

A Note on Bayesian Analysis of Error Function Distribution under Different Loss Functions

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Abstract The Bayesian analysis of the scale parameter of error function distribution has been considered in this paper. A class of informative and non-informative priors has been assumed to derive the corresponding posterior distributions. The Bayes estimators and associated risks have calculated under different loss functions. The Bayesian credible intervals have been constructed under each prior. The performance of the Bayes estimators have been evaluated and compared under a comprehensive simulation study. The purpose is to find out the combination of a loss function and a prior having the minimum Bayes risk and hence producing the best results. The study depicts that in order to estimate the said parameter use of entropy loss function under informative priors can be preferred.

Keywords Squared Error Loss Function (SELF), Quadratic Loss Function (QLF), Entropy Loss Function (ELF), Precautionary Loss Function (PLF), Credible Intervals

1. Introduction

The error function distribution is one of the most widely used distributions in statistics. Estimating its parameter using Bayesian inference is extremely useful. Eberly and Casella[1] discussed the construction of Bayesian credible intervals using Rao-Blackwellized construction which offers smallest standard error of estimate. Korsgaard et al.[2] considered the multivariate normal distribution and concentrated on the model where residuals associated with liabilities of the binary traits have been assumed to be independent. A Bayesian analysis using Gibbs sampling has been outlined for the model where this assumption has been relaxed. Wang[3] proposed a criterion to choose a loss function in Bayesian analysis. Liang[4] introduced and derived Dempster EM-Algorithm for the two-component normal mixture models to obtain the iterative computation estimates, also used data augmentation and general Gibbs sampler to get the sample from posterior distribution under conjugate prior. Wang[5] developed the new method, called matrix-variate graphical models (MGGMs), which involves simultaneously modeling variable and sample dependencies with the matrix-normal distribution. Khan and Islam[6] evaluated the maintenance performance of the system when time is continuous and consider half-normal failure lifetime model as well as repair time model.

However, error function distribution has rarely received the attention of the analysts. But it is always of interest to study the behaviour and properties of the estimators for the parameters of the new/deprived distributions. So, the problem of estimation of the parameter of the error function distribution under a Bayesian framework has been addressed in this paper. A class of priors have been assumed under various loss functions to estimate the parameter of the distribution.

2. Model and Likelihood Function

The probability density function of error function distribution is:

$$f(x) = \frac{\omega}{\sqrt{\pi}} e^{-\omega^2 x^2}; -\infty < x < \infty, \omega > 0$$

It is a special case of normal distribution with mean zero and

considering $\frac{1}{\sqrt{2}\sigma} = \omega$ where σ is the standard deviation of the normal distribution.

The likelihood function for a random sample of size 'n' is

$$L(\omega|x) \propto \omega^n e^{-\omega^2 \sum_{i=1}^n x_i^2}$$

3. Bayesian Analysis under the Assumption of Uniform Prior

The uniform prior is assumed to be: $p(\omega) \propto 1$

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Published online at <http://journal.sapub.org/ijps>

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The posterior distribution under the assumption of uniform prior is:

$$p(\omega|x) = \frac{2 \left(\sum_{i=1}^n x_i^2 \right)^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} \omega^n e^{-\omega^2 \sum_{i=1}^n x_i^2}; \omega > 0$$

The Bayes estimators and risks under SELF, QLF, ELF and PLF are respectively presented in the following.

$$\begin{aligned} \omega_{SELF} &= \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \\ R(\omega_{SELF}) &= \frac{n+1}{2 \left(\sum_{i=1}^n x_i^2 \right)} - \frac{1}{\left(\sum_{i=1}^n x_i^2 \right)} \left\{ \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)} \right\}^2 \\ \omega_{QLF} &= \frac{(n-1) \Gamma\left(\frac{n}{2}\right)}{2 \Gamma\left(\frac{n+1}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \\ R(\omega_{QLF}) &= 1 - \frac{(n-1)}{2} \left\{ \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)} \right\}^2 \\ \omega_{ELF} &= \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \\ R(\omega_{ELF}) &= E(\ln \omega) - \ln(\omega_{ELF}) \\ \omega_{PLF} &= \left\{ \frac{n+1}{2 \sum_{i=1}^n x_i^2} \right\}^{\frac{1}{2}} \\ R(\omega_{PLF}) &= 2 \left[\left\{ \frac{n+1}{2 \sum_{i=1}^n x_i^2} \right\}^{\frac{1}{2}} - \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \right] \end{aligned}$$

