac{v^2}{c^2}}}$ was not derived based on or covering universal situations; it was merely based on a particular motion relationship between light pulse speed and moving vehicle speed.

2.1. Depiction of Universal Situation

According to the time dilation thinking in above two citations, Einstein and his proponents considered only a particular situation that the propagation direction of light pulse is perpendicular to the direction of moving v. They did not consider the universal situation that the direction of light pulse always has an arbitrary angle φ to the mirrors and moving speed v, as shown in **Fig.2**.

Considering universal situation (we represent $\dfrac{t}{2}$ as t for convenient), from the triangle **ABC** of **Fig.2**, we have

$$L^2 = {L_0}^2 + (vt)^2 - 2L_0 vt \cos(\pi - \varphi) \qquad (2.1)$$

Since $t_0 = \frac{L_0}{c}$ is the time interval t_0 between ticks on the clock on the ground, so

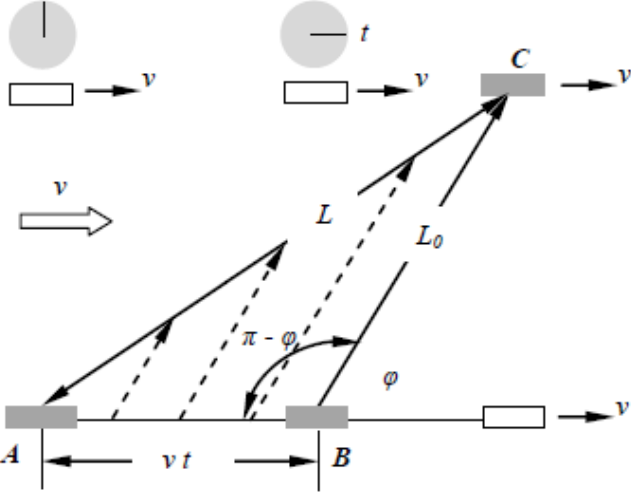$$L_0 = ct_0 \text{ and } L = ct \qquad (2.2)$$

Figure 2

Substituting (2.2) into (2.1) obtains

$$L^2 = (ct)^2 = (ct_0)^2 + (vt)^2 - 2ct_0vt\cos(\pi - \varphi)$$
$$= (ct_0)^2 + (vt)^2 + 2ct_0vt\cos\varphi \qquad (2.3)$$
$$= (ct_0)^2 + (vt)^2 + 2ct_0vt - 2ct_0vt + 2ct_0vt\cos\varphi \qquad (2.4)$$

Simplified, then the universal relationship of t and t_0 is expressed as

$$(ct)^2 = (ct_0 + vt)^2 + 2ct_0vt(\cos\varphi - 1) \qquad (2.5)$$

2.2. Analyzing the Case When $\varphi = 0$

When φ approaches zero, the limiting value of $2ct_0vt(\cos\varphi - 1)$ is zero or 0, therefore (2.5) is reduced to
$$(ct)^2 = (ct_0 + vt)^2$$

Find the square root of both sides, obtains $ct - vt = ct_0$, i. e.,

$$t = t_0\frac{c}{c-v}. \qquad (2.6)$$

2.3. Analyzing the Case When $\varphi = \frac{\pi}{2}$

When φ approaches $\frac{\pi}{2}$, $2ct_0vt\cos\varphi$ equals zero (0), (2.3) is reduced to

$$(ct)^2 = (ct_0)^2 + (vt)^2$$

Thus Einstein obtained his time dilation formula,

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}. \qquad (2.7)$$

2.4. Analyzing the Case When $\varphi = \pi$

When φ approaches π, the limiting value of

$2ct_0vt(\cos\varphi - 1)$ is $-ct_0vt$ therefore (2.5) is reduced to $(ct)^2 = (ct_0 - vt)^2$.

