

# Statistical Inferences on Uniform Distributions: The Cases of Boundary Values Being Parameters

Ismail Erdem

Baskent University Faculty of Science and Letters Department of Statistics and Computer Science Baglica, Ankara, 06530, Turkey

**Abstract** If a continuous random variable X is uniformly distributed over the interval and if any of the two boundary values is unknown, it is necessary to make inferences related to the unknown parameter. In this work, for the unknown boundary values of X, some unbiased estimators based on certain order statistics and sample mean are suggested. These estimators are compared in terms their efficiencies. The most efficient unbiased estimator is used to provide confidence intervals and tests of hypotheses procedures for the unknown parameter (the unknown boundary value).

**Keywords** Uniform Distribution, Order Statistics, Unbiased Estimators, Efficiency, Confidence Intervals, Tests of Hypotheses

## 1. Background

The books and articles, listed in the reference list of this study with the reference numbers from[1] to[18], and many more not in the list, studied order statistics in such a way that every aspects of the topic has already been well explored.

However, there is no inferential study, as far as I am aware of, on the boundary values of the uniform distributions. This work aims at the determination of good estimators for the boundary values of uniform distributions. Based on the determined good estimator, construction of confidence intervals and procedures for the test of hypotheses are established.

For illustration, a simulation study is conducted and summaries of the simulation study are provided. The raw data and computations are provided in the appendix of this paper.

## 2. Introduction

A uniformly distributed continuous random variable, assuming real values in the interval  $(\theta_1, \theta_2)$ , has the following probability density function (pdf)

$$f(x) = \frac{1}{\theta_2 - \theta_1}, \theta_1 < x < \theta_2 \quad (1)$$

For this distribution, the Maximum Likelihood Estimators

(MLE) of  $\theta_1$  and  $\theta_2$  will be  $Y_1$  and  $Y_n$ , the smallest and the largest order statistics, respectfully.

$\theta_1$  and  $\theta_2$  need to be estimated for that each moment of the random variable X is , as shown below, a function of these parameters.

$$E(X^k) = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} x^k dx = \frac{(\theta_2^{k+1} - \theta_1^{k+1})}{(k+1)(\theta_2 - \theta_1)}, k = 1, 2, 3, \dots \quad (2)$$

Let  $X_1, X_2, \dots, X_n$  be a random sample of size n from a uniform distribution over the interval  $(\theta_1, \theta_2)$ , and let  $Y_i$  = ith order statistic,  $i=1, 2, \dots, n$ .

The pdf of ith order statistic is obtained by the following general formula, as given in almost all mathematical statistics textbooks, like the ones with the reference numbers[1],[2],[3],[4],[6], and[7].

$$f_{Y_i}(y) = \frac{n!}{(i-1)!(n-i)!} [F(y)]^{(i-1)} [1-F(y)]^{(n-i)} f_Y(y) \quad (3)$$

Where,  $n$ = size of the random sample,  $F(y)$  is the cumulative distribution function (cdf) of the distribution, and  $f_{Y_i}(y)$  is the pdf of the random variable ( ith order statistic)  $Y_i$ .

Specifically, if the pdf given in (1) is used we obtain the following cdf.

$$F(y) = \int_{\theta_1}^y \frac{1}{\theta_2 - \theta_1} dx = \frac{y - \theta_1}{\theta_2 - \theta_1}, \theta_1 < y < \theta_2 \quad (4)$$

Using the equations (3) and (4), we obtain the following pdfs for  $Y_1$  and  $Y_n$ .

\* Corresponding author:

iserdem@baskent.edu.tr (Ismail Erdem)

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$$f_{Y_1}(y) = n \left[ 1 - \frac{y_1 - \theta_1}{\theta_2 - \theta_1} \right]^{(n-1)} \left( \frac{1}{\theta_2 - \theta_1} \right) = \frac{n}{(\theta_2 - \theta_1)^n} (\theta_2 - y_1)^{(n-1)}, \theta_1 < y_1 < \theta_2 \quad (5)$$

$$f_{Y_n}(y_n) = n \left[ \frac{y_n - \theta_1}{\theta_2 - \theta_1} \right]^{(n-1)} \left( \frac{1}{\theta_2 - \theta_1} \right) = \frac{n}{(\theta_2 - \theta_1)^n} (y_n - \theta_1)^{(n-1)}, \theta_1 < y_n < \theta_2 \quad (6)$$

Use of the above pdfs will enable us to obtain the expected values and the variances of  $Y_1$ , and  $Y_n$ .

To take advantage of the computational simplicity lets introduce the following transformations.

$$\text{Let } U_1 = \theta_2 - Y_1 \rightarrow f_{U_1}(u_1) = \frac{n}{(\theta_2 - \theta_1)^n} u_1^{n-1}, 0 < u_1 < (\theta_2 - \theta_1),$$

and let

$$(7)$$

$$U_n = Y_n - \theta_1 \rightarrow f_{U_n}(u_n) = \frac{n}{(\theta_2 - \theta_1)^n} u_n^{n-1}, 0 < u_n < (\theta_2 - \theta_1)$$

$$E(U_1) = \int_0^{\theta_2 - \theta_1} u_1 \frac{n}{(\theta_2 - \theta_1)^n} u_1^{n-1} = \frac{n}{(\theta_2 - \theta_1)^n} \left( \frac{u^{n+1}}{(n+1)} \right) \Big|_{u=0}^{(\theta_2 - \theta_1)} = \frac{n(\theta_2 - \theta_1)}{(n+1)} \quad (8)$$

$$E(U_1^2) = \int_0^{\theta_2 - \theta_1} u_1^2 \frac{n}{(\theta_2 - \theta_1)^n} u_1^{n-1} = \frac{n}{(\theta_2 - \theta_1)^n} \left( \frac{u^{n+2}}{(n+2)} \right) \Big|_{u=0}^{(\theta_2 - \theta_1)} = \frac{n(\theta_2 - \theta_1)^2}{(n+2)} \quad (9)$$

$$Var(U_1) = \frac{n(\theta_2 - \theta_1)^2}{(n+2)} - \frac{n^2(\theta_2 - \theta_1)^2}{(n+1)^2} = \frac{n(\theta_2 - \theta_1)^2}{(n+1)^2(n+2)} \quad (10)$$

From (7) we get  $Y_1 = \theta_2 - U_1$ . It then follows that

$$E(Y_1) = \theta_2 - E(U_1) = \theta_2 - \frac{n(\theta_2 - \theta_1)}{(n+1)} = \frac{n\theta_1 + \theta_2}{(n+1)}$$

$$Var(Y_1) = Var(U_1) = \frac{n(\theta_2 - \theta_1)^2}{(n+1)^2(n+2)} \quad (11)$$

From (7) we observe that the pdfs of  $U_1$  and  $U_n$  are

exactly the same.

Hence,

$$E(U_n) = \frac{n(\theta_2 - \theta_1)}{(n+1)}, \text{ and}$$

$$Var(U_n) = \frac{n(\theta_2 - \theta_1)^2}{(n+1)^2(n+2)}$$

From (7) we get  $Y_n = U_n + \theta_1$ . It then follows that

$$E(Y_n) = \theta_1 + E(U_n)$$

$$= \theta_1 + \frac{n(\theta_2 - \theta_1)}{(n+1)} = \frac{n\theta_2 + \theta_1}{(n+1)}$$

$$Var(Y_n) = Var(U_n) = \frac{n(\theta_2 - \theta_1)^2}{(n+1)^2(n+2)} \quad (12)$$

Here we established the distributions, expected values, and variances of the smallest (the first) and the largest (the last) ordered statistics that are to be used in subsequent sections of this study.

### 3. Statistical Inferences Related to the

**Parameter of**  $X \sim \text{Uniform}(\theta_1, b)$

**Distribution**

#### 3.1. Estimation of the Parameter $\theta_1$ of the Distribution

$X \sim \text{Uniform}(\theta_1, b)$  by the First Order Statistic

$Y_1$

A uniformly distributed continuous random variable  $X$ , over the interval  $(\theta_1, b)$ , where  $b$  is given constant, has the following pdf

$$f(x) = \frac{1}{b - \theta_1}, \theta_1 < x < b$$

The MLE of  $\theta_1$  of  $X \sim Uniform(\theta_1, b)$  will be  $Y_1$  and its expected value and variance are from Equations given in (11),

$$E(Y_1) = \frac{n\theta_1 + \theta_2}{(n+1)} = \frac{n\theta_1 + b}{(n+1)}, \text{ and} \\ Var(Y_1) = \frac{n(\theta_2 - \theta_1)^2}{(n+1)^2(n+2)} = \frac{n(b - \theta_1)^2}{(n+1)^2(n+2)} \quad (3.1.1)$$

An unbiased estimator of  $\theta_1$ , based on the first order statistic is

$$T_1 = \frac{(n+1)Y_1 - b}{n}, \\ Var(T_1) = \frac{(n+1)^2}{n^2} Var(Y_1) = \frac{(b - \theta_1)^2}{n(n+2)} \quad (3.1.2)$$

### 3.2. Estimation of the Parameter $\theta_1$ of the Distribution

$X \sim Uniform(\theta_1, b)$  by the Last Order Statistic  $Y_n$

Another estimator of  $\theta_1$  of  $X \sim Uniform(\theta_1, b)$  will be  $Y_n$  and its expected value and variance are from Equations given in (12),

$$E(Y_n) = \frac{n\theta_2 + \theta_1}{(n+1)} = \frac{nb + \theta_1}{(n+1)} \quad (3.2.1) \\ Var(Y_n) = \frac{n(\theta_2 - \theta_1)^2}{(n+1)^2(n+2)} = \frac{n(b - \theta_1)^2}{(n+1)^2(n+2)}$$

By the utilization of the first equation of (3.2.1) we can obtain an unbiased estimator for  $\theta_1$  as a function of the last order statistic  $Y_n$

$$T_2 = (n+1)Y_n - nb \\ Var(T_2) = (n+1)^2 Var(Y_n) = \frac{n(b - \theta_1)^2}{(n+2)} \quad (3.2.2)$$

### 3.3. Estimation of the Parameter $\theta$ of the Distribution

$X \sim Uniform(\theta_1, b)$  by  $\bar{X}$

If  $X \sim Uniform(\theta_1, b)$  then its pdf is  $f(x) = \frac{1}{b - \theta_1}$ ,  $\theta < x < b$ . Then by equation (1), as given below:

$$E(X^k) = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} x^k dx = \frac{(\theta_2^{k+1} - \theta_1^{k+1})}{(k+1)(\theta_2 - \theta_1)}, k = 1, 2, 3, \dots$$

For  $k=1$  and  $\theta_2 = b$

$$E(X) = \frac{(b^2 - \theta_1^2)}{(2)(b - \theta_1)} = \frac{b + \theta_1}{2} \quad (3.3.1)$$

For  $k=2$  and  $\theta_2 = b$ ,

$$E(X^2) = \frac{(b^3 - \theta_1^{31})}{(3)(b - \theta_1)} = \frac{b^2 + b\theta_1 + \theta_1^2}{3} \quad (3.3.2)$$

From (1.3.1) and (1.3.2) and  
 $Var(X) = \sigma^2 = E(X^2) - [E(X)]^2$  we obtain

$$Var(X) = \frac{(b - \theta_1)^2}{12}.$$

For any distribution, if the sample mean is  $\bar{X}$ , for any random sample of size n, the followings hold true.

$$E(\bar{X}) = E(X), \\ \text{and}$$

$$Var(\bar{X}) = \frac{Var(X)}{n}$$

Hence, for  $X \sim Uniform(\theta_1, b)$ ,

$$E(\bar{X}) = \frac{(b + \theta_1)}{2} \quad (3.3.3)$$

$$Var(\bar{X}) = \frac{(b - \theta_1)^2}{12n} \quad (3.3.4)$$

From (3.3.3) an unbiased estimator of  $\theta_1$  will be

$$T_3 = 2(\bar{X}) - b, \text{ and } Var(T_3) = 4Var(\bar{X}) = \frac{(b - \theta_1)^2}{3n} \quad (3.3.5)$$

### 3.4. Estimation of the Parameter $\theta_1$ of the Distribution

$X \sim Uniform(\theta_1, b)$  by the Sample Median M

If  $X \sim Uniform(\theta_1, b)$  then  $W = X - \theta_1$  is uniformly distributed over the interval  $(0, (b - \theta_1) = \eta_1)$

To take advantage of computational simplicity, the parameter  $\eta_1$  (and in turn  $\theta_1$  of X), of the distribution  $W \sim Uniform(0, \eta_1)$ , is to be estimated by the sample median.

**Sample Median (M):**

Let  $W_1, W_2, \dots, W_n$  is a random sample of size n from  $f(w) = \frac{1}{\eta_1}$ ,  $0 < w < \eta_1$

The sample median M, depending on if n is odd or even, is obtained as follows.

If  $W_1, W_2, \dots, W_n$  are ordered, in the order of their magnitude, we obtain the following order statistics.  $U_1 < U_2 < \dots < U_n$ .

**Note:**

If a random sample of size  $n$  is taken from  $X \sim \text{Uniform}(\theta_1, b)$ , then the order statistics are  $[Y_1, Y_2, \dots, Y_n]$ .

If a random sample of size  $n$  is taken from  $W \sim \text{Uniform}(0, \eta_1)$ , then the order statistics are  $[W_1, W_2, \dots, W_n] = [(Y_1 - \theta_1), (Y_2 - \theta_1), \dots, (Y_n - \theta_1)]$

From this observation, we can see that, obtaining sample median's distribution and its mean and variance from  $[W_1, W_2, \dots, W_n]$  will

enable us to obtain distribution of the sample median and its mean and variance of  $[Y_1, Y_2, \dots, Y_n]$

If  $n$  is odd:  $M = U_{(n+1)/2}$ ;

If  $n$  is even:  $M = \frac{U_{n/2} + U_{(n+2)/2}}{2}$ .

$$\begin{aligned} f_M(u) &= \frac{3!}{1!1!} \left[ \frac{u}{\eta_1} \right]^1 \left[ 1 - \frac{u}{\eta_1} \right]^1 \frac{1}{\eta_1} \\ &= \frac{6}{\eta_1^3} (u)(\eta_1 - u), 0 < u < \eta_1 \end{aligned} \quad (3.4.1)$$

### 3.4.1. Estimation of the Parameter $\theta_1$ of the Distribution

$X \sim \text{Uniform}(\theta_1, b)$  by the Sample Median  $n$  is

Odd

**Theorem 3.4.1.** If  $W \sim \text{Uniform}(0, \eta_1)$  and if an odd sized random sample is taken from this distribution then for the sample median  $M$ ,

$$E(M) = \frac{\eta_1}{2}, \quad E(M^2) = \frac{\eta_1^2(n+3)}{4(n+2)} \quad \text{and}$$

$$\text{Var}(M) = \frac{\eta_1^2}{4(n+2)} \quad \text{dir.}$$

Proof:

Proof will be given by induction.

$$\text{for } n=1: \quad E(M) = E(W) = \frac{\eta_1}{2}, \quad \text{and}$$

$$\text{Var}(M) = \text{Var}(W) = \frac{\eta_1^2}{12} = \frac{\eta_1^2}{4(n+2)}.$$

For  $n=3$ : the sample median is  $M = U_2$ . The pdf of  $M = U_2$  is to be obtained, by the use of (3), as follows.

$$\text{Since,} \quad f(w) = \frac{1}{\eta_1}, 0 < w < \eta_1 \quad \text{and}$$

$$F(u) = \int_0^u \frac{1}{\eta_1} dw = \frac{u}{\eta_1}, 0 < u < \eta_1, \text{ then}$$

$$\begin{aligned} E(M) &= \int_0^{\eta_1} u \frac{6}{\eta_1^3} (u)(\eta_1 - u) du \\ &= \frac{6}{\eta_1^3} \left[ \eta_1 \frac{u^3}{3} - \frac{u^4}{4} \Big|_0^{\eta_1} \right] = \frac{6\eta_1}{12} = \frac{\eta_1}{2}, \end{aligned}$$

$$\begin{aligned} E(M^2) &= \int_0^{\eta_1} u^2 \frac{6}{\eta_1^3} (u)(\eta_1 - u) du \\ &= \frac{6}{\eta_1^3} \left[ \eta_1 \frac{u^4}{4} - \frac{u^5}{5} \Big|_0^{\eta_1} \right] = \frac{6\eta_1^2}{20} = \frac{\eta_1^2(n+3)}{4(n+2)}, \\ \text{Var}(M) &= \frac{3\eta_1^2}{10} - \frac{\eta_1^2}{4} = \frac{\eta^2}{20} = \frac{\eta_1^2}{4(n+2)}. \end{aligned}$$

Similarly, for  $n=5$ ,  $M = U_3$  and from (3)

$$\begin{aligned} f_M(u) &= \frac{5!}{2!2!} \left[ \frac{u}{\eta_1} \right]^2 \left[ 1 - \frac{u}{\eta_1} \right]^2 \frac{1}{\eta_1} \\ &= \frac{30}{\eta_1^5} (u^2)(\eta_1 - u)^2, 0 < u < \eta_1 \end{aligned}$$

$$\begin{aligned} E(M) &= \int_0^{\eta_1} u \frac{30}{\eta_1^5} (u^2)(\eta_1 - u)^2 du \\ &= \frac{30}{\eta_1^5} \left[ \frac{\eta_1^2 u^4}{4} - \frac{2\eta_1 u^5}{5} + \frac{u^6}{6} \Big|_0^{\eta_1} \right] = \frac{\eta_1}{2} \end{aligned}$$

$$E(M^2) = \int_0^{\eta_1} u^2 \frac{30}{\eta_1^5} (u^2)(\eta_1 - u)^2 du = \frac{30}{\eta_1^5} \left[ \frac{\eta_1^2 u^5}{5} - \frac{2\eta_1 u^6}{6} + \frac{u^7}{7} \right]_0^{\eta_1} = \frac{2\eta_1^2}{7} = \frac{\eta_1^2(n+3)}{4(n+2)},$$

$$Var(M) = \frac{2\eta_1^2}{7} - \frac{\eta_1^2}{4} = \frac{\eta_1^2}{28} = \frac{\eta_1^2}{4(n+2)}.$$

For  $n=2m+1$ , then  $M = U_{(m+1)}$ .

$$f_M(u) = \frac{(2m+1)!}{(m)!(m)!} \left[ \frac{u}{\eta_1} \right]^{(m)} \left[ 1 - \frac{u}{\eta_1} \right]^{(m)} \frac{1}{\eta_1} = \frac{(2m+1)!}{(m)!(m)!\eta_1^{2m+1}} (u^{(m)}) (\eta_1 - u)^{(m)}, 0 < u < \eta_1$$

Now, let's assume that the following hold true for any  $n=2m+1$ .

$$E(M) = \int_0^{\eta_1} u \frac{(2m+1)!}{(m)!(m)!\eta_1^{2m+1}} (u^m) (\eta_1 - u)^m = \frac{\eta_1}{2} \quad (3.4.2)$$

$$E(M^2) = \int_0^{\eta_1} u^2 \frac{(2m+1)!}{(m)!(m)!\eta_1^{2m+1}} (u)^m (\eta_1 - u)^m du = \frac{\eta_1^2(2m+4)}{4(2m+3)} \quad (3.4.3)$$

We need to show that for any m and for  $n=2m+3$  the statements of the Theorem 3.4.1 are true.

For  $n=(2m+3)$   $M = U_{(m+2)}$ .

$$f_M(u) = \frac{(2m+3)!}{(m+1)!(m+1)!} \left[ \frac{u}{\eta_1} \right]^{(m+1)} \left[ 1 - \frac{u}{\eta_1} \right]^{(m+1)} \frac{1}{\eta_1} = \frac{(2m+1)!}{(m+1)!(m+1)!\eta_1^{2m+2}} (u^{(m+1)}) (\eta_1 - u)^{(m+1)}, 0 < u < \eta_1$$

$$E(M) = \int_0^{\eta_1} u \frac{(2m+3)!}{(m+1)!(m+1)!\eta_1^{2m+3}} (u^{(m+1)}) (\eta_1 - u)^{(m+1)} du.$$

If we let  $(m+1)=k$  then in accordance with (3.4.2) and (3.4.3) we conclude the following.

$$E(M) = \int_0^{\eta_1} u \frac{(2k+1)!}{(k)!(k)!\eta_1^{2k+1}} (u^k) (\eta_1 - u)^k du = \frac{\eta_1}{2}.$$

$$E(M^2) = \int_0^{\eta_1} u^2 \frac{(2m+3)!}{(m+1)!(m+1)!\eta_1^{2m+3}} (u^{(m+1)}) (\eta_1 - u)^{(m+1)} du = \int_0^{\eta_1} u^2 \frac{(2k+1)!}{(k)!(k)!\eta_1^{2k+1}} (u^k) (\eta_1 - u)^k du$$

$$E(M^2) = \frac{\eta_1^2(2k+4)}{4(2k+3)} = \frac{\eta_1^2(2m+6)}{4(2m+5)} = \frac{\eta_1^2(n+3)}{4(n+2)}, \text{ (For that } n = (2m+3)) \text{, and}$$

$$Var(M) = \frac{\eta_1^2(2m+6)}{4(2m+5)} - \frac{\eta_1^2}{4} = \frac{\eta_1^2}{4(2m+5)} = \frac{\eta_1^2}{4(n+2)}.$$

For the case of n being an odd number, the proof is completed.

### 3.4.2. Estimation of the Parameter $\theta$ of the Distribution $X \sim Uniform(\theta_1, b)$ by the Sample Median: n is even

**Theorem 3.4.2** If  $W \sim Uniform(0, \eta_1)$  and if an even sized random sample is taken from this distribution then for the sample median M,

$$E(M) = \frac{\eta_1}{2}, \quad E(M^2) = \frac{(n^2 + 4n + 2)\eta_1^2}{4(n+1)(n+2)}, \text{ and } Var(M) = \frac{n\eta_1^2}{4(n+1)(n+2)}.$$

**Proof:**

**Note:**

If n is even, then the sample median is  $M = \frac{U_{n/2} + U_{(n+2)/2}}{2}$ .

To compute the expected value and the variance of M we need to have the joint pdf of  $U_{n/2}, U_{(n+2)/2}$ .

For any random sample taken from the distribution of  $f(y)$ , joint pdf of the ordered statistics  $U_r$ , and  $U_t$ , ( $r < t$ ) can be obtained by the use of the following general formulation as given in[6].

$$f_{U_r, U_t}(u_r, u_t) = \frac{n!}{(r-1)!(t-r-1)!(n-t)!} [F(u_r)]^{r-1} [F(u_t) - F(u_r)]^{t-r-1} [F(u_t)]^{n-t} f(u_r) f(u_t) \quad (3.4.4)$$

$$\text{For } n=2: M = \frac{U_1 + U_2}{2} = \frac{Y_1 + Y_2}{2} = \bar{Y}$$

$$\text{If } W \sim \text{Uniform}(0, \eta_1) \text{ then, } E(Y) = E(\bar{Y}) = E(M) = \frac{\eta_1}{2}, \text{ and}$$

$$\text{Var}(\bar{Y}) = \text{Var}(M) = \frac{\text{Var}(Y)}{n} = \frac{\eta_1^2}{12n} = \frac{\eta_1^2}{24} = \frac{n\eta_1^2}{4(n+1)(n+2)}.$$

For  $n=4: M = \frac{U_2 + U_3}{2}$ . The joint pdf of  $U_2$  and  $U_3$  is obtained as given below.

