

# Einstein's Theory of Gravity: Misunderstood for a Century

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**Abstract** In a time-independent gravitational field, Emmy Noether's theorem implies that light moves upward with constant energy. The gravitational red shift is therefore to be understood as resulting from a reduction of the energy states of the emitting source. Applied to the annihilation of para-positronium, in which the entire rest mass of the positronium system is converted into gamma rays, this understanding of the red shift proves the rest masses are reduced in a gravitational field. Going back to the emission by simple atoms, whose emission frequencies are inversely proportional to the Rydberg period,  $P_R = h^3/e^4 m_e$ , it is clear that the red shift must be attributed to rest mass reduction, rather than the imagined *gravitational time dilation* effect, according to which, *the flow of time itself is reduced*. We are indebted to Niels Bohr's quantum theory of the hydrogen atom not only for the above expression for the Rydberg period, but also for the Bohr radius,  $a_0 = h^2/e^2 m_e$ . Clearly, gravitational rest mass reduction causes an increase in the size of the hydrogen atom. But Logic demands, and observation confirms, that all entities of the same dimensionality must be affected in exactly the same way. Thus all material bodies, including measuring rods, must suffer an increase in size, indicating that the presently accepted geometry of space in a gravitational field is incorrect. This paper details how *gravitational rest mass reduction* and the heretofore unimagined phenomenon, *gravitational size dilation*, play a vital role in the correct understanding of the gravitational field, quantum gravity, black hole structure, and cosmology.

**Keywords** Gravitational rest mass reduction, Noether's Theorem, Bohr's theory of the hydrogen atom

## 1. Introduction

Considering the magnificent success of the 'Standard Model' in describing the existence and interactions of the incredible zoo of elementary and composite quantum particles that inhabit (or can be forced to materialize in) our universe, it is very difficult to imagine that there is anything in physics that has been misunderstood. Nevertheless, regarding the most fundamental elements of gravitation theory, serious errors have persisted for over a century. This is understandable in view of the fact that the editors of physics journals must deal with a flood of submissions by 'cranks' who characteristically address issues that mainstream physicists consider to be settled once and for all, having been so thoroughly studied by the dominant physics community that no further discussion need be considered. In fact, it seems that any submission addressing any one of these *settled issues* will be rejected out of hand without any real review, even unread. But certain of these issues have actually not received careful consideration, with the result that several incorrect concepts in the theory of gravitation are

protected as settled issues, allowing errors to survive, even for a century. The purpose of this paper is to identify these errors, to show how they entered the theory, and finally to present a new interpretation of Einstein's 1916 theory of gravity<sup>1</sup> based upon the concept of *Gravitational Rest Mass Reduction (GRMR)*. A brief review of the history of the development of Einstein's theory of gravity will reveal how certain errors regarding the interpretation of the theory entered mainstream thinking. The most egregious error was, and is, the failure of Einstein's followers to attempt to discover the *cause* of the gravitational red shift. This phenomenon was predicted by Einstein in a very remarkable 1911 paper [1]. In this paper Einstein boldly proposed his now famous *principle of equivalence*, according to which acceleration is equivalent to gravity: thus, for instance, experiments conducted in a rocket ship accelerating in the absence of gravity at one 'g', are predicted to give results identical to the same experiments conducted on the surface of the earth, where the acceleration of gravity has the same value.

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<sup>1</sup> The interpretation offered in this paper presents evidence that only those elements of the matter tensor possessing non-zero rest mass should be included as sources. Otherwise, Einstein's theory is accepted as is.

## 2. Einstein's 1911 Paper

Section (1) of the paper introduces the principle of equivalence, following which Section (2) uses the principle to prove that the addition of energy to a mass results in identical increases of inertial and gravitational mass. Einstein actually presents two arguments: the first is quite prolix, but the second is short and convincing, employing simple spring balances, one in the rocket ship, one on earth. Section (3) is vital to the point of this article, dealing as it does with the gravitational red shift. Curiously and revealingly, Einstein titles this section, "*Time and the Velocity of Light in a Gravitational Field*". According to the well-understood Doppler Effect, a continuous single-frequency light signal sent 'upward' in the accelerating rocket ship will be measured by the receiver to be reduced in frequency (red-shifted) because the receiver will have a higher velocity when the signal arrives than the emitter had when it launched the signal. Invoking his new principle of equivalence, Einstein immediately inferred the existence of a *gravitational red shift effect*: light sent upward in a gravitational field would be received red shifted as compared with the frequency observed in the absence of gravity. Straightaway, Einstein noted that this seemed to imply an absurdity: if fewer wavecrests are received above than are emitted below, the number of wavecrests in between would continually increase, contrary to the assumption of time-independence. If the number between is constant, it has to be true that each time a wavecrest is emitted below, one is detected above: the frequencies must be equal, and the frequency must be constant in transit. More fundamentally, according to Noether's Theorem, energy is conserved in every time-independent system [2]. It follows that the frequency was already reduced when emitted by the source. But measurements at the source would show no such reduction – how could this be the case? The measurement of frequency requires a clock, and Einstein settled this issue by asserting that there is no reason to assume that clock rates are not influenced by gravity. He postulated that in a gravitational field, clock rates are reduced by the same factor by which emission frequencies are reduced. (Today, in the era of 'atomic' clocks, this would be taken for granted.)

