

Regarding the Special Theory of Relativity

Toshiaki Ishikawa

Retired Seiya-Ward, Yokohama-City, Kanagawa-Pref, Japan

Abstract The Special Theory of Relativity and the Light-quantum Hypothesis were proposed in 1905 by Albert Einstein, who later introduced the General Theory of Relativity to supplement the Special Theory of Relativity's shortcomings. The study reported here was conducted to address these shortcomings using the Light-quantum Hypothesis to improve the theory itself. Expansion of the improved Special Theory of Relativity enabled the discussion of motion equations for a light quantum and planet, which further led to a formula regarding planetary perihelion advancement. Related analysis has indicated that the General Theory of Relativity has no significance in physics.

Keywords Special Relativity Theory, General Relativity Theory, Motion Equation of Planet, Motion Equation of Light, Corpuscular Character of Light, Planetary Perihelion, Lorentz's Ether Theory, Space Distortion, Escape from Gravity, Quantum Mechanics

1. Introduction

The Special Theory of Relativity (STR) and the Light-quantum Hypothesis were proposed by Albert Einstein in 1905. The STR suggests that time and space are interrelated, which is supported by results from a number of experiments.

However, Einstein developed the General Theory of Relativity (GTR) because he considered the STR insufficient. As discussions on the STR usually start with the Special Principle of Relativity and the Principle of the Constancy of the Velocity of Light, the mathematical roles of the two principles need to be considered. The first principle (i.e., that of relativity) deals with the existence of a function maintained in a constant form via Lorentz transformation [1-6], and the second involves the determination of a function form. For the constancy of the velocity of light (as the second principle) to contain a function form, this velocity must not be tangible. This is clearly a self-contradiction. The constancy of the velocity of light has a sufficient condition to determine a function form, but does not have a necessary condition (i.e., it does not have to be the velocity of light). This study's discussion is based on the relativity of space (premised on the concept that the recession velocity of the coordinate origins of two inertial systems are equal), which satisfies the necessary and sufficient conditions to determine a function form instead of the constancy of the velocity of light.

Expanding the improved STR, which incorporates the

Light-quantum Hypothesis, allows handling of the equations of motion for a light quantum and planet. It further highlights how a light quantum and particle can escape from any gravitational field, and allows derivation of the formula for the advance of the planetary perihelion. The subsequent detailed analysis indicated that the GTR lacks significance.

2. STR

The Theory of Relativity generally relates to the relationship between independent measurements of particle motion in two separate coordinate systems. The STR incorporates the static coordinate system K_0 and the inertial coordinate system K_1 , which moves at a constant velocity v to K_0 . In line with the principle of special relativity, a function that always expresses the laws of physics in all inertial systems is needed. This is generally expressed as

$$F = F(t, x, y, z).$$

Here, the variants are time t and the coordinates x , y and z . With this, the Special Principle of Relativity is given as [7]

$$F(t_0, x_0, y_0, z_0) \equiv F(t_1, x_1, y_1, z_1),$$

where the measured values of a particle in the K_0 system are labeled 0, and 1 is used for those in the K_1 system. When relativity in space is applied, differentiation of both sides of the equation with v is necessary. The relationship between the time t_0 in the K_0 system and the time t_1 in the K_1 system is expressed as

$$\frac{t_1}{t_0} = \frac{1}{A}, \quad (1)$$

where A is a function of v . The differential equation to be

* Corresponding author:

tf-ishikawa@jcom.home.ne.jp (Toshiaki Ishikawa)

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satisfied by A is determined as [8]

$$\frac{A'}{A} = \frac{(A^2 - 1)}{v}. \quad (2)$$

This is solved as

$$\left| \frac{A^2 - 1}{A^2} \right| = \left(\frac{v}{c} \right)^2 \equiv \beta^2,$$

where c is a universal constant with the dimension of velocity, and is therefore referred to as universal velocity. Three conditions are set for the solution above: first, if $A = 1$, $c \rightarrow \infty$. Therefore,

$$F(t, x, y, z) = t, \quad (3)$$

and if $A > 1$, $A = 1/\sqrt{1 - \beta^2}$. Therefore,

$$F(t, x, y, z) = c^2 t^2 - (x^2 + y^2 + z^2), \quad (4)$$

and if $A < 1$, $A = 1/\sqrt{1 + \beta^2}$. Therefore,

$$F(t, x, y, z) = c^2 t^2 + x^2 + y^2 + z^2. \quad (5)$$

Einstein used the speed of light in a vacuum as the universal velocity (called the principle of the constancy of the velocity of light), expressed as