Since,  $f(u) = \frac{1}{\eta_1}$ , and  $F(u) = \int_0^u \frac{1}{\eta_1} dy = \frac{u}{\eta_1}$ ,  $0 < u < \eta_1$  according to (3.4.5) the joint pdf is

$$f_{U_2, U_3}(u_2, u_3) = \frac{4!}{(1)!(0)!(1)!} \left[ \frac{u_2}{\eta_1} \right]^1 \left[ 1 - \frac{u_3}{\eta_1} \right]^1 \frac{1}{\eta_1^4} = \frac{24}{\eta_1^4} (u_2)(\eta_1 - u_3), \quad 0 < u_2 < u_3 < \eta_1 \quad (3.4.5)$$

$$\begin{aligned} E(M) &= E\left(\frac{U_2 + U_3}{2}\right) = \int_0^{\eta_1} \int_0^{u_2} \frac{u_2 + u_3}{2} \left( \frac{24}{\eta_1^4} \right) (u_2)(\eta_1 - u_3) du_2 du_3 = \\ &\frac{12}{\eta_1^4} \int_0^{\eta_1} \left( \frac{\eta_1 u_2^3}{3} + \frac{\eta_1 u_3 u_2^2}{2} - \frac{u_3 u_2^3}{3} - \frac{u_3^2 u_2^2}{2} \Big|_{u_2=0}^{u_3} \right) du_3 = \frac{12}{\eta_1^4} \int_0^{\eta_1} \left( \frac{\eta_1 u_3^3}{3} + \frac{\eta_1 u_3^3}{2} - \frac{u_3^4}{3} - \frac{u_3^4}{2} \right) du_3 = \\ &\frac{12}{\eta_1^4} \left( \frac{\eta_1 u_3^4}{12} + \frac{\eta_1 u_3^4}{8} - \frac{u_3^5}{15} - \frac{u_3^5}{10} \Big|_{u_3=0} \right) = 12\eta_1 \left( \frac{1}{12} + \frac{1}{8} - \frac{1}{15} - \frac{1}{10} \right) = \frac{\eta_1}{2} \end{aligned}$$

$$E(M^2) = E\left[\left(\frac{U_2 + U_3}{2}\right)^2\right] = \int_0^{\eta_1} \int_0^{u_2} \left( \frac{u_2 + u_3}{2} \right)^2 \left( \frac{24}{\eta_1^4} \right) (u_2)(\eta_1 - u_3) du_2 du_3 =$$

$$\frac{6}{\eta_1^4} \int_0^{\eta_1} \left( \frac{\eta_1 u_2^4}{4} + \frac{2\eta_1 u_3 u_2^3}{3} + \frac{\eta_1 u_3^2 u_2^2}{2} - \frac{u_3 u_2^4}{4} - \frac{2u_3^2 u_2^3}{3} - \frac{u_3^3 u_2^2}{2} \Big|_{u_2=0}^{u_3} \right) du_3$$

$$= \frac{6}{\eta_1^4} \int_0^{\eta_1} \left( \eta_1 u_3^4 \left( \frac{1}{4} + \frac{2}{3} + \frac{1}{2} \right) - u_3^5 \left( \frac{1}{4} + \frac{2}{3} + \frac{1}{2} \right) \right) du_3 = \frac{6}{\eta_1^4} \left( \frac{17}{12} \right) \int_0^{\eta_1} (\eta_1 u_3^4 - u_3^5) du_3$$

$$= \frac{17}{2\eta_1^4} \left( \frac{\eta_1 u_3^5}{5} - \frac{u_3^6}{6} \Big|_{u_3=0} \right) = \frac{17\eta_1^2}{2} \left( \frac{1}{5} - \frac{1}{6} \right) = \frac{17\eta_1^2}{60}$$

$$E(M^2) = \frac{17\eta_1^2}{60} \rightarrow E(M^2) = \frac{(n^2 + 4n + 2)\eta_1^2}{4(n+1)(n+2)}, \text{ and}$$

$$\text{Var}(M) = \frac{17\eta_1^2}{60} - \frac{\eta_1^2}{4} = \frac{\eta_1^2}{30} \rightarrow \text{Var}(M) = \frac{n\eta_1^2}{4(n+1)(n+2)}.$$

For  $n=6: M = \frac{U_3 + U_4}{2}$ , the joint pdf of  $U_3$  and  $U_4$  is obtained as given below.:

$$\begin{aligned}
f_{U_3, U_4}(u_2, u_3) &= \frac{6!}{(2)!(0)!(2)!} \left[ \frac{u_3}{\eta_1} \right]^2 \left[ 1 - \frac{u_4}{\eta_1} \right]^2 \frac{1}{\eta_1^2} \\
&= \frac{180}{\eta_1^6} (u_3^2)(\eta_1 - u_4)^2, 0 < u_3 < u_4 < \eta_1 \\
E(M) &= E\left(\frac{U_3 + U_4}{2}\right) = \int_0^{\eta_1} \int_0^{u_4} \frac{u_3 + u_4}{2} \left( \frac{180}{\eta_1^6} \right) (u_3^2)(\eta_1 - u_4)^2 du_2 du_3 = \\
\frac{90}{\eta_1^6} \int_0^{\eta_1} &\left( \frac{\eta_1^2 u_3^4}{4} + \frac{\eta_1^2 u_4 u_3^3}{3} - \frac{2\eta_1 u_4 u_3^4}{4} - \frac{2\eta_1 u_4^2 u_3^3}{3} + \frac{u_4^2 u_3^4}{4} + \frac{u_4^3 u_3^3}{3} \right) du_3 = \quad \rightarrow \text{for } n=6, E(M) = \frac{\eta_1}{2} \\
\frac{90}{\eta_1^6} \left( \frac{7}{12} \right) \int_0^{\eta_1} &(\eta_1^2 u_4^4 - 2\eta_1 u_4^5 + u_4^6) du_3 = \frac{105}{2\eta_1^6} \left( \frac{\eta_1^2 u_4^5}{5} - \frac{2\eta_1 u_4^6}{6} + \frac{u_4^7}{7} \right) \Big|_0^{\eta_1} \\
\frac{105\eta_1}{2} \left( \frac{1}{5} - \frac{1}{3} + \frac{1}{7} \right) &= \frac{\eta_1}{2} \\
E(M^2) &= E\left[\left(\frac{U_3 + U_4}{2}\right)^2\right] = \int_0^{\eta_1} \int_0^{u_4} \left( \frac{u_3 + u_4}{2} \right)^2 \left( \frac{180}{\eta_1^6} \right) (u_3^2)(\eta_1 - u_4)^2 du_2 du_3 = \\
\frac{45}{\eta_1^6} \int_0^{\eta_1} &\left( \frac{\eta_1^2 u_3^5}{5} + \frac{2\eta_1^2 u_4 u_3^4}{4} + \frac{\eta_1^2 u_4^2 u_3^3}{3} - \frac{2\eta_1 u_4 u_3^5}{5} - \frac{4\eta_1 u_4^2 u_3^4}{4} - \frac{2\eta_1 u_4^3 u_3^3}{3} + \frac{u_4^2 u_3^5}{5} + \frac{2u_4^2 u_3^4}{4} + \frac{u_4^4 u_3^3}{3} \Big|_{u_3=0} \right) du_4 = \\
\frac{45}{\eta_1^6} \left( \frac{31}{30} \right) \int_0^{\eta_1} &(\eta_1^2 u_4^5 - 2\eta_1 u_4^6 + u_4^7) du_4 = \frac{93}{2\eta_1^6} \left( \frac{\eta_1^2 u_4^6}{6} - \frac{2\eta_1 u_4^7}{7} + \frac{u_4^8}{8} \right) \Big|_0^{\eta_1} \\
\frac{93\eta_1^2}{2} \left( \frac{1}{6} - \frac{2}{7} + \frac{1}{8} \right) &= \frac{31\eta_1^2}{112} \\
E(M^2) &= \frac{31\eta_1^2}{112} \rightarrow E(M^2) = \frac{(n^2 + 4n + 2)\eta_1^2}{4(n+1)(n+2)}, \text{ and} \\
Var(M) &= \frac{31\eta_1^2}{112} - \frac{\eta_1^2}{4} = \frac{3\eta_1^2}{112} \rightarrow Var(M) = \frac{n\eta_1^2}{4(n+1)(n+2)} \\
\text{For } n=2m+2: M &= \frac{U_{(m+1)} + U_{(m+2)}}{2} \\
f_{U_{(m+1)}, U_{(m+2)}}(u_{m+1}, u_{m+2}) &= \frac{(2m+2)!}{(m)!(0)!(m)!} \left[ \frac{u_{m+1}}{\eta_1} \right]^m \left[ 1 - \frac{u_{m+2}}{\eta_1} \right]^m \frac{1}{\eta_1^2} \\
&= \frac{(2m+2)!}{m! m! \eta_1^{2m+2}} (u_{m+1}^m) (\eta_1 - u_{m+2})^m, 0 < u_{m+1} < u_{m+2} < \eta_1
\end{aligned} \tag{3.4.6}$$

Now, let's assume that the following, given in (3.4.7) and (3.4.8), hold true.

$$E(M) = \int_0^{\eta_1} \int_0^{u_{m+2}} \frac{u_{m+1} + u_{m+2}}{2} \frac{(2m+2)!}{m! m! \eta_1^{2m+2}} (u_{m+1}^m) (\eta_1 - u_{m+2})^m du_{m+1} du_{m+2} = \frac{\eta_1}{2} \tag{3.4.7}$$

$$\begin{aligned} E(M^2) &= \int_0^{\eta_1} \int_0^{u_{m+2}} \left( \frac{u_{m+1} + u_{m+2}}{2} \right)^2 \frac{(2m+2)!}{m!m!\eta_1^{2m+2}} (u_{m+1}^m)(\eta_1 - u_{m+2})^m du_{m+1} du_{m+2} = \\ &= \frac{[(2m+2)^2 + 4(2m+2)+2] \eta_1^2}{4[(2m+2)+1][(2m+2)+2]} = \frac{[n^2 + 4(n+2)+2] \eta_1^2}{4(n+1)(n+2)} \end{aligned} \quad (3.4.8)$$

We need to show that the statements of the Theorem 3.4.2 hold true for  $n=2m+4$ .

$$\text{For } n=2m+4, M = \frac{U_{(m+2)} + U_{(m+3)}}{2}$$

The joint pdf of  $U_{(m+2)}$  and  $U_{(m+3)}$  is obtained as follows.

$$\begin{aligned} f_{U_{(m+2)}, U_{(m+3)}}(u_{m+2}, u_{m+3}) &= \frac{(2m+4)!}{(m+1)!(0)!(m+1)!} \left[ \frac{u_{m+1}}{\eta_1} \right]^{m+1} \left[ 1 - \frac{u_{m+2}}{\eta_1} \right]^{m+1} \frac{1}{\eta_1^2} \\ &= \frac{(2m+4)!}{(m+1)!(m+1)!\eta_1^{2m+4}} (u_{m+1}^{m+1})(\eta_1 - u_{m+2})^{m+1}, 0 < u_{m+2} < u_{m+3} < \eta_1 \\ E(M) &= \int_0^{\eta_1} \int_0^{u_{m+3}} \frac{u_{m+2} + u_{m+3}}{2} \frac{(2m+4)!}{(m+1)!(m+1)!\eta_1^{2m+4}} (u_{m+1}^{m+1})(\eta_1 - u_{m+2})^{m+1} du_{m+2} du_{m+3} \end{aligned} \quad (3.4.9)$$

In (3.4.9) take  $(m+1) = k$ , observing the identity between (3.4.7) and (3.4.10) we conclude the following.

$$E(M) = \int_0^{\eta_1} \int_0^{u_{k+2}} \frac{u_{k+1} + u_{k+2}}{2} \frac{(2k+2)!}{k!k!\eta_1^{2k+2}} (u_{k+1}^k)(\eta_1 - u_{k+2})^k du_{k+1} du_{k+2} = \frac{\eta_1}{2} \quad (3.4.10)$$

Similarly,

$$E(M^2) = \int_0^{\eta_1} \int_0^{u_{m+3}} \left( \frac{u_{m+2} + u_{m+3}}{2} \right)^2 \frac{(2m+4)!}{(m+1)!(m+1)!\eta_1^{2m+4}} (u_{m+1}^{m+1})(\eta_1 - u_{m+2})^{m+1} du_{m+2} du_{m+3} \quad (3.4.11)$$

In (3.4.11), if we let  $(m+1) = k$ , observing the identity between (3.4.8) and (3.4.12) we conclude the following.

$$\begin{aligned} E(M^2) &= \int_0^{\eta_1} \int_0^{u_{k+2}} \left( \frac{u_{k+1} + u_{k+2}}{2} \right)^2 \frac{(2k+2)!}{k!k!\eta_1^{2k+2}} (u_{k+1}^k)(\eta_1 - u_{k+2})^k du_{k+1} du_{k+2} = \\ &= \frac{[(2k+2)^2 + 4(2k+2)+2] \eta_1^2}{4[(2k+2)+1][(2k+2)+2]} = \frac{[(2m+4)^2 + 4(2m+4)+2] \eta_1^2}{4[(2m+4)+1][(2m+4)+2]} = \frac{(n^2 + 4n+2)\eta_1^2}{4(n+1)(n+2)} \end{aligned} \quad (3.4.12)$$

For the case of  $n$  being an even number, the proof is completed.

### 3.4.3. Unbiased Estimators of $\theta_1$ for $X \sim \text{Uniform}(\theta_1, b)$ by M

$$\text{if } n \text{ is odd: } E(M) = \frac{\eta_1}{2}, \quad E(M^2) = \frac{\eta_1^2(n+3)}{4(n+2)}, \text{ and } \text{Var}(M) = \frac{\eta_1^2}{4(n+2)}$$

Where  $\eta_1 = b - \theta_1$ , hence an unbiased estimator of  $\theta_1$  as function of is M

$$T_4 = b - 2M, \quad \text{Var}(T_4) = 4\text{Var}(M) = \frac{(b - \theta_1)^2}{(n+2)} \quad (3.4.13)$$

$$\text{if } n \text{ is even: } E(M) = \frac{\eta_1}{2}, \quad E(M^2) = \frac{(n^2 + 4n+2)\eta_1^2}{4(n+1)(n+2)}, \text{ and } \text{Var}(M) = \frac{n\eta_1^2}{4(n+1)(n+2)}.$$

**Table 1.** Unbiased estimators of  $\theta_1$  for the Distribution  $X \sim Uniform(\theta_1, b)$  and their variances

	Unbiased estimators ( $T_i$ )				
	$T_1 = \frac{(n+1)Y_1 - b}{n}$	$T_2 = (n+1)Y_n - nb$	$T_3 = 2(\bar{X}) - b$	$T_4 = b - 2M$ (n is odd)	$T_5 = b - 2M$ (n is even)
$E(T_i)$	$\theta_1$	$\theta_1$	$\theta_1$	$\theta_1$	$\theta_1$
$Var(T_i)$	$\frac{(b-\theta_1)^2}{n(n+2)}$	$\frac{n(b-\theta_1)^2}{(n+2)}$	$\frac{(b-\theta_1)^2}{3n}$	$\frac{(b-\theta_1)^2}{(n+2)}$	$\frac{n(b-\theta_1)^2}{(n+1)(n+2)}$

Where  $\eta_1 = b - \theta_1$ , hence an unbiased estimator of  $\theta$  as function of is M  $T_5 = a - 2M$ ,

$$Var(T_5) = 4Var(M) = \frac{n(b-\theta_1)^2}{(n+1)(n+2)} \quad (3.4.14)$$

If we give the above comparisons in a tabulated form, we obtain the following:

For  $n > 1$ ;

$$Var(T_1) < Var(T_3) < Var(T_5) < Var(T_4) < Var(T_2)$$

We see that, for  $n > 1$ , the most efficient unbiased estimator, among the ones as given above, is  $T_1 = \frac{(n+1)Y_1 - b}{n}$ .

## 4. Confidence Interval for the Parameter

### $\theta_1$ of the Distribution $X \sim Uniform(\theta_1, b)$

The most efficient unbiased estimator of  $\theta_1$  is seen to be  $T_1 = \frac{(n+1)Y_1 - b}{n}$ . By the use of the pdf of  $Y_1$  we can construct a  $100(1-\alpha)\%$  confidence interval for  $\theta_1$ . As it is shown before

$$f_{Y_1}(y) = \frac{n}{(b-\theta_1)^n} (b-y)^{(n-1)}, \theta_1 < y < b.$$

By the use of following probability statement we can obtain a confidence interval for  $\theta$ .

$$P(y_{1L} < Y_1 < y_{1U}) = (1-\alpha) \quad (4.1)$$

$$\int_{\theta_1}^{y_{1L}} \frac{n}{(b-\theta)^n} (b-y)^{(n-1)} dy = \alpha/2 \quad \text{and}$$

$$\int_{\theta_1}^{y_{1U}} \frac{n}{(b-\theta_1)^n} (b-y)^{(n-1)} dy = 1 - \alpha/2$$

If we let  $(b-y) = w$  then the following are true:

$$dw = -dy; \quad y = \theta_1 \rightarrow w = b - \theta_1;$$

$$y = y_{1L} \rightarrow w = b - y_{1L}, \text{ and } y = y_{1U} \rightarrow w = b - y_{1U}.$$

$$\int_{\theta_1}^{y_{1L}} \frac{n}{(b-\theta_1)^n} (b-y)^{(n-1)} dy =$$

$$\int_{b-y_{1L}}^{b-\theta_1} \frac{n}{(b-\theta)^n} w^{n-1} dw = 1 - \left( \frac{b-y_{1L}}{b-\theta_1} \right)^n = \alpha/2$$

$$y_{1L} = b - (b-\theta_1) \left( 1 - \frac{\alpha}{2} \right)^{1/n} \quad (4.2)$$

Similarly,

$$\int_{\theta_1}^{y_{1U}} \frac{n}{(b-\theta_1)^n} (b-y)^{(n-1)} dy =$$

$$\int_{b-y_{1U}}^{b-\theta_1} \frac{n}{(b-\theta)^n} w^{n-1} dw = 1 - \left( \frac{b-y_{1U}}{b-\theta_1} \right)^n = 1 - \alpha/2$$

$$y_{1U} = b - (b-\theta_1) \left( \frac{\alpha}{2} \right)^{1/n} \quad (4.3)$$

If the results in (4.2) and (4.3) are substituted in (4.1)

$$P\left(b - (b - \theta_1)\left(1 - \frac{\alpha}{2}\right)^{1/n} < Y_1 < b - (b - \theta_1)\left(\frac{\alpha}{2}\right)^{1/n}\right) = (1 - \alpha)$$

Solving the above inequalities for  $\theta_1$  we obtain the following Confidence Interval.

$$P\left(b - \frac{b - Y_1}{\left(\frac{\alpha}{2}\right)^{1/n}} < \theta_1 < b - \frac{b - Y_1}{\left(1 - \frac{\alpha}{2}\right)^{1/n}}\right) = (1 - \alpha) \quad (4.4)$$

A  $(1 - \alpha) * 100\%$  confidence interval for  $\theta_1$ :

$$\left(b - \frac{b - Y_1}{\left(\frac{\alpha}{2}\right)^{1/n}}, b - \frac{b - Y_1}{\left(1 - \frac{\alpha}{2}\right)^{1/n}}\right) \quad (4.5)$$

## 5. Tests of Hypotheses Related to the Parameter $\theta_1$ of the Distribution

$$X \sim Uniform(\theta_1, b)$$

**Table 2.** Tests of Hypotheses Related to the Parameter  $\theta_1$  for the Distribution  $X \sim Uniform(\theta_1, b)$

$H_0 : \theta_1 = \theta_0$	$H_0 : \theta_1 \leq \theta_0$	$H_0 : \theta_1 \geq \theta_0$
$H_1 : \theta_1 \neq \theta_0$	$H_1 : \theta_1 > \theta_0$	$H_1 : \theta_1 < \theta_0$
If $Y_1 \geq y_{1U}$ or $Y_1 \leq y_{1L}$ $H_0$ is rejected	If $Y_1 \geq y_{1U}$ $H_0$ is rejected	If $Y_1 \leq y_{1L}$ $H_0$ is rejected
Don't reject $H_0$ otherwise	Don't reject $H_0$ otherwise	Don't reject $H_0$ otherwise
Where, $y_{1L} = b - (b - \theta_0)\left(1 - \frac{\alpha}{2}\right)^{1/n}$ and $y_{1U} = b - (b - \theta_0)\left(\frac{\alpha}{2}\right)^{1/n}$	Where, $y_{1U} = b - (b - \theta_0)(\alpha)^{1/n}$	Where $y_{1L} = b - (b - \theta_0)(1 - \alpha)^{1/n}$

If  $H_0 : \theta_1 = \theta_0$  is to be tested against to any proper alternative hypothesis, a plausible test statistic is to be  $Y_1 = X_{Min}$  for that the most efficient unbiased estimator is  $T_1 = \frac{(n+1)Y_1 - b}{n}$ , which is a linear function of  $Y_1 = X_{Min}$ .

If the level of significance is chosen to be  $\alpha$ , then the decision rules, as given in the following table, are applicable.

It is concluded that the best unbiased estimator, among the ones suggested, of the parameter  $\theta_1$  for the uniform distribution over  $(\theta_1, b)$  is  $T_1 = \frac{(n+1)Y_1 - b}{n}$ .

Since  $T_1$  is a linear function of the first order statistic  $Y_1$ , construction of confidence interval and tests of hypotheses procedures are related to and dependent upon the observed value of the first order statistic  $Y_1$  and the chosen level of significance  $\alpha$ .

## 6. Statistical Inferences Related to the Parameter of $X \sim Uniform(a, \theta_2)$ Distribution

### 6.1. Estimation of the Parameter $\theta_2$ of the Distribution $X \sim Uniform(a, \theta_2)$ by the First Order Statistic

$$Y_1 = X_{(Min)}$$

A uniformly distributed continuous random variable  $X$ , over the interval  $(a, \theta_2)$ , has the following pdf.

$$f(x) = \frac{1}{\theta_2 - a}, a < x < \theta_2$$

If  $Y = X - a$  then,  $Y \sim Uniform(0, (\theta_2 - a) = \eta)$  and its pdf is as given below.

$$f(y) = \frac{1}{\eta}, 0 < y < \eta.$$

If a random sample of size is taken from the distribution of  $Y$ , then the ordered statistics will be denoted by  $U_1 < U_2 < \dots < U_n$ .

The parameter of this distribution,  $\eta$ , can be estimated by  $U_1$ . The pdf of  $U_1$  is given below.

$$f_{U_1}(y) = n \left[ 1 - \frac{y}{\eta} \right]^{(n-1)} \frac{1}{\eta} = \frac{n}{(\eta)^n} (\eta - y)^{(n-1)}, 0 < y < \eta \quad (6.1.1)$$

$$E(U_1) = \frac{n}{(\eta)^n} \int_0^{\eta} y(\eta - y)^{(n-1)} dy \quad (6.1.2)$$

If we let  $(\eta - y) = t$  then the following will hold true.

$$dt = -dy; \quad y = \eta - t; \quad y = \theta \rightarrow t = \eta - \theta_2 \quad y = a \rightarrow t = 0.$$

$$\begin{aligned} E(U_1) &= \frac{n}{(\eta)^n} \int_0^{\eta} y(\eta - y)^{(n-1)} dy = \frac{n}{(\eta)^n} \int_0^{\eta} (\eta - t)t^{(n-1)} dt = \\ &= \frac{n}{(\eta)^n} \left[ \frac{\eta t^n}{n} - \frac{t^{n+1}}{n+1} \right]_{t=0}^{\eta} = n\eta \left[ \frac{1}{n} - \frac{1}{n+1} \right] \\ E(U_1) &= \frac{\eta}{n+1} \end{aligned} \quad (6.1.3)$$

Hence, an estimator of  $\theta_2$  of  $X \sim Uniform(a, \theta_2)$  will be  $X_{(1)}$  and its expected value, from the transformation  $Y = X - a$ , is obtained as given below.

$$E(Y_1) = E(U_1) + a = \frac{\eta}{(n+1)} + a = \frac{(\theta_2 - a)}{(n+1)} + a = \frac{na + \theta_2}{(n+1)} \quad (6.1.4)$$

Similarly,

$$\begin{aligned} E(U_1^2) &= \frac{n}{(\eta)^n} \int_0^\eta y^2 (\eta - y)^{(n-1)} dy = \frac{n}{(\eta)^n} \int_0^\eta (\eta - t)^2 t^{(n-1)} dt = \\ &= \frac{n}{(\eta)^n} \left[ \frac{\eta^2 t^n}{n} - \frac{2\eta t^{n+1}}{n+1} + \frac{t^{n+2}}{n+2} \right]_{t=0}^{(\eta)} = n\eta^2 \left[ \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2} \right] \rightarrow E(U_1^2) = \frac{2\eta^2}{(n+1)(n+2)} \end{aligned} \quad (6.1.5)$$

By using (6.1.3) and (6.1.5) we obtained the variance of  $U_1$  and  $Y_1$ .

$$\begin{aligned} Var(U_1) &= E(U_1^2) - [E(U_1)]^2 = \frac{2\eta^2}{(n+1)(n+2)} - \frac{(\eta)^2}{(n+1)^2} = \\ Var(Y_1) &= \frac{n(\eta)^2}{(n+1)^2(n+2)} \end{aligned} \quad (6.1.6)$$

Since,  $Y = X - a$ , then  $Var(Y_1) = Var(Y) = Var(U_1)$ .

$$Var(Y_1) = \frac{n(\eta)^2}{(n+1)^2(n+2)} = \frac{n(\theta_2 - a)^2}{(n+1)^2(n+2)} \quad (6.1.7)$$

By the utilization of (6.1.3) we can obtain an unbiased estimator for  $\theta$  as a function of  $Y_1$ .

$$W_1 = (n+1)Y_1 - na, \text{ and } Var(W_1) = (n+1)^2 Var(Y_1) = \frac{n(\theta_2 - a)^2}{n(n+2)} \quad (6.1.8)$$

## 6.2. Estimation of the Parameter $\theta$ of the Distribution $X \sim Uniform(a, \theta_2)$ by the Last Order Statistic

$$Y_n = X_{(Max)}$$

If we want to estimate the parameter  $\eta$  of  $Y \sim Uniform(0, \eta)$  by  $U_n$ ,

$$f_{U_n}(y) = n \left[ \frac{y}{\eta} \right]^{(n-1)} \frac{1}{\eta} = \frac{n}{(\eta)^n} (y)^{(n-1)}, 0 < y < \eta \quad (6.2.1)$$

$$E(U_n) = \frac{n}{(\eta)^n} \int_0^\eta y (y)^{(n-1)} dy \quad (6.2.2)$$

$$E(U_n) = \frac{n}{(\eta)^n} \int_0^\eta y (y)^{(n-1)} dy = \frac{n}{(\eta)^n} \left[ \frac{y^{n+1}}{n+1} \right]_{t=0}^{(\eta)} = \left[ \frac{n\eta}{n+1} \right] \quad (6.2.3)$$

The Maximum Likelihood estimator for the parameter  $\theta_2$  is  $Y_n$ .