Einstein did not attempt to understand the cause of this clock slowing effect – his immediate concern was that slowing clock rates would (and here he made an unwarranted assumption – that distance measurements would not be influenced by gravity) cause local observers to overvalue the speed of light – contrary to an enshrined principle of special relativity, that all observers will measure the speed of light to be the same universal constant. Einstein immediately jumped to the conclusion that the *true* speed of light must be reduced by gravity so as to insure that the *locally measured* speed of light be constant. One can only marvel at the remarkable philosophical flexibility exercised by Einstein in this matter. Regarding the reduction of the speed of light, Einstein's title for Section (3) suggests that he might have agreed with most

modern theorists, who subscribe to the rather mystical idea that *the flow of time itself* is reduced by gravity<sup>2</sup>, thus 'explaining' not only light's speed reduction, but also the slowing of clocks and the gravitational red shift. In any event, it did not occur to Einstein, nor has it occurred to a century of his followers, that perhaps the measurement of distances might be influenced by gravity. Section (4) treats the bending of light rays in a gravitational field, and here again Einstein presents two arguments. The first is based upon the incorrect belief that the speed of light is reduced by gravity, but the second argument returns to the rocket ship: a light ray moving horizontally according to stationary observers would, to observers in the rocket ship, appear to move in a straight line deflected downward if the rocket ship were cruising at a constant speed, but will follow a curving bent-down path if the ship is accelerating. In accordance with the principle of equivalence, light rays must 'bend' except when moving parallel to the direction of the acceleration of gravity. Einstein's prediction was just half the value later predicted in his great 1916 treatise on General Relativity, which added the curvature inherent in the non-Euclidean geometry of the solutions to the field equations. This extremely important effect involves a subtle consideration of what is meant by 'bent' in non-Euclidean geometry: it will be dealt with in due course.

## 3. The Cause of the Red Shift: Gravitational Rest Mass Reduction

Regarding the cause of the gravitational red shift, the answer to this unasked question was not long in coming, but was ignored. In 1911 the frequencies emitted by atoms were known, in every case, to be proportional to a single number, the Rydberg constant, but it was only determined empirically. Without a real theory of atomic spectra, it is understandable that Einstein did not attempt to determine the cause of the gravitational red shift. But in 1913, Niels Bohr put forward his quantum theory of the hydrogen atom [3]. Bohr not only expressed the Rydberg in terms of fundamental constants, he did the same for the size of the hydrogen atom: Bohr gave us the Rydberg period,  $P_R = h^3/m_e e^4$ , and the Bohr radius,  $a_0 = h^2/m_e e^2$ . The frequency spectrum emitted by every species of atom is determined by a formula involving species-specific quantum numbers, but in every case the leading factor in the formula is the reciprocal of the Rydberg period,  $P_R$ . This formula holds regardless of the location in the field, and the factor by which frequencies are red shifted is the same for all species of atoms. Thus the gravitational field somehow causes an increase in the Rydberg period,  $P_R$ . Clearly, the imagined '*time dilation effect*' cannot be held responsible, since none of the factors in  $P_R$  is affected by the '*flow of time*.' This suggests a simple decrease in the electron rest mass,  $m_e$ , rather than some

<sup>2</sup> This imagined phenomenon is commonly referred to as '*gravitational time dilation*'.

unimaginable change in the factor  $h^3/e^4$ . (Detailed proof is presented in APPENDIX A) Of course, all quantities of the same kind, i.e., of the same dimensionality, must vary together, so all masses must be similarly reduced. Thus, as early as 1914, the cause of the gravitational red shift might have been understood to be a decrease of rest masses in a gravitational field. Bohr's theory also resolves the issue regarding the speed of light. The Bohr radius,  $a_0 = h^2/m_e e^2$ , is inversely proportional to the electron rest mass. Rest mass reduction thus implies that gravity causes the hydrogen atom increase in size, and again, since all entities of the same dimensionality must be affected in the same manner, measuring rods must suffer elongation in a gravitational field. Returning to the issue of the measured speed of light, this means that distances are underestimated by the same factor by which time measurements are reduced, so Einstein's angst regarding the measured speed of light is assuaged, the *true* speed of light is restored as a constant, and finally, idea that *the flow of time itself* is reduced by gravity is vitiated.

## 4. Proof of Rest Mass Reduction

We have seen that gravitational rest mass reduction may reasonably be said to be the cause of the gravitational red shift, yet the fundamental question remains: why *are* rest masses be reduced by gravity? The answer involves a reconsideration of the nature of gravitational potential energy. One possible, if seemingly naïve, understanding of gravitational potential energy simply invokes the celebrated equation,  $E=mc^2$ : when a mass is raised against gravity, the increase in potential energy,  $\Delta w$ , is stored in the body itself as an increase in rest mass:  $\Delta m = \Delta w / c^2$ . Unfortunately, this simple idea runs counter to one of the *settled issues* of physics. Before there was any hint of the connection between mass and energy, the consensus was that gravitational potential energy resided in the gravitational field itself. Awkwardly, since the absolute value of gravitational field strength increases as gravitating masses are brought together, this model, in order to account for the attractive nature of gravity, required that the energy density of the gravitational field must be *negative*! Fortunately, this difficult concept may be safely discarded since there is a simple thought experiment proving that rest masses *are* reduced in a gravitational field, in accord with the above naïve interpretation of gravitational potential energy.

### A. Positronium Proof of rest mass reduction

Consider the annihilation of a ground-state para-positronium system deep in a time-independent gravitational field. In this process, the entire combined rest mass of the electron and the positron will be converted to gamma ray photons. In a time-independent field, energy is conserved [2], so the photons will move upward with constant energy. Nevertheless, because of the gravitational red shift, the total energy of the photons emitted from the annihilation event will be measured by distant observers,

who are less affected by the field, to be reduced as compared with the rest mass energy of such a system measured locally by these observers. The unavoidable conclusion is that the rest mass of the positronium system was reduced by the action of the gravitational field, upsetting another of the *settled issues* of physics, namely, that rest mass is unaffected by gravity.

### B. Kinematic proof of rest mass reduction

Crucially, there is an equation of motion [4] for a test mass moving freely in a static field of gravity: it expresses the conservation of energy:

$$m^* c^2 (1 - v^2/c^2)^{-1/2} = \text{constant}.$$

Here  $m^* = m |g_{00}|^{1/2}$  is the true rest mass,  $m$  is the proper rest mass, a constant, and  $g_{00}$  is the time-time component of the metric tensor ( $|g_{00}| < 1$ ), a function of position, that characterizes the gravitational field. Significantly, energy conservation is expressed here as a product, rather than a sum. The equation implies that gravity converts rest mass energy to kinetic energy and vice versa. An obvious inference is that any change in the gravitational potential energy of the test mass exists as a change in its rest mass energy: no energy is stored in the gravitational field.