$$c = c_0. \quad (6)$$

It is known that the STR can be used to explain experimental results effectively based on Equation 4. In fact, the theory became so historically entrenched that its validity was rarely questioned. This is because Equation 6 is somewhat self-contradictory; that is, the propagation speed of physical phenomena cannot reach universal velocity, with only the speed of light as an exception ($\beta = 1$). If this is accepted,

$$A = \frac{1}{0}, \quad (7)$$

is true, or $t_1 = 0$ in Equation 1 is simply a remote action in and of itself (which would clearly be contradictory because there is no time lapse in light's own coordinate system). This devalues the validity of Equation 4 and throws the theory's *raison d'être* into question. To address the contradiction, discussion without exclusion of the speed of light as an exception is required.

3. Mass and Speed of Light Quantum

In regard to the Light-quantum Hypothesis, a primary consideration is that the mass of a light quantum may not be zero, as light quanta and particles interact in reality. Here, the mass of a light quantum is provisionally labeled δm . The physical quantity opposing mass in the STR approaches the universal velocity $\delta\eta$ (known as the mass-conversion factor). When this is applied, the light quantum velocity v ($= c_0$) can be expressed as

$$c_0 = c \cdot \sqrt{1 - \delta\eta^2}. \quad (8)$$

This replaces Equation 7 and gives

$$A = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\delta\eta},$$

in which, to ensure validity, $\delta\eta$ needs to be a limited definite value of a minute quantity and cannot be infinitesimal. The Theory of Relativity's mass m^* for a light quantum is expressed as [9]

$$m^* = \frac{m}{\sqrt{1 - \beta^2}} = \left(\frac{\delta m}{\delta\eta} \right),$$

which can be further derived as

$$\left(\frac{\delta m}{\delta\eta} \right) = \frac{E}{c^2}. \quad (9)$$

As the right-hand side of this formula is a settled value, neither δm nor $\delta\eta$ equates to zero. These values have the same sign and represent minute quantities with an order of the same degree. Thus, the following is true:

$$\delta m > 0, \quad \sqrt{\delta\eta^2} = \delta\eta > 0.$$

The former provides the theoretical basis for the Light-quantum Hypothesis (i.e., the light quantum is now material rather than hypothetical), and the mass-conversion factor of the latter is applied to Equation 8 to render the following true:

$$c > c_0. \quad (10)$$

Here, Equation 9 is further examined. First, when $E \rightarrow 0$, $\delta m \rightarrow 0$ applies; as both δm and $\delta\eta$ are minute quantities of the same degree, $\delta\eta \rightarrow 0$ is true at the same time. Also, in accordance with the relational expression of an energy quantum, if $E \rightarrow 0$, then $v \rightarrow 0$. Accordingly, when $\delta\eta \rightarrow 0$, $v \rightarrow 0$ is true; using Equation 8, the principle of the constancy of the velocity of light is revised as follows:

$$c = \lim_{v \rightarrow 0} c_0. \quad (11)$$

This equation also relates to the gravitation red-shift of light. Specifically, when a light quantum with mass is accelerated by gravity, its frequency decreases (i.e., its wavelength increases) as universal velocity approaches.

Additionally, the relationship of energy momentum in the STR regarding light quanta is expressed as

$$\left(\frac{E}{c} \right)^2 - p^2 = \delta m^2 \cdot c^2.$$

Previously, light quanta were considered to have no mass, so the solution for the right side of this formula was zero. The wave number k of a light quantum and the formula of an energy quantum derive

$$p = hk \cdot (1 - \delta\eta^2).$$

$\delta\eta$ here can be considered to represent the dispersion of momentum.

4. Single-Dimension Motion Equation

Unlike the mass for a light quantum, the mass m for a particle is a fixed-quantity value. Accordingly, the mass-conversion factor for a particle is greater than that for a light quantum. Therefore, applying η as the mass-conversion factor for a particle, the particle mass m^* in the Theory of Relativity is expressed as

$$m^* = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m}{\eta}.$$

Additionally, the relationship between the mass-conversion factor and the velocity of the physical substance is expressed as

$$\left. \begin{aligned} 0 < (\delta\eta, \eta) \leq 1, \\ c > (c_0, |v|) \geq 0. \end{aligned} \right\}$$

The single-dimension motion of a particle involves the issue of escape velocity. Here, a gravitational field with a mass M exists at the origin of the horizontal axis, and a particle placed at a positive coordinate x moves along the axis in the positive direction while being pulled by universal gravity from the particle at the origin. This motion is expressed by the equation of motion in the STR as

$$\frac{d}{dt} \left(\frac{\dot{x}}{\eta} \right) = -\frac{GM}{x^2}, \quad \eta = \sqrt{1 - \frac{\dot{x}^2}{c^2}}.$$