Since,  $Y = X - a$ , then the following will be true.

$$E(Y_n) = E(U_n) + a = \frac{n\eta}{(n+1)} + a = \frac{n(\theta - a)}{(n+1)} + a = \frac{n\theta_2 + a}{(n+1)} \quad (6.2.4)$$

$$E(U_n^2) = \frac{n}{(\eta)^n} \int_0^\eta y^2 (y)^{(n-1)} dy = \frac{n}{(\eta)^n} \left[ \frac{y^{n+2}}{n+2} \right]_{t=0}^{(\eta)} = \left[ \frac{n\eta^2}{n+2} \right] \quad (6.2.5)$$

$$\begin{aligned} Var(U_n) &= E(U_n^2) - [E(U_n)]^2 = \frac{n\eta^2}{(n+2)} - \frac{(n\eta)^2}{(n+1)^2} = \\ &= \frac{n(\eta)^2}{(n+1)^2(n+2)} = Var(Y_n) \\ Var(Y_n) &= Var(Y) = Var(U_n). \end{aligned} \quad (6.2.6)$$

From (6.2.4) we obtain an unbiased estimator for  $\theta$  as a function of  $Y_n$ .

$$W_2 = \frac{(n+1)Y_n - a}{n}, \text{ and } Var(W_2) = \frac{(n+1)^2 Var(X_{(n)})}{n^2} = \frac{(\theta_2 - a)^2}{n(n+2)} \quad (6.2.7)$$

### 6.3. Estimation of the Parameter $\theta_2$ of the Distribution $X \sim Uniform(a, \theta_2)$ by the Sample Mean $\bar{X}$

$$\begin{aligned} \text{If } X \sim Uniform(a, \theta) \text{ then the pdf is} \\ f(x) &= \frac{1}{\theta - a}, a < x < \theta_2, \quad \text{and} \\ E(X) &= \int_a^\theta x \frac{1}{\theta - a} dx = \frac{(\theta^2 - a^2)}{2(\theta - a)} = \frac{(a + \theta_2)}{2}, \\ E(X^2) &= \int_a^\theta x^2 \frac{1}{\theta - a} dx = \frac{(\theta^3 - a^3)}{3(\theta - a)} = \frac{(a^2 + a\theta_2 + \theta_2^2)}{3}, \\ Var(X) &= \frac{(\theta - a)^2}{12}. \end{aligned}$$

For any distribution, if the sample mean is  $\bar{X}$ , for any random sample of size n, the following hold true.

$$\begin{aligned} E(\bar{X}) &= E(X), \text{ and } Var(\bar{X}) = \frac{Var(X)}{n} \\ E(\bar{X}) &= \frac{(a + \theta_2)}{2} \end{aligned} \quad (6.3.1)$$

$$\text{And } Var(\bar{X}) = \frac{(\theta_2 - a)^2}{12n} \quad (6.3.2)$$

From (4.3.1) we can obtain an unbiased estimator for  $\theta$ , as function of the sample mean  $\bar{X}$ .

$$W_3 = 2(\bar{X}) - a, \text{ and}$$

$$Var(W_3) = 4Var(\bar{X}) = \frac{(a - \theta_2)^2}{3n} \quad (6.3.3)$$

### 6.4. Estimation of the Parameter $\theta_2$ of the Distribution

#### $X \sim Uniform(a, \theta_2)$ by the Median M

A uniformly distributed continuous random variable X, over the interval  $(a, \theta_2)$ , has the following pdf

$$f(x) = \frac{1}{\theta_2 - a}, a < x < \theta_2$$

If  $Y = X - a$ , then as it is stated in Note 1,  $Y \sim Uniform(0, (\theta_2 - a)) = \eta$  and its pdf is as given below.

$$f(y) = \frac{1}{\eta}, 0 < y < \eta.$$

Estimation procedures and the findings, related to the parameter  $\theta_2$  of  $X \sim Uniform(a, \theta_2)$ , will be exactly the same as the one given in sections 1.4.1 and 1.4.2, with the exception that  $\eta = (\theta_2 - a)$

In other words, the statements of Theorem 1.4.1 and Theorem 1.4.2 hold true.

$$\begin{aligned} \text{That is: if n is odd then, } E(M) &= \frac{\eta}{2}, \\ E(M^2) &= \frac{\eta^2(n+3)}{4(n+2)}, \text{ and } Var(M) = \frac{\eta^2}{4(n+2)}, \end{aligned}$$

If n is even, then

$$\begin{aligned} E(M) &= \frac{\eta}{2}, E(M^2) = \frac{(n^2 + 4n + 2)\eta^2}{4(n+1)(n+2)} \text{ and,} \\ Var(M) &= \frac{n\eta^2}{4(n+1)(n+2)}, \text{ where} \\ \eta &= (\theta_2 - a) \end{aligned}$$

The unbiased estimators of  $\theta_2$  as functions of the sample median M are as follows:

$W_4 = a + 2M$  (when n is odd),  $W_5 = a + 2M$  (when n is even).

The variances of these unbiased estimators are:

$$Var(W_4) = \frac{(\theta_2 - a)^2}{(n+2)}, \text{ and } Var(W_5) = \frac{n(\theta_2 - a)^2}{(n+1)(n+2)}$$

### 6.5. Comparisons of Unbiased Estimators in Terms of Their Efficiencies

The unbiased estimators and their comparisons are given in Table 3.

We see that, for  $n > 1$ , the most efficient unbiased estimator among the ones given above, is  $W_2 = \frac{(n+1)Y_n - a}{n}$ .

The most efficient unbiased estimator of  $\theta$  is seen to be  $T_2 = \frac{(n+1)Y_n - a}{n}$ . By the use of the pdf of  $Y_n$  we can construct a  $100(1-\alpha)\%$  confidence interval for  $\theta$ . As it is shown before

$$f_{Y_n}(y) = \frac{n}{(\theta_2 - a)^n} (y - a)^{(n-1)}, a < y < \theta_2.$$

By the use of following probability statement we can obtain a confidence interval for  $\theta$ .

## 7. Confidence Interval for the Parameter

$\theta$  of the Distribution  $X \sim \text{Uniform}(a, \theta_2)$

$$\begin{aligned} P(y_{nL} < Y_n < y_{nU}) &= (1-\alpha) \\ \int_a^{y_{nL}} \frac{n}{(\theta_2 - a)^n} (y - a)^{(n-1)} dy &= \alpha / 2 \quad \text{And,} \quad \int_a^{y_{nU}} \frac{n}{(\theta_2 - a)^n} (y - a)^{(n-1)} dy = 1 - \alpha / 2 \end{aligned} \quad (7.1)$$

If we let  $(y - a) = w$  then the following are true:

$$\begin{aligned} dw &= dy; \quad y = a \rightarrow w = 0; \quad y = y_{nL} \rightarrow w = y_{nL} - a, \text{ and } y = y_{nU} \rightarrow w = y_{nU} - a. \\ \int_a^{y_{nL}} \frac{n}{(\theta_2 - a)^n} (y - a)^{(n-1)} dy &= \int_0^{y_{nL}-a} \frac{n}{(\theta_2 - a)^n} w^{n-1} dw = \left( \frac{y_{nL} - a}{\theta_2 - a} \right)^n = \alpha / 2 \\ y_{nL} &= a + (\theta_2 - a) \left( \frac{\alpha}{2} \right)^{1/n} \end{aligned} \quad (7.2)$$

Similarly,

$$\begin{aligned} \int_a^{y_{nU}} \frac{n}{(\theta_2 - a)^n} (y - a)^{(n-1)} dy &= \int_0^{y_{nU}-a} \frac{n}{(\theta_2 - a)^n} w^{n-1} dw = \left( \frac{a - y_{nU}}{\theta_2 - a} \right)^n = 1 - \alpha / 2 \\ y_{nU} &= a + (\theta_2 - a) \left( 1 - \frac{\alpha}{2} \right)^{1/n} \end{aligned} \quad (7.3)$$

**Table 3.** Unbiased estimators of  $\theta_2$  for the Distribution  $X \sim \text{Uniform}(a, \theta_2)$  and their variances

	Unbiased estimators ( $T_i$ )					
	$W_1 = (n+1)Y_1 - na$	$W_2 = \frac{(n+1)Y_n - a}{n}$	$W_3 = 2(\bar{X}) - a$	$W_4 = a + 2M$ (n is odd)	$W_5 = a + 2M$ (n is even)	
$E(W_i)$	$\theta_2$	$\theta_2$	$\theta_2$	$\theta_2$	$\theta_2$	$\theta_2$
$Var(W_i)$	$\frac{n(\theta_2 - a)^2}{(n+2)}$	$\frac{(\theta_2 - a)^2}{n(n+2)}$	$\frac{(\theta_2 - a)^2}{3n}$	$\frac{(\theta_2 - a)^2}{(n+2)}$	$\frac{n(\theta_2 - a)^2}{(n+1)(n+2)}$	

**Table 4.** Tests of Hypotheses Related to the Parameter  $\theta$  of  $X \sim Uniform(a, \theta_2)$ 

$H_0 : \theta = \theta_0$	$H_0 : \theta \leq \theta_0$	$H_0 : \theta \geq \theta_0$
$H_1 : \theta \neq \theta_0$	$H_1 : \theta > \theta_0$	$H_1 : \theta < \theta_0$
If $Y_n \geq y_{nU}$ or $Y_n \leq y_{nL}$ $H_0$ is rejected	If $Y_n \geq y_{nU}$ $H_0$ is rejected	If $Y_n \leq y_{nL}$ $H_0$ is rejected
Don't reject $H_0$ otherwise	Don't reject $H_0$ otherwise	Don't reject $H_0$ otherwise
Where, $y_{nL} = a + (\theta_0 - a) \left( \frac{\alpha}{2} \right)^{1/n}$ and $y_{nU} = a + (\theta_0 - a) \left( 1 - \frac{\alpha}{2} \right)^{1/n}$	Where, $y_{nU} = a + (\theta_0 - a) (1 - \alpha)^{1/n}$	Where, $y_{nL} = a + (\theta_0 - a) (\alpha)^{1/n}$

If the results in (7.2) and (7.3) are substituted in (7.1)

$$P\left(a + (\theta_2 - a) \left( \frac{\alpha}{2} \right)^{1/n} < Y_n < a + (\theta_2 - a) \left( 1 - \frac{\alpha}{2} \right)^{1/n}\right) = (1 - \alpha).$$

Solving the above inequalities for  $\theta_2$ , the following Confidence Interval is obtained.

$$P\left(a + \frac{Y_n - a}{\left(1 - \frac{\alpha}{2}\right)^{1/n}} < \theta_2 < a + \frac{Y_n - a}{\left(\frac{\alpha}{2}\right)^{1/n}}\right) = (1 - \alpha) \quad (7.4)$$

A  $100(1 - \alpha)\%$  confidence interval for  $\theta_2$  is given as follows.

$$\left( a + \frac{Y_n - a}{\left(1 - \frac{\alpha}{2}\right)^{1/n}}, \quad a + \frac{Y_n - a}{\left(\frac{\alpha}{2}\right)^{1/n}} \right) \quad (7.5)$$

alternative hypothesis, a plausible test statistic is to be  $Y_n = X_{Max}$  for that the most efficient unbiased estimator is

$$W_1 = \frac{(n+1)Y_n - a}{n}, \text{ which is a linear function of } Y_n = X_{Max}.$$

If the level of significance is chosen to be  $\alpha$ , then the decision rules, as given in the following table, are applicable.

## 8. Tests of Hypotheses Related to the Parameter $\theta_2$ of the Distribution

$X \sim Uniform(a, \theta_2)$

If  $H_0 : \theta = \theta_0$  is to be tested against to any proper

## 9. Conclusions

For uniform distributions, of the types  $X \sim Uniform(\theta_1, b)$ , and  $X \sim Uniform(a, \theta_2)$ , some unbiased estimators for the unknown boundary values  $\theta_1$  and  $\theta_2$  are suggested. Among the suggest estimators, the most efficient estimator is selected. By the use the most efficient estimators, confidence interval and tests of hypotheses procedures are established.

## 10. Simulation Study

To see the match between the established theoretical findings and the empirical results, a simulation study on a uniform distribution over the interval ( $\theta_1 = 5$ ,  $\theta_2 = 10$ ) is carried out. 100 independent samples of size 100 are drawn from this uniform distribution. From each sample the first order statistic  $Y_1$ , the last order statistic  $Y_n$ , sample mean  $\bar{X}$ , sample medians  $M(n \text{ is even})$  (for  $n=100$ ), and  $M(n \text{ is odd})$  for  $n=99$  are computed. Unbiased estimator values are computed from each sample. Summary of the simulation results are given in the following Table 5.

**Table 5.** Simulation summaries

	Mean	Variance
Sample Mean ( $\bar{X}$ )	7,502283	0,021271
Min ( $Y_1$ )	5,053689	0,003032
Max ( $Y_n$ )	9,955695	0,001421
Sample Median, M(n even)	7,506121	0,061029
Sample Median, M(n odd)	7,501759	0,063832
T1	5,004226	0,003093
T2	5,525145	14,49897
T3	5,004565	0,085083
T4	5,003519	0,255329
T5	5,012243	0,244115
W1	10,42261	30,93163
W2	10,00525	0,00145
W3	10,00457	0,085083
W4	10,00352	0,255329
W5	10,01224	0,244115

A Simulation Study On A Uniform Distribution Over the Interval (5,10)

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Sample 7	Sample 8	Sample 9	Sample 10
6.76805	5.54758	8.71645	7.32947	6.26586	7.69478	8.62869	5.77136	9.49987	7.23937
6.64879	9.24694	5.11766	9.21047	6.51333	7.24894	9.61452	5.0222	9.40268	8.33267
6.18835	8.82961	5.65464	7.76501	6.00813	7.04285	5.98805	8.67661	6.39686	6.45668
8.92565	8.61381	6.33152	5.77686	5.65716	9.11983	5.47428	6.81726	8.08379	5.80943
9.27706	5.19226	9.14137	8.91613	5.50441	6.1482	9.15237	5.74133	5.70093	6.37832
9.69805	7.447	5.27074	8.73209	9.52611	9.74396	8.92942	8.69419	8.11978	7.46212
9.56789	9.11269	5.96345	9.34895	9.44353	8.04405	6.18012	7.64807	9.53177	7.13468
7.44082	7.77596	5.3617	6.7841	6.32015	5.99846	8.07746	8.43545	9.36648	5.17614
9.72813	9.11189	7.05884	6.63384	9.70685	8.54253	9.96564	9.85792	7.12269	8.9224
7.46514	7.06914	9.44628	5.96065	8.5456	6.99726	7.58934	6.05904	7.17869	5.70564
8.86561	5.7497	9.80677	6.44889	5.60188	6.28214	7.08043	8.17612	9.03821	6.99094
5.64825	8.37879	9.60301	9.26988	5.2388	6.00553	6.61076	9.29307	5.87516	9.03501
7.75676	6.23922	7.51729	8.14453	5.49739	9.02683	5.50518	5.32239	6.76773	6.67583
8.03505	5.17559	8.4334	6.97945	7.26849	9.3344	7.00371	5.67347	6.34599	9.72847
7.91383	5.82104	6.11075	8.0797	7.72618	5.98542	6.14096	9.38147	9.80645	6.04649

Based on a simulation of 100 independent samples of size n=100 from a uniform distribution over the interval ( $\theta_1 = 5$ ,  $\theta_2 = 10$ ), summary statistics,  $\bar{X}$ ,  $Y_1$ ,  $Y_n$ ,  $M(n \text{ even})$ , and  $M(n \text{ odd})$  of 100 observations are obtained.

Values of five different estimators for  $\theta_1$ , namely T1, T2, T3, T4, and T5 are computed from each sample by the use of proper summary statistics. The second and third columns of Table 5 contain the computed means and the variances of each summary statistics and of estimate values for  $\theta_1$ .

From Table 5, we observe that T1, T2, T3, and T5 are unbiased estimators of  $\theta_1$ . Among these unbiased estimators T1 is the best (the most efficient) estimator, for that it has the smallest variance.

In the same line, values of five different estimators for  $\theta_2$ , namely W1, W2, W3, W4, and W5 are computed from each sample by the use of proper summary statistics.

From the second and the third columns of Table 5, we observe that W1, W2, W3, W4 and W5 are unbiased estimators of  $\theta_2$ . Among these unbiased estimators W2 is the best estimator, for that it has the smallest variance.

By using the formula (2.5), a 95% confidence interval is computed for  $\theta_1$  from each sample. Exactly 95 out of 100 confidence intervals contained the parameter value 5 of  $\theta_1$ .

Similar procedures are followed for  $\theta_2$ . By the use of the formula (5.5), 95% confidence intervals for  $\theta_2$  are computed. 94 out of 100 confidence intervals contained the parameter value 10 of  $\theta_2$ .

Simulation study results are seen to be in accordance with the theoretical findings.

5.27165	7.36576	9.04968	7.10611	5.02197	6.07066	9.12631	6.74326	7.08122	6.08235
7.5777	9.18829	5.01803	5.2689	7.79532	5.91644	6.42044	8.48295	8.37441	7.84362
8.67416	7.68485	9.87295	5.12895	8.99236	7.01728	7.97886	7.58623	6.07359	7.71055
5.27403	5.27586	7.36725	6.06629	7.00971	7.9662	9.61821	5.2543	6.21058	8.19278
7.0574	6.73858	7.3245	9.21873	7.81031	9.38243	7.71026	8.78718	9.44682	5.20389
9.87328	8.86696	9.23279	9.24976	5.99653	8.43504	8.92682	5.74824	6.78444	6.5277
7.16373	6.29781	8.06776	9.20781	8.69283	8.152	5.33519	7.59396	9.83634	6.81062
5.57791	8.07136	9.18987	7.23794	9.60694	7.59491	7.82591	6.98435	6.19687	5.69536
5.60227	6.57299	8.33378	9.75946	9.20069	6.46122	6.62164	9.47102	9.08496	8.82302
7.92689	8.40137	8.25053	7.68828	6.22544	7.94492	7.92489	8.54114	6.19162	7.24041
5.27507	6.7398	5.5761	7.31552	9.7343	6.7126	5.60909	9.00667	5.96167	8.85088
5.23583	6.70567	6.15485	8.7925	9.4837	5.44594	6.9962	6.8876	7.89363	6.62394
8.41925	5.45209	5.42727	6.77572	5.70472	8.80198	8.95848	9.03918	7.95832	5.53407
6.42925	7.48792	6.29917	5.4144	8.12612	8.09951	8.49912	6.17323	7.83436	9.80579
7.73869	5.0058	8.84326	8.7735	5.68494	9.73225	8.84631	8.54916	5.94561	5.7347
9.05125	6.25505	6.88755	5.56231	6.21997	6.3683	8.07337	9.86432	9.2897	7.34417
5.17675	5.41977	7.20424	6.90153	5.44206	8.83918	9.85975	8.25992	6.71686	6.21734
8.53267	9.87429	6.23478	5.78054	7.99312	8.95462	7.59182	6.21742	5.45427	7.47042
5.00688	5.11108	5.7408	5.67876	7.41248	6.71499	6.45063	5.04365	8.61655	9.66729
6.55685	6.49044	5.53936	6.65629	6.14796	8.12447	6.35537	5.14893	7.84643	8.22793
9.36141	7.79091	9.08092	8.26076	9.67676	7.07278	8.97439	8.51532	9.32552	5.68898
9.25425	9.76699	6.01214	7.98605	7.08992	8.91317	5.99515	5.83984	9.94706	6.31333
5.70783	8.29902	5.9022	6.96476	6.68713	8.66273	9.94635	5.76362	6.10951	9.43733
5.09335	8.38556	6.65674	7.80429	8.94538	6.10271	5.29402	5.33342	9.79042	6.1095
5.13942	6.21019	5.46105	6.39426	8.47895	6.67935	7.11159	5.19055	9.01288	8.00078
9.58752	6.82679	8.83202	6.98508	9.38232	7.49847	7.80716	7.75974	5.75598	5.06975
5.5504	5.11758	5.56772	9.0693	7.68216	9.3875	5.40705	8.41356	6.83629	5.95721
9.71618	5.02069	7.66213	9.0033	9.966	7.78309	5.51729	9.07735	7.51673	5.17899
9.03292	8.47602	8.83159	6.20887	5.47519	7.15126	8.86561	6.3694	8.66828	7.03376
9.18015	7.62615	8.6341	8.45812	9.32312	7.55031	5.57368	8.06073	7.46861	8.53911
5.79433	5.94207	5.24463	6.38624	5.97973	8.57801	9.05113	6.08959	6.99081	5.73397
5.71019	8.28413	9.88012	7.85481	9.80912	8.95461	6.62755	7.00377	5.91353	8.6948
6.90311	7.49966	8.89027	9.93639	5.57294	7.61461	9.62354	5.84489	6.86733	7.33916
6.01093	9.88304	8.05946	7.993	7.72002	6.92338	9.15678	7.42699	5.98298	6.47172
8.29117	5.24312	5.46054	6.17231	9.17399	7.52364	9.02832	9.30765	8.3826	9.01742
7.21463	9.3925	5.50457	9.61379	8.59673	9.09117	9.92957	9.79773	9.06485	9.59982
5.69122	5.14753	9.94336	7.83586	9.71874	7.21083	9.225	6.64117	5.16821	7.76513
7.39651	6.56979	8.79823	9.2966	5.71389	5.44684	9.49124	8.38406	9.94748	7.52972
6.21596	6.77723	5.91663	7.29684	9.18548	8.2596	7.60838	9.81333	9.8305	9.63577
9.3906	5.99472	7.03362	9.1611	8.75165	7.10397	7.34638	6.99033	7.57368	5.04623
7.59387	8.75147	7.98545	5.99185	6.43094	9.14706	5.94997	6.82112	5.62793	7.73832
9.52858	8.41175	6.10377	5.98491	5.12037	7.84236	6.95173	8.25666	8.2186	7.19077
8.45732	7.6749	9.07446	6.99203	5.66462	8.24431	6.9998	8.93805	6.90706	7.50859
8.72879	7.28557	8.83935	9.4945	8.69578	6.51762	7.88476	9.19904	5.10719	6.49467
9.24246	7.91863	9.83302	9.28537	8.35988	7.37135	6.33597	6.92714	6.14444	9.00411
5.69165	9.39004	8.42245	5.71779	8.40625	7.5988	9.4064	5.49549	6.13181	9.79494
5.10031	6.06409	6.50803	7.26941	6.18215	8.75021	9.68249	9.29264	8.51029	5.562
9.58141	6.30627	6.53334	5.14218	9.72103	8.15898	8.96704	9.89307	5.02439	8.57001
6.91086	9.94315	6.16404	9.90682	5.80761	8.60099	9.79025	8.48124	7.23672	6.01227
6.20842	5.40688	6.97988	6.79329	5.48799	5.01892	5.09225	9.47962	5.92262	5.8503
6.69155	6.27316	8.22989	6.74831	7.72835	6.54153	5.78193	6.57418	6.2449	8.86547



<i>UCL for LB</i>	5.005615692	5.004535418	5.016768515	5.030321946	5.020709513	5.01765874	5.077763956	5.020939571	5.023130125	5.044975655
<i>LCL for UB</i>	9.95429416	9.944401656	9.962096135	9.937639944	9.967257441	9.997775167	9.96689735	9.99091343	9.948732752	9.916874687
<i>UCL for UB</i>	10.13916351	10.12890186	10.14725661	10.12188784	10.15261051	10.18426701	10.15223699	10.17714922	10.13339458	10.10034773