4. Bayesian Analysis under the Assumption of Jeffreys Prior

The Jeffreys prior is defined as:

$$p_j \propto \sqrt{|I(\omega)|} \text{ here; } p_j \propto \sqrt{|(\omega)|} = \frac{1}{\omega}$$

The posterior distribution under Jeffreys prior is:

$$p(\omega|x) = \frac{2 \left(\sum_{i=1}^n x_i^2 \right)^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)} \omega^{n-1} e^{-\omega^2 \sum_{i=1}^n x_i^2}; \omega > 0$$

The Bayes estimators and risks under SELF, QLF, ELF and PLF are respectively given in the following.

$$\begin{aligned} \omega_{SELF} &= \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \\ R(\omega_{SELF}) &= \frac{n}{2 \left(\sum_{i=1}^n x_i^2 \right)} - \frac{1}{\left(\sum_{i=1}^n x_i^2 \right)} \left\{ \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \right\}^2 \\ \omega_{QLF} &= \frac{(n-2) \Gamma\left(\frac{n-1}{2}\right)}{2 \Gamma\left(\frac{n}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \\ R(\omega_{QLF}) &= 1 - \frac{(n-2)}{2} \left\{ \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \right\}^2 \\ \omega_{ELF} &= \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right) \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}} \\ R(\omega_{ELF}) &= E(\ln \omega) - \ln(\omega_{ELF}) \\ \omega_{PLF} &= \left\{ \frac{n}{2 \sum_{i=1}^n x_i^2} \right\}^{\frac{1}{2}} \end{aligned}$$

$$R(\omega_{PLF}) = 2 \left[\left\{ \frac{n}{2 \sum_{i=1}^n x_i^2} \right\}^{\frac{1}{2}} - \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) \left(\sum_{i=1}^n x_i^2\right)^{\frac{1}{2}}} \right]$$

5. Bayesian Analysis under the Assumption of Maxwell Prior

The Maxwell prior is assumed to be:

$$p(\omega) \propto \omega^2 e^{-\omega^2/a} \quad \omega > 0, a > 0$$

Where 'a' is hyper-parameter

The posterior distribution under Maxwell prior is:

$$p(\omega|\underline{x}) = \frac{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{a} \right)^{\frac{n+3}{2}}}{\Gamma\left(\frac{n+3}{2}\right)} \omega^{n+2} e^{-\omega^2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{a} \right)} \quad ; \omega > 0$$

The Bayes estimators and risks under SELF, QLF, ELF and PLF are respectively shown in the following.

$$\begin{aligned} \omega_{SELF} &= \frac{\Gamma\left(\frac{n+4}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)^{\frac{1}{2}}} \\ R(\omega_{SELF}) &= \frac{n+4}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)} - \frac{1}{\left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)} \left\{ \frac{\Gamma\left(\frac{n+4}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)} \right\}^2 \\ \omega_{QLF} &= \frac{(n+1) \Gamma\left(\frac{n+2}{2}\right)}{2 \Gamma\left(\frac{n+3}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)^{\frac{1}{2}}} \\ R(\omega_{QLF}) &= 1 - \frac{(n+1)}{2} \left\{ \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right)} \right\}^2 \\ \omega_{ELF} &= \frac{\Gamma\left(\frac{n+3}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)^{\frac{1}{2}}} \end{aligned}$$

$$R(\omega_{ELF}) = E(\ln \omega) - \ln(\omega_{ELF})$$

$$\omega_{PLF} = \left\{ \frac{n+3}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)} \right\}^{\frac{1}{2}}$$

$$R(\omega_{PLF}) = 2 \left[\left\{ \frac{n+3}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)} \right\}^{\frac{1}{2}} - \frac{\Gamma\left(\frac{n+4}{2}\right)}{\Gamma\left(\frac{n+3}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)^{\frac{1}{2}}} \right]$$