As the escape velocity of a particle varies with its position, if this velocity at the coordinate x_0 is labeled as v_0 , the following is derived:

$$v_0 = \sqrt{\frac{2GM}{x_0}} \cdot \frac{\sqrt{1 + \frac{GM}{2c^2 x_0}}}{\left(1 + \frac{GM}{c^2 x_0}\right)}. \tag{12}$$

Next, for a light quantum, the equation of motion is expressed as

$$\frac{d}{dt} \left(\frac{\delta m}{\delta \eta} \cdot \dot{x} \right) = -G \cdot \frac{M \cdot \delta m}{x^2}.$$

Due to the characteristic of $\delta\eta$, $1 \gg \delta\eta$. Applying $\dot{x} = c_0$ and Equation 8, and T as the integral constant for integration with time, the following formula is derived:

$$\frac{d}{dt} \left(\frac{\delta m}{\delta \eta} \right) = \frac{GM \cdot \delta m}{c^3} \cdot \frac{d}{dt} \left\{ \frac{1}{(t+T)} \right\}. \tag{13}$$

As the wavelengths of both δm and $\delta \eta$ change together, iteration stops here.

5. Motion Equation in Two Dimensions

Assuming that a source of gravity with a mass M exists at the origin of a plane of coordinates, and the coordinates of the light quantum are labeled (x, y) , the equation of motion for the light quantum is split into two coordinate components. They are developed by including the polar coordinate (r, θ) to give

$$\left. \begin{aligned} \frac{d}{dt} \left(\frac{\delta m}{\delta \eta} \cdot \dot{x} \right) &= -G \frac{M \cdot \delta m}{r^2} \cdot \cos \theta, \\ \frac{d}{dt} \left(\frac{\delta m}{\delta \eta} \cdot \dot{y} \right) &= -G \frac{M \cdot \delta m}{r^2} \cdot \sin \theta. \end{aligned} \right\} \tag{14}$$

Also in this case, both δm and $\delta \eta$ change together with wavelength.

Next, particulate objects such as planets have constant mass, so the equation of motion is expressed as

$$\left. \begin{aligned} \frac{d}{dt} \left(\frac{\dot{x}}{\eta} \right) &= -\frac{GM}{r^2} \cdot \cos \theta, \\ \frac{d}{dt} \left(\frac{\dot{y}}{\eta} \right) &= -\frac{GM}{r^2} \cdot \sin \theta. \end{aligned} \right\} \tag{15}$$

In Equation 15, placing K as the constant via integration leads to the derivation of $\delta\eta = Kr^2\dot{\theta}$, and

$$\frac{d}{dt} \left(\frac{\dot{r}}{\eta} \right) - \frac{\eta}{K^2 r^3} = -\frac{GM}{r^2}.$$

Using $\dot{r}^2 + r^2\dot{\theta}^2 = v^2 = c^2 \cdot (1 - \eta^2)$ as the condition for the velocity of a particle also gives

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} \sqrt{\frac{\left(1 - \frac{\dot{r}^2}{c^2}\right)^3}{\left(1 + \frac{1}{c^2 K^2 r^2}\right)}}. \tag{16}$$

6. Advance of the Planetary Perihelion

In Newtonian mechanics, the perihelion advances due to gravitational interaction between planets and fluctuation of the vernal equinox. However, the advance is still observed even after correction for these factors; this is an issue with the advance of Mercury’s perihelion that astronomers discussed in the mid-19th century. Here, the issue of the advance of the planetary perihelion is examined using the equations of motion obtained.

In Equation 16 relating to planetary motion, the value of the second term in each set of parentheses on the right-hand side is much smaller than 1. The three terms are therefore derived by an approximate expansion of the right-hand side. When the major radius of an orbit for a planet is expressed as

a and the eccentricity of the orbit is expressed as e in the equation, the following is derived:

$$-\frac{\dot{\theta}}{Ka \cdot (1-e^2)} \approx -\frac{GM}{r^2} + \frac{3GM}{2c^2} \cdot \frac{\dot{r}^2}{r^2} + \frac{GM}{2c^2 K^2} \cdot \frac{1}{r^4}.$$

The initial term is associated with rotation based on Newtonian mechanics, and the other two are related to the advance of the perihelion for a planet. In consideration of the angular velocity of the left-hand side in terms of orbital motion and the advance of the perihelion, the angular velocity for the advance of the perihelion as an average value for one rotation is expressed as

$$-\dot{\theta} = \frac{(GM)^{3/2} \cdot (1 + \frac{9}{2} \cdot e^2 + \frac{3}{4} \cdot e^4)}{2c^2 \cdot a^{5/2} \cdot (1-e^2)^{5/2}}. \tag{17}$$

Based on this equation, the coordinate system associated with planetary rotation takes the direction opposite to that of rotation. Meanwhile, the coordinate system maintains a constant direction in space, so the planetary perihelion advances in the direction of rotation.