## 5. Gravity Couples Exclusively to Rest Mass

This suggests, as will be further argued below, that gravity couples exclusively to rest mass. This fact is of inestimable importance, since it undermines another *settled issue*, namely, that every element of the stress-energy tensor contributes to the gravitational field. In particular, freely moving<sup>3</sup> massless quanta such as photons and gravitons are presently thought to act as sources of the gravitational field and to be affected by it. The experimental evidence supporting this concept is the observation of the 'bending of light rays' by the gravitational field, first observed during the eclipse of May 1919 by teams organized by Frank Dyson and Arthur Eddington. More accurate modern observations corroborate the fact that indeed light does *not* follow the spatial geodesics of the *proper* non-Euclidean geometry inherent to the gravitational field. But this geometry is incorrect in that it is based upon distance measurements made directly with elongated measuring rods (or equivalently, by echo ranging using slow cocks). Correcting for these effects gives rise to a new, *corrected*,  $\Theta$ ,<sup>4</sup> metric and corresponding geometry, in which light rays *do* follow geodesics paths (the straightest paths possible in a

3 It must be noted that *confined* radiation, as for instance in the body of a star, exhibits an equivalent rest mass and does interact with, and contribute to, the gravitational field. Even more striking is the fact that nearly all of the rest mass of a baryon derives from the kinetic energy of *confined* massless gluons, and nearly massless quarks:  $m = E/c^2$ .

4 Please forgive the introduction of  $\Theta$ , a special symbol intended to suggest *corrected* or *conformal*. It is formed by typing three consecutive strokes: (.,c.).

non-Euclidean geometry), as will be shown directly.

### A. Light moving in the new $\Theta$ geometry

Because of gravitational rest mass reduction, the proper metric under-values distance intervals and time intervals by the factor  $|g_{00}|^{1/2}$ . Correcting both is easily accomplished by merely dividing  $ds^2$  by  $g_{00}$ , yielding the  $\Theta$  metric,

$$ds^{*2} = ds^2/g_{00} = (dx^0)^2 - g_{ab}/g_{00} dx^a dx^b$$

Regarding the bending of light rays, the path followed by a light ray may be determined from the proper metric,  $ds^2 = g_{00} (dx^0)^2 + g_{ab} dx^a dx^b$ , by setting  $ds^2 = 0$ , whence  $(dx^0)^2 = -g_{ab}/g_{00} dx^a dx^b$ . Then, according to Fermat's principle of least time, the ray path may be found by minimizing the integral of  $(dx^0)^2$  between specified end points. But this is same as finding the path of minimum distance in the *corrected*  $\Theta$  spatial metric,  $dl^{*2} = -g_{ab}/g_{00} dx^a dx^b$ . Thus light rays are merely following the spatial geodesics (the straightest possible paths) of the *correct* geometry, proving that there is no coupling with the gravitational field. This result is in line with the conclusion regarding the gravitational red shift, that light moves upward with constant energy, indicating again that there is no coupling of gravity to the free electromagnetic field.

### B. Optical Geometry

Importantly, the identical geometry has been proposed on the basis of a truly profound discovery regarding the kinematics of massive bodies in a gravitational field. Abramowicz *et al.* [5] proved that in a static gravitational field, a body will not experience speed-dependent forces (centrifugal and Coriolis forces) if it is constrained to move along a path that a light ray might follow. By analogy to Newtonian mechanics, in which such forces vanish for bodies moving in straight lines, the authors defined a new geometry by identifying light ray paths as the geodesics of the new geometry, which they named, appropriately, 'Optical Geometry'. Again, light rays follow spatial geodesics, proving there is no coupling to gravity. This geometry is identical to our  $\Theta$  geometry, which was posited by taking account of the *gravitational size dilation* effect.

## 6. Quantum Implications: Saving the Spin-Zero Graviton

By far the most important consequence of this heretofore unsuspected phenomenon, *gravitational size dilation*, is its implications regarding quantum gravity. The 'bending of light rays,' *i.e.*, the failure of light rays to follow the geodesics of the (incorrect) *proper* geometry, has been used to rule out the spin-zero graviton, since spinless quanta can only couple to the trace of the energy-momentum tensor of the target field, and the energy-momentum tensor of the free electromagnetic field has a zero trace. But light rays *do* follow the geodesics of the *correct*  $\Theta$  geometry, so theorists have wrongly rejected the spin-zero graviton, which should be accepted, and wrongly accepted the spin-two graviton,

which must be rejected, since a spin-two graviton of *would* couple to the complete energy-momentum tensor the electromagnetic field.

The fact that gravity does not couple massless photons implies that photons and other massless quanta do not act as sources of gravity. Gravity's inverse square law requires the graviton to be massless: thus, according to the theory just developed, gravitons are not affected by gravity, they do not act as sources of gravity, and they do not interact with one another! In this respect, the situation is analogous to the relation of photons to the electromagnetic field and to one another. Within the same analogy, rest mass acts as the 'charge' for the gravitational field. But the analogy breaks down since electric charge is invariant and the electromagnetic field possesses energy, whereas in the case of gravity, potential energy resides not in the field, but rather in the variable rest mass energy of the 'charges.' The fact that gravitons interact neither with the field itself, nor with one another, eliminates dreaded non-linearities, and gives promise of a linear, spin-zero scalar theory – a radical and welcomed simplification of the problem of formulating a quantum theory of gravity.

## 7. Black Hole Structure

We restrict our attention to the Schwarzschild field. The proper metric may be written:

$$ds^2 = f^2 c^2 dt^2 - f^{-2} dr^2 - r^2 [d\theta^2 + \sin^2\theta d\phi^2],$$

where  $f = (1 - r_s/r)^{1/2}$  in which  $r_s = 2GM/c^2$ .