6. Bayesian Analysis under the Assumption of Rayleigh Prior

The Rayleigh prior is assumed to be:

$$p(\omega) \propto \omega e^{\omega^2/b^2} \quad \omega > 0, b > 0$$

Where 'b' is hyper-parameter.

The posterior distribution under Rayleigh prior is:

$$p(\omega|\underline{x}) = \frac{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2} \right)^{\frac{n+2}{2}}}{\Gamma\left(\frac{n+2}{2}\right)} \omega^{n+1} e^{-\omega^2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2} \right)} \quad ; \omega > 0$$

The Bayes estimators and risks under SELF, QLF, ELF and PLF are respectively presented in the following.

$$\begin{aligned} \omega_{SELF} &= \frac{\Gamma\left(\frac{n+3}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2}\right)^{\frac{1}{2}}} \\ R(\omega_{SELF}) &= \frac{n+3}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2}\right)} - \frac{1}{\left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2}\right)} \left\{ \frac{\Gamma\left(\frac{n+3}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)} \right\}^2 \\ \omega_{QLF} &= \frac{(n) \Gamma\left(\frac{n+1}{2}\right)}{2 \Gamma\left(\frac{n+2}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2}\right)^{\frac{1}{2}}} \end{aligned}$$

$$\begin{aligned}
R(\omega_{QLF}) &= 1 - \frac{(n)}{2} \left\{ \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right)} \right\}^2 \\
\omega_{ELF} &= \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2} \right)^{\frac{1}{2}}} \\
R(\omega_{ELF}) &= E(\ln \omega) - \ln(\omega_{ELF}) \\
\omega_{PLF} &= \left\{ \frac{n+2}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2} \right)} \right\}^{\frac{1}{2}} \\
R(\omega_{PLF}) &= 2 \left[\left\{ \frac{n+2}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2} \right)} \right\}^{\frac{1}{2}} - \frac{\Gamma\left(\frac{n+3}{2}\right)}{\Gamma\left(\frac{n+2}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2} \right)^{\frac{1}{2}}} \right]
\end{aligned}$$

7. Bayesian Analysis under the Assumption of Chi Prior

The chi prior is assumed to be:

$$p(\omega) \propto \omega^{k-1} e^{\omega^2/2} \quad \omega > 0, \quad k > 0$$

Where ‘ k ’ is hyper-parameter.

The posterior distribution under chi prior is:

$$p(\omega|\underline{x}) = \frac{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)^{\frac{n+k}{2}}}{\Gamma\left(\frac{n+k}{2}\right)} \omega^{n+k} e^{-\omega^2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)} \quad ; \omega > 0$$

The Bayes estimators and risks under SELF, QLF, ELF and PLF are respectively derived in the following.

$$\begin{aligned}
\omega_{SELF} &= \frac{\Gamma\left(\frac{n+k+1}{2}\right)}{\Gamma\left(\frac{n+k}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)^{\frac{1}{2}}} \\
R(\omega_{SELF}) &= \frac{n+k}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)} - \frac{1}{\left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)} \left\{ \frac{\Gamma\left(\frac{n+k+2}{2}\right)}{\Gamma\left(\frac{n+k+1}{2}\right)} \right\}^2
\end{aligned}$$

$$\omega_{QLF} = \frac{(n+k-2) \Gamma\left(\frac{n+k-1}{2}\right)}{2 \Gamma\left(\frac{n+k}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)^{\frac{1}{2}}}$$

$$R(\omega_{QLF}) = 1 - \frac{(n+k-2)}{2} \left\{ \frac{\Gamma\left(\frac{n+k-1}{2}\right)}{\Gamma\left(\frac{n+k}{2}\right)} \right\}^2$$

