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Replay: On a Common Logical Error in Calculation and Applying the Complex Conductivities of Collisionless Plasmas

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Abstract Damping/growing of electron waves in collisionless plasmas (kinetic equation with zero collision terms) does not exist contrary to the established among plasma physicists more than half a century ago the view about theoretical discovery of its existence as a natural and experimentally confirmed physical phenomenon.

Keywords Electron waves in collisionless plasma, Plasma waves damping/growing

1. Commentary

The recent discussion [1-4] (with not principal amendment in improper integral (18) in [2]) clearly confirms the trivial logical (and ultimately mathematical) cause of widespread (see e.g. textbooks [5-7] and numerous in literature other articles and textbooks on plasma physics) erroneous description of non-existing collisionless damping of waves in both non-magnetoactive and in general case magnetoactive collisionless (in the sense of neglecting the collision energy-exchange terms in kinetic equation) plasmas. The reason is that all the quantitative laws of the nature are characterized as relations between the so-called physically detectable values (PhDVs), which in the case of their complex representation (before all further non-linear complex transformations!) have to be substituted into the real initial wave equations in the form of real combinations of complex conjugate sets as it is done, for example, in the form of direct substitution of sought PhDVs into the wave equations in the works [1-4] with the final results also in the natural form of relations real PhDVs. Moreover, a direct unambiguous relation is derived between the wave damping/growing parameters (if present) with the defined collision terms of kinetic equation.

It is wrong, for example, using generalization to damping/growing plasma waves by moving on to a complex wave number $k \to k_1 - ik_2$ in the intermediate steps or the final result (in the form of the dispersion equation) in the case of a real k, because in complex nonlinear equations mixing occurs of terms with k_1 and k_2 in the imaginary and real parts of the resulting complex equations and expressions, and it becomes impossible to extract from them the true relations between PhDVs. In these cases, there arise also wrong side complex roots of the dispersion equation and a variety of side terms of complex expressions ("spirits"). It is absolutely wrong detection the true relations of PhDVs by separating the real part of final complex dispersion equation with dropping the imaginary part [2] which leads to collisionless damping.

A typical case is nonlinear with respect to combinations of k_1 and k_2 resulting dispersion equation with complex wave functions in the initially linear wave equations. And it is due to initial complex nonlinearity between combinations of the real and imaginary values. Complex nonlinearity arises, for example, in the simplest case of derivative $d\left(e^{i(\omega t-kx)}\right)\Big/ dx = -ike^{i(\omega t-kx)}$ at complex k with its corresponding inequalities at extending $k \to k_1 - ik_2$: as an arbitrary but typical illustrative simple example of possible mixing real and imaginary parts in final real expressions (cf. also complex conjugated sums (13) with mixing real and imaginary terms):

$$\begin{aligned}
&\mathrm{Re}\frac{d}{dx}\exp\left[i\left(\omega t - kx\right)\right] \\
&= -\mathrm{Re}\{\left(ik_1 + k_2\right)\exp\left(-k_2 x\right) \\
&\quad \left[\cos\left(\omega t = k_1 x\right) + i\sin\left(\omega t - k_1 x\right)\right]\} \\
&= -\exp\left(-k_2 x\right)\left[k_2\cos\left(\omega t - k_1 x\right) - k_1\sin\left(\omega t - k_1 x\right)\right] \\
&\neq \mathrm{Re}\left(-ik\right)\times\mathrm{Re}\{\exp\left[i\left(\omega t - kx\right)\right]\},
\end{aligned} \tag{1}$$

i.e. there are inadmissible anyway generalizations in the

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transition from real to complex c and d of the type $\mathrm{Re}\, cd \to \mathrm{Re}\, c \times \mathrm{Re}\, d$ (cf. also below Eq. (13)).