## A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 11	Sample 12	Sample 13	Sample 14	Sample 15	Sample 16	Sample 17	Sample 18	Sample 19	Sample 20
7.39785	6.20763	7.25944	9.16618	9.736	7.62824	9.22624	5.55963	5.08188	5.26447
7.94401	5.69129	7.50581	8.4911	9.7161	5.81821	6.55312	6.8276	6.4105	5.84608
7.06894	6.13494	8.63681	8.31787	9.2237	6.7997	8.20013	7.32031	5.91184	6.74863
7.57285	9.33397	5.82982	9.98078	7.7035	8.83509	6.88446	9.09341	7.54447	6.9885
6.8369	6.91514	6.94853	8.68123	7.8618	6.14139	5.85179	6.44702	9.17696	7.19409
8.35096	5.78391	8.38171	5.26161	8.4084	9.44126	5.92579	9.7605	9.8743	6.69552
7.50176	8.27563	9.6654	9.23336	5.0226	5.61973	7.416	6.37405	8.16673	9.13798
8.39866	8.39755	5.27332	5.64921	7.5933	5.58701	8.18447	6.0201	9.40008	5.74566
6.99428	9.86785	5.83636	7.22776	6.3804	5.97779	9.33531	9.44215	6.2974	7.6175
9.99918	6.54547	9.38957	7.7293	8.943	5.09222	8.41037	6.13657	5.47198	6.15335
7.6526	7.64054	7.09923	5.3689	6.4189	7.78039	7.83215	7.2793	5.94635	9.80417
5.494	7.74346	8.93599	5.6543	9.0726	5.41968	8.61613	6.90328	6.73894	7.67689
7.91504	5.53583	7.62648	7.04771	8.3337	6.16025	7.40354	7.1197	8.80171	8.36973
5.07712	7.69366	8.5506	5.84612	8.9331	8.12448	9.4728	9.04545	5.75443	7.29942
7.23817	6.42892	6.04477	7.41749	8.7025	6.53184	5.26031	6.27699	6.05236	7.35986
7.85828	7.3348	6.84577	9.4722	5.6618	5.55453	9.36014	7.49751	7.64806	7.32806
7.36382	7.52933	5.97317	7.69992	9.4438	9.06078	5.07384	6.3336	5.74785	8.92414
8.06353	5.43307	9.85969	5.32654	7.3743	7.48679	5.44947	8.89453	6.38419	6.05716
6.70263	8.26209	8.86738	6.79479	5.3812	5.87593	8.76083	5.54261	5.43776	8.72389
9.04834	9.53827	7.7689	9.47768	8.397	6.64335	6.27355	6.19124	8.96379	6.41558
6.36979	5.64731	8.53807	7.15427	6.9409	8.11397	7.56165	5.23458	6.63205	5.37156
6.92077	9.54783	8.50996	7.55985	7.0424	8.24892	6.18634	6.22211	8.88804	8.69459
5.53717	7.53904	7.70199	5.07621	9.8026	8.84672	6.3397	9.15922	9.13591	7.05403
9.40802	6.04497	9.09425	7.09306	9.4289	5.23032	7.42046	8.00369	7.64426	6.63196
5.06715	8.48232	9.98652	5.22271	8.2933	9.3684	7.02774	7.83541	8.26621	6.79833
7.4118	9.02372	6.62458	7.81491	6.9305	8.16785	8.04377	7.17165	7.08211	7.78966
9.99531	8.48427	8.45951	8.10252	7.6984	9.73513	7.26051	9.52155	7.63931	7.27974
7.31508	6.23923	6.37046	6.29922	7.6716	7.67403	7.88992	8.02092	8.41438	9.06916
6.67168	8.03332	5.23003	6.37034	5.9381	9.13418	8.22346	6.65097	9.16668	7.24098
7.23515	5.9435	6.05655	8.4317	5.4536	6.23276	8.4368	9.56282	9.92996	5.52407
6.51806	7.85592	5.33177	6.11586	8.1983	7.76704	8.73137	5.62925	6.65516	7.26843
9.6771	9.82559	7.16112	8.128	9.2463	9.37293	8.17856	6.45708	5.79967	8.18539
7.4341	8.74451	6.276	6.58604	5.6991	9.0135	6.82261	9.98926	7.97095	6.91902
9.64631	8.92125	6.44299	5.29139	7.7926	8.18592	9.19378	9.35941	6.49807	9.36072
5.29207	9.77339	6.03218	5.36048	5.6051	5.12744	8.19658	7.43825	5.10703	8.14276
5.13412	8.19317	5.5289	7.38909	8.4656	5.3014	5.18899	7.05525	7.60794	8.6865
6.75019	6.76985	5.88927	6.19036	5.6172	5.87572	5.82793	6.10128	9.87141	6.64686
8.62868	7.84538	6.68348	5.76894	6.4758	9.24376	9.81656	7.8525	5.87902	8.25264
9.93939	5.24889	7.7888	9.05191	6.17	7.16321	9.88333	6.04879	9.54144	9.32253
9.28682	5.15208	5.71131	8.56557	9.2971	8.34593	5.52996	8.22105	5.72501	5.05149
8.30137	8.35834	5.14256	7.73196	8.9883	5.19708	6.27565	5.27873	9.90038	9.36698
9.59388	7.56078	8.47397	7.29452	8.9028	5.65104	7.5154	8.06941	6.37073	6.71419
8.06009	6.38888	9.37852	6.01287	9.2642	6.26467	9.93527	5.13417	6.36996	5.92047

8.51556	6.42493	6.23675	7.77947	8.5727	6.6845	5.60262	6.71558	8.53929	6.16471
7.02528	5.1305	8.50583	5.33739	9.4496	8.6685	9.1879	5.15035	5.04674	6.08422
8.86225	6.24247	9.80768	8.82819	6.1882	8.66858	8.57314	7.42402	6.37984	6.29987
8.27566	7.0474	6.18524	7.24026	6.6582	7.32217	5.79064	9.77784	8.10408	8.81119
9.93162	8.09323	8.91552	9.93509	8.6645	8.86669	6.34789	8.81902	5.46736	7.28297
8.37572	5.61883	8.63409	5.99632	9.3585	8.32184	7.40013	7.18373	6.22218	7.1285
8.65388	8.48995	9.34811	7.46079	5.4136	9.07994	8.38419	7.27919	8.81753	9.30363
8.69014	8.20422	7.06179	5.12608	7.1828	5.56435	6.79376	7.74926	9.51067	9.80786
9.53102	6.56193	7.58618	6.02701	6.6931	6.68482	5.44141	9.43515	8.79169	5.29862
8.46926	5.40281	6.30829	7.31449	6.2906	9.01565	5.32804	9.7633	7.19075	8.51201
5.34265	6.97902	5.36909	5.36397	5.0331	5.74077	8.18694	8.02474	6.70091	8.38229
9.06678	6.98001	9.36352	5.81136	5.2576	8.11496	5.05098	9.06684	9.47276	5.63144
9.23981	9.00532	7.21196	8.55793	6.5879	6.94823	9.46631	8.2726	6.20838	8.1244
5.66355	6.4169	6.78388	6.58622	8.7434	9.90656	8.57594	5.19805	7.23249	7.75632
7.22543	6.30703	7.30567	9.05109	9.9999	6.99865	8.70659	8.44717	9.91768	7.04789
7.81943	5.0073	8.04645	5.68206	7.5186	5.87804	6.50219	7.57899	9.60546	6.60042
8.93109	7.81565	7.13441	6.70723	5.4204	9.76292	8.91562	5.3944	8.32549	8.09718
9.65457	8.73707	7.7928	5.64265	5.8887	6.09697	7.21258	6.42296	9.48355	5.27488
8.5874	6.92289	8.07765	9.18122	7.552	6.21786	7.72036	5.08256	7.6745	5.29402
8.29896	9.66623	6.01848	6.25046	8.7183	6.38391	5.65233	7.37934	6.69772	6.0828
8.17236	5.2656	9.38141	5.34474	8.8397	5.92125	9.14714	7.48546	7.66778	8.18654
8.83367	6.66726	8.27641	9.39273	8.0676	6.65641	6.95142	7.07142	6.10548	6.89464
5.34083	6.6406	8.60935	7.92246	5.8396	9.05802	7.37061	8.07589	5.35419	5.64123
8.0906	7.76244	9.72225	5.5272	6.9759	9.56182	5.56989	9.81599	8.53576	6.97032
7.58749	9.66608	5.05439	7.15307	6.3469	9.56614	6.69554	7.47203	9.95491	9.11285
6.7169	9.16153	7.13055	6.82499	7.4483	6.94713	8.70693	6.42977	8.92406	7.38939
9.81761	9.68075	5.92115	9.92838	9.4644	9.09148	8.08565	5.02457	6.3539	6.19471
9.01024	6.79721	9.46717	5.93763	9.9809	8.86075	8.73073	5.89269	7.81358	8.45515
6.86641	8.20738	7.35658	8.13622	9.0001	7.34992	9.39392	7.37536	5.60076	8.03869
7.35323	8.74021	7.61242	8.42627	9.7001	8.0541	6.56241	6.8519	7.36086	6.54893
7.61803	6.14829	8.63496	8.8758	7.9314	9.60707	5.00066	7.58104	5.66064	9.93244
5.57868	5.24022	5.72411	6.26774	8.5974	5.7903	7.58716	5.77288	5.09175	8.00952
8.12611	8.39449	8.11326	6.8039	8.3532	7.67104	6.30606	5.45776	7.22413	6.9665
5.75729	7.44862	7.50849	7.644	8.2103	7.36702	7.35778	7.11629	6.27391	6.1399
7.2263	7.95143	5.89386	9.66861	5.9897	9.25666	7.40101	8.41466	8.2625	8.25507
6.4719	8.81848	7.11683	7.84421	8.3013	5.77341	9.59417	6.11447	8.26719	8.2262
8.25635	7.97871	8.26657	6.052	6.1524	5.90646	6.29579	7.28931	9.12862	9.76343
7.51749	5.81842	7.451	8.28588	9.4695	6.95256	6.12368	5.36801	8.03506	6.80507
6.8456	8.53818	8.42014	9.15292	5.9858	7.41112	7.8785	7.60355	9.95089	7.14803
7.46963	6.60374	8.86763	6.0438	8.804	7.80519	7.81696	7.96352	7.8157	6.97771
5.12147	7.4682	5.39092	6.26042	9.4348	7.26604	7.43905	9.63743	5.49536	8.62098
5.90402	5.93774	9.24483	8.04131	7.4353	5.46798	8.953	9.56017	9.80873	8.75339
7.26553	9.14071	6.7082	9.19207	5.0085	8.63621	9.71324	5.33261	6.608	6.61845
6.69629	5.42626	7.93621	7.49468	6.562	8.36234	6.31127	7.08913	9.29109	6.91747
6.22081	7.37053	8.67953	7.06509	7.426	8.90101	5.2426	6.2947	6.72996	7.27462
9.69949	5.51725	9.0679	6.41517	8.6747	9.46768	7.47333	8.89542	7.17449	7.19151
5.73183	6.3377	5.29299	7.00573	6.2077	9.07925	8.31147	9.63636	8.7862	8.12202
6.81631	8.57068	6.78897	7.74131	5.1666	9.86332	7.72579	8.10177	8.54152	5.05227
5.52755	9.28046	8.1645	6.69749	7.8452	5.53572	5.87355	6.78089	6.07446	5.74197
6.62158	5.44138	7.23956	9.61705	7.1616	6.22685	6.72456	8.43433	9.84138	6.82374
9.72252	8.27665	8.09126	6.84545	7.3639	7.77444	7.78668	5.32349	7.39328	9.89427

5.32557	7.14012	7.89246	7.02957	8.7941	5.63638	7.73279	5.52783	8.55309	5.77849
7.44546	7.66052	5.47908	6.17161	9.1419	6.64577	7.6545	6.74936	8.00272	5.11153
7.89587	7.11172	8.53037	5.68772	7.6342	5.5546	8.43349	9.9478	5.4087	9.52389
6.83903	5.14611	6.73316	7.30832	7.605	6.56172	5.58905	6.02784	6.02485	8.2161
5.63146	5.39738	9.97097	9.10903	9.7046	8.55917	5.71887	7.70168	6.16477	7.02549
8.32613	9.37987	7.02287	7.08365	8.4832	9.86445	8.88024	5.78705	8.95264	9.63638
7.5828072	7.351035	7.4916724	7.2433333	7.675299	7.4377727	7.4395483	7.3125242	7.5052924	7.3758685
5.06715	5.0073	5.05439	5.07621	5.0085	5.09222	5.00066	5.02457	5.04674	5.05149
9.99918	9.86785	9.98652	9.98078	9.9999	9.90656	9.93527	9.98926	9.95491	9.93244
7.509625	7.45841	7.50715	7.15367	7.74805	7.38907	7.494365	7.279245	7.576205	7.217535
7.509625	7.45841	7.547335	7.15367	7.70095	7.35847	7.45619	7.284305	7.576205	7.217535
5.0178215	4.957373	5.0049339	5.0269721	4.958585	5.0431422	4.9506666	4.9748157	4.9972074	5.0020049
9.91718	-3.34715	8.63852	8.05878	9.9899	0.56256	3.46227	8.91526	5.44591	3.17644
5.1656144	4.70207	4.9833448	4.4866666	5.350598	4.8755454	4.8790966	4.6250484	5.0105848	4.751737
5.01925	4.91682	5.09467	4.30734	5.4019	4.71694	4.91238	4.56861	5.15241	4.43507
5.01925	4.91682	5.0143	4.30734	5.4961	4.77814	4.98873	4.55849	5.15241	4.43507
10.0491718	9.9165285	10.0363852	10.0305878	10.049899	9.9556256	9.9846227	10.0391526	10.0044591	9.9817644
11.78215	5.7373	10.49339	12.69721	5.8585	14.31422	5.06666	7.48157	9.72074	10.20049
10.1656144	9.70207	9.9833448	9.4866666	10.350598	9.8755454	9.8790966	9.6250484	10.0105848	9.751737
10.01925	9.91682	10.09467	9.30734	10.4019	9.71694	9.91238	9.56861	10.15241	9.43507
10.01925	9.91682	10.0143	9.30734	10.4961	9.77814	9.98873	9.55849	10.15241	9.43507
4.881785185	4.819686164	4.868545695	4.891185639	4.820931257	4.907797257	4.812796649	4.83760513	4.860608226	4.86553672
5.065900953	5.006035798	5.053137722	5.074963247	5.007236102	5.090977301	4.999394117	5.023310171	5.045485785	5.050236987
10.00044584	9.869082589	9.987782637	9.982041184	10.00116603	9.907802391	9.93651966	9.990523331	9.956164633	9.933688944
10.18703734	10.05077227	10.17390161	10.16794591	10.18778439	10.0909369	10.12072575	10.17674457	10.14110378	10.11778941

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 21	Sample 22	Sample 23	Sample 24	Sample 25	Sample 26	Sample 27	Sample 28	Sample 29	Sample 30
8.32569	5.75325	6.13597	7.53555	5.11664	9.08224	9.99042	6.58396	7.01388	7.2635
7.46212	5.33049	5.68595	7.10432	6.19317	5.77521	9.98285	5.6621	8.41163	6.76243
9.59227	5.16567	6.52314	8.00479	5.28679	5.8814	6.61661	6.21335	9.86478	8.85571
8.39591	6.02589	8.67845	7.88271	8.92604	8.0815	7.0593	8.38374	7.44662	7.85309
8.3823	9.62654	7.39604	6.10528	5.07929	6.00122	5.65181	7.54661	7.17612	7.47679
5.7627	5.79353	7.57989	7.81238	5.0122	8.893	7.58773	5.40331	8.23479	7.57756
7.60945	6.29223	6.09938	6.84769	5.15296	9.68669	6.75935	8.80203	5.16679	8.16733
8.15443	6.53956	7.80029	8.98717	7.0657	9.57708	5.17218	7.92756	6.41966	7.34651
6.91606	7.18033	8.8917	7.64828	5.33753	5.28144	8.62498	6.78056	5.21995	7.71554
7.39946	6.8039	6.81733	6.9729	5.72004	5.59521	8.73899	9.45348	8.13322	8.72174
6.37475	9.04566	6.50417	5.16776	5.63193	9.43085	8.14344	5.93248	6.97278	6.21404
8.45752	6.48389	5.3978	9.86514	6.7095	6.21591	9.3044	8.95939	5.72177	9.72006
6.53484	8.202	9.93562	9.32914	7.70935	7.23305	5.49412	5.76469	9.14226	7.42846
5.16489	7.20354	7.53374	9.35638	6.59767	7.89046	6.77294	9.83523	7.40732	7.95444
9.47808	6.22206	5.58332	9.59478	7.99221	6.43917	5.89302	9.99247	6.9696	9.40442
9.23631	6.72295	8.34206	5.1359	6.68982	5.0244	6.02004	6.71983	6.41408	9.16651
5.81773	9.97844	8.1384	8.93292	5.36105	6.86311	8.6027	7.84583	8.10285	6.17235
9.45194	7.1549	6.49622	6.78082	7.38425	8.18952	8.76817	8.62527	7.40127	7.8021
6.58326	7.84476	9.88987	6.47946	8.34488	9.66706	5.26086	8.0864	9.83865	5.92278
7.37079	5.95003	7.18485	9.11627	6.98097	5.31315	8.45788	5.00991	5.44174	7.21229
9.77103	6.43369	9.70458	5.54783	9.44731	7.05563	8.59299	5.8155	6.67589	5.6197
9.92275	9.19744	9.79383	9.83196	9.80004	9.38907	8.59939	7.13723	8.11409	9.57908
8.63247	6.82916	9.66182	6.18629	7.51857	6.00483	5.62462	8.37601	8.01362	6.6603

7.85437	5.12673	7.61048	9.86566	6.05878	9.01645	8.80219	6.26343	7.85254	8.15468
9.85044	8.78166	9.54995	9.11415	7.20682	8.13201	5.20044	6.22073	5.5691	7.24427
8.35803	7.53815	7.9768	7.10602	6.23081	9.8711	5.00736	7.50726	5.88478	6.58255
5.21047	5.74051	8.55309	9.48773	7.79278	8.48275	9.79649	5.33199	8.9342	8.24775
8.8132	6.39609	5.44752	5.73028	9.52593	8.56755	7.04515	6.68785	8.18693	8.33047
8.0482	8.69522	6.08674	9.55419	9.87047	9.34426	6.79469	5.27031	7.55688	9.12237
9.4708	9.06963	8.42509	5.92478	5.52015	6.31504	6.41845	6.69683	7.66046	6.37926
8.37135	6.00814	8.77829	7.92755	6.22218	7.69534	7.95063	7.22383	5.67456	5.48947
9.35921	8.63166	7.10174	6.6094	7.27742	8.72676	6.90121	8.3384	8.44562	8.29378
8.13584	8.66406	7.05679	6.99214	8.53729	7.11442	6.47795	5.38802	8.41168	5.82682
9.70577	9.51162	6.79132	5.17175	6.7689	6.9018	5.32815	9.06795	9.07235	8.12531
6.30491	6.89919	9.57747	7.65716	5.16844	7.19381	5.5668	9.33529	6.08724	6.84524
6.84694	8.09432	8.32904	8.74449	8.89727	6.79455	5.153	7.97058	7.67589	7.95755
9.42759	9.86337	7.76314	6.66112	5.57856	8.14185	9.19218	7.50049	8.70145	9.26928
8.44692	9.38785	9.37161	6.63592	9.18742	7.59484	5.88403	7.24741	9.72363	5.62789
6.37371	9.42335	9.34725	8.29872	9.85391	5.69642	5.69619	8.42181	7.74294	6.05066
5.08498	8.52938	6.14002	7.36046	6.28566	8.81382	5.32175	7.98748	6.66847	8.60827
9.2007	7.24265	6.98993	9.79685	6.47539	8.7651	8.62978	7.87737	8.08682	9.34668
5.09686	6.75718	7.2148	6.28526	5.9538	7.2025	8.78464	7.52633	7.38987	9.86544
6.48897	7.24311	5.67578	6.59609	7.08098	5.97434	8.39302	7.84144	5.43232	8.13497
8.0231	9.37684	8.38881	9.14406	6.11128	7.31805	6.17675	8.30275	9.40497	5.23083
9.55717	7.9431	7.27392	6.21884	6.5125	8.70587	9.80154	6.98046	9.82169	9.87062
9.28342	8.84346	5.15286	9.65465	7.21903	7.85329	8.03664	6.97107	5.14758	7.83208
8.50767	6.95314	7.45182	8.6744	7.61861	7.82214	5.48193	8.27924	6.72037	7.42882
8.28484	8.80668	9.05997	5.92591	8.72074	7.43228	6.00952	5.312	9.66738	6.76058
6.57427	6.89189	8.5574	5.78803	8.39316	7.54053	9.95541	6.19337	6.21624	8.43987
8.70755	8.29218	9.46167	6.70254	9.69068	8.41251	6.43353	9.86517	9.9937	5.16463
5.95035	9.11406	8.32017	5.18303	7.51086	8.16715	7.39361	8.7723	7.97123	5.4851
9.47106	6.90114	6.62021	6.92079	7.82887	8.96076	7.76304	5.006	5.59473	5.74209
5.21334	7.91109	5.90649	5.6749	7.61417	8.77656	9.42714	7.91209	5.91617	8.24382
5.08345	6.73298	9.46502	5.59773	6.33785	8.03829	6.6409	8.52277	7.77127	8.0021
9.93153	5.9156	7.58929	7.70522	5.58872	9.29319	9.13734	5.27659	5.24541	6.10749
6.72426	8.7903	8.00611	9.48693	7.75885	9.44562	7.66871	9.4551	5.14162	8.62176
8.70934	8.26347	5.99589	6.88425	6.02726	8.07495	9.88452	7.10123	9.46598	6.69237
6.63213	5.32896	9.22016	9.79235	8.26484	7.65199	7.94584	5.68903	7.0186	5.45011
8.57051	5.35863	5.85238	6.22104	6.38446	5.79889	5.44857	7.31848	8.71663	7.20317
8.80074	5.34427	8.79146	5.5691	9.72043	5.81353	8.40854	5.6765	6.85618	8.19874
6.59686	7.80487	9.63725	6.29436	8.02882	5.55195	8.87062	8.73813	8.89183	7.3207
9.28042	7.09206	8.95505	8.63971	8.69459	8.34788	9.11057	5.93923	8.88901	6.43554
7.4718	9.70124	7.71218	9.22338	7.42276	5.80467	6.80978	5.3872	6.14053	9.00922
5.36482	6.39633	5.20002	9.13003	6.68784	7.53523	5.44916	7.43923	6.73776	6.59053
9.2361	5.36731	8.49363	6.19681	8.7512	6.38547	6.70923	9.57723	5.93882	6.84534
7.76462	7.72537	6.11152	7.43202	6.53933	8.44879	6.75255	9.52321	5.65836	6.57105
9.554	6.96495	9.97953	9.11587	7.46974	9.20809	6.30623	9.30795	9.57838	7.02036
5.90198	7.28968	8.73938	6.21479	8.55203	9.28329	6.29008	5.15888	8.02672	7.32017
9.56169	9.6815	8.74877	5.26943	6.62279	7.01126	6.66614	6.16267	7.01844	7.79521
7.74022	6.33242	9.29239	6.29484	5.76628	9.56238	8.53247	5.87328	7.33305	5.90003
7.94114	8.28761	8.31355	7.24938	5.22979	5.48585	5.15516	8.16651	6.85857	8.33192
5.00681	9.0607	6.82394	8.66481	6.99464	7.13776	7.79786	7.52859	9.36286	5.62292
9.41515	7.35261	8.15893	9.64423	6.52943	5.29951	8.9449	5.88656	5.92009	8.80299
7.68255	8.11154	5.51103	8.84758	8.84668	5.1523	8.89712	8.69061	8.56185	8.68377