Since, as argued above, proper time intervals and distance intervals are underestimated by the same factor,  $f$ , the correction is effected by multiplying by  $f^{-2}$ . Thus the correct  $\Theta$  metric is

$$ds^{*2} = c^2 dt^2 - f^{-4} dr^2 - f^{-2} r^2 [d\theta^2 + \sin^2\theta d\phi^2].$$

Note that this metric does not represent a solution to the field equations: it simply introduces a system for the measurement of time and distance that is not influenced by the field. Time is measured using signals from a remote clock (the 'clock at infinity'), while distances are measured by electromagnetic echo ranging calculated using the same time system. Note that it is not being suggested that the proper metric should be discarded: the proper metric correctly describes the dynamical behavior of non-zero rest mass matter, which the  $\Theta$  metric does *not* describe.

Before considering how the true geometry of a black hole is revealed in the *GRMR* interpretation, a review of Schwarzschild static black hole structure according to the mainstream interpretation seems appropriate.

### A. The Black Hole: Mainstream view

The basic structure is a singularity hidden behind a surface called the event horizon, from which the escape velocity is equal to the speed of light. This surface is very peculiar in that light sent from a finite proper distance directly toward the event horizon never reaches that surface! Proponents of

the accepted interpretation ‘explain’ that because of ‘*the slowing of the flow of time itself*’, the speed of light goes to zero as light approaches the event horizon. Another puzzling fact is that light does not follow the spatial geodesics (shortest paths) of the proper geometry. Again, proponents ‘explain’ that the speed of light is greater along a path outside of the proper geodesic path, where there is less ‘*slowing of the flow of time itself*’. But light *does* follow the geodesics of the correct geometry. Also, as argued in APPENDIX A, the claim that light’s speed is reduced by a factor  $f$  in a gravitational field also requires, unacceptably, that Planck’s constant and rest masses *increase* by factors of  $f^{-1}$  and  $f^{-2}$  respectively. Another mystery concerns the behavior of centrifugal force. According to the proper metric, every sphere centered on a black hole is convex when viewed from the outside, except for the event horizon itself, which, in the proper metric, appears to have zero curvature. Nevertheless, for any location inside the locus of photon orbits at  $3/2 r_s$ , centrifugal force acts *inwardly*! Clearly, this phenomenon, the Abramowicz Effect [6-8] cannot be accounted for in the context of the proper geometry of the conventional interpretation.

### B. The Black Hole: The GRMR interpretation

The puzzling phenomena that appear in the conventional interpretation are easily understood in terms of the GRMR interpretation. First of all, the geometry is very different. In the  $\Theta$  metric, the area of a centered sphere is equal to

$$4\pi r^2 f^{-2} = 4\pi r^2 (1 - r_s/r)^{-1}.$$

Differentiating with respect to  $r$ , one has

$$d/dr [r^2(1 - r_s/r)^{-1}] = (2r - 3r_s)(1 - r_s/r)^{-2}$$

Thus the area of a sphere is not a monotone function of  $r$ : it has a minimum at  $r = 3/2 r_s$ , the locus of photon orbits. Furthermore, for  $3/2 r_s > r > r_s$ , the area of a sphere increases without limit as  $r$  approaches  $r_s$ . The sphere of minimum area, the *stenosphere*, is the throat of a wormhole-like structure that magically connects our familiar universe to another infinite three-dimensional space, which may reasonably be called ‘*innerspace*.’ In the  $\Theta$  metric, the stenosphere has zero curvature, which explains why centrifugal force vanishes there. This vanishing, in turn, confirms the fact that photons do not feel the force of gravity, since on the stenosphere there can be no centrifugal force to counter an imagined force of gravity acting on photons.

Note that inside the stenosphere, the surface of a centered sphere, viewed from the ‘outside,’ will be concave rather than convex. Thus, regarding the Abramowicz Effect, the correct geometry shows that the direction of centrifugal force obeys the usual pattern inside the stenosphere: the force is directed from the concave side to the convex side of the circle on which a body is constrained to move. That proper geodesics lie inside  $\Theta$  geodesics is easily understood: proper geodesics ‘cheat’ by taking advantage of the elongation of measuring rods implied by the gravitational size dilation effect.

Regarding the other puzzles, it is obvious that nothing, not

even light can reach the event horizon since that “surface” is just a name for the infinity of ‘*innerspace*’ (the seeming finite proper distance to the event horizon is an artifact of the limitless elongation of measuring rods as  $r \rightarrow r_s$ ). Thus the “no-hair” theorem is invalid, and the controversy regarding the supposed loss of information and entropy when matter “disappears” into a black hole is settled: nothing disappears; nothing is lost. Another significant feature of the (GRMR) interpretation is the non-existence of the baleful singularity, which, according to the usual interpretation, is thought to lurk behind the event horizon.

## 8. Cosmology

The most surprising implications of the variable rest mass concept relates to cosmology. Hubble’s discovery of the systematic cosmological red shift immediately suggested that galaxies were flying through space away from us and from one another – the further, the faster. This in turn suggested that the universe grew from an incredibly hot and dense condition (a singularity!) billions of years ago. Later, it became clear that the galaxies were not actually moving *through* space, but rather that *space itself* was expanding. Thus the conventional understanding holds that the wavelength of light is continually stretched in flight by the expansion of space. In the present understanding, the redshift parameter,  $(Z+1) = \lambda_{obs}/\lambda_{emit}$ , shown by a galaxy is held to be proportional to the ratio of the scale of the present universe to that of the universe at the time of emission.

Neither astronomers nor cosmologists seem to be concerned with the fact that this explanation of the cosmological red shift fails to conserve the momentum of the observed radiation, as Noether’s Theorem demands in a spatially homogeneous universe [2]. Observation shows that, on a sufficiently large scale, the universe is homogeneous and isotropic to a very high degree, and virtually every cosmological model assumes this at the outset. Perhaps no one has even considered this problem since, as every observation of the red shift appears to have demonstrated, momentum seems manifestly to be *not* conserved. The photons of the cosmic microwave background radiation field that we detect today were born in a  $3000^\circ K$  hydrogen-helium plasma at ‘recombination’ time, when the plasma first became transparent. Their wavelength has seemingly increased by a factor of about 1000 – how is it possible that their momentum has not changed? The only possibility is that over the aeons, *measuring instruments* have changed, and are changing, decreasing their characteristic wavelengths – for example, diffraction gratings have shrunk and are shrinking. And the only way this might occur is if all rest masses have been and are increasing in proportion to  $a(t) = A(t)/\tilde{A}$ , in which  $A(t)$  is the function that is presently interpreted as representing the increasing scale of the universe, and  $\tilde{A}$  is its present value.