In line with the GTR, applying δ as the advance of the planetary perihelion for every rotation (where T is an orbital period) gives the following [10-12]:

$$\frac{\delta}{2\pi} = \frac{3GM}{c^2 a \cdot \sqrt{1-e^2}}.$$

As the form of this equation is not appropriate for the expression of a total, an angular velocity is applied to express the advance of the planetary perihelion in the form of $-\dot{\theta} = \delta / T$. This derives the following equation:

$$-\dot{\theta} = \left(\frac{2\pi}{T}\right) \cdot \frac{3GM}{c^2 a \cdot \sqrt{1-e^2}}. \tag{18}$$

The two observation values for the advance of the planetary perihelion are angular velocity $\dot{\theta}_0$, which cannot be estimated in terms of Newtonian mechanics, and $\dot{\Theta}$, which is the sum of an estimated angular velocity $\dot{\theta}'$ and $\dot{\theta}_0$; that is, $\dot{\Theta} = \dot{\theta}' + \dot{\theta}_0$. The three variables share logical relations; this is expressed as $\dot{\Theta} = \dot{\theta}'$ in Newtonian mechanics and as $\dot{\Theta} = \dot{\theta}_0$ in other fields. Accordingly, no theory can be used to predict $\dot{\Theta}$ for the actual planet. The $\dot{\Theta}$ value for Mercury presented in the middle of the 19th century was around $40''$ for a 100-year total.

Examples of calculation with Equations 17 and 18 are shown in Table 1 for the three inner planets [10-16]. The angular velocity of the STR and the GTR labeled as $\dot{\theta}_S$ and $\dot{\theta}_G$, respectively, and the column on the right shows the angular velocity $\dot{\theta}_G$ of the GTR multiplied by the orbital eccentricity e for the planet. Predicted values based on the GTR are shown in parentheses. The numerical values are 100-year totals.

Table 1

Planet	STR $-\dot{\theta}_S$	GTR $-\dot{\theta}_G$	Observation $-\dot{\theta}_0$	$e \cdot (-\dot{\theta}_G)$
Mercury	9".107	42".06 (43".03)	8".863±0".093	8".648 (8".847)
Venus	1".438	8".625 (8".63)	0".057±0".033	0".059
Earth	0".641	3".838 (3".84)	0".084±0".020	0".064

According to the GTR, the observation values in Table 1 are proportionate to planetary orbital eccentricity because the ratio of the observation value $\dot{\theta}_0$ to the angular velocity value $\dot{\theta}_G$ for Mercury was relatively close to the orbital eccentricity e for Mercury. That is, orbital eccentricity appears only in the interaction between $\dot{\theta}_0$ and (43".03). In other words, the observation value for Mercury is a physical quantity independent of orbital eccentricity.

Focusing on the angular velocity of Mercury (43".03) in terms of the GTR, no theory should exist (as previously mentioned) to predict this value, as it is close to $\dot{\Theta}$ for Mercury. Since Equation 18 therefore loses its significance, all values in the three columns in Table 1 are meaningless except the observation for Mercury.

In conclusion, in fields other than Newtonian mechanics, the STR can be used to predict the observation value for the advance of Mercury's perihelion; this value $\dot{\theta}_0$ and the angular value $\dot{\theta}_S$ match to within 3%. This means that Equation 17 represents the first answer to the argument on the advance of the perihelion for Mercury since related discussions began in the mid-19th century.

7. Discussion

The motion equation for a particle and a light quantum in a gravitational field was obtained by developing the improved STR via the incorporation of a new physical quantity (the mass-conversion factor) based on the light quantum hypothesis. As the equation cannot be solved for the light quantum in a strict sense, this section examines how the equation can be utilized.

Motion in one dimension is an issue of escape velocity. This velocity for a particle is determined using Equation 12; in the region near $x_0 \rightarrow 0$, it becomes

$$v_0 \approx c \cdot \sqrt{1 - \left(\frac{c^2 x_0}{GM}\right)^2}, \quad \lim_{x_0 \rightarrow 0} v_0 = c.$$

With this, the particle can escape from any gravitational field by approaching universal velocity.