$$\omega_{ELF} = \frac{\Gamma\left(\frac{n+k}{2}\right)}{\Gamma\left(\frac{n+k-1}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)^{\frac{1}{2}}}$$

$$R(\omega_{ELF}) = E(\ln \omega) - \ln(\omega_{ELF})$$

$$\omega_{PLF} = \left\{ \frac{n+k}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)} \right\}^{\frac{1}{2}}$$

$$R(\omega_{PLF}) = 2 \left[\left\{ \frac{n+k}{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)} \right\}^{\frac{1}{2}} - \frac{\Gamma\left(\frac{n+k+1}{2}\right)}{\Gamma\left(\frac{n+k}{2}\right) \left(\sum_{i=1}^n x_i^2 + \frac{1}{2} \right)^{\frac{1}{2}}} \right]$$

8. Bayesian Analysis under the Assumption of Normal Prior Considering Location Parameter to be Zero

The normal prior is assumed to be:

$$p(\omega) \propto \omega^{k-1} e^{\omega^2/2h^2} \quad \omega > 0; \quad h > 0$$

Where ‘ h ’ is hyper-parameter.

The posterior distribution under normal prior is:

$$p(\omega|\underline{x}) = \frac{2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2} \right)^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)} \omega^n e^{-\omega^2 \left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2} \right)} \quad ; \omega > 0$$

The Bayes estimators and risks under SELF, QLF, ELF and PLF are respectively presented in the following.

$$\omega_{SELF} = \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)^{\frac{1}{2}}}$$

$$R(\omega_{SELF}) = \frac{n+1}{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)} - \frac{1}{\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)} \left\{ \frac{\Gamma\left(\frac{n+2}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)} \right\}^2$$

$$\omega_{QLF} = \frac{(n-1)\Gamma\left(\frac{n}{2}\right)}{2\Gamma\left(\frac{n+1}{2}\right)\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)^{\frac{1}{2}}}$$

$$R(\omega_{QLF}) = 1 - \frac{(n-1)}{2} \left\{ \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)} \right\}^2$$

$$\omega_{ELF} = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)^{\frac{1}{2}}}$$

$$R(\omega_{ELF}) = E(\ln \omega) - \ln(\omega_{ELF})$$

$$\omega_{PLF} = \left\{ \frac{n+1}{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)} \right\}^{\frac{1}{2}}$$

9. Bayesian Credible Intervals under Different Priors

The Bayesian credible intervals, as discussed by Saleem and Raza[7], under uniform, Jeffreys, Maxwell, Rayleigh, chi and normal priors are respectively constructed in the following.

$$\frac{2\left(\sum_{i=1}^n x_i^2\right)}{\chi^2_{2\left(\frac{n+1}{2}\right),\left(\frac{\alpha}{2}\right)}} < \omega_U < \frac{2\left(\sum_{i=1}^n x_i^2\right)}{\chi^2_{2\left(\frac{n+1}{2}\right),\left(1-\frac{\alpha}{2}\right)}}$$

$$\frac{2\left(\sum_{i=1}^n x_i^2\right)}{\chi^2_{2\left(\frac{n}{2}\right),\left(\frac{\alpha}{2}\right)}} < \omega_J < \frac{2\left(\sum_{i=1}^n x_i^2\right)}{\chi^2_{2\left(\frac{n}{2}\right),\left(1-\frac{\alpha}{2}\right)}}$$

$$\frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)}{\chi^2_{2\left(\frac{n+3}{2}\right),\left(\frac{\alpha}{2}\right)}} < \omega_M < \frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{a}\right)}{\chi^2_{2\left(\frac{n+3}{2}\right),\left(1-\frac{\alpha}{2}\right)}}$$

$$\frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2}\right)}{\chi^2_{2\left(\frac{n+2}{2}\right),\left(\frac{\alpha}{2}\right)}} < \omega_R < \frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{b^2}\right)}{\chi^2_{2\left(\frac{n+2}{2}\right),\left(1-\frac{\alpha}{2}\right)}}$$