9.16417	7.00771	5.0861	5.06758	5.69542	9.75588	7.57315	9.75201	6.54216	7.16394
6.58181	5.37791	8.30916	6.1962	8.55578	9.70515	6.12634	9.11282	9.50384	8.09805
6.88542	9.96593	9.59138	8.58082	9.87487	5.59857	5.64299	9.57104	7.14321	7.11006
7.05921	6.86049	8.2182	6.85334	8.45328	5.07638	8.66818	9.81014	9.2333	5.44467
9.348	9.26233	5.08559	6.75299	6.25372	8.38338	6.14941	9.83879	5.6177	6.8766
6.15159	8.62186	5.55241	6.67253	8.33838	9.27209	9.85426	5.53322	8.97406	7.55892
5.36697	7.70981	7.08366	6.84166	9.12528	7.79355	5.44239	9.11704	7.02016	5.17327
5.67579	9.12153	5.957	6.52146	7.12082	8.13858	5.76206	8.2026	9.90825	6.42041
8.26166	5.19902	9.58008	7.51785	7.92083	9.45899	7.30978	9.3532	8.51319	9.97358
6.9554	7.96893	6.71263	5.93681	7.52358	9.4103	6.44313	5.71085	6.03751	7.08
9.82566	7.733	7.61559	8.76653	6.35292	9.91858	6.72181	6.90514	5.38461	8.02552
9.72762	6.63478	9.24999	6.13954	7.73999	8.36853	8.74962	6.49554	9.98583	8.80602
8.043	9.03275	5.35129	7.17005	7.29054	8.46986	6.81885	8.00299	5.51275	9.64817
6.92699	6.98176	6.86259	8.46919	5.85317	7.35454	9.39212	8.71494	5.65092	5.71094
9.93362	5.78553	6.32483	8.88344	9.8555	5.20436	6.41424	8.26311	8.21389	9.79654
7.76587	5.04796	8.08696	5.56381	9.6022	8.87437	6.83705	8.4564	7.94055	9.94701
6.59597	5.90034	5.60437	5.94579	9.24471	6.03496	7.86729	5.6491	8.40195	8.91079
5.94233	5.76951	5.31754	9.59859	7.35535	7.45189	8.1489	8.11002	5.15887	5.3046
7.61879	6.59192	9.35701	6.82095	6.54584	9.11542	7.05358	8.74775	7.70798	7.67933
6.145	9.31874	7.06257	8.60142	7.06093	8.81227	9.15257	8.27367	6.69281	6.72126
8.71196	9.84293	6.83098	6.25536	6.61502	9.38405	8.17532	8.17673	9.06596	6.21218
9.51708	7.98002	9.44205	5.99122	6.21299	5.01264	8.36691	7.75685	7.50646	7.32819
6.61961	7.94988	5.00158	8.55421	7.88629	7.11178	8.58084	6.90012	5.48261	7.49717
9.17986	5.09501	8.62308	6.22316	6.98922	6.48201	7.70858	5.7019	5.69332	6.62124
9.49229	5.3171	9.72923	6.56501	9.68348	5.50995	5.32583	5.07636	9.15882	9.05931
9.90961	6.35248	7.7217	5.64547	7.99413	5.77287	7.50229	6.77848	5.30039	6.16587
7.8319418	7.4174899	7.6170964	7.3924143	7.3113334	7.6170689	7.3575578	7.4658598	7.4299821	7.4715701
5.00681	5.04796	5.00158	5.06758	5.0122	5.01264	5.00736	5.006	5.14162	5.16463
9.93362	9.97844	9.97953	9.86566	9.87487	9.91858	9.99042	9.99247	9.9937	9.97358
8.0456	7.223095	7.663885	6.98252	7.212925	7.807845	7.18454	7.5376	7.42697	7.42864
8.03305	7.24288	7.663885	6.98252	7.212925	7.807845	7.05644	7.65173	7.47654	7.452805
4.9568781	4.9984396	4.9515958	5.0182558	4.962322	4.9627664	4.9574336	4.95606	5.0930362	5.1162763
3.29562	7.82244	7.93253	-3.56834	-2.63813	1.77658	9.03242	9.23947	9.3637	7.33158
5.6638836	4.8349798	5.2341928	4.7848286	4.6226668	5.2341378	4.7151156	4.9317196	4.8599642	4.9431402
6.0661	4.48576	5.32777	3.96504	4.42585	5.61569	4.11288	5.30346	4.95308	4.90561
6.0912	4.44619	5.32777	3.96504	4.42585	5.61569	4.36908	5.0752	4.85394	4.85728
9.9829562	10.0282244	10.0293253	9.9143166	9.9236187	9.9677658	10.0403242	10.0423947	10.043637	10.0233158
5.68781	9.84396	5.15958	11.82558	6.2322	6.27664	5.74336	5.606	19.30362	21.62763
10.6638836	9.8349798	10.2341928	9.7848286	9.6226668	10.2341378	9.7151156	9.9317196	9.8599642	9.9431402
11.0661	9.48576	10.32777	8.96504	9.42585	10.61569	9.11288	10.30346	9.95308	9.90561
11.0912	9.44619	10.32777	8.96504	9.42585	10.61569	9.36908	10.0752	9.85394	9.85728
4.819177751	4.861874071	4.81375122	4.882231344	4.824770295	4.825226829	4.819748419	4.818337313	4.959053591	4.982928253
5.005545674	5.046706093	5.00031435	5.066331061	5.010937039	5.01137715	5.006095813	5.004735469	5.140389809	5.163405635
9.934869242	9.979700591	9.980790867	9.866892034	9.876104366	9.919825434	9.991683625	9.993734144	9.994964455	9.974839361
10.11901375	10.16551798	10.16664894	10.04849997	10.05805606	10.10340858	10.17794816	10.18007519	10.18135141	10.16047535

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 31	Sample 32	Sample 33	Sample 34	Sample 35	Sample 36	Sample 37	Sample 38	Sample 39	Sample 40
7.50035	5.24713	9.8954	8.80255	8.8218	6.59927	9.76463	6.64735	9.00522	8.29796
9.39137	9.34341	9.58363	6.57885	7.122	5.22716	9.74086	5.31393	7.85267	9.30098
5.10678	8.88935	6.40569	8.50099	6.08443	9.05548	5.31031	5.81083	8.48287	8.96436

6.93517	6.03544	5.52368	5.7983	7.48132	7.5783	8.14952	7.92606	9.73019	7.86092
8.43921	8.46033	5.95257	7.20855	9.91838	7.85581	7.56823	6.4932	5.23424	9.52892
8.47548	9.28135	7.33526	6.60288	6.83983	9.78701	7.11568	9.57081	9.11385	6.90428
8.6267	8.89018	5.46819	9.79767	7.40833	8.58843	5.92564	7.16664	8.66639	6.22338
6.16086	6.53802	5.32322	6.88292	8.2209	9.24749	5.9092	6.47112	8.98785	8.95951
6.34905	8.34066	5.07714	7.48032	8.28914	9.1217	7.68114	9.83668	9.82768	9.0615
6.36305	9.86537	5.17876	7.57962	5.06465	9.04939	9.92067	5.20862	7.54751	9.08712
5.26247	7.6131	9.12762	9.04396	6.90057	5.38525	6.12297	5.275	5.05169	7.04878
9.43292	9.65217	9.90705	6.54693	6.74766	7.84515	6.64887	5.14738	5.68784	7.15085
5.10103	8.12979	8.61176	7.15245	7.79231	7.44	5.01203	6.88381	9.1343	8.0252
8.36157	5.95266	6.90353	7.47758	5.73901	5.96571	5.75286	7.6891	5.99593	5.8456
6.91612	8.41876	8.81934	8.39008	6.29432	8.80102	7.25997	9.86909	8.03123	9.21003
7.31561	9.99863	8.20564	6.68536	6.72675	9.26345	7.23124	7.84323	7.41001	5.29254
7.88559	5.52625	9.87828	5.73659	7.30404	5.73118	6.21262	9.99663	9.97311	9.6172
8.75016	5.59702	8.89966	7.08961	7.71737	9.8482	6.99923	8.34049	6.04299	9.34101
5.46755	7.50067	7.87756	6.4981	5.93183	7.43826	8.91837	9.84959	5.44741	6.37042
8.78039	5.12196	5.25938	6.31369	5.18612	7.30355	8.7283	9.75394	9.80027	9.02646
8.9755	6.3974	9.40572	8.536	8.01396	6.13468	7.41033	5.45015	6.90211	7.75888
9.52359	8.49771	8.20356	5.22742	5.96029	7.67899	5.15104	6.18517	9.49309	7.19216
8.19882	6.89299	6.7108	8.98305	5.76676	6.04506	9.14157	5.87919	9.42711	5.82084
6.98004	9.3487	8.95231	8.31513	6.53649	9.72419	7.06336	9.17046	8.38205	9.94284
5.61322	8.9436	6.26396	6.98272	7.14582	5.89077	7.97074	8.60369	9.18188	5.06491
9.79137	5.79373	5.07575	6.28582	8.05085	7.78088	5.45468	5.49343	5.83014	8.67372
6.22033	6.80948	5.91316	8.39016	7.26063	7.66103	9.6192	7.43953	9.73846	7.72983
9.39838	8.97061	9.88289	7.07068	5.07646	5.07269	8.97085	5.81337	7.77438	9.63095
7.39763	9.88705	6.60971	7.49315	8.75298	6.87313	7.52418	5.70439	5.93907	5.13677
6.95253	7.52177	6.72597	9.05985	6.7729	9.3604	9.25821	5.07968	8.20215	7.81762
8.75217	5.73752	8.77873	5.80632	9.00841	5.34608	5.63368	6.90172	9.47084	9.39468
5.34833	6.46865	6.24724	5.57895	5.10476	7.38875	6.62421	6.46137	9.82975	6.13191
8.502	9.21032	8.34483	9.87609	7.86116	6.27031	8.40452	8.82969	8.37235	8.57947
5.61407	6.29874	5.7808	6.74338	7.70448	5.18031	5.63428	6.31334	9.30713	6.57546
7.18887	6.50371	9.72908	5.75651	6.47396	6.80489	9.56272	6.96724	7.1687	6.33079
8.08008	7.1872	6.66952	9.22244	6.24408	7.25462	5.54531	6.24603	8.92379	6.49622
9.39534	8.63983	6.09715	5.44615	5.95364	8.11025	7.1534	7.50291	9.77039	7.03312
7.36883	5.60201	6.13864	6.41581	7.9705	8.26183	5.86545	8.36105	6.44638	8.18014
5.03076	6.5941	7.11916	7.6769	7.25937	6.27984	9.9761	9.44285	6.98838	5.64943
7.48446	6.41826	6.8072	6.57661	5.74379	9.63576	5.07302	9.98667	6.14888	6.64932
6.60067	5.36763	9.22689	6.26972	7.76929	8.41126	9.79231	5.16177	7.73309	6.69538
9.87631	6.52021	6.68256	9.3851	7.88302	6.40151	8.59067	6.01476	9.065	5.3806
6.93997	5.16642	6.81449	5.61354	8.11308	7.27037	9.49548	9.7027	9.46306	9.92547
8.97711	6.75733	6.65952	8.77258	7.10883	6.69459	9.8485	9.17647	6.92013	8.2229
9.01033	8.63543	9.31001	9.80727	6.28439	8.81035	6.22439	8.46489	8.44683	7.22277
6.95119	7.676	7.10224	6.56012	9.11109	7.5507	8.46004	7.77055	7.19512	6.10208
8.27998	8.28741	9.90079	7.56961	9.82754	7.57486	5.40428	5.63512	5.67407	5.21846
7.52858	9.9158	5.50121	9.11535	5.63298	7.76186	8.69459	6.93366	7.44959	5.25495
6.92018	7.95367	8.1118	7.62766	8.54468	6.59354	6.16161	5.60092	8.87817	7.11485
7.698	5.39568	6.82866	7.16574	7.262	5.77043	7.10444	9.56848	7.29469	6.63726
5.49964	6.30695	7.14232	6.97931	7.68499	7.56297	5.24927	6.16807	8.19343	9.20254
5.80615	7.97755	6.29881	6.7675	7.38663	9.62251	5.33776	6.10377	9.51476	5.79857
8.42811	5.91292	5.96541	7.37976	6.52555	5.27592	9.75478	6.4022	7.16116	6.77476
9.40245	6.98585	5.50676	5.32682	9.12917	6.7105	8.67354	9.2487	5.61845	9.51702

5.10785	9.54979	6.37485	7.34524	5.01416	5.06697	5.22	6.52407	9.78722	6.35788
5.79286	7.63631	7.14124	9.19054	5.44655	9.55481	8.31342	8.24977	8.58574	8.7989
6.00535	6.79061	6.69654	8.05914	8.89574	8.87154	5.66521	7.83562	5.21879	7.30689
8.30135	7.94624	8.25281	5.27925	9.90575	8.61063	8.65035	8.92262	7.94457	6.81333
5.8929	9.94946	6.47397	5.687	9.5043	6.8656	9.74509	5.18851	7.28214	8.57533
5.12307	9.03214	6.56241	8.694	7.54733	9.32573	7.08729	5.54121	5.28525	7.85561
9.09673	9.20852	5.63572	8.50909	5.74765	6.94556	7.71247	9.27144	8.69491	6.41712
7.09896	7.1381	9.65995	7.07506	6.56871	7.11553	7.68939	6.15319	5.38714	9.03766
6.70307	7.81267	7.3581	7.23356	7.5603	9.30252	6.37883	7.29669	9.64474	6.46283
5.74119	9.65478	9.60155	7.77059	5.55502	7.4182	6.27061	8.99578	5.59539	8.09088
5.63535	7.51114	7.80895	9.50728	6.28914	7.02888	8.29441	9.4568	9.36215	6.2991
8.14417	9.07637	7.75014	8.67529	7.9587	5.98642	8.26501	7.13449	9.49481	9.57277
5.0483	7.24265	8.55692	9.59615	7.70646	7.28453	5.34746	5.0828	7.90193	9.21734
9.04833	8.72591	7.5159	8.79805	8.30448	7.46856	6.83016	6.2488	7.98167	8.42516
5.49696	7.65268	9.10518	5.49842	6.38216	7.12359	5.64852	6.74658	5.78611	8.29595
6.90834	5.12963	6.64125	6.34092	7.73763	7.05909	8.96289	5.39704	8.93044	9.99625
7.72702	7.28399	8.16153	9.91849	9.47267	6.17608	5.08349	6.52262	5.57163	8.30925
9.48972	7.25995	7.32795	6.41024	6.46112	6.35974	6.09765	7.58295	7.9226	8.25202
8.15677	7.71819	7.66333	6.31399	8.45629	6.38887	6.11225	5.10857	8.51196	8.93596
9.43355	8.2281	5.77918	5.34246	7.68658	7.09204	7.95314	7.36108	8.06914	9.01574
8.57609	6.20476	9.18717	5.87335	9.88776	7.97843	8.792	7.35794	9.63873	8.24092
6.65405	8.65733	7.66848	7.85896	9.91895	9.34843	6.69237	9.96433	7.56576	9.12607
9.53108	8.47913	5.83892	6.07332	5.35934	5.71528	9.78231	8.48638	7.70619	9.43349
7.90557	7.61717	7.67768	8.91528	8.44884	6.30982	7.49963	7.61643	9.52309	5.12784
8.60646	9.79069	9.70913	5.49767	7.58413	6.0019	8.24264	9.08876	9.14056	6.11842
6.98536	5.44063	7.7044	8.87764	7.12364	8.91802	7.78517	8.29459	5.30002	7.69069
6.0887	6.75514	9.3903	5.93204	5.18948	9.99587	5.19588	6.36054	6.35852	7.12462
5.1886	5.64261	5.80165	8.53785	9.71369	9.85396	9.6512	5.81206	5.97774	6.56621
6.31121	6.67925	9.34305	5.37089	7.89786	5.99516	5.58288	8.43911	9.35734	7.08135
6.392	6.07415	6.96554	5.02307	5.53499	5.89981	9.12609	5.07862	8.29857	9.68776
7.7541	8.46677	6.76784	9.39449	6.22619	8.45301	9.12479	8.46252	6.7461	8.73015
6.48263	9.76504	5.36303	6.56167	6.16899	9.06327	7.78414	7.43133	8.82988	7.50803
6.22499	7.84079	7.9608	5.02901	6.94961	8.33897	5.37405	9.62391	9.31231	8.01537
6.63061	6.31841	9.77621	5.84552	7.26944	7.84017	8.09963	8.85667	9.23926	6.73126
9.79488	6.1229	9.91444	5.83367	8.80545	6.28899	5.90027	7.79591	6.09259	8.51372
8.85675	6.81318	7.85959	6.27168	9.31587	7.51534	9.88815	9.76111	9.39197	9.90941
8.4955	7.61686	5.12982	9.27654	5.41725	9.66808	9.84191	5.55896	8.4537	8.91896
5.83202	7.30084	8.99787	5.61518	5.8903	8.57491	9.3326	8.4068	7.92917	7.74168
7.2049	7.77536	7.74656	9.77974	7.71381	6.25353	9.5249	9.76764	9.629	9.91017
6.62744	7.4541	6.16705	8.05253	8.89099	5.8907	8.08662	8.02769	7.79993	7.12726
7.70593	9.29906	6.04552	9.03362	6.75863	8.08085	7.18897	8.7436	6.15168	7.0943
5.36637	8.00875	8.70935	5.40784	6.81813	9.39201	7.8435	7.44429	8.12506	8.24254
6.95076	8.39484	5.6508	7.47934	5.93032	5.95169	7.09552	8.56172	9.65427	8.64521
8.6586	7.77499	8.46304	9.12469	6.87499	5.61357	7.33172	8.18191	7.13528	5.85543
6.30913	7.79716	5.62634	6.97933	8.79659	5.61594	7.56762	6.6486	5.80945	7.8774
7.26166	7.84167	9.04309	5.98448	5.08924	7.77815	5.48356	8.0683	7.06436	8.12468
7.3470365	7.5746329	7.4431615	7.3082438	7.2629051	7.4527839	7.4694661	7.4333784	7.8811669	7.731526
5.03076	5.12196	5.07575	5.02307	5.01416	5.06697	5.01203	5.07862	5.05169	5.06491
9.87631	9.99863	9.91444	9.91849	9.91895	9.99587	9.9761	9.99663	9.97311	9.99625
7.23328	7.62674	7.235135	7.12103	7.261315	7.42823	7.511905	7.43543	8.050185	7.858265
7.196885	7.62674	7.14178	7.12103	7.261315	7.42823	7.511905	7.43543	8.050185	7.836615

4.9810676	5.0731796	5.0265075	4.9733007	4.9643016	5.0176397	4.9621503	5.0294062	5.0022069	5.0155591
-2.49269	9.86163	1.35844	1.76749	1.81395	9.58287	7.5861	9.65963	7.28411	9.62125
4.694073	5.1492658	4.886323	4.6164876	4.5258102	4.9055678	4.9389322	4.8667568	5.7623338	5.463052
4.39377	5.25348	4.28356	4.24206	4.52263	4.85646	5.02381	4.87086	6.10037	5.67323
4.46656	5.25348	4.47027	4.24206	4.52263	4.85646	5.02381	4.87086	6.10037	5.71653
9.9250731	10.0486163	9.9635844	9.9676749	9.9681395	10.0458287	10.025861	10.0465963	10.0228411	10.0462125
8.10676	17.31796	12.65075	7.33007	6.43016	11.76397	6.21503	12.94062	10.22069	11.55591
9.694073	10.1492658	9.886323	9.6164876	9.5258102	9.9055678	9.9389322	9.8667568	10.7623338	10.463052
9.39377	10.25348	9.28356	9.24206	9.52263	9.85646	10.02381	9.87086	11.10037	10.67323
9.46656	10.25348	9.47027	9.24206	9.52263	9.85646	10.02381	9.87086	11.10037	10.71653
4.844027735	4.938654815	4.890708353	4.836048763	4.826803947	4.881598421	4.824593906	4.893686201	4.865744235	4.879461011
5.029501738	5.120724831	5.07450313	5.021809791	5.012897535	5.065720907	5.010766996	5.077373857	5.050437038	5.063660385
9.877544731	9.999895704	9.915684386	9.919735411	9.920195528	9.997135005	9.977359999	9.997895197	9.974369242	9.997515101
10.05955018	10.18646667	10.09911301	10.1033152	10.10379249	10.18360296	10.16309005	10.18439152	10.15998769	10.18399724

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 41	Sample 42	Sample 43	Sample 44	Sample 45	Sample 46	Sample 47	Sample 48	Sample 49	Sample 50
7.43951	8.24635	6.9857	9.16292	6.89561	7.05886	7.51524	6.7313	5.95412	6.07004
7.57102	6.56106	8.23664	7.00731	5.59381	8.01754	7.0963	7.57462	9.82414	6.72801
6.77325	5.08852	7.98663	9.61054	8.49726	6.19844	9.21696	8.84272	5.92339	7.96327
9.79135	8.41533	5.04852	9.38471	6.89829	9.90602	6.06974	5.90194	6.96043	5.49204
6.57138	5.62135	6.74215	8.10656	5.95605	9.29961	5.79917	7.64465	7.42631	8.85193
9.09014	5.55481	6.01842	7.29108	6.05688	5.97353	8.62974	6.39139	6.2272	5.3403
5.51461	8.87642	5.60244	5.28958	5.46722	6.99885	5.37584	6.79277	6.13681	8.23701
7.8468	8.89368	7.24821	8.91456	8.31677	7.62211	6.27116	9.86692	5.34224	5.56701
8.6792	6.3491	5.62216	5.37401	6.01975	5.19198	8.29397	9.70763	5.75499	6.55636
6.87539	7.61707	9.59302	8.56879	8.14354	7.35214	8.67519	8.25598	5.13631	9.40228
6.18459	5.94701	5.5542	5.79957	8.90951	7.75874	5.79837	6.51989	9.80317	8.57684
5.23315	7.54473	8.16906	9.75204	8.43854	5.53767	7.69683	8.41953	8.12961	5.312
8.90708	7.6082	8.72935	6.24769	9.70217	8.3059	5.17356	7.97678	6.79428	5.3863
5.83295	9.99723	9.49626	7.50843	9.83234	9.10314	6.51401	6.45117	7.75835	9.38284
9.32006	9.63741	5.70256	8.53486	7.29334	7.78092	8.35213	9.38985	5.96788	7.80611
9.3104	8.40734	7.62382	8.57436	7.67231	5.87987	5.76888	8.0549	9.74297	5.70827
5.67176	7.42233	5.60536	9.96176	9.27612	6.77939	5.30697	7.36826	9.06778	8.97023
9.79551	7.14915	6.50241	9.04957	8.81878	7.91764	5.65608	5.25155	5.74336	8.56599
5.21052	5.88984	9.82599	5.37411	5.8502	8.07446	6.6818	8.20686	8.95578	8.86164
5.9953	8.86643	9.24647	6.23347	8.56669	9.12914	5.33939	7.31624	7.48825	5.50116
6.65791	6.78691	6.25997	6.60135	7.94366	7.50505	7.18998	7.8403	8.35399	9.19841
8.56078	8.90699	6.64405	9.17714	7.94625	7.57455	7.17983	8.69869	6.37955	9.5397
6.73062	5.20994	9.31602	9.05614	9.44758	9.03226	8.52391	7.94971	6.63326	5.89869
9.41529	8.17317	7.07624	5.47399	7.36347	6.31873	8.94606	6.31982	9.9166	9.01191
6.07554	9.62733	9.58128	6.70003	8.14372	8.76728	8.5526	5.69285	6.89756	9.84497
8.01566	8.71194	6.23515	6.0835	9.74107	6.5938	5.55883	6.67754	5.19569	6.8125
6.83475	9.07548	6.13443	5.9505	9.18959	5.89701	5.87925	6.99587	7.23445	8.32718
7.9589	7.65341	7.72308	5.95331	7.29536	7.89416	9.46205	6.49586	8.3046	5.76099
8.51384	5.63138	8.96095	9.78814	9.09645	6.34093	6.77067	7.38832	7.46259	8.61816
8.92311	7.69951	6.51392	5.03922	5.66252	9.50021	9.15549	9.35	8.93735	7.82585
5.1192	6.56533	9.03573	6.73994	6.30135	6.24742	8.20792	7.84861	9.00198	7.86626
9.4041	5.73872	5.11939	7.01022	8.02802	7.16684	7.16019	8.01223	8.73716	7.42041
9.94098	9.7951	6.69433	7.07338	8.9592	8.07521	6.66061	6.96828	7.05565	5.14813
5.90571	7.03409	5.65142	8.92985	7.9998	8.93696	7.60167	5.83995	7.42758	9.59906