It is rather unclear how to allocate the true relation between the sought real PhDVs after transformations to final complex dispersion equation with complex wave functions for the case of the usual complex wave functions with the damping $\sim e^{-k_2 x}$:

$$f_1(v_x, x, t) = f_1(v_x) e^{i(\omega t - kx)},$$
$$E(x,t) = iE_0 e^{i(\omega t - kx)}, \quad k \equiv k_1 - ik_2, \tag{2}$$

unlike initial substitutionin real form:

$$\mathrm{Re}\, f_1(v_x, x, t) = f_1(v_x) \cdot e^{-k_2 x} \cos(\omega t - k_1 x), \tag{3}$$

$$\mathrm{Re}\, E(x,t) = -E_0 \sin(\omega t - k_1 x);$$
$$a \equiv \cos(\omega t - k_1 x); \quad b \equiv \sin(\omega t - k_1 x) \tag{4}$$

where $f_1(v_x, x, t)$ is perturbation of the background electron Maxwellian function $f_0(v)$; $E(x,t)$ is the field perturbation from external field $-E_0(\omega t, x = 0)$.

Substituting real expressions (3) and (4) into real wave equations (kinetic and Maxwell equation)

$$\frac{\partial f_1(v_x, x, t)}{\partial t} + v_x \frac{\partial f_1(v_x, x, t)}{\partial x} + \frac{e}{m_e} E(x,t) \frac{\partial f_0(v)}{\partial v_x} = 0 \tag{5}$$

$$\frac{\partial E(x,t)}{\partial x} = \frac{e}{m_e} \int f_1(v_x, x, t)\, dv_x \tag{6}$$

where $e = -|e|$ is electron charge, m_e is electron mass and v, v_x are electron velocities, leads to the real dispersion relation [1], [2]:

$$1 = -\omega_0^2 \frac{ab}{k_1 a + k_2 b} \int \frac{(\partial f_0 / \partial v_x)}{(\omega - k_1 v_x) b + k_2 v_x a} dv_x \tag{7}$$

where $a \equiv \cos(\omega t - k_1 x)$, $b \equiv \sin(\omega t - k_1 x)$, ω_0 is Langmuir frequency $\omega_0^2 = 4\pi |e| / m_e$.

It means that hypothetical solutions of Eq. (7) with $k_1, k_2 \neq 0$ that would not depend on x, t do not exist. However, one can try to get some average values $k_1, k_2 \neq 0$ over the wave period introducing the collision term S on the right hand side of the kinetic equation with the dispersion equation of the form

$$1 = \frac{-ab}{ak_1 + bk_2} \cdot \omega_0^2 \int \frac{\partial f_0 / \partial v_x}{(\omega - k_1 v_x) b + a k_2 v_x - A} dv_x \tag{8}$$

taking

$$A = a k_2 v_x; \quad S = A f_1(v_x) e^{-k_2 x}. \tag{9}$$

In this case dispersion relation takes the form [1], [2]:

$$1 = \frac{-a}{ak_1 + bk_2} \cdot \omega_0^2 \int \frac{\partial f_0 / \partial v_x}{(\omega - k_1 v_x)} dv_x;$$

$$\left(\frac{a}{ak_1 + bk_2} \right)_{av} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{k_1 + k_2 \tan y} dy = \frac{1}{2\pi} \int_0^{2\pi} \frac{k_1}{k_1^2 - k_2^2 (\tan y)^2} \tag{10}$$

with

$$f_1(v_x) = \frac{e}{m_e} \frac{\partial f_0 / \partial v_x}{(\omega - k_1 v_x)} E_0 \tag{11}$$

where all integrals can be taken, including improper integrals (10), in the principal value sense. Besides that, due to singularity of the function $f_1(v_x)$ at the point $v_x = \omega / k_1$ in Eq. (11) near which the kinetic equation does not apply, it is necessary cutting off $f_1(v_x)$ in (11) nearby this point in accordance with the condition of positivity the total distribution function $f = f_0(v) + f_1(v_x)$:

$$|f_1(v_x, x, t)| < f_0(v). \tag{12}$$

The results obtained are in principle different from those obtained when ubiquitous, without any exceptions, using the dispersion equations in the case $k_2 \neq 0$ after substitution therein complex wave functions (2) (see [1]). In this case, the complex roots of the dispersion equation contain an artifact when appearing imaginary part due to mixed, real and imaginary terms with k_1 and k_2 entails respectively two equations for the real and imaginary parts of dispersion equation that does not correspond in no way to the law of energy conservation or any other physical law. In contrast, the real dispersion equation is single equation containing both parameters k_1 and k_2, while the second equation of the system for to find k_1 and k_2 is directly the new second equation of energy conservation that independently binds the sought parameters k_1 and k_2 included in collisional energy-exchange terms A and S according to (9) (see, for example, [1], [2], where also are pointed out numerous contradictions of the so-called collisionless damping in the current formulation of the problem). From the above, it also follows that the commonly used expressions for complex tensors of electrical conductivity and dielectric permittivity of collisionless plasma should be used directly only for the real ω and $\vec{k}$ or with substitution of initially complex conjugate expressions for PhDVs which significantly reduces the advantages of using complex variables.