To prove that momentum conservation requires rest masses to increase in proportion to  $a(t)$ , one need only

consider the motion of a test mass *through* space. In this case, there is an integral of motion (developed in APPENDIX B) for the Robertson-Walker metric, namely,  $a(t) \cdot \beta (1-\beta^2)^{-1/2} = \text{constant}$ . Since momentum is given by  $m^* c \beta (1-\beta^2)^{-1/2}$ , it is clear that conservation of momentum requires that the true rest mass,  $m^*$ , must be proportional to  $a(t)$ , that is,  $m^* = m \cdot a(t)$  where  $m$  is the proper rest mass, a constant.

The solution of Einstein's field equations for cosmology under the assumption that rest masses increase in proportion to  $a(t)$  (so as to insure momentum conservation) is presented in APPENDIX C. The surprising result is that  $a(t)$  turns out to be a simple exponential function of world time:

$$a(t) = \exp(-\omega(t_{\text{now}} - t)).$$

The frequency parameter,  $\omega$ , is easily identified as the Hubble constant,  $H_0$ : For nearby galaxies, Hubble's 'law' implies  $\lambda_{\text{obs}}/\lambda_{\text{std}} \approx 1 + V/c = 1 + H_0 D/c = 1 + H_0(t_{\text{now}} - t_{\text{emit}})$ . On the other hand, the exponential function gives,  $\lambda_{\text{obs}}/\lambda_{\text{std}} = 1/a = \exp(\omega(t_{\text{now}} - t_{\text{emit}})) \approx 1 + \omega(t_{\text{now}} - t_{\text{emit}})$ . Thus,  $\omega = H_0$ .

Since we know that the measured wavelength of the cosmic microwave background radiation field has increased by a factor of  $10^3$ , we can determine when 'recombination' occurred:

$$\lambda_{\text{obs}}/\lambda_{\text{std}} = 10^3 = 1/a = \exp(H_0(t_{\text{now}} - t_{\text{recom}})) \text{ so} \\ (t_{\text{now}} - t_{\text{recom}}) = 3 \ln(10) H_0^{-1} \approx 2.3 H_0^{-1} \approx 32.2 \times 10^9 \text{ yrs}$$

Regarding the evolution of proper time,  $T$ , one has  $dT/dt = m^*/m = a(t)$ , hence

$$T = \int_{-\infty}^t a(t') dt' = H_0^{-1} a(t) \Rightarrow a = H_0 T.$$

Thus the rest mass evolution function is a linear function of proper time,  $T$ .

Since  $a(T_{\text{now}}) = 1$ , the age of the universe is

$$T_{\text{now}} = H_0^{-1} \approx 14 \times 10^9 \text{ yrs.}$$

Also, since  $a(T_{\text{recom}}) = 10^{-3}$ ,

$$T_{\text{recom}} = H_0^{-1} a_{\text{recom}} \approx (14 \times 10^9) \times 10^{-3} = 14 \times 10^6 \text{ yr}$$

Proper time is that kept by physical clocks whose rates, looking backward in time, slow in proportion to  $a(t)$ . World time,  $t$ , may be thought of as being defined with reference to any observable free electromagnetic radiation – in particular, the cosmic microwave background radiation – whose true frequency is constant. Paradoxically, it is quite clear that the universe is infinitely old in terms of world time,  $t$ : yet it is no less true that the universe suddenly came into existence with a 'Big Bang' some  $H_0^{-1}$  seconds ago ( $\sim 14$  billion years) in terms of proper time,  $T$ .

## 9. Looking Back: Is Gravity a Field Theory?

In retrospect, the concept of gravity presented here is clearly more in line with Newton's (self-berated) action-at-a-distance formulation, than with Einstein's field theory. Indeed, what role is left for the field if gravitational

'charges' (masses) appear to be capable of exchanging momentum and energy directly? The role of the gravitational field is further diminished by the fact that any truly tensorial formulation of an energy-momentum tensor of the gravitational field itself must, in Einstein's theory of gravity, be identically zero, since the field and the proposed energy-momentum tensor will vanish locally in any freely-falling frame of reference. The tensor character then guarantees that the proposed energy-momentum tensor vanishes in every frame of reference. Despite these facts, which would seem to render the gravitational field almost irrelevant, it must be recognized that the non-Euclidean geometry of three-space inherent to the gravitational field is indispensable to the correct understanding of the dynamical behavior of both massive particles and light, as well as the structure of black holes. The problem remaining is to discover how the presence of mass induces such profound changes in the geometry of three-space. But underlying all, the field seems somehow to be responsible for the reduction of rest masses, which is the cause of the two fundamental gravitational phenomena: *gravitational clock slowing* (the red shift) and *gravitational size dilation*.

## Appendix A: Proof of Gravitational Rest Mass Reduction

It is proved that rest mass reduction is necessary and sufficient to account for the gravitational red shift in a manner that preserves the invariance of the measured speed of light and reveals the correct geometry of space in the neighborhood of a gravitating body. Consideration of the *Strong Equivalence Principle (SEP)* will guide our search for the cause of the reduction of emission frequencies in a gravitational field. According to Richard Dicke [9] the *SEP* asserts that

*In a freely falling, non-rotating laboratory, the local laws of physics take on some standard form, including a standard numerical content, independent of the position of the laboratory in space and time.*

Many modern-day physicists misunderstand this as implying that nothing can actually change in the laboratory as it falls in a real gravitational field. This certainly is not the case since, as we already know, clock rates are slowed. The *SEP* only requires that any changes must occur in concert so as to be *undetectable* by observers in the falling lab. One thing the *SEP* certainly demands is that quantities of the same dimensionality must vary together, if they do vary, by exactly the same factor. Thus all quantities having the dimension of time must vary exactly as does the Rydberg period,  $P_R$ , and all quantities having the dimension of length must vary as does the Bohr radius,  $a_0$ . Going further, all quantities having the dimension of velocity must vary as  $a_0/P_R = e^2/h$ . This must apply to the velocity of light, so it must be true that  $(e^2/h)/c$  is unaffected by gravity. But this is just the fine structure constant, which is an observed constant,

not just in Dicke's falling lab, but as observed for instance in the spectra of distant stars.