5.38793	9.25165	5.04133	5.51345	8.69423	5.53007	7.197	5.49402	5.13618	7.88444
9.07852	5.97285	9.82933	5.29632	6.7193	8.02162	9.6299	6.24232	9.40428	6.01784
9.75015	8.81171	5.31653	6.94139	7.83926	8.14834	5.57836	9.22661	5.47605	9.75275
6.76399	9.1946	9.32238	8.47283	5.35729	5.60662	7.7814	8.73219	7.61303	7.46602
6.02291	5.11834	5.91528	9.12003	8.71052	9.90778	9.39268	5.95808	6.63507	6.41813
5.25138	8.81288	5.9573	5.49332	6.87346	5.64291	5.07731	5.60835	9.47975	7.35195
6.25045	7.1705	7.26168	6.15644	5.92243	8.75308	6.50225	5.93999	7.38841	5.44004
7.33451	6.53746	9.63119	9.74086	6.03286	5.42392	8.05853	7.09627	8.34702	8.67129
9.20082	7.83462	9.31918	9.26425	7.06266	8.93879	9.39374	9.63792	6.30204	8.00283
9.97497	7.09639	7.92596	5.88119	6.85013	6.01892	6.99948	9.70543	7.45196	8.67672
9.53287	9.41457	9.11055	8.86196	5.03096	6.46716	5.10722	7.29534	8.3244	7.7181
5.84091	7.65869	5.44099	9.90768	7.47179	9.37263	6.66334	7.57088	7.95822	7.39396
9.18119	7.21974	8.51952	5.89365	5.88786	6.52954	6.95884	5.76143	6.84519	9.2574
6.96248	6.3832	9.70854	5.70285	5.24924	9.15932	7.35008	9.51421	5.76095	7.19165
5.24717	6.48257	6.62552	8.78999	6.20273	5.52388	7.71603	5.84104	9.59808	6.978
6.41804	9.61701	9.65573	6.44594	5.06977	8.21815	7.5542	5.66472	7.65665	8.02003
5.66935	8.12661	7.20189	6.86661	6.76592	6.13275	8.78086	5.73257	6.02749	9.12578
7.47293	5.9308	8.20636	5.09093	5.0075	7.94199	5.88725	5.21135	9.67117	9.2848
6.57728	6.9568	9.94126	7.16476	9.93408	9.88816	5.27527	6.09702	9.18018	6.79695
5.11535	7.28443	5.64122	9.61041	5.81708	9.63919	9.11772	8.69824	8.41551	6.03511
9.5647	6.64926	6.33472	9.07005	8.39516	8.86788	7.67557	8.43853	9.04269	8.1958
5.35054	6.61546	8.239	9.00267	6.06365	9.02672	8.31096	9.10896	7.50541	8.59496
9.34019	8.65791	6.64103	9.51511	6.48266	7.0065	5.40455	7.42353	9.82435	7.5418
6.95715	6.33275	9.39788	8.53616	7.71581	6.27857	6.87064	7.22672	8.38683	9.87279
8.555	9.39175	9.05629	9.31881	6.08377	8.03252	8.79059	8.26784	9.44529	6.7277
5.51522	8.65433	6.59797	7.79202	5.82398	7.00401	6.32255	6.61919	7.32542	7.63007
6.98757	5.39166	6.97215	6.52214	9.80779	8.902	8.36381	7.18559	8.63179	7.93261
8.72711	6.21024	7.63766	8.87125	8.26993	9.9263	7.644	7.96486	5.08055	6.36137
9.9309	5.12694	6.95474	6.35751	9.14504	6.60382	6.32103	5.89882	5.65283	7.62126
7.13662	9.22069	8.40073	8.81758	8.9091	6.74108	5.85272	5.13041	9.11331	5.09419
8.51499	9.92393	8.20698	8.82897	8.11063	6.18418	8.03629	9.6953	9.97321	9.41404
6.08532	9.06559	5.81364	6.48222	9.83561	7.95791	5.81131	9.81686	5.28384	5.26181
7.9058	7.03432	5.36941	6.31882	6.13581	6.9675	6.07276	5.78281	8.53467	9.8859
8.96789	7.67053	8.83847	7.64882	8.6043	9.35202	8.08925	6.17718	5.58029	6.45991
8.22032	9.57666	8.98342	5.79114	7.02505	9.10283	5.18413	5.70371	6.85401	6.72302
6.27195	8.22608	7.36932	5.56267	6.39519	6.76235	5.13912	5.18073	6.08412	8.43374
9.96244	7.96592	5.40733	5.57619	6.68363	6.26084	7.68457	5.04036	9.17757	8.13467
8.28219	6.41225	7.00406	9.32312	6.84285	9.86887	8.0585	7.24348	9.72408	7.39015
5.88895	5.03994	6.60156	6.67636	6.82369	5.47815	7.0661	9.36929	6.5768	5.37066
7.34311	9.75218	8.88316	5.26618	8.6049	6.27582	5.14785	8.36003	5.03886	8.05789
8.97547	9.71587	9.3962	9.65488	5.55818	6.35421	7.37417	8.68283	6.32769	8.28047
5.08764	8.62187	7.78397	5.70435	8.50117	8.23416	6.96361	8.50785	6.34794	5.05514
8.34194	8.76304	9.21736	9.33399	9.68556	6.18819	6.93515	5.56871	8.41116	7.58874
7.5373	7.68996	6.42551	8.80661	7.07111	5.92674	9.76143	8.65908	9.61686	7.11489
7.94285	5.71637	7.79367	5.27705	9.51942	5.86369	5.65145	6.85014	8.1267	5.81606
8.05711	7.86758	8.41685	7.60925	8.59745	5.57227	5.42199	9.42663	6.74462	9.74792
9.82766	7.72996	8.79661	7.00486	5.18294	8.47841	7.12482	9.22974	7.83256	5.55829
8.09716	7.31975	7.75975	7.13936	8.29423	9.9513	5.81243	7.28782	5.23318	9.08026
8.61072	9.23133	5.7962	8.28836	5.00868	5.38611	7.28355	7.09029	5.32676	6.14384
7.20674	7.78688	5.68589	9.00133	9.56598	7.97488	7.78226	5.05105	5.1623	9.97439
7.7513	7.95667	5.4114	8.85065	7.28274	5.19199	5.75819	7.66748	5.3924	9.95346

9.40413	8.94034	5.5852	9.99187	5.37086	6.81897	8.73733	7.86887	9.95678	9.62016
5.77088	5.53858	6.27369	6.48469	6.09167	5.50229	9.99484	5.17116	5.11626	7.69289
5.23131	5.44623	6.08715	6.07144	9.142	5.57742	7.46513	5.84182	9.05438	9.0399
9.99467	9.89436	9.15455	8.05391	8.86884	9.12612	7.9147	9.11843	9.85325	9.4755
9.19098	9.97195	8.06076	9.69721	5.42229	8.01647	9.3464	5.48021	9.51128	6.28591
7.49843	5.13288	9.63548	6.36749	5.84569	7.47162	8.90698	6.34547	7.9542	9.87033
7.50273	7.78533	5.61616	7.97225	9.77321	6.64285	6.17817	8.69537	5.60874	9.81662
5.90043	9.26598	6.68498	7.95444	8.23315	5.50794	8.61171	5.0466	5.76493	9.56114
6.11802	7.16928	7.13987	6.7704	6.07435	6.71233	6.38886	9.68132	6.73007	9.03472
7.25812	6.67925	7.27082	6.39514	8.22222	8.51572	8.83161	9.69489	5.34264	8.60396
8.34532	5.19177	6.70078	8.97641	8.99011	7.90163	8.92595	5.87858	8.58469	6.57678
7.4234	5.35588	7.41259	7.92591	7.30127	9.71997	6.73075	7.7946	5.51657	5.25574
5.36923	8.01188	7.1629	6.66586	5.49855	6.34009	7.96571	5.30967	5.05268	6.83756
6.10584	7.90877	8.89931	8.93062	5.34218	7.85024	8.8389	9.78126	9.30543	8.08921
8.46396	8.70763	9.60679	8.30677	6.70285	9.51303	8.00947	9.86259	6.94866	8.39986
7.5122486	7.5949586	7.4454103	7.5605038	7.4072754	7.4553969	7.2466371	7.3699762	7.4200566	7.6781372
5.08764	5.03994	5.04133	5.03922	5.0075	5.19198	5.07731	5.04036	5.03886	5.05514
9.99467	9.99723	9.94126	9.99187	9.93408	9.9513	9.99484	9.86692	9.97321	9.97439
7.48568	7.680245	7.254945	7.55884	7.298315	7.488335	7.19349	7.30579	7.426945	7.846055
7.48568	7.66461	7.254945	7.399755	7.33237	7.488335	7.184905	7.30579	7.43977	7.846055
5.0385164	4.9903394	4.9917433	4.9896122	4.957575	5.1438998	5.0280831	4.9907636	4.9892486	5.0056914
9.46167	9.72023	4.06726	9.17887	3.34208	5.0813	9.47884	-3.44108	7.29421	7.41339
5.0244972	5.1899172	4.8908206	5.1210076	4.8145508	4.9107938	4.4932742	4.7399524	4.8401132	5.3562744
4.97136	5.32922	4.50989	4.79951	4.66474	4.97667	4.36981	4.61158	4.87954	5.69211
4.97136	5.36049	4.50989	5.11768	4.59663	4.97667	4.38698	4.61158	4.85389	5.69211
10.0446167	10.0472023	9.9906726	10.0417887	9.9834208	10.000813	10.0447884	9.9155892	10.0229421	10.0241339
13.85164	9.03394	9.17433	8.96122	5.7575	24.38998	12.80831	9.07636	8.92486	10.56914
10.0244972	10.1899172	9.8908206	10.1210076	9.8145508	9.9107938	9.4932742	9.7399524	9.8401132	10.3562744
9.97136	10.32922	9.50989	9.79951	9.66474	9.97667	9.36981	9.61158	9.87954	10.69211
9.97136	10.36049	9.50989	10.11768	9.59663	9.97667	9.38698	9.61158	9.85389	10.69211
4.903045151	4.853552698	4.854994931	4.852805642	4.81989368	5.011306001	4.892326974	4.853988481	4.852432114	4.869323878
5.086396141	5.038684063	5.040074415	5.03796388	5.006235849	5.190762561	5.076063525	5.039104169	5.037603789	5.053887912
9.995934701	9.998495349	9.942511177	9.993133992	9.935329359	9.952553719	9.996104744	9.868152353	9.974469267	9.975649566
10.18235786	10.18501406	10.12694084	10.17945265	10.11949104	10.13735812	10.18253425	10.04980732	10.16009145	10.16131579

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 51	Sample 52	Sample 53	Sample 54	Sample 55	Sample 56	Sample 57	Sample 58	Sample 59	Sample 60
6.18441	9.77813	8.39031	9.65477	5.86795	8.38988	6.12309	8.57511	7.95065	5.16081
7.52938	8.11911	5.41477	5.11005	8.66089	8.90848	7.80516	8.0271	8.08311	9.85174
5.7298	7.77614	5.75094	7.25955	8.35071	6.34104	7.45682	8.94139	7.65799	8.71809
5.65407	6.21417	7.48042	9.61853	6.2657	5.50628	5.15738	8.23405	5.0672	7.01167
6.5132	7.78595	9.72915	9.85962	9.95137	7.0434	5.32205	6.69915	9.42174	7.80554
9.15284	8.32317	7.41974	6.26129	8.59548	8.90681	8.82022	5.16808	5.96558	6.9585
6.89479	6.23736	7.28764	8.06862	7.65048	8.65787	7.43745	8.99241	5.19813	7.17669
6.18415	5.26976	5.31304	8.36516	7.54241	8.16157	8.14659	5.7749	5.26771	5.08625
5.09403	8.29187	5.5948	6.40877	6.96558	9.45925	7.6921	7.50975	9.2622	9.73171
5.33161	8.53094	8.40545	7.06276	8.58653	8.99499	8.0419	8.66307	7.2801	7.30383
9.7353	8.62911	6.89256	8.69667	6.03224	6.58653	9.93124	5.46704	9.76989	8.63773
6.15844	9.44765	9.463	8.25846	6.04034	7.40368	6.68005	5.10564	6.69407	6.60476
8.6694	8.23546	7.00588	9.23245	7.61833	5.96345	6.88131	7.93937	7.49138	9.76985
5.11945	7.36746	6.33881	9.8266	8.858	9.70288	6.76729	9.0635	9.56054	7.13157

6.23517	8.16178	7.25906	6.74165	9.75388	6.12064	5.58344	6.87522	7.58093	8.05827
7.459	9.85782	5.42105	7.91471	5.29669	7.81313	5.53362	6.29903	7.54815	5.36101
6.15282	6.12379	8.10728	9.70583	9.26756	6.41615	9.18958	7.03666	7.1987	5.93602
9.04239	7.09456	6.04791	5.88338	9.59331	6.60862	6.76693	5.99125	6.27664	7.29312
9.03538	9.69882	9.82677	5.72132	9.03176	9.57339	6.97337	7.90027	7.77838	7.97781
7.6511	8.3023	8.18709	6.88171	5.95244	9.5369	5.28247	6.55677	7.28901	8.93189
9.09275	7.85247	9.18903	5.85384	5.91196	8.43459	6.64867	9.75891	6.79576	5.64979
9.75152	9.62568	8.47905	7.79395	9.8703	8.12302	8.84026	8.61556	5.74694	8.86874
9.68698	9.98707	7.59699	6.1366	8.92272	6.54508	6.90207	7.0073	5.50345	7.58442
9.05751	5.42718	7.37152	8.79267	6.81113	7.50593	8.33312	9.12354	7.27304	7.31634
8.91537	6.86456	8.01292	7.35785	7.20574	8.8904	5.75667	5.57835	8.59046	7.94288
9.97589	9.21938	9.62568	6.36609	6.08722	7.97244	5.71647	6.68945	6.11918	6.63354
8.27349	9.7458	5.75795	7.79634	7.13689	8.4753	7.53441	5.6029	8.30421	5.18881
9.71917	5.41594	9.31332	8.58671	6.83474	8.97424	9.89658	6.8172	8.73106	9.23008
7.60242	8.57215	9.75298	7.68112	7.80213	7.05231	7.74282	6.40577	9.84369	5.15754
5.68316	5.83958	5.47859	6.92632	5.58554	5.19551	7.44682	5.19873	8.94484	9.31304
9.71791	7.00554	8.3692	8.45708	7.74591	6.45849	9.84977	6.91658	9.29768	9.33689
9.46993	5.51622	8.4123	9.65682	8.2902	6.75682	7.54487	7.99848	8.70995	6.43359
7.20811	6.91121	5.49088	8.82913	9.56287	7.67468	5.95003	5.31318	6.39004	9.40255
7.47061	5.69742	9.38112	8.34972	7.04619	6.63872	9.08162	6.81247	9.54834	8.34652
9.60131	8.62992	8.85575	9.7735	9.68311	8.02502	9.32612	8.00922	7.39809	5.01968
8.03263	7.5373	7.12965	6.4923	8.59243	6.24727	7.88883	5.11067	8.45522	8.00539
8.25582	5.46626	8.6622	5.62291	6.38463	9.26127	8.33275	7.99774	7.79466	8.03671
9.25829	8.41607	9.73467	7.69665	7.40422	7.86851	9.63212	6.09523	6.48612	7.50582
9.37412	7.437	8.5678	8.45972	7.98068	5.0729	8.47921	5.10394	7.90408	5.00043
8.67304	6.79355	7.89504	6.77702	7.6832	9.479	7.09153	5.27925	8.31455	8.14435
6.91964	9.50101	9.64066	8.56392	9.18037	6.60794	9.17862	7.56436	9.43766	8.46242
5.44453	9.21792	9.83241	8.5164	6.45945	6.89076	5.71788	8.09432	6.92333	8.59856
9.65478	5.55052	9.23615	8.99227	8.73246	9.51937	7.25563	9.00115	6.81002	5.37905
8.48216	9.58579	5.0599	8.42569	7.91049	5.47587	5.77437	7.82368	9.80094	7.23265
7.84001	7.9974	7.89337	7.97349	9.97144	8.57502	9.02201	5.83154	9.124	9.02202
8.02645	7.43669	8.99841	5.82342	7.19501	6.65335	7.64623	9.47926	7.8075	5.01488
8.75322	9.70789	8.17372	8.88025	7.19618	9.69254	7.99807	5.4627	6.46246	9.15872
8.07078	5.08714	5.98375	8.73024	8.87049	6.2613	9.58124	8.68369	7.81575	8.32141
6.36179	7.86828	7.98099	5.4947	9.4247	9.97324	5.01158	9.41777	9.51792	6.86094
7.01405	5.14188	9.98821	7.14922	7.63093	7.99576	9.5162	8.58829	9.70551	5.38763
8.73944	6.71643	9.54226	7.91017	6.23884	8.38594	5.28207	7.3914	9.91407	5.34603
6.68406	9.69955	9.07839	6.20943	8.43589	8.60686	8.63083	5.0309	6.33025	7.08278
6.73343	5.52304	5.09884	6.18011	9.73911	9.54162	5.58246	7.42872	8.92022	7.4024
8.13158	8.06549	9.81985	5.53223	6.35963	6.95308	6.91183	7.69949	8.84785	9.97761
9.36368	9.57943	7.61263	8.48922	5.90058	9.02667	9.49098	8.6505	6.27251	9.92686
9.1309	5.43246	5.14575	5.94565	8.13965	5.32671	6.5524	5.95253	9.57085	9.51143
9.0303	9.8424	8.99437	5.88844	9.00089	5.33275	8.37706	9.71518	5.21969	6.87416
8.45743	8.78069	7.37784	6.54968	5.81084	7.72496	5.30164	7.65904	8.38205	8.47167
8.63807	7.21463	7.46972	7.21539	8.74623	7.28296	9.98552	7.16415	9.75178	5.55264
7.79227	5.4264	9.90541	8.2935	5.33786	9.49702	5.22564	8.38841	7.50006	9.59656
6.05257	9.60174	9.95031	9.3927	7.29248	5.68345	5.41758	6.73481	5.14385	9.47486
7.94833	6.82022	9.76159	6.24078	6.71651	9.72959	9.75539	8.28903	9.93662	8.88633
6.11134	5.43339	9.56345	5.74677	6.45726	9.8596	9.23968	5.89602	7.43733	8.05441
6.07547	5.69045	6.67957	9.81997	9.18793	8.37938	7.93922	8.69046	9.55115	6.03941
7.91549	8.96779	6.30639	5.58669	9.4096	8.4019	7.34804	8.62317	7.18481	7.02732

8.06379	8.24225	6.75746	9.62221	7.84568	7.45376	5.34074	6.12272	5.98859	5.00399
7.84529	8.52141	8.80603	7.27211	6.58523	6.54437	6.46756	6.22431	5.79068	6.99358
6.45022	7.44071	7.13344	6.08751	8.90721	5.83905	7.00844	6.18728	8.2809	8.33084
8.52234	6.98186	9.83986	7.46406	6.08123	8.96479	9.59466	8.76763	9.35811	7.36208
5.88729	6.89956	5.58168	6.9496	9.26844	9.2954	5.26689	5.00364	9.74425	9.12353
5.77397	9.0892	8.8798	8.21308	9.85811	5.23356	8.7956	7.17591	9.33207	8.2156
5.69758	9.89164	9.35386	8.88636	7.32812	6.38681	9.9186	7.68886	5.53153	7.41705
9.08985	6.41588	7.70854	9.24809	5.74499	9.13703	7.18093	9.963	6.58589	8.65935
9.94847	6.33626	6.05126	5.84515	5.81283	6.49815	6.20016	7.44447	8.21266	8.8851
7.10877	5.2211	8.27271	5.7172	8.44278	5.49426	6.826	8.35574	7.15198	5.04134
6.7451	6.52118	8.5777	7.07207	8.44876	7.01084	6.52394	6.06689	8.91099	6.2181
5.60918	7.01379	7.90532	7.68582	9.75914	5.82628	6.52274	7.42524	5.45111	6.97763
5.93603	9.35532	7.96868	8.10172	8.51139	6.48598	7.02645	8.40771	7.29297	7.1425
5.9479	6.24595	6.3582	6.35956	9.0359	6.77507	5.47117	6.32987	7.15242	8.19139
8.49392	8.35634	8.64854	8.74886	6.90622	5.44751	7.06064	5.26536	5.91695	9.73825
9.0644	5.97243	9.50727	8.08037	8.4277	6.43048	9.13293	5.75442	6.0637	7.92024
9.17538	7.7193	7.7696	9.60945	9.48412	7.72512	8.02189	9.31978	6.69851	6.98321
7.26528	9.90616	5.2074	5.73383	8.96302	5.51179	9.9519	5.31713	6.59807	6.25088
9.89833	9.70411	6.02907	6.56174	5.80383	5.76764	8.04739	9.70067	6.24077	6.38908
6.07587	9.48139	7.22251	7.48862	9.28034	8.53885	9.82253	5.00517	7.68063	5.04631
7.48619	6.67631	8.52172	8.45061	7.29266	9.69726	6.91933	8.20324	7.24522	8.52147
9.41638	6.76598	5.67395	5.11093	5.56395	6.71959	8.03859	6.72585	7.71852	6.0629
5.03674	7.97037	8.47232	8.68307	5.32326	6.87318	5.51448	5.98817	6.55812	8.39528
6.17799	5.50626	6.83114	5.39618	6.46491	5.90433	8.43892	7.13247	9.12011	6.48164
8.73003	8.24729	6.55087	8.86766	6.88535	7.25075	9.8524	6.44582	5.80534	8.23346
6.07742	9.83916	8.36661	5.69918	7.70954	5.83577	7.51845	8.33377	9.22778	9.99889
6.13614	9.85828	5.34886	7.74584	7.00925	8.65152	7.2892	5.44521	9.83676	8.69858
6.99859	7.06494	5.21203	5.98081	5.99386	8.39643	7.58576	6.46653	6.00821	8.00733
6.00444	5.61538	9.96913	7.2796	9.8521	9.39541	8.95235	8.94116	5.94177	6.94007
8.79535	5.19535	8.30057	9.69229	7.7574	7.23484	5.5718	5.51957	5.18564	7.06545
8.4928	8.593	8.66926	8.98707	8.25488	9.19378	6.20976	5.54518	6.39157	5.7834
6.14125	6.84861	5.95863	6.53861	8.98047	7.56509	6.75978	9.15693	7.75378	8.9516
5.89027	8.47176	5.69369	6.96692	7.70542	6.92462	6.33232	5.00285	7.14195	8.84821
7.40216	5.35465	8.01998	9.8892	6.41571	8.75031	9.48606	9.06446	8.4417	8.79506
8.65471	7.53784	7.83349	9.24779	8.98193	6.70062	8.34643	7.31231	8.06331	8.35888
7.6375795	7.619553	7.7650743	7.5877174	7.7445401	7.5558432	7.5032576	7.2032412	7.622842	7.5631899
5.03674	5.08714	5.0599	5.11005	5.29669	5.0729	5.01158	5.00285	5.0672	5.00043
9.97589	9.98707	9.98821	9.8892	9.97144	9.97324	9.98552	9.963	9.93662	9.99889
7.81614	7.74772	7.974835	7.691235	7.727725	7.479845	7.45182	7.17003	7.56454	7.69498
7.81614	7.74772	7.974835	7.68347	7.727725	7.479845	7.45182	7.14831	7.524105	7.69498
4.9871074	5.0380114	5.010499	5.0611505	5.2496569	5.023629	4.9616958	4.9528785	5.017872	4.9504343
7.56489	8.69407	8.80921	-1.1908	7.11544	7.29724	8.53752	6.263	3.59862	9.88789
5.275159	5.239106	5.5301486	5.1754348	5.4890802	5.1116864	5.0065152	4.4064824	5.245684	5.1263798
5.63228	5.49544	5.94967	5.36694	5.45545	4.95969	4.90364	4.29662	5.04821	5.38996
5.63228	5.49544	5.94967	5.38247	5.45545	4.95969	4.90364	4.34006	5.12908	5.38996
10.0256489	10.0369407	10.0380921	9.938092	10.0211544	10.0229724	10.0353752	10.01263	9.9859862	10.0488789
8.71074	13.80114	11.0499	16.11505	34.96569	12.3629	6.16958	5.28785	11.7872	5.04343
10.275159	10.239106	10.5301486	10.1754348	10.4890802	10.1116864	10.0065152	9.4064824	10.245684	10.1263798
10.63228	10.49544	10.94967	10.36694	10.45545	9.95969	9.90364	9.29662	10.04821	10.38996
10.63228	10.49544	10.94967	10.38247	10.45545	9.95969	9.90364	9.34006	10.12908	10.38996
4.85023245	4.902526362	4.874262748	4.926297266	5.119950755	4.887751257	4.824126996	4.815068944	4.881837064	4.812558006

5.035483252	5.085896014	5.058649117	5.108811815	5.295499074	5.071652409	5.010316882	5.001584671	5.065950965	4.999164058
9.977149946	9.988332776	9.989473065	9.890437995	9.972698819	9.974499275	9.986782384	9.964256682	9.937870002	10.00015577
10.16287216	10.17447227	10.17565511	10.07292455	10.15825494	10.16012258	10.17286403	10.14949778	10.12212648	10.18673644