Motion for a light quantum is expressed by Equations 13 (one dimension) and 14 (two dimensions). Although δm and $\delta \eta$ change with time, a light quantum whose velocity

is infinitely close to universal velocity gives $\delta m \rightarrow 0$ immediately regardless of temporal changes. Accordingly, for a light quantum with a velocity extremely close to universal velocity, the equation of motion can be transformed assuming that $\delta\eta$ changes with time. With this, for one dimension, Equation 13 is derived as

$$\frac{d}{dt} \left(\frac{1}{\delta\eta} \right) = \frac{GM}{c^3} \cdot \frac{d}{dt} \left\{ \frac{1}{(t+T)} \right\}.$$

With the temporal differentiation of $\delta\eta$ labeled $\delta\eta'$, and $\delta\eta$ and $\delta\eta'$ labeled as $\delta\eta_0$ and $\delta\eta'_0$, respectively, when $t = 0$, the following is derived:

$$\frac{\delta\eta_0}{\delta\eta} = 1 - \sqrt{\frac{GM \cdot \delta\eta'_0}{c^3}} \cdot \frac{t}{(t+T)}.$$

This equation shows the situation when a light quantum escapes from a gravitational field. That is, $\delta\eta$ does not change greatly after the escape because a light quantum whose velocity is near universal velocity has an extremely small $\delta\eta'_0$ value. With very intense gravity, the light quantum may not be able to escape from the gravitational field in some cases. However, it is more likely that a light quantum with this velocity will escape.

In similar analysis for a light quantum in two dimensions, motion is expressed by Equation 15 for a planet. Accordingly, by Equation 16 it is expressed as

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM \cdot \delta\eta}{r^2} \cdot \left(1 - \frac{\dot{r}^2}{c^2} \right)$$

or

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM \cdot \delta\eta}{r^2} \cdot \left(\delta\eta^2 + \frac{r^2 \dot{\theta}^2}{c^2} \right).$$

When a light quantum is distant from the gravity source, $r\dot{\theta} \rightarrow 0$ and $\dot{r} \rightarrow c_0$. Near the gravitational field $r\dot{\theta} = c_0$ and $\dot{r} = 0$. When the variable n , which changes between 1 and 3 (including $n = 1$) is applied, the equation of motion for a light quantum is expressed as

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM \cdot \delta\eta^n}{r^2}. \tag{19}$$

This equation of motion expresses how much the path of a light quantum near the surface of the Sun is bent (angle φ) by the Sun's gravity, with R_0 as the radius of the sun, as

$$\sin \frac{\varphi}{2} = \frac{1}{e} \approx \frac{GM \cdot \delta\eta^n}{R_0 \cdot c_0^2}.$$

Here, as in the single-dimension analysis, the angle φ depends on the wavelength, and a light quantum moving at close to universal velocity is hardly affected (i.e., its orbit is a straight line).

According to the differential Equation 2, the concept of common velocity for all inertial systems (universal velocity)

supports the derivation of Equations 3 - 5 as transformation formulae. Among these, Equation 5 appears to represent the simplest solution when applied to the principle of the constancy of the speed of sound. However, as the speed of sound varies with the medium, universal velocity cannot be reached. That is, the existence of universal velocity itself puts an end to the argument regarding ether (the absolute rest system) that began in the 19th century.

Next, Equations 3 and 4 remain as transform expressions, but the Galilei transformation for the former (universal velocity is equivalent to infinity) is not supported by any experimental evidence. As a result, only Equation 4 is left as a possible solution. Here the issue of universal velocity arises, and further development incorporating the following is needed; 1) consideration of universal velocity as the velocity of light (this practically means the principle of the constancy of the velocity of light as expressed by Equation 6); 2) consideration of universal velocity (which has no substance) as greater than the velocity of light for the derivation of Equation 10, or Equation 11, which is characterized by stricter expression. In the history of physics, as the GTR emerged during the first step, the second step has been left unaddressed.

8. End Notes

Conditional on the velocity of light being less than universal velocity and changing with wavelength, and a particle being able to escape from any gravitational field at close-to-universal velocity, faster-than-light particles (tachyons) may exist.

Analysis has yielded a motion equation for a light quantum, but conditions remain that do not allow its complete solution. However, related motion has been clarified by limiting the case to a light quantum moving at extremely close to universal velocity. This means that if visible rays meet such a condition (i.e., being significantly close to universal velocity), their motions in a gravitational field are clarified. In this case, the behavior of visible rays in the Solar System is expressed by Equation 19, which indicates that their motions are hardly influenced by the gravity of the Sun ($\varphi = 0$) at all, suggesting that spatial distortion does not exist.

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