Use of complex wave functions can lead to complex dispersion equations with complex non-physical roots. The analytical mathematical formalism of complex tensor conductivity/permeability is inapplicable in the presence of any significant energy exchange processes, with rather non-sinusoidal form of propagating waves.

In general, the solution for PhDVs can be obtained by using the complex conductivity tensor $\sigma_{ij}\left(\omega,\vec{k}\right)$ with $\omega \geq 0$ and complex $\vec{k}$ as a real solution of the dispersion equation of the form

$$\left[j_{\vec{v}_i}\left(\omega,\vec{k}\right)e^{i\left(\omega t-\vec{k}\vec{r}\right)}\right]+\left[\text{compl. conjug.}\right]$$
$$=\left[\sigma_{ij}\left(\omega,\vec{k}\right)E_j\left(\omega,\vec{k}\right)e^{i\left(\omega t-\vec{k}\vec{r}\right)}\right]_{\vec{v}_i} \tag{13}$$
$$+\left[\text{compl. conjug.}\right]_{\vec{v}_i}$$

which should be followed by integration over $\vec{v}$, where $j_{\vec{v}_i}\left(\omega,\vec{k}\right)$, $E_j\left(\omega,\vec{k}\right)$ are complex components of integral Fourier expansion correspondingly for electrical current and tension of electrical field. In this approach, the consideration of imaginary part of such form of dispersion equation which should lead to the collisionless damping becomes meaningless.

Thus, in general, the real dispersion equation and its solution may contain the angular coordinates of the real and imaginary parts of the initial wave function with $\cos\theta$ and $\sin\theta$, which makes however impossible to use the solution of dispersion equation (cf. (7)).

In the derivation of Eq.(13) it was assumed solution in the form of Fourier transform $F\left(\vec{r},t\right)=\int C\left(\omega\right)e^{-i\vec{k}\vec{r}}\cdot e^{i\omega t}d\omega$ with one-valued dependence $\vec{k}=f\left(\omega\right)$ at general case of complex $\vec{k}$. But in general, solutions have form of the double (in ω and $\vec{k}$) Fourier transform

$$F\left(\vec{r},t\right)=\iint\limits_{\infty}C\left(\omega,\vec{k}\right)e^{i\left(\omega t-\vec{k}\vec{r}\right)}d\omega d\vec{k} \tag{14}$$

with real $\omega,\vec{k}$, and such one-valued dependence $\vec{k}=f\left(\omega\right)$ can be not existing, i.e. to each ω there corresponds the whole spectrum of values $\vec{k}$, and solutions of the form $\sim e^{i\left(\omega t-\vec{k}\vec{r}\right)}$ of the type (13) with complex $\vec{k}$ do not exist at all (as evidenced by the above example of the dispersion equation (7)). Also very relation (13) becomes invalid.

Moreover, theoretical contradictions are accompanied with poorly compatible conditions of proposed experiments on collisionless damping (1).

2. Conclusions

Using a two-parametric k_1, k_2 non-physical complex root of the complex dispersion equation for damping complex plasma waves in a collisionless plasma instead of detecting the two real observable parameters of the real dispersion equation with adding two-parametric real energy conservation equation is common fundamental logical and, by inference, mathematical error available in the literature on Plasma Physics, which leads to an erroneous conclusion about the existence of collisionless damping of plasma waves with derived non physical decrement.

Procedure (13) of the transition to real dispersion equation complemented by integration over $\vec{v}$ entirely changes the normal procedure for the application of the complex conductivity tensor of collisionless and collision plasma and can lead to in principle different from existing dispersion equations and their solutions with appearing collisionless damping.

Exponentially damping sinusoidal plasma wave solutions for both collisionless and collision plasma do not exist.

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