$$\frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{2}\right)}{\chi^2_{2\left(\frac{n+k}{2}\right),\left(\frac{\alpha}{2}\right)}} < \omega_C < \frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{2}\right)}{\chi^2_{2\left(\frac{n+k}{2}\right),\left(1-\frac{\alpha}{2}\right)}}$$

$$\frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)}{\chi^2_{2\left(\frac{n+1}{2}\right),\left(\frac{\alpha}{2}\right)}} < \omega_N < \frac{2\left(\sum_{i=1}^n x_i^2 + \frac{1}{2h^2}\right)}{\chi^2_{2\left(\frac{n+1}{2}\right),\left(1-\frac{\alpha}{2}\right)}}$$

10. Simulation Study

Simulation study has been carried out using $n = 50, 100, 200$ and 300 for $\omega \in (1, 2, 3, 4, 5, 6)$; however, the results are presented for $\omega = 1$. While, the discussions have been made for all $\omega \in (1, 2, 3, 4, 5, 6)$. Different values of the hyper-parameters have been used and the results for the values giving better convergence and the minimum risks have been presented. In order to have more precise estimates, the results have been replicated sufficiently. The risks associated with Bayes estimates have been underlined in the tables. Similarly, the differences between lower and upper limits of credible intervals have been underlined.

Table 1. Bayes estimates and risks under uniform prior

n	Loss Functions			
	SELF	QLF	ELF	PLF
50	1.38112	1.35350	1.36738	1.38790
	0.01879	0.01015	0.00503	0.01357
100	1.15111	1.13960	1.14537	1.15396
	0.00657	0.00504	0.00250	0.00570
200	1.00946	1.00441	1.00693	1.01071
	0.00254	0.00251	0.00124	0.00251
300	1.00377	1.00043	1.00210	1.00461
	0.00167	0.00167	0.00083	0.00167

Table 2. Bayes estimates and risks under Jeffreys prior

n	Loss Functions			
	SELF	QLF	ELF	PLF
50	1.36738 0.01879	1.33948 0.01036	1.35349 0.00513	1.37423 0.01371
100	1.14537 0.00658	1.13380 0.00509	1.13960 0.00252	1.14824 0.00574
200	1.00693 0.00254	1.00187 0.00252	1.00441 0.00125	1.00819 0.00252
300	1.00210 0.00168	0.99875 0.00168	1.00043 0.00083	1.00294 0.00167

Table 3. Bayes estimates and risks under Maxwell prior

n	Loss Functions			
	SELF	QLF	ELF	PLF
50	1.31254 0.04914	1.28730 0.00975	1.29998 0.00484	1.31874 0.01241
100	1.13302 0.01877	1.12191 0.00493	1.12747 0.00245	1.13577 0.00550
200	1.00432 0.00747	0.99935 0.00248	1.00184 0.00123	1.00556 0.00248
300	1.00042 0.00496	0.99711 0.00166	0.99877 0.00082	1.00125 0.00165

Table 4. Bayes estimates and risks under Rayleigh prior

n	Loss Functions			
	SELF	QLF	ELF	PLF
50	1.22224 0.04344	1.19827 0.00995	1.21032 0.00493	1.22813 0.01178
100	1.10025 0.01788	1.08936 0.00499	1.09482 0.00247	1.10295 0.00540
200	0.99200 0.00732	0.98707 0.00250	0.98954 0.00124	0.99323 0.00246
300	0.99222 0.00490	0.98893 0.00167	0.99058 0.00083	0.99304 0.00164

Table 5. Bayes estimates and risks under chi prior

n	Loss Functions			
	SELF	QLF	ELF	PLF
50	1.36911 0.01829	1.34226 0.00995	1.35575 0.00493	1.37571 0.01320
100	1.14927 0.00653	1.13789 0.00499	1.14360 0.00247	1.15209 0.00564
200	1.00941 0.00253	1.00439 0.00250	1.00690 0.00124	1.01066 0.00250
300	1.00376 0.00167	1.00043 0.00167	1.00210 0.00083	1.00459 0.00166