We are now in a position to summarize what we know regarding the possible dependence of the 'constants' upon the gravitational potential. First, there is a red shift, implying that  $P_R \sim f^{-1} > 1$ , where  $f = |g_{00}|^{1/2}$ . In what follows the dependence of any quantity,  $X$ , on  $f$  will be indicated by writing  $X \sim f^{(X)}$ . Accordingly, the red shift implies  $(P_R) = -1$ . This may be expanded to  $3(h) - (m_e) - 2(e^2) = -1$ . Next,  $(a_0) = 2(h) - (m_e) - (e^2)$ . Finally, from the finding regarding the fine structure constant, we have  $(c) = (e^2) - (h)$ . Clearly, these equations are satisfied by  $(m_e) = 1$ ,  $(h) = 0$ ,  $(e^2) = 0$ , implying  $(a_0) = -1$ , and  $(c) = 0$ . These relations characterize the gravitational *rest mass reduction* (GRMR) interpretation of gravity. Of course, according to the SEP, if the electron rest mass,  $m_e$ , is reduced, all masses are similarly reduced. For the same reason, the increase of  $a_0$  implies that the dimension of all objects, including measuring rods, is increased, and the increase in  $P_R$  implies that all time periods are similarly increased (or, what is the same thing, frequencies are decreased by the reciprocal factor).

Following Einstein, modern day physicists have unconsciously assumed that that distance measurements are unaffected by gravity, implying  $(a_0) = 0$ . Then since  $(P_R) = -1$ , it would follow that  $(a_0) - (P_R) = 1$ . But from the original definitions,  $(a_0) - (P_R) = (e^2) - (h)$ , which is equal to  $(c)$ . This would imply that  $(c) = 1$ , just as Einstein had assumed in order that the locally measured speed of light be independent of location in a gravitational field. The same equation,  $(c) = (e^2) - (h)$ , would then imply that  $(h) = (e^2) - 1$ . Finally inserting this into the original equation for  $(a_0)$ , and setting  $(a_0) = 0$  would give  $(m_e) = (e^2) - 2$ . But charge invariance implies  $(e^2) = 0$ , so that finally, Einstein's assumption that distance measurements are unaffected by gravity requires not only that  $(c) = 1$ , but also  $(h) = -1$ , and  $(m_e) = -2$ . Mainstream physicists will perhaps be surprised and puzzled by the full implications of Einstein's seemingly necessary and harmless assumption regarding the speed of light in a gravitational field. Notice that the correct choice implies that measuring rods are elongated, so that distance measurements are undervalued by the same factor as clock rates are slowed, guaranteeing that local measurements of the speed of light are unaffected by gravity. And of course the presently accepted idea that the true speed of light is reduced in a gravitational field must be rejected.

## Appendix B: Momentum Conservation Implies Increasing Rest Mass

The equation of motion for a test mass moving through space with constant momentum is derived, showing that this requires that the rest mass of the particle must increase in proportion to  $A(t)$ .

The Robertson Walker metric may be written [10]

$$ds^2 = c^2 dt^2 - A^2(t) [d\chi^2 + S^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)],$$

where  $S(\chi)$  is  $\sin(\chi)$ ,  $\chi$ , or  $\sinh(\chi)$ , depending on whether the curvature of the universe is positive, zero, or negative, respectively.

Introducing a new time variable,  $\eta = ct / A$  the metric becomes

$$ds^2 = A^2(\eta) [d\eta^2 - d\chi^2 - S^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)]$$

In a homogeneous, isotropic universe, one may, without loss of generality, consider the motion to be in the  $\chi$  direction, for which the geodesic equation is

$$d^2\chi/ds^2 + \Gamma_{\eta\eta}^{\chi}(d\eta/ds)^2 + 2\Gamma_{\eta\chi}^{\chi}(d\eta/ds)(d\chi/ds) + \Gamma_{\chi\chi}^{\chi}(d\chi/ds)^2 = 0$$

$$\text{But } \Gamma_{\eta\eta}^{\chi} = 0, \Gamma_{\eta\chi}^{\chi} = A^{-1}dA/d\eta, \text{ and } \Gamma_{\chi\chi}^{\chi} = 0.$$

Thus the geodesic equation reduces to

$$d^2\chi/ds^2 + 2A^{-1}(dA/d\eta)(d\eta/ds)(d\chi/ds) = 0$$

Inserting  $(dA/d\eta)(d\eta/ds) = dA/ds$  and integrating, gives

$$A^2(d\chi/ds) = \text{constant}$$

The proper velocity,  $v$ , is defined by  $v = dl/dt$ , where  $dl = A d\chi$ , and  $dt = c^{-1} A d\eta$ .

Thus  $v/c = d\chi/d\eta$  and the integral may be written,

$$A^2(d\chi/ds) = A^2(d\chi/d\eta)/(ds/d\eta) = A^2(v/c)/(ds/d\eta) = \text{constant}.$$

But directly from the line element, for the motion considered,

$$ds/d\eta = A\sqrt{1 - (d\chi/d\eta)^2} = A\sqrt{1 - (v/c)^2}$$

Inserting this and introducing  $\beta = v/c$ , the integral of motion may be written

$$A(\eta)\beta/\sqrt{1 - \beta^2} = \text{constant}$$

## Appendix C: Solving the Gravitational Field Equations with Momentum Conserved

Here we develop and solve the equations for the function  $A(t)$ , under the assumption that all rest masses increase in proportion to  $A(t)$  insuring the conservation of momentum, which is implied by the assumption of the large scale homogeneity of the universe.

### Preliminaries

We shall limit our consideration to the case of a single type of massive particle.

Let  $a = A(t) / \tilde{A}$  where  $\tilde{A}$  is the present value of  $A$ , and let  $u = \beta / \sqrt{1 - \beta^2}$ .