Find the square root of both sides, obtains $ct = ct_0 - vt$, i.e.,

$$t = t_0\frac{c}{c+v} \qquad (2.8)$$

2.5. Summarize the All Cases of Extreme Conditions

The time dilation formulas manifest completely different and self-confliction as

$$(ct)^2 = (ct_0 + vt)^2 + 2ct_0vt(\cos\varphi - 1)$$
$$\text{when } \varphi \text{ is arbitrary value} \qquad (2.9)$$

$$t = t_0\frac{c}{c-v} \quad \text{when } \varphi = 0 \qquad (2.10)$$

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{when } \varphi = \frac{\pi}{2} \qquad (2.11)$$

$$t = t_0\frac{c}{c+v} \quad \text{when } \varphi = \pi \qquad (2.12)$$

The prevailing formula $t = \dfrac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is only account of a particular relationship of the directions of the light pulse and motion of the moving vehicle (light source); it is not general formula of universal situation and is contradicting to other formulas of altered arbitrary situations.

This self-confliction completely and firmly falsifies Einstein's time dilation assertion.

Therefore, the prevailing time dilation formula of Einstein's SRT is invalid and untenable, i.e.,

$$t \neq \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (\textbf{2.9}) \text{ and } t = t_0 \qquad (2.13)$$

(The conclusive formulas of situations when $\varphi = 0$ and when $\varphi = \pi$ are also contradicting the formulas of Einstein's Lorentz –Transformation, we do not discuss the details in this paper.)

3. (Point of view 2): The Two Measured Light Speeds by Stationary Observer and Moving Observer have no Geometrical Relationship to Meet the Pythagorean Theorem

3.1. Slope Direction or Hypotenuse Light Pulse for Stationary Observer is not Existed

As a light pulse, suppose it consists of N wave cycles, its duration time must last a time interval say $N\Delta t$ which is larger than zero $(0 < \Delta t < NT)$ and before end of the emission, the light source moves away from the initial 0 time position.
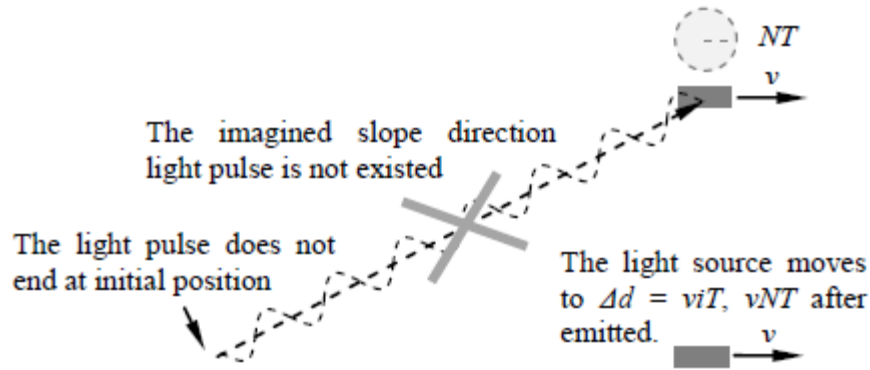As shown in **Fig.3.1a** and **Fig.3.1b**.
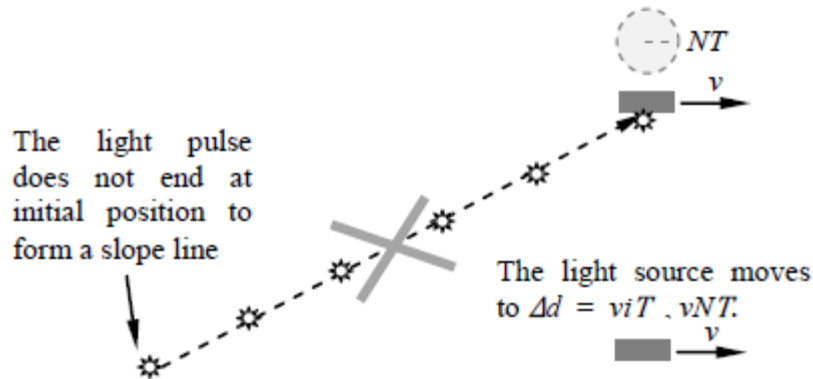
Figure 3.1a

Figure 3.1b

The light pulse should not continue to emit wave cycles from a position which is not or no more a light source. After *ith* cycles or *Nth* cycles are emitted, the light pulse must be continuously emitted from the moved position a distance $\Delta d = viT$ or vNT away from the initial position. Therefore, the imagined slope (or hypotenuse) direction light pulse is not existed for stationary no matter wave property or particle property is considered.