From the above study it can be seen that by increasing the sample size the estimated value of the parameter converges to the true value of the parameter and magnitude of risk associated with each estimate decreases. The increasing values of the parameter impose a negative impact on rate of convergence under each prior; similarly, the performance of squared error loss function and precautionary loss function is badly affected. However, the performance of quadratic loss function and entropy loss function is independent of choice

of parametric value. In comparison of non-informative priors the uniform prior gives the better estimates as the corresponding risks are smaller for each loss function. While in case of informative priors the Maxwell prior for QLF and ELF, Chi prior for SELF and Rayleigh prior for PLF provide the best results. Similarly, estimates under entropy loss function give the minimum risks among all loss functions for each prior. It can also be assessed that the performance of estimates under informative priors is better than those under non-informative priors. Some prior elicitation technique may further strengthen this argument. Hence, the use of Maxwell prior under entropy loss function can be preferred to estimate the parameter of the error function distribution using a Bayesian framework.

In case of interval estimation, the credible intervals under uniform prior are again narrower than those under Jeffreys prior. Using informative priors, the intervals under chi prior are having the minimum width. So for Bayesian interval estimation of the parameter of error function distribution, the use of chi prior can be preferred.

Table 6. Bayes estimates and risks under normal prior

n	Loss Functions			
	SELF	QLF	ELF	PLF
50	1.28730 0.01632	1.26155 0.01015	1.27449 0.00503	1.29362 0.01265
100	1.12191 0.00624	1.11069 0.00504	1.11631 0.00250	1.12469 0.00556
200	0.99935 0.00249	0.99435 0.00251	0.99685 0.00124	1.00059 0.00249
300	0.99711 0.00165	0.99379 0.00167	0.99545 0.00083	0.99794 0.00166

Table 7. 95% credible intervals under uniform and Jeffreys priors

n	Uniform Prior		Jeffreys Prior	
	lower Limit	Upper Limit	lower Limit	Upper Limit
50	0.57460	1.01838	0.56071	1.00824
	0.44378		0.44754	
100	0.58031	1.01017	0.58541	1.02189
	0.42986		0.43648	
200	0.81254	1.20248	0.81624	1.20915
	0.38994		0.39290	
300	0.85001	1.16999	0.85244	1.17460
	0.31999		0.32216	

Table 8. 95% credible intervals under Maxwell and Rayleigh priors

n	Maxwell Prior		Rayleigh Prior	
	lower Limit	Upper Limit	lower Limit	Upper Limit
50	0.55633	1.02635	0.46709	1.01495
	0.47001		0.54786	
100	0.60046	1.03955	0.63598	1.10403
	0.43908		0.46805	
200	0.82161	1.21354	0.84176	1.24451
	0.39193		0.40275	
300	0.85652	1.17645	0.87033	1.19668
	0.31993		0.32636	

Table 9. 95% credible intervals under chi and normal priors

n	Chi Prior		Normal Prior	
	lower Limit	Upper Limit	lower Limit	Upper Limit
50	0.57225	1.00888	0.51969	1.01900
	0.43662		0.49931	
100	0.58289	1.01186	0.61092	1.06344
	0.42897		0.45253	
200	0.81299	1.20197	0.82906	1.22693
	0.38898		0.39787	
300	0.85043	1.16933	0.86141	1.18569
	0.31890		0.32428	

11. Conclusions and Recommendations

The study has been conducted to estimate the parameter of the error function distribution using four different loss functions and under six informative and non-informative priors. The study indicates that for Bayesian point estimation, the use of entropy loss function under Maxwell prior can be preferred. While for interval estimation, the chi prior can affectively be employed.

The study can be extended by using more priors and loss functions. Some censoring procedures and finite mixture of components of error function distribution can also be used.

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