From APPENDIX B,

$$p^* = m^* c \beta / \sqrt{1 - \beta^2} = mc (A / \tilde{A}) \beta / \sqrt{1 - \beta^2} =$$

$$= mc a u = mc \tilde{u} = \text{const.}$$

and

$$E^* = m^* c^2 / \sqrt{1 - \beta^2} = mc^2 a \sqrt{1 + u^2} =$$

$$= mc^2 \sqrt{a^2 + (au)^2} = mc^2 \sqrt{a^2 + \tilde{u}^2}$$

### Some kinetic theory

For the moment we suppress the asterisks. Consider a cubical box,  $L$  on a side. When a particle bounces off the side normal to the direction  $x$ , the momentum imparted is equal to twice the  $x$ -component of the particles momentum:

$$\Delta p = 2mc\beta_x / \sqrt{1 - \beta^2} \quad \text{the time between hits on this wall is}$$

$$\Delta t = 2L / c\beta_x, \text{ so the force imparted is}$$

$$f_x = \Delta p / \Delta t = mc^2 \beta_x^2 / L \sqrt{1 - \beta^2}$$

Now  $\beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2$  and for the isotropic case assumed,  $\beta_x^2 = \frac{1}{3} \beta^2$ . Thus

$$f_x = \frac{mc^2 \beta^2}{3L \sqrt{1 - \beta^2}}$$

$$\text{The pressure is then } \wp = \frac{f_x}{L^2} = \frac{mc^2 \beta^2}{3L^3 \sqrt{1 - \beta^2}} = \frac{mc^2 \beta^2}{3V \sqrt{1 - \beta^2}}$$

The pressure due to  $N$  particles is  $N$  times this, and the number density is  $n = N / V$

$$\text{Thus finally we have } \wp = \frac{nmc^2 \beta^2}{3\sqrt{1 - \beta^2}}. \text{ Now}$$

$$\frac{\beta^2}{\sqrt{1 - \beta^2}} = \frac{u^2}{1 + u^2} \sqrt{1 + u^2} = \frac{u^2}{\sqrt{1 + u^2}}$$

Restoring the asterisks and tildes,  $u = \tilde{u} / a$  and

$$\frac{\beta^2}{\sqrt{1 - \beta^2}} = \frac{\tilde{u}^2}{a \sqrt{a^2 + \tilde{u}^2}} \text{ and the pressure is}$$

$$\wp^* = \frac{nm^* c^2 \beta^2}{3\sqrt{1 - \beta^2}} = \frac{a n m c^2}{3} \frac{\tilde{u}^2}{a \sqrt{a^2 + \tilde{u}^2}} = \frac{n m c^2}{3} \frac{\tilde{u}^2}{\sqrt{a^2 + \tilde{u}^2}}$$

And the Energy density is

$$\varepsilon^* = \frac{n m^* c^2}{\sqrt{1 - \beta^2}} = a n m c^2 \sqrt{1 + u^2} = n m c^2 \sqrt{a^2 + \tilde{u}^2}$$

The energy-momentum tensor  $T_{ik}$  is diagonal with

$$T_{00} = \varepsilon^*, \quad T_{kk} = -\wp^* \quad \text{for } k = 1, 2, 3$$

### Calculating the Einstein Tensor

For the sake of simplicity and since space actually appears

to be Euclidean, the metric is assumed to be

$$ds^2 = a^2(x_0) [(dx_0)^2 - (dx_1)^2 - (dx_2)^2 - (dx_3)^2]$$

Note that this form assumes that clock rates and distance measurements will be affected in the in exactly the same way, a fact that an inherent consequence of variable rest mass. The metric tensor is diagonal with

$$g_{00} = a^2, g_{kk} = -a^2; g^{00} = a^{-2}, g^{kk} = -a^{-2}$$

The only non-zero Christoffel symbols are

$$\Gamma_{00}^0 = \frac{1}{2} g^{00} \frac{\partial g_{00}}{\partial x^0} = \frac{1}{2} a^{-2} (a^2)' = \frac{\dot{a}}{a}$$

$$\Gamma_{k0}^k = \frac{1}{2} g^{kk} \frac{\partial g_{kk}}{\partial x^0} = \frac{1}{2} (-a^{-2}) (-a^2)' = \frac{\dot{a}}{a}$$

$$\Gamma_{kk}^0 = \frac{1}{2} g^{00} \left( -\frac{\partial g_{kk}}{\partial x^0} \right) = \frac{1}{2} (a^{-2}) (a^2)' = \frac{\dot{a}}{a}$$

$$\text{and } g = \det(g_{**}) = -a^8, \sqrt{-g} = a^4$$

The Ricci tensor is may be expressed as

$$R_{ik} = 1 / \sqrt{-g} \frac{\partial}{\partial x^j} (\Gamma_{ik}^j \sqrt{-g}) - \frac{\partial^2}{\partial x^i \partial x^k} [\ln(\sqrt{-g})] - \Gamma_{il}^m \Gamma_{km}^l$$

For the case at hand, we have

$$R_{ik} = a^{-4} \frac{\partial}{\partial x^j} (\Gamma_{ik}^j a^4) - \frac{\partial^2}{\partial x^i \partial x^k} [\ln a^4] - \Gamma_{il}^m \Gamma_{km}^l =$$

$$= 4(\dot{a} / a) \Gamma_{ik}^0 + \frac{\partial}{\partial x^0} \Gamma_{ik}^0 - 4\delta_{0i} \delta_{0k} \frac{\partial}{\partial x^0} (\dot{a} / a) - R_{ik}^{(3)}$$

Where the third terms,  $R_{ik}^{(3)} = \Gamma_{il}^m \Gamma_{km}^l$ , will be evaluated below.