Therefore, there is no a geometrics relationship of hypotenuse and opposite sides of a right triangular to apply Pythagorean's theorem to derive the formula $t = \dfrac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

Figure 3.2

3.2. Accurate Analyses Considering Wave Property of the Light Pulse

Suppose the light pulse consists of *N* wave cycles, as shown in **Fig.3.2**.

In $N\Delta t$ time interval, the *N* cycles of light pulse complete the emission and the head of the first cycle approaches the up mirror.

3.2.1

When the pulse starts off from the down light source it is born with horizontal speed *v*, due to the moving speed *v* of light source. Hence, whenever we begin to calculate the propagation distance of the light pulse, the light source is moving away from the initial position concretely.

When time is Δt, $i\Delta t$, and $N\Delta t$ successively, the down light source moves forward to new positions with the concrete delta distances away from the initial 0 time position,

$\Delta d_1 = v\Delta t$, $\Delta d_i = vi\Delta t$, and $\Delta d_N = vN\Delta t$ respectively.

After the first cycle is emitted, there is a time interval $\Delta t = T$ delay, the light source is moved to position of $\Delta d_1 = vT$ to emit the second cycle.

After the *ith* cycle is emitted, there is a time interval $\Delta t = iT$ delay, the light source is moved to position of $\Delta d_i = viT$ to emit next cycle and so on.

Thus, **the successive wave cycle series of the light pulse form a vertical line rather than any slope line to the stationary observer**, as shown in **Fig.3.2**.
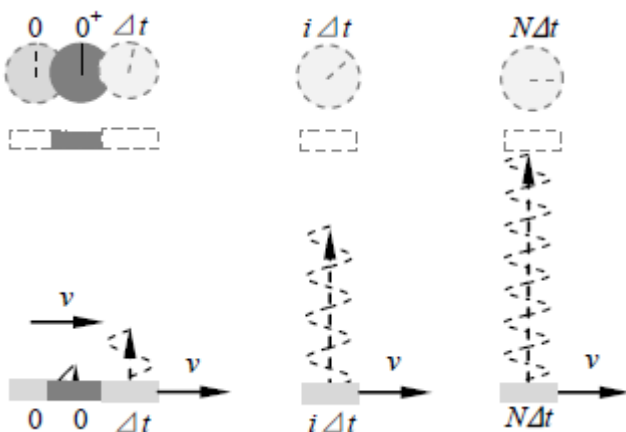
Therefore, both the stationary observer and moving observer measure vertically light pulse. Thus, there is no a right triangular to apply Pythagorean's theorem to derive the formula $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ and thus $t \neq \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$. The prevailing time dilation formula $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is invalid.

Therefore, for the light pulse, the light speeds measured by stationary observer and moving observer are the same

$$c = \frac{\lambda}{\Delta t} = \frac{\lambda}{\Delta t_0} \text{ reduce to } c = \frac{\lambda}{t} = \frac{\lambda}{t_0} \text{ when } \Delta d_1 = v\Delta t$$

$$c = \frac{i\lambda}{i\Delta t} = \frac{i\lambda}{i\Delta t_0} \text{ reduce to } c = \frac{\lambda}{t} = \frac{\lambda}{t_0} \text{ when } \Delta d_i = vi\Delta t$$

$$c = \frac{N\lambda}{N\Delta t} = \frac{N\lambda}{N\Delta t_0} \text{ reduce to } c = \frac{\lambda}{t} = \frac{\lambda}{t_0} \text{ when } \Delta d_N = vN\Delta t$$

Hence all the equations of light pulse speed (for both stationary observer and moving observer) are the same

$$c = \frac{\lambda}{t} = \frac{\lambda}{t_0} \text{ reduce to } t = t_0$$

Therefore $t = t_0$ (for the whole light pulse emitting time interval of $\Delta t > 0$)

3.3. Accurate Analyses Considering Particle Property of the Light Pulse

As a light pulse, suppose it consists of N particles, the light pulse (particles) must continuously moves position away a distance $\Delta d_i = vi\Delta t$ from the initial position.