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 61	Sample 62	Sample 63	Sample 64	Sample 65	Sample 66	Sample 67	Sample 68	Sample 69	Sample 70
8.33386	8.68324	6.06569	5.71666	9.47765	9.64388	7.85257	8.53535	6.9447	7.51992
5.22478	6.66672	9.24647	8.52855	7.42982	5.91314	9.79971	6.09779	9.84141	9.88765
7.48819	5.68176	6.86125	6.59853	7.1176	8.90473	9.98461	8.18308	6.11857	8.85597
5.5882	7.30471	6.48496	6.38709	8.08738	7.4811	6.21575	8.14733	8.65565	5.94725
5.96085	7.61	7.11843	9.27331	5.5232	6.13069	9.88163	5.6077	9.21344	6.15824
8.56892	8.28776	6.39237	8.06907	5.24572	6.37339	8.76857	8.04366	8.47271	5.91623
8.97896	5.08579	5.9067	6.5079	7.25708	6.73493	5.04228	8.13771	5.64076	5.0382
8.63581	7.31938	7.69707	5.39935	8.38331	8.88774	9.96271	5.64265	5.75171	9.21659
8.41999	8.35568	8.50346	8.05188	5.6169	5.58187	8.41413	8.78208	9.89354	9.00375
9.78084	5.67967	5.90973	6.18974	5.29252	6.17131	8.56936	8.25032	8.54631	8.04344
7.66588	8.25381	7.05925	5.09671	5.25951	7.08656	5.81537	6.7704	9.90095	8.30653
6.92399	6.8226	8.71176	9.89	8.64355	8.75183	9.91843	6.24668	8.8615	8.68202
9.69364	6.14229	6.24036	7.22335	9.51305	8.18923	8.28014	6.94117	8.95644	9.05494
8.07553	8.2379	6.89813	6.1323	5.20797	5.4124	8.24999	6.88675	5.75946	6.48923
7.13678	7.19783	7.17464	6.53436	7.77828	8.99647	9.93548	5.47309	9.73718	7.18899
5.46275	6.68039	9.61244	9.95482	6.4129	7.81069	7.59375	8.57916	5.68632	7.90829
6.66038	9.71753	8.51766	8.26063	6.53211	7.87249	9.16573	6.43857	7.87597	7.84278
9.62912	7.73406	9.11933	8.01549	7.97769	7.60608	5.79438	6.26158	9.68509	7.04703
8.83282	9.94711	8.43322	7.78444	6.06722	7.6925	9.85857	9.88778	9.53893	5.43782
7.03567	7.52122	8.1134	7.92752	5.06866	9.39389	6.54179	8.64031	7.60922	5.00381
6.42294	5.56357	9.88218	5.26424	5.97506	9.10147	7.95108	7.06334	8.93199	6.85971
5.98707	8.87558	5.85202	9.65681	6.80356	6.43161	8.64349	9.90858	5.11364	8.55038
5.70708	9.87361	5.3156	9.31189	8.02122	6.86439	7.59885	8.81479	6.84802	7.67056
7.04436	7.07852	6.29023	6.4295	8.1344	7.75136	8.06189	6.2419	6.03001	7.14341
8.02537	8.66816	6.84858	8.40387	7.65541	7.20096	6.42981	7.09314	6.9006	6.04064
8.33408	9.05791	7.1658	9.71832	5.9763	7.16651	9.71811	6.69745	8.735	7.49907
9.42628	8.03253	9.34731	8.91699	5.66612	7.47388	9.6291	8.63777	5.11638	8.67552
9.00173	8.99835	8.32397	5.98463	5.33446	5.86943	9.62623	9.87488	7.67189	8.22861
9.05446	6.00548	8.77921	6.77116	5.99397	9.22114	5.73214	7.75917	8.59905	9.7921
6.69387	9.41124	6.02981	7.00684	6.0964	8.86051	7.0662	9.13609	8.35805	5.10605
7.67799	9.72483	9.95558	8.23076	6.76612	5.81739	9.38686	6.73117	5.72705	5.02207
6.16507	6.0107	6.70852	7.44478	6.21869	6.8646	5.21042	8.92047	8.23157	9.92693
6.58252	7.89609	5.5003	8.53104	5.69627	8.77941	7.47998	8.73833	5.95011	9.61862
5.81891	8.85887	9.18572	8.81609	6.10179	5.06915	8.14501	6.62071	9.63842	7.77983
6.65354	5.58709	8.74049	7.1361	7.39563	9.53942	6.69487	8.76193	8.53949	6.5275
8.76265	9.31455	6.54088	7.28148	7.12965	7.73246	8.48712	5.51892	5.79084	6.75708
5.33782	8.09744	6.18606	5.98701	5.07749	6.28199	7.2714	6.32978	8.89635	5.996
6.18056	6.14572	5.53232	8.04446	9.80377	6.44218	8.50149	6.16586	5.3252	6.64179
5.24491	8.81609	8.39977	9.23049	6.00862	5.34227	7.52978	9.9794	9.12934	9.57183
8.12176	9.36103	7.62953	7.55252	7.50491	5.46081	5.83626	7.39493	6.97651	6.31754
8.46138	5.54397	7.60296	6.51692	8.83076	6.72617	5.41345	5.35451	7.37735	9.29448
7.73663	8.01723	9.6759	7.55959	5.89533	8.91035	8.85853	8.22519	9.72636	6.76011
7.691	7.16922	9.5918	6.82171	9.23448	5.95217	5.84331	5.54358	7.73501	7.78302
9.6719	9.63236	6.66733	5.16886	8.06586	9.43375	8.80321	9.69027	5.44459	8.32776
5.95845	8.62762	8.8266	7.10326	6.40169	9.85486	9.02197	5.81885	7.15705	9.67611

9.6524	7.98828	5.33652	8.88738	6.48217	8.15836	7.70143	9.91359	6.43033	5.59025
8.57258	9.67486	5.3964	7.99901	9.44993	6.7269	5.66166	8.8892	5.9972	7.22957
6.71769	7.51228	8.44983	8.91037	6.1252	9.2289	6.59422	8.22895	7.44405	5.30495
7.10466	9.44375	9.23458	5.7317	6.08369	8.10902	5.56866	9.85803	8.35293	8.20143
7.37815	7.97606	6.45987	9.7838	6.94581	5.47738	9.60638	5.24008	6.69014	8.3159
6.89403	9.90241	9.37102	5.91808	9.29558	7.23408	8.59309	9.45139	8.45893	8.27796
8.07724	5.11798	6.25738	8.99032	9.18478	9.72427	8.68152	7.43384	9.52185	5.16785
9.26927	5.80991	6.73841	7.21946	5.07955	9.26349	6.64582	7.55995	6.61926	5.80286
7.87828	5.86586	6.70286	9.79774	9.65022	7.41293	5.0511	6.71409	5.60402	7.60797
9.08609	5.12224	7.77928	8.84423	7.32385	5.63107	5.40632	9.53479	6.0485	8.9394
6.45729	8.85233	9.25147	7.11062	5.39416	6.61645	5.52122	7.56912	5.92528	5.96898
9.82111	8.33257	8.00819	8.30246	9.57709	8.45468	5.06487	8.60241	6.35751	7.7519
5.10976	7.49392	9.20066	7.00983	8.68987	6.56298	7.7176	7.06828	8.23075	6.63957
5.56465	8.90742	5.56905	7.77406	8.05317	9.82249	6.45705	8.01073	9.04162	7.23699
9.68031	7.0323	7.74441	6.99614	7.24063	9.27084	5.83705	9.13224	8.9959	7.68466
9.09526	9.29029	7.70087	8.3534	5.62464	5.09632	8.94895	5.9616	9.68823	6.34422
6.06885	7.20337	7.60429	7.75066	5.13508	9.11684	8.83786	5.5959	8.85802	9.76222
5.6508	7.47264	6.49127	9.87601	9.02235	9.26011	7.84568	8.04311	6.4105	9.43895
8.7526	8.41672	5.86018	5.39356	6.74129	6.42119	8.73636	7.9601	6.46076	8.89411
6.03644	5.51168	9.45856	6.21985	5.39194	7.28389	5.43208	6.13526	5.60217	9.9389
7.14275	8.02242	6.92747	8.18737	6.02965	6.11066	7.47058	8.68924	6.10157	9.09118
6.62505	5.80263	7.83477	5.88136	5.77446	6.97625	6.49447	5.66641	8.75575	8.08966
7.68236	7.55089	8.41003	8.4743	7.75783	9.69339	8.2436	7.72624	6.15137	7.6
9.45172	7.32735	7.37356	6.70466	8.93945	7.59987	8.35586	6.76774	6.02844	8.19393
6.80846	5.63651	9.36106	7.67725	9.67556	5.68883	5.15961	8.96271	6.7943	9.72937
6.94879	6.85435	8.45135	8.32445	6.15589	7.16063	6.13695	7.86882	9.95457	6.36688
8.26429	9.78912	8.92117	7.39889	6.80224	8.74128	9.40541	5.02507	6.51274	5.54716
7.04287	6.50473	8.63012	7.90356	8.45161	7.76939	6.76933	6.48022	5.28502	8.65768
7.39116	9.08849	9.43567	8.86959	9.91994	7.66516	7.47777	9.54715	5.75461	5.72837
9.35261	7.51313	8.9968	6.17384	8.62934	7.00489	9.77151	7.08121	5.0327	7.58114
8.3701	7.01248	8.26069	8.77181	5.60648	6.16315	9.77179	9.59441	5.5967	6.07523
5.22734	8.57884	7.23122	7.42776	6.8791	6.07049	7.39752	8.22299	9.83072	8.35479
9.88564	5.24448	8.49086	9.50934	6.52388	8.48588	8.50987	6.42765	8.33584	7.70413
8.82171	6.27211	7.36852	9.26275	7.70492	8.16528	8.53653	6.5852	9.39095	6.74791
7.4494	5.56244	7.28072	9.14924	8.77848	5.95936	7.40984	8.51579	7.09682	8.46664
7.54878	8.99541	7.26423	5.75228	5.502	8.62439	8.9575	6.86228	6.76772	9.48803
5.40644	8.73727	6.89228	6.90719	6.31984	7.31535	8.33738	7.66226	7.72646	7.8487
5.09355	5.16451	5.73235	9.22466	7.76055	7.44975	9.55632	5.57597	9.28418	5.86881
7.48512	9.85053	8.05566	9.59532	8.48906	9.73986	7.39477	8.89912	8.1804	9.76484
6.95781	9.22177	9.34607	6.85383	6.28402	9.03032	5.56995	6.90687	5.06927	5.71306
8.42643	5.16787	5.51558	5.72018	5.67606	6.7517	6.76827	9.87248	7.63406	5.3556
7.56975	9.78318	8.04863	9.59108	9.70017	7.41699	5.6624	9.01987	9.4891	7.51072
7.71975	9.21024	9.78256	5.12396	5.59603	8.501	5.03121	7.93347	8.5607	8.38311
8.29868	6.57621	7.73692	9.94762	9.98483	8.57225	8.96507	9.5054	8.07876	9.58421
6.13798	7.70321	8.81989	7.52177	6.64854	8.88791	9.69224	6.75112	8.19998	7.53558
8.9742	9.79037	9.33454	6.93985	9.50659	9.18756	8.78541	5.00258	7.04669	5.47221
7.82483	7.80348	7.25222	9.15928	6.09933	5.63777	9.06008	9.3655	6.91488	5.51542
5.97426	6.62957	9.47866	8.8637	6.24502	8.3928	7.21228	6.65438	5.48703	7.40783
7.50132	8.98723	6.62637	6.8612	8.44655	5.4352	6.83272	7.53405	8.19802	8.50561
8.73236	6.57317	7.14614	8.86055	9.10463	7.65611	9.46024	7.78907	6.04717	9.76321
5.59234	7.2335	7.93035	9.76666	6.554	9.39217	5.09808	8.90024	7.67306	6.23286

6.98817	6.63232	5.9989	6.58405	5.91854	5.60315	9.11419	7.60936	7.77501	9.83863
5.02637	6.44252	6.80486	7.21482	5.11322	6.00649	5.70567	7.35227	7.35518	6.06713
9.84123	7.0811	7.99674	7.61441	7.64477	6.36329	9.84873	8.20754	5.66138	7.11614
8.2317	7.89465	8.17666	8.26099	6.81355	7.71657	5.32406	8.2478	7.6208	8.39845
7.5052207	7.6446216	7.6617351	7.6937927	7.1320721	7.5259049	7.6778164	7.6428931	7.4876963	7.5551826
5.02637	5.08579	5.3156	5.09671	5.06866	5.06915	5.03121	5.00258	5.0327	5.00381
9.88564	9.94711	9.95558	9.95482	9.98483	9.85486	9.98461	9.9794	9.95457	9.9389
7.52505	7.656605	7.69897	7.713955	6.8029	7.47749	7.849125	7.742705	7.61501	7.639265
7.494755	7.580445	7.69897	7.713955	6.78418	7.461815	7.89838	7.69425	7.62164	7.639265
4.9766337	5.0366479	5.268756	5.0476771	5.0193466	5.0198415	4.9815221	4.9526058	4.983027	4.9538481
-1.55036	4.65811	5.51358	5.43682	8.46783	-4.65914	8.44561	7.9194	5.41157	3.8289
5.0104414	5.2892432	5.3234702	5.3875854	4.2641442	5.0518098	5.3556328	5.2857862	4.9753926	5.1103652
4.98951	5.16089	5.39794	5.42791	3.56836	4.92363	5.79676	5.3885	5.24328	5.27853
5.0501	5.31321	5.39794	5.42791	3.6058	4.95498	5.69825	5.48541	5.23002	5.27853
9.9344964	9.9965811	10.0051358	10.0043682	10.0346783	9.9034086	10.0344561	10.029194	10.0041157	9.988289
7.66337	13.66479	36.8756	14.76771	11.93466	11.98415	8.15221	5.26058	8.3027	5.38481
10.0104414	10.2892432	10.3234702	10.3875854	9.2641442	10.0518098	10.3556328	10.2857862	9.9753926	10.1103652
9.98951	10.16089	10.39794	10.42791	8.56836	9.92363	10.79676	10.3885	10.24328	10.27853
10.0501	10.31321	10.39794	10.42791	8.6058	9.95498	10.69825	10.48541	10.23002	10.27853
4.83947277	4.901125632	5.139571348	4.91245598	4.883351928	4.883860341	4.844494645	4.814788798	4.846040636	4.816065018
5.025110627	5.084545672	5.314413863	5.095468437	5.067411335	5.067901459	5.029951852	5.001314603	5.031442229	5.002544914
9.886877093	9.948362658	9.956834803	9.95607461	9.986092209	9.8560893	9.985872154	9.980660834	9.955824547	9.940150579
10.06923077	10.13301067	10.14179895	10.1410104	10.1721481	10.03729414	10.17191983	10.16651405	10.140751	10.12449216

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 71	Sample 72	Sample 73	Sample 74	Sample 75	Sample 76	Sample 77	Sample 78	Sample 79	Sample 80
5.29534	5.4051	7.56133	9.96908	7.53051	8.40439	9.44397	8.96029	7.45155	5.21829
8.13335	8.64351	6.27441	9.64264	5.01363	9.65657	8.45304	7.76633	5.66789	8.14464
8.40473	7.07537	6.75974	9.26804	9.55263	6.24114	9.85276	6.6641	5.73502	7.88671
6.42903	9.7191	8.46917	5.33117	8.09644	7.67821	7.15306	9.30058	9.63164	8.03104
7.35789	7.35105	6.01939	9.11603	9.53862	6.39918	9.87777	6.37648	9.99317	8.3246
9.07897	7.43879	6.43919	8.78856	7.98232	6.28716	5.43209	8.01208	5.36995	6.85959
5.19804	7.19301	6.06461	9.7701	8.14539	7.11155	7.20384	7.92299	9.99581	7.60249
9.3609	9.69603	9.43531	9.99537	8.61534	5.23147	7.56725	7.12656	7.62644	8.53846
7.32324	8.08261	8.69594	5.5765	6.54041	5.59485	7.65413	6.99637	6.86903	5.08915
6.49733	9.70384	5.80952	9.18232	9.37328	6.36741	6.33203	6.42279	8.59711	6.35949
8.2815	8.06264	8.25435	7.31431	6.05274	8.3838	7.62851	6.35988	6.27148	8.37625
7.08181	6.73147	7.39243	5.56838	9.20858	5.42332	9.84077	6.5585	8.82488	5.43078
8.22865	9.55817	8.86825	6.11046	8.70642	5.46827	8.27236	7.72211	7.35678	7.11766
6.86189	9.02345	6.80547	7.11598	9.04168	9.10847	7.49522	5.0822	6.85479	5.02253
9.20473	5.46532	6.4591	5.58171	8.50471	9.07961	8.12461	9.31261	8.39975	7.63533
8.82527	8.59435	7.22584	7.95044	8.47582	8.88789	8.73111	7.02911	9.67798	6.35092
6.24431	6.49082	7.59455	9.8452	5.98017	8.59638	7.40956	5.47912	9.96034	5.99075
7.3967	8.70318	8.04278	7.32597	5.70607	5.76139	5.73786	9.07153	5.551	7.04882
8.45104	7.43454	5.98673	8.97214	5.06472	5.72077	6.36085	7.52864	5.69736	6.36641
8.29331	9.44632	5.2041	6.91434	8.56621	5.9415	8.58338	8.45853	8.31169	9.32085
5.69956	6.75171	7.9774	8.9045	8.58249	6.93127	6.92932	6.96405	8.13372	5.41516
5.64788	9.6392	9.73921	6.1765	6.1255	6.0984	6.74669	8.81287	7.67933	8.0871
8.76535	7.59807	7.07306	9.5935	6.51772	9.22505	8.75101	9.44544	5.35319	6.24701
6.17504	9.83485	9.55313	6.68909	5.90225	8.10396	6.06039	9.08815	5.63077	5.62519
5.46677	5.17498	8.69409	6.84149	7.12229	5.49817	7.52367	7.18283	9.24113	5.15548

9.30876	9.23359	6.7705	6.87347	5.13853	9.25815	9.84215	6.82382	6.26363	6.6716
5.01886	9.18277	7.43306	6.11176	5.99072	7.22315	9.37119	8.65214	7.63824	5.00687
6.78122	8.23928	5.29579	8.17785	6.02131	7.74553	5.4462	6.29907	9.74905	5.68926
6.32199	8.35536	5.22901	8.50337	5.97678	9.52302	6.97347	6.65162	6.2988	7.71424
9.19983	9.29061	7.81481	6.2644	7.24424	7.08041	7.07182	6.03148	5.29219	9.65871
7.43294	6.77657	7.64186	5.41858	8.88235	8.45258	8.06013	5.00188	8.14174	7.36088
7.24019	6.9096	6.07778	5.55532	9.49193	9.3221	6.85112	9.30964	9.79674	5.69013
6.39516	9.2279	6.75409	5.0645	7.9859	9.95811	6.06917	5.41132	9.07126	9.94433
9.43418	6.20271	5.08832	6.26543	7.33277	6.90591	8.80166	7.72543	9.21388	6.42476
5.39403	8.01287	9.03566	9.75881	5.44664	5.04796	7.886	9.22552	6.01976	6.36335
9.49724	9.1196	7.08968	5.16515	5.41742	6.44899	6.27698	8.03042	8.7927	8.15088
6.68395	8.48663	7.25289	9.67279	6.6861	9.68156	8.70112	5.46472	8.97166	9.53605
9.48292	9.56944	8.9656	5.20857	7.92094	5.85746	9.97093	7.80934	8.39553	9.02484
7.69481	5.52114	9.72543	6.95667	6.36609	8.44411	9.18805	9.90656	5.4816	9.78365
6.97065	5.86121	7.48428	8.96358	8.62657	5.69585	9.66506	7.44959	9.47504	7.52832
7.84423	5.97416	6.38899	6.01195	6.18012	6.66692	5.72219	8.07083	8.04304	5.65626
8.67949	5.83938	8.02349	8.15237	7.08351	7.65652	7.21907	8.33413	7.58703	7.63994
7.59177	5.42143	6.7227	6.29164	8.29775	5.8229	5.28866	9.01964	7.797	7.57027
5.83353	5.66829	5.75984	7.9705	9.27981	8.07329	7.90288	6.90913	7.73381	8.5249
7.57259	8.71823	7.68973	5.17854	5.48182	6.29955	9.29068	5.53494	9.04982	8.44965
8.72939	8.84012	5.26645	8.74519	7.58978	5.11911	6.0014	6.88681	5.60474	8.34064
7.61519	6.71546	8.03027	7.77736	9.46543	5.11278	8.00496	7.18887	6.44058	7.30406
6.61552	7.09991	7.25714	6.12316	5.79652	7.72331	9.40073	5.45892	7.79656	9.41226
7.50569	7.62317	5.57397	9.19012	8.3348	7.68349	9.49536	5.07539	7.69256	6.05531
7.40309	6.92016	5.87198	5.45002	5.2735	5.71202	9.77996	7.40817	7.93739	6.72863
7.13899	5.14875	6.63998	7.23377	7.90643	8.35001	7.01171	7.63708	9.28511	9.9608
7.32827	5.68753	7.11705	5.43795	6.82817	6.72536	6.93648	5.97862	5.42306	5.10638
8.37098	9.23052	7.50225	9.40986	9.7416	8.24445	5.92922	5.37878	9.15302	9.84264
6.26015	9.42748	7.81313	5.673	6.60734	9.9132	9.48578	5.75398	9.69997	7.18737
7.26795	7.48105	6.27431	5.48448	6.98519	8.55639	9.54436	7.62911	6.27758	6.62238
6.39533	9.02117	9.54595	5.28121	9.74012	5.34607	9.14299	7.52073	5.95138	9.0428
6.94118	5.38779	9.53261	7.22368	9.09579	6.1336	9.1414	7.79254	9.20972	6.21807
9.24588	6.50671	9.95373	8.23706	6.93321	6.28683	7.54942	8.97603	9.89806	6.68969
7.18791	6.2994	7.84705	5.05981	9.03845	5.07346	8.01857	7.97768	7.84839	5.08693
6.55135	7.19112	9.36113	7.01901	7.65375	5.23822	9.39849	9.40422	8.45605	6.53516
8.92805	6.08762	5.79588	6.70499	6.46709	6.80786	7.30701	8.88747	8.72765	8.15458
9.41864	5.48922	6.24726	8.41119	6.74134	5.82158	7.84158	5.4823	7.58557	6.04208
9.2922	8.6476	5.40192	8.90916	6.26551	9.62164	5.32289	9.40668	9.9148	9.36673
8.13926	5.44262	5.20668	5.04055	5.51972	5.19939	6.69828	7.99052	6.84556	7.19024
9.84548	9.98231	7.16447	9.30051	5.64217	5.85639	9.07011	9.1436	9.50644	7.72295
8.67036	8.00156	5.29431	9.26896	5.38636	9.80744	9.60842	8.81366	5.98002	9.12609
8.40559	6.43515	9.5363	7.20404	8.64601	6.07981	6.25599	9.46481	5.16439	8.17782
5.29963	6.6301	7.56778	5.67911	7.85268	6.98859	6.96277	6.06743	5.9182	9.13599
6.76764	7.13432	5.93226	8.49848	5.07038	6.54919	6.47136	6.84704	7.86795	5.49791
9.51112	5.32433	7.59292	6.52796	5.01114	6.23841	5.35863	9.75385	9.88559	8.64466
5.31713	8.5887	5.52494	5.21867	6.02918	7.02132	5.16717	7.54588	5.51899	9.64977
7.15063	9.75176	9.62152	6.38017	5.68669	9.83771	9.65791	9.4539	9.27902	5.33515
6.3164	6.99577	6.83246	5.7129	7.62097	8.81163	5.18835	9.44224	6.30472	6.27389
6.13585	5.63179	9.86054	6.56385	9.99129	8.48545	6.24447	5.67577	7.77488	6.33034
6.61098	8.03654	6.97019	9.04619	6.47781	7.72583	9.63305	8.63675	5.79467	6.38956
5.16782	5.54582	8.91361	8.69885	7.7997	5.67967	5.19931	9.79648	9.81905	9.76787

9.16174	8.92778	8.74183	8.93347	9.36532	9.8992	8.14076	7.90319	5.38274	5.53606
6.37182	6.70585	7.40478	9.83507	7.10131	7.91901	7.2765	9.69637	8.21908	9.7551
6.53468	5.95207	9.07178	5.23307	7.11999	8.66149	7.31296	8.70332	5.88623	9.15518
5.71213	5.64929	7.97101	6.31872	5.67335	9.56325	6.10776	8.41779	6.14299	5.76685
6.46098	7.71468	9.0266	9.02425	7.4055	7.71833	7.08935	9.87207	8.7474	5.75401
8.76856	9.64445	6.92235	6.83029	9.65275	5.57511	5.5319	7.38721	7.30692	5.05861
6.46077	9.95203	9.81265	6.93899	8.26572	7.0478	7.88366	6.75284	7.42808	7.34822
7.46201	5.16277	8.00212	6.51628	7.01669	7.30998	5.59729	6.89631	5.26573	6.3456
9.26282	9.40311	6.9458	7.95059	6.9822	9.04668	7.81417	7.84466	9.63677	5.14765
8.0931	6.4848	9.70243	5.27327	5.69362	5.33762	5.90408	6.80198	8.58794	5.56341
7.80421	6.68389	9.01429	5.67524	6.54805	7.21557	9.28523	9.78314	5.91936	9.07652
5.24953	5.36996	7.28766	7.36066	6.77175	9.23456	5.79048	8.33375	7.59024	9.95954
6.19756	8.29288	6.70654	7.82556	7.38983	9.21878	9.0311	5.39392	5.21681	6.73822
5.18959	6.86035	9.25425	8.17053	6.4326	7.98001	7.25519	8.93437	8.11309	9.23046
7.73343	6.09152	5.17068	7.27204	5.05283	8.0638	9.66958	8.56582	5.91282	6.88254
7.2929	5.96746	5.21514	7.24319	6.74777	9.21517	7.55252	7.77992	9.99945	9.04506
9.89837	7.81718	7.4805	6.01874	7.59569	7.20281	7.44432	5.46889	8.58833	5.81268
6.83321	5.5267	7.69138	9.66579	5.3794	9.10553	6.04799	7.19692	9.03809	6.38052
9.92199	9.99422	8.23611	9.60179	6.93213	9.81284	6.03293	9.38214	7.37826	6.17689
8.47842	7.83244	5.77491	9.24496	7.82089	7.04397	7.29091	8.97133	6.52339	6.90199
8.05264	8.28855	6.24129	9.25083	7.08711	9.53095	6.37915	6.59197	6.66995	9.40555
6.60906	5.8014	8.29275	6.61656	6.71756	5.52905	6.10093	9.26536	7.02692	5.39336
6.81741	7.12191	7.66751	7.42123	6.55887	8.11209	5.1328	5.77956	8.31003	8.93714
7.40876	8.64889	7.81112	6.70283	6.98624	8.79452	5.3233	5.43018	6.09286	7.15431
7.4339031	7.4862798	7.4113813	7.3654428	7.2229253	7.4025258	7.5258677	7.6293368	7.6223947	7.2909781
5.01886	5.14875	5.08832	5.04055	5.01114	5.04796	5.1328	5.00188	5.16439	5.00687
9.92199	9.99422	9.95373	9.99537	9.99129	9.95811	9.97093	9.90656	9.99945	9.9608
7.34308	7.392795	7.41892	7.21386	7.0501	7.21936	7.42694	7.72377	7.713185	7.135985
7.34308	7.392795	7.398605	7.21386	7.0501	7.20919	7.42694	7.72377	7.754345	7.152515
4.9690486	5.1002375	5.0392032	4.9909555	4.9612514	4.9984396	5.084128	4.9518988	5.1160339	4.9569387
2.12099	9.41622	5.32673	9.53237	9.12029	5.76911	7.06393	0.56256	9.94445	6.0408
4.8678062	4.9725596	4.8227626	4.7308856	4.4458506	4.8050516	5.0517354	5.2586736	5.2447894	4.5819562
4.68616	4.78559	4.79721	4.42772	4.1002	4.41838	4.85388	5.44754	5.50869	4.30503
4.68616	4.78559	4.83784	4.42772	4.1002	4.43872	4.85388	5.44754	5.42637	4.27197
9.9712099	10.0441622	10.0032673	10.0453237	10.0412029	10.0076911	10.0206393	9.9556256	10.0494445	10.010408
6.90486	20.02375	13.92032	9.09555	6.12514	9.84396	18.4128	5.18988	21.60339	5.69387
9.8678062	9.9725596	9.8227626	9.7308856	9.4458506	9.8050516	10.0517354	10.2586736	10.2447894	9.5819562
9.68616	9.78559	9.79721	9.42772	9.1002	9.41838	9.85388	10.44754	10.50869	9.30503
9.68616	9.78559	9.83784	9.42772	9.1002	9.43872	9.85388	10.44754	10.42637	9.27197
4.831680562	4.96645152	4.903750704	4.85418562	4.823670462	4.861874071	4.949902157	4.814062493	4.982679234	4.819240006
5.017598725	5.147521615	5.087076313	5.039294217	5.00987677	5.046706093	5.131567576	5.000614426	5.163165575	5.005605689
9.923236298	9.995484587	9.954984334	9.996634878	9.992553845	9.959365444	9.97218869	9.907802391	10.00071591	9.962056125
10.10694672	10.18189095	10.13987944	10.18308417	10.17885085	10.14442403	10.15772577	10.0909369	10.18731748	10.14721511