For  $i \neq k$ ,  $\Gamma_{ik}^0 = 0$ , and

$$R_{ik} = -R_{ik}^{(3)}$$

For  $i = k = 0$

$$R_{00} = 4(\dot{a} / a) \Gamma_{00}^0 + \frac{\partial}{\partial x^0} \Gamma_{00}^0 - 4 \frac{\partial}{\partial x^0} (\dot{a} / a) - R_{00}^{(3)}$$

$$= 4(\dot{a} / a)^2 - 3[\ddot{a} / a - (\dot{a} / a)^2] - R_{00}^{(3)}$$

$$R_{00} = 7(\dot{a} / a)^2 - 3\ddot{a} / a - R_{00}^{(3)}$$

For  $i = k \neq 0$

$$R_{kk} = 4(\dot{a} / a) \Gamma_{kk}^0 + \frac{\partial}{\partial x^0} \Gamma_{kk}^0 - R_{kk}^{(3)}$$

$$= 4(\dot{a} / a)^2 + \frac{\partial}{\partial x^0} (\dot{a} / a) - R_{kk}^{(3)} = 3(\dot{a} / a)^2 + \ddot{a} / a - R_{kk}^{(3)}$$

$$R_{kk} = 3(\dot{a} / a)^2 + \ddot{a} / a - R_{kk}^{(3)}$$



Now regarding the products of Christoffel symbols,

$$R_{ik}^{(3)} = \Gamma_{il}^m \Gamma_{km}^l$$

$$R_{0k}^{(3)} = \Gamma_{0l}^m \Gamma_{km}^l = \Gamma_{00}^0 \Gamma_{k0}^0 + \Gamma_{0k}^0 \Gamma_{k0}^k + \Gamma_{00}^k \Gamma_{kk}^0 + \Gamma_{0k}^k \Gamma_{kk}^k$$

$$(l, m) \quad 0, 0 \quad k, 0 \quad 0, k \quad k, k$$

$$R_{0k}^{(3)} = 0 \text{ since } \Gamma_{k0}^0 = \Gamma_{0k}^0 = \Gamma_{00}^k = \Gamma_{kk}^k = 0$$

$$R_{00}^{(3)} = \Gamma_{0l}^m \Gamma_{0m}^l = \Gamma_{00}^0 \Gamma_{00}^0 + \Gamma_{0j}^0 \Gamma_{00}^j + \Gamma_{00}^j \Gamma_{0j}^0 + \Gamma_{0j}^j \Gamma_{0j}^j$$

$$(l, m) \quad 0, 0 \quad j, 0 \quad 0, j \quad j, j$$

Now  $\Gamma_{0j}^0 = \Gamma_{00}^j = 0$ , so

$$R_{00}^{(3)} = \Gamma_{00}^0 \Gamma_{00}^0 + \sum_{j=1,2,3} \Gamma_{0j}^j \Gamma_{0j}^j = 4(\dot{a}/a)^2$$

$$R_{00}^{(3)} = 4(\dot{a}/a)^2$$

$$R_{kk}^{(3)} = \Gamma_{kl}^m \Gamma_{km}^l = \Gamma_{k0}^0 \Gamma_{k0}^0 + \Gamma_{kk}^0 \Gamma_{k0}^k + \Gamma_{k0}^k \Gamma_{kk}^0 + \Gamma_{kk}^k \Gamma_{kk}^k$$

$$(l, m) \quad 0, 0 \quad k, 0 \quad 0, k \quad k, k$$

Now  $\Gamma_{k0}^0 = \Gamma_{kk}^k = 0$ , so

$$R_{kk}^{(3)} = 2\Gamma_{kk}^0 \Gamma_{k0}^k = 2(\dot{a}/a)^2$$

$$R_{kk}^{(3)} = 2(\dot{a}/a)^2$$

Collecting the above results, we have  $R_{0k} = 0$  for the off diagonal components, while

$$R_{00} = -3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} \text{ and}$$

$$R_{kk} = \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \text{ for } k = 1, 2, 3$$

The scalar 'curvature',  $R$ , is given by

$$R = g^{ik} R_{ik} = g^{ii} R_{ii} = g^{00} R_{00} + 3g^{kk} R_{kk} = a^{-2} \left[ -3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} \right] - 3a^{-2} \left[ \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right] = -6a^{-2} \frac{\ddot{a}}{a}$$

The Einstein tensor  $G_{ik} = R_{ik} - \frac{1}{2}g_{ik}R$  is diagonal with components

$$G_{00} = 3\frac{\dot{a}^2}{a^2} \text{ and } G_{kk} = -2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}$$

The gravitational field equations

$$G_{ik} = \frac{8\pi G}{c^4} T_{ik}, \text{ are then}$$

$$3\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{c^4} nmc^2 \sqrt{a^2 + \tilde{u}^2}$$

$$\text{and } -2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{8\pi G}{3c^4} nmc^2 \frac{\tilde{u}^2}{\sqrt{a^2 + \tilde{u}^2}}$$

Introduce a new time variable  $\tau = \Omega (x_0 / c)$ , where

$$\Omega = \sqrt{\frac{8}{3}} \pi G n m.$$

The equations may be written as (reusing the dot as  $\tau$ )

$$\frac{\dot{a}^2}{a^2} = \sqrt{a^2 + \tilde{u}^2} \text{ and } 2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{\tilde{u}^2}{\sqrt{a^2 + \tilde{u}^2}}$$

The product of the right hand members of the two equations equals the constant,  $\tilde{u}^2$ ; so obviously the product of the members of the left hand side must also equal the same constant:

$$\frac{\dot{a}^2}{a^2} \left[ 2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right] = \tilde{u}^2.$$

The form of this expression suggests an exponential function:  $a(\tau) = \exp(\varpi \tau)$ : inserting this

$$\frac{\dot{a}^2}{a^2} \left[ 2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right] = \varpi^2 (2\varpi^2 - \varpi^2) = \varpi^4$$

Thus  $\varpi = \sqrt{\tilde{u}}$  and  $a(\tau) = \exp(\sqrt{\tilde{u}} \tau)$

Actually, we have no way to make a reasonable estimate of  $\tilde{u} = p^*/mc$ , so let's just set

$$a(t) = \exp(-\omega(t_{\text{now}} - t))$$

where  $\omega$  is to be determined from observational data.

Note that  $a \rightarrow 0$  only for  $\tau \rightarrow -\infty$ .

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