The particles of light pulse must be continuously emitted from a moved position a distance $\Delta d_i = vi\Delta t$ ($1 < i < N$) away from the initial position of the light source when $\Delta t > 0$ as shown in **Fig.3.3**.

3.3.1

When the pulse starts off from the down light source, it is born with horizontal speed v, due to the moving of light source in speed v. As shown in **Fig.3.3**.
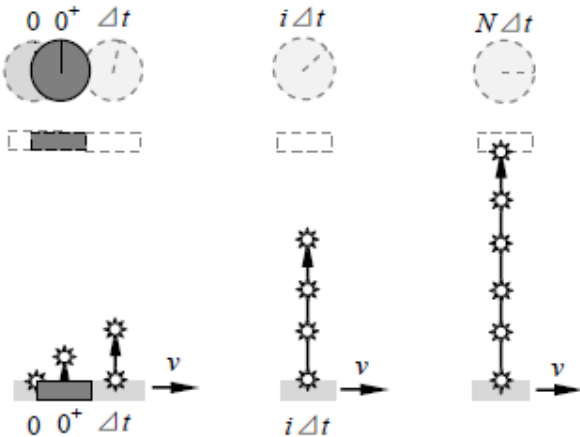

Figure 3.3

When time $= \Delta t$, $= i\Delta t$, and $= N\Delta t$, successively, the down light source moves forward to new positions with the concrete delta distances or displacements

$$\Delta d_1 = v\Delta t, \ \Delta d_i = vi\Delta t, \text{ and } \Delta d_N = vN\Delta t \text{ respectively.}$$

3.3.2

The first particle is starting off from the initial 0 time position. As shown in **Fig.3.3**.

The second particle is starting off from the $\Delta d_1 = v\Delta t$ position away from the 0 time position. It is no way for the *2th* particle to start off from the initial 0 time position since the light source is moving away. As shown in **Fig.3.3**.

Inductively, the *ith* particle must be starting off from the position of $\Delta d_i = vi\Delta t$ away from the 0 time position.

The *Nth* particle must be starting off from the position of $\Delta d_N = vN\Delta t$ away from the 0 time position respectively. Of course, it is no way for the *Nth* particle to start off from the initial 0 time position to form a hypotenuse direction light pulse speed at all.

Thus, **the successive particle series of the light pulse form a vertical line rather than any slope line to the stationary observer,** as shown in **Fig.3.3**.

Hence, there is no a geometrics relationship of hypotenuse and opposite sides to form a right triangular to apply Pythagorean's theorem to derive the formula $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$. Therefore, for all the light pulse emitting time interval of Δt, $i\Delta t$ and $N\Delta t > 0$,

$$t \neq \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad t = t_0 .$$

The prevailing time dilation formula $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ is invalid.

4. Conclusions

From decisive points of view to accurately reanalyze and re-clarify the details of derivation of the time dilation formula, the conclusions and deduced formulas show that the prevailing time dilation formula $t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ of Einstein's SRT is self-confliction, untenable and invalid.

REFERENCES

[1] B. Arthur: Concepts of Modern Physics, 6th edition. McGraw-Hill Higher Education.

[2] J. Walker: Fundamental of Physics, 8th edition. HALLIDAY · RESNICK.

[3] Einstein, Albert: Relativity, the Special and the General Theory. The Project Gutenberg eBook.

[4]   K. Liu: Refuting Einstein's special relativity theory systematically. American Academic Press.