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 81	Sample 82	Sample 83	Sample 84	Sample 85	Sample 86	Sample 87	Sample 88	Sample 89	Sample 90
9.07633	6.22709	8.93142	5.47879	6.10295	8.43147	7.2103	6.31087	9.48339	7.79791
7.46731	8.72773	5.75841	9.42928	9.83704	9.53598	8.15276	6.07963	7.78439	8.53862
9.54953	6.3204	7.05294	5.08947	9.129	7.80631	8.99794	6.12574	6.73973	5.4044
8.66916	6.54669	9.65735	8.9706	8.08936	8.84479	5.2176	8.43857	7.93393	8.05026
7.51807	8.65697	7.78018	9.56596	9.241	6.38432	7.57764	8.03424	8.46759	5.00375

9.96887	5.04526	9.53446	8.64266	7.15972	7.0115	7.83808	6.70294	9.18522	8.87111
8.38447	8.38428	8.45902	9.86237	5.54774	5.17332	8.92935	5.85837	7.97042	7.83184
9.38132	6.99789	9.25827	9.4047	8.60551	9.31576	6.95524	9.38951	6.30501	7.14173
8.36909	5.31679	8.42918	6.30254	7.89914	8.26666	8.75799	7.99675	9.14121	8.11533
7.77458	8.37596	5.97245	8.10285	7.79995	9.42587	9.51766	7.26872	9.86295	8.04864
6.90223	8.95205	9.77229	7.80857	9.80924	6.70765	9.91338	7.35814	5.009	6.08199
5.82569	9.4997	9.09686	8.74921	9.60492	5.60625	5.32293	8.13181	6.07887	7.19997
7.18163	7.20617	6.23193	8.92224	9.94783	8.37003	6.53386	5.96027	8.94818	5.16562
8.05876	8.4156	8.61153	6.18517	5.20607	7.26223	8.90202	7.8827	5.79591	6.77074
5.87638	7.28407	7.19134	6.04779	9.52896	5.91006	9.66275	5.54028	6.20219	7.7689
9.10323	8.84522	8.4695	9.078	8.89641	7.40409	6.57756	7.35032	9.74279	6.11562
9.90434	9.42639	9.9566	9.83461	9.88311	5.71111	8.74113	5.5964	8.56765	8.07186
8.5548	9.00009	8.08694	5.61178	8.26934	7.72908	7.07537	6.99856	6.9398	7.65009
9.6689	9.37422	8.89694	7.5591	7.09802	5.69839	8.76348	6.16627	9.72531	8.38185
9.41526	7.44896	8.10825	5.51063	9.25145	7.51541	7.30225	9.98795	5.50996	6.3232
7.95134	7.79711	5.10071	6.09557	7.74172	5.94703	6.43373	6.03625	7.4332	5.91577
6.38257	6.69303	8.5814	9.62237	5.10002	5.14785	9.04101	8.30514	8.19413	6.38583
9.328	5.34615	9.44272	8.90035	7.74304	5.6909	5.44873	9.05099	9.89202	8.88471
8.63928	7.43681	8.18744	5.89068	8.72103	5.17332	5.76046	8.09659	5.8363	8.34229
6.36035	5.93942	6.74264	8.29132	9.87873	5.68508	6.05138	6.26417	6.53717	7.18014
9.1778	7.34451	8.38875	5.70707	6.09284	8.0057	8.72497	9.57313	8.0811	6.88493
7.84861	9.60068	6.06075	5.63802	9.32194	5.63218	5.61786	7.04585	5.54464	5.2163
7.42323	7.14332	6.73482	7.12835	5.29601	9.64634	7.94475	7.99345	5.24456	9.67143
8.38202	6.26412	9.76778	7.3215	7.91254	6.75369	9.53715	8.36737	8.36967	9.402
8.59521	5.1949	7.71135	5.92932	6.72905	6.66515	7.30941	7.41085	7.22859	5.09414
9.14654	5.70358	6.94673	7.97583	8.30065	7.25863	8.9871	6.10168	5.22925	9.04202
6.48628	9.7911	8.64303	9.98503	7.96757	9.18777	7.13643	5.74402	9.18795	6.21718
9.72725	6.04504	6.15704	6.06796	5.79659	8.32379	5.51952	6.67743	9.22809	6.2912
9.86748	6.26325	8.65568	8.59173	5.00006	8.74246	8.50563	8.06583	6.79274	9.79693
7.78651	6.64519	9.82113	5.63412	7.50837	8.60388	8.36815	7.63878	8.30585	9.00874
5.33484	9.06685	7.7032	7.25081	6.83956	6.92026	6.74168	7.2335	5.54603	9.94135
5.61297	7.24213	5.7096	8.694	7.07634	7.93477	5.71694	8.19622	6.66306	7.8148
7.65203	8.21584	8.1421	5.74423	9.88209	7.33768	9.06258	9.91354	8.76814	6.50362
8.54449	9.56977	5.7193	6.16355	6.8599	8.43264	8.57104	6.77622	8.88427	6.22265
5.60707	9.10991	5.31822	5.22034	9.89144	5.24844	8.68051	8.13552	7.51829	7.05126
8.67859	6.90821	7.81467	9.83428	5.78034	8.35468	6.40224	5.7264	9.55292	5.62758
5.10407	7.28666	5.55307	7.11502	9.58049	8.44556	8.87585	7.38827	7.81382	8.15318
9.75566	7.65528	9.63027	5.88568	6.24805	8.20787	5.63741	8.69578	8.71738	8.83658
7.70969	9.88787	5.97344	5.23668	8.66446	5.19056	9.77257	9.38541	7.02226	6.37527
8.00113	8.60451	7.56841	8.46417	6.78324	8.15205	6.46249	9.45046	5.2789	6.67137
8.54376	7.26556	9.45059	7.24374	6.10959	8.01725	6.82612	6.00114	9.018	5.23073
8.57918	8.50753	6.11825	5.04661	8.89777	9.38925	8.19598	7.53332	6.68263	8.92021
9.07103	7.97208	5.77974	6.20954	9.06049	5.37434	7.77542	7.54264	6.76916	8.79001
9.26989	7.73261	7.14549	6.76127	6.52816	8.37528	6.8716	5.30078	8.10329	5.84675
5.56392	8.52953	8.75197	7.62647	9.63035	7.71954	5.03854	6.76783	6.69635	9.04063
7.18519	8.48438	5.79384	9.12714	8.79869	8.59304	5.48052	9.83521	9.55592	5.26622
8.93471	9.51167	9.86274	7.45957	7.4274	8.81161	5.75054	9.13476	7.4893	7.40221
7.23625	9.13255	8.91991	9.03222	6.7582	7.68769	8.8713	5.32851	6.29813	6.11422
7.30418	9.70645	7.00452	5.11274	9.93808	6.3142	7.21214	6.25381	8.79968	8.78798
5.70224	7.12702	5.70682	9.64889	7.44847	6.63399	6.27472	6.68976	9.02834	7.3203
7.8031	9.26786	9.00692	7.05711	5.37352	5.44855	5.96876	6.8745	8.90409	9.49288

8.22638	9.99282	5.99494	7.98889	5.80891	9.84553	9.91278	9.39287	5.21566	6.76083
8.98805	5.33242	6.29257	6.61824	5.03523	8.11681	9.22959	9.48408	9.17563	8.75787
9.18522	7.26111	5.63285	5.38287	8.97046	9.3195	9.35613	8.79452	8.61452	9.60473
5.37536	7.18578	7.1684	9.36659	7.06847	8.24906	6.87939	5.35755	7.6583	9.85092
8.69515	9.27406	8.42417	6.15972	8.5622	9.09463	9.76368	5.58893	5.77243	6.57097
6.18468	6.67535	8.82219	9.80014	7.28625	9.92803	7.46238	8.487	9.92058	8.59864
9.015	7.41978	5.10039	9.17704	5.06473	5.11896	6.98448	8.92485	7.82697	9.20877
6.23037	6.52546	7.27584	9.11296	8.22946	7.45643	7.73959	7.05418	7.0088	8.24658
9.34539	8.41991	5.33893	8.64336	8.37211	7.97283	9.43204	6.66904	8.72435	7.84374
8.48799	8.40256	8.45211	8.46259	5.94704	5.2207	6.55709	5.22487	9.85847	7.80304
7.29593	7.37494	8.2561	6.75092	7.94508	7.91946	8.33038	7.13965	9.0289	6.9178
8.67699	7.08657	7.12942	9.03059	8.87441	5.25413	6.12891	9.54423	7.47211	6.71345
7.22994	7.42033	6.68438	6.61835	9.28037	7.6746	8.20667	9.26325	6.48228	6.87201
7.78658	9.35449	5.25175	8.4785	6.78095	9.40123	5.56376	5.88447	7.66836	8.73826
5.80629	6.34893	6.05232	7.49887	6.8357	5.75933	9.72439	8.37426	7.98882	6.40707
7.30855	8.53711	6.31937	8.79807	8.26445	6.65658	6.90127	5.04827	7.39614	8.36559
8.41501	5.77193	8.65249	9.96891	8.69938	6.99918	7.78425	6.39136	8.00812	6.41539
5.98222	5.1469	9.71738	5.93387	5.10037	5.12661	8.92279	8.63533	8.98703	9.44222
8.18762	7.95081	8.33455	7.31054	7.23103	8.19221	5.45183	7.4644	7.19731	9.62582
9.21356	7.90997	9.68697	9.06071	9.2253	6.41638	5.1527	7.31179	8.54084	5.32475
7.0458	6.62475	7.03171	6.55648	7.14657	6.08043	8.42496	6.84821	9.4327	5.7495
5.5853	5.4036	9.48263	8.36331	5.9696	8.73767	5.10864	7.11747	8.14308	9.81236
8.12823	8.13646	8.55005	7.41726	8.02554	9.05935	8.95674	6.87275	8.39471	8.17648
9.56013	8.10491	8.64901	8.69259	7.8158	7.57775	9.75613	7.33403	5.03069	5.27356
6.77433	7.00529	9.77188	9.04303	6.7092	7.16666	7.45141	5.79434	7.29223	5.45911
8.12497	5.61858	7.8484	9.57103	5.62544	6.58559	6.34629	7.48521	8.14389	9.36212
8.60444	5.44335	7.1199	6.53212	5.50608	5.19065	6.37115	6.97049	8.85598	8.77562
7.61063	7.29401	7.30566	7.65378	8.95417	8.50668	6.69215	5.037	8.87605	5.01347
5.39954	8.69618	7.3053	9.25626	7.27645	8.8889	5.82267	9.86805	8.90112	8.66835
9.42613	7.43183	7.10642	7.62359	6.76323	9.17422	6.24667	5.59745	5.62909	9.16497
9.57625	9.58287	5.35302	7.1753	6.65114	5.35956	5.59339	5.4509	6.65904	8.12805
9.95597	7.2754	8.39801	5.11721	7.33697	8.85338	8.58702	5.04665	5.6132	8.20784
7.31365	8.58626	9.68817	5.57323	5.95172	9.25482	8.73844	5.93622	6.93194	8.23233
5.24302	9.94828	6.47925	5.73817	7.96237	8.05379	6.69024	9.32954	7.22746	9.64555
9.96533	7.12839	7.67128	7.38048	9.1061	7.37199	9.51538	5.92175	6.49251	8.13069
7.34558	8.00808	6.31561	5.76741	6.90026	5.02683	9.65726	9.8701	6.62672	9.11493
6.71969	6.36953	9.81843	8.09429	8.34533	8.98084	5.5026	8.20076	8.83492	9.18321
5.4686	9.56864	6.76206	5.09924	7.9987	6.67188	5.23925	8.71601	6.2656	9.31354
7.41439	9.87928	5.30882	8.61303	7.39773	9.78339	9.65924	6.22194	9.36869	5.01302
9.00601	5.08797	9.66356	8.11113	8.22486	7.66119	5.12186	9.0265	7.63114	8.17037
5.44717	8.8959	6.73766	5.52687	5.91976	8.06607	9.04503	8.5356	6.61399	7.3298
5.30261	8.55193	9.20313	8.29471	9.38746	6.28996	9.92739	5.42419	5.12521	5.42682
9.4693	6.03858	7.75878	6.44164	7.99109	9.06713	8.60666	7.29492	5.51489	6.56011
7.01584	8.98609	9.00099	7.42043	6.81574	7.5904	5.18286	8.72836	7.54818	8.53305
<b>7.840856</b>	<b>7.691014</b>	<b>7.6778775</b>	<b>7.5009392</b>	<b>7.6395676</b>	<b>7.4588462</b>	<b>7.5225205</b>	<b>7.3637649</b>	<b>7.6330737</b>	<b>7.5759014</b>
<b>5.10407</b>	<b>5.04526</b>	<b>5.10039</b>	<b>5.04661</b>	<b>5.00006</b>	<b>5.02683</b>	<b>5.03854</b>	<b>5.037</b>	<b>5.009</b>	<b>5.00375</b>
<b>9.96887</b>	<b>9.99282</b>	<b>9.9566</b>	<b>9.98503</b>	<b>9.94783</b>	<b>9.92803</b>	<b>9.92739</b>	<b>9.98795</b>	<b>9.92058</b>	<b>9.94135</b>
<b>8.029945</b>	<b>7.55212</b>	<b>7.76948</b>	<b>7.47922</b>	<b>7.771495</b>	<b>7.681145</b>	<b>7.456895</b>	<b>7.303355</b>	<b>7.726375</b>	<b>7.82332</b>
<b>8.029945</b>	<b>7.55212</b>	<b>7.735065</b>	<b>7.528985</b>	<b>7.807875</b>	<b>7.681145</b>	<b>7.52001</b>	<b>7.303355</b>	<b>7.726375</b>	<b>7.82332</b>
<b>5.0551107</b>	<b>4.9957126</b>	<b>5.0513939</b>	<b>4.9970761</b>	<b>4.9500606</b>	<b>4.9770983</b>	<b>4.9889254</b>	<b>4.98737</b>	<b>4.95909</b>	<b>4.9537875</b>
<b>6.85587</b>	<b>9.27482</b>	<b>5.6166</b>	<b>8.48803</b>	<b>4.73083</b>	<b>2.73103</b>	<b>2.66639</b>	<b>8.78295</b>	<b>1.97858</b>	<b>4.07635</b>

5.681712	5.382028	5.355755	5.0018784	5.2791352	4.9176924	5.045041	4.7275298	5.2661474	5.1518028
6.05989	5.10424	5.47013	5.05797	5.61575	5.36229	5.04002	4.60671	5.45275	5.64664
6.05989	5.10424	5.53896	4.95844	5.54299	5.36229	4.91379	4.60671	5.45275	5.64664
10.0185587	10.0427482	10.006166	10.0348803	9.9973083	9.9773103	9.9766639	10.0378295	9.9697858	9.9907635
15.51107	9.57126	15.13939	9.70761	5.00606	7.70983	8.89254	8.737	5.909	5.37875
10.681712	10.382028	10.355755	10.0018784	10.2791352	9.9176924	10.045041	9.7275298	10.2661474	10.1518028
11.05989	10.10424	10.47013	10.05797	10.61575	10.36229	10.04002	9.60671	10.45275	10.64664
11.05989	10.10424	10.53896	9.95844	10.54299	10.36229	9.91379	9.60671	10.45275	10.64664
4.920092551	4.859072611	4.916274266	4.860473341	4.812174102	4.839950055	4.852100089	4.85050222	4.821450046	4.816002764
5.102830301	5.04400541	5.099149369	5.045355752	4.998793965	5.025570743	5.037283708	5.035743318	5.007736228	5.002484899
9.970128168	9.994084232	9.957855061	9.98629226	9.949082841	9.929277827	9.928637665	9.989212999	9.921825941	9.9426012
10.15558836	10.18043835	10.14285728	10.17235562	10.13375773	10.11321369	10.11254964	10.17538534	10.10548374	10.12703422

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 91	Sample 92	Sample 93	Sample 94	Sample 95	Sample 96
5.86572	9.13582	9.92336	9.02606	8.32079	8.35625
9.04363	5.6464	8.95248	6.20083	7.81803	5.15256
8.66985	9.32944	9.54133	6.31972	6.33105	5.18741
8.65893	6.23357	6.21029	6.07032	5.156	5.24429
7.31984	9.96533	7.1128	6.08748	9.92731	8.64478
6.13777	7.84763	8.37369	8.29732	5.56943	7.89334
5.10856	6.2574	9.3278	9.06103	6.48706	5.16776
8.3243	5.34829	6.04274	8.5992	8.47424	7.36028
9.69419	6.37833	8.50027	8.37377	9.53429	8.6986
7.33633	6.98037	8.411	6.71552	6.46917	7.80143
8.43794	6.00619	6.10002	7.45449	5.59396	8.04843
8.26051	7.41828	7.8871	9.89547	9.74559	6.92853
8.35463	6.14928	9.62454	8.61974	8.01633	6.35648
8.83131	5.92827	5.42501	8.09416	7.94183	8.87483
8.47461	7.24523	7.95075	8.72966	6.82337	9.20996
8.45167	9.26223	9.42486	8.51323	7.862	8.08856
7.9084	6.13401	8.1216	8.31812	7.25036	8.75127
7.50068	5.69459	6.18948	8.40196	8.81836	8.11296
7.51844	8.6257	8.21662	7.221	8.87436	8.12392
7.48881	5.28283	6.86155	5.97045	5.96877	8.68363
8.9915	5.6318	6.04671	6.83479	6.12325	5.8468
8.93084	6.88242	7.93402	6.53323	9.82366	8.93326
6.7716	9.90217	9.31641	9.50011	5.39753	8.28776
5.93379	7.26842	5.81641	6.92787	8.29695	8.69209
8.10334	8.95291	7.82231	9.47038	7.45759	8.75926
9.91705	8.11124	5.57305	7.79777	8.39773	6.67366
9.18388	6.17943	5.67291	9.40558	8.84358	9.78651
8.35432	5.15284	5.50311	5.44203	5.72602	6.91125
7.51931	8.72996	8.43469	9.3203	9.64756	6.87958
5.68985	7.14541	8.00684	5.64337	6.88585	5.84195
9.04888	9.5941	7.00703	9.90737	8.348	8.11501
7.33399	8.23909	5.56518	5.54059	6.08967	9.52512
7.96985	7.04011	6.82435	5.59351	5.43279	7.42408
9.26438	7.30276	9.85036	6.61649	7.64762	5.70815
6.89913	8.74842	8.37378	8.67246	9.49837	7.95319
7.65159	6.20507	9.23769	5.91164	6.26615	5.2339

9.71752	7.43167	8.39145	5.43253	6.73935	9.32309
8.15593	8.19124	6.00996	5.46631	7.32051	6.62743
7.78713	5.17834	5.15968	8.07533	6.37032	9.47532
7.84396	7.27345	9.77918	6.82975	5.95845	6.16939
6.1413	7.46559	8.38298	5.74795	5.58061	5.88661
8.81012	7.18406	9.68431	5.49727	6.05596	5.70989
9.74056	9.09032	5.07699	8.9039	8.03717	9.37978
7.5583	8.27282	8.92244	6.36074	7.30441	6.36012
9.4902	6.5142	9.78872	6.9561	7.2033	8.63422
6.60529	6.66159	6.96212	9.4697	7.3725	6.55171
5.87705	7.892	5.33919	6.51344	6.46308	6.25676
6.08031	8.28815	9.11105	8.2719	7.1645	9.85544
7.84844	5.47504	6.16136	8.93055	8.52803	8.57674
9.09614	8.07122	7.14447	9.2174	9.72313	9.20112
7.70841	5.21424	9.35447	8.0174	9.65122	9.69076
6.51518	7.69133	6.51391	8.00974	7.16047	8.55198
5.07027	8.65295	5.1765	9.47664	7.93638	6.73614
6.64277	7.02979	9.97077	7.6124	6.19742	8.39844
8.44305	6.01492	8.14348	7.66167	5.29104	8.80795
6.97692	8.53164	8.61289	8.8483	6.38396	6.81859
5.5855	8.89811	8.58016	7.25241	9.33989	6.56283
8.82062	5.1131	8.16859	5.63221	5.27245	7.67655
8.33591	5.12164	7.05508	6.44184	7.02534	7.88164
9.2979	8.68988	9.27228	9.34844	9.335	7.57438
9.71805	6.04889	7.72917	5.06334	5.31684	8.79569
7.6126	7.75411	5.57882	7.00005	8.99788	7.25569
9.65659	7.94325	5.49581	6.0681	7.17909	5.34649
6.71937	9.28436	6.81028	6.18412	8.20189	9.34453
5.71262	9.02465	6.57436	8.25482	9.17856	8.40376
8.39577	7.25704	8.6201	5.9777	7.34209	8.45199
7.39871	5.14884	9.28114	7.78681	6.5373	5.00843
5.22456	8.58504	9.6408	9.27244	8.08614	7.96656
8.89719	8.81921	7.58059	5.10885	5.35376	6.98271
6.84458	8.92073	6.55887	8.27278	5.29776	7.29433
5.92116	8.37494	7.85216	9.68456	7.49348	8.25419
8.56835	8.01447	5.30782	7.6975	9.61352	6.63628
5.91579	9.57159	7.9622	9.60373	8.84222	6.83221
5.10213	9.26724	9.67652	8.55332	5.24183	5.74419
7.02156	8.08377	7.17982	6.10646	7.33902	6.95325
6.21123	5.67665	5.05615	5.5194	7.92216	8.10212
6.71114	9.27757	7.30917	6.88397	8.96757	9.10216
6.36315	9.13549	7.71975	6.47669	6.02399	8.84095
6.95813	9.89787	9.53642	8.83447	8.47992	9.13579
5.10689	5.73173	5.97953	6.50463	6.33939	6.58964
5.67011	9.23312	6.90265	8.1055	7.59999	5.02374
8.53045	7.50885	5.77513	9.95609	5.09864	8.37032
5.08424	5.09385	7.02799	7.33499	6.40422	7.95074
9.75538	9.40583	7.72641	7.47409	5.49103	7.84894
7.4949	7.93969	7.07998	7.16771	6.36073	8.6891
6.43358	6.798	7.11137	6.3902	7.81645	9.46907
9.31756	9.75418	7.67978	8.21611	6.76287	7.81748

5.73522	8.71442	7.82624	8.38104	7.01808	8.26516
6.53467	6.85715	8.85128	9.32542	7.40197	7.84465
5.74598	8.78821	5.21513	5.60309	6.2175	8.49354
8.22455	5.90085	8.03437	5.88735	9.68126	8.26842
9.68162	8.91189	7.49925	6.36036	8.17312	5.74127
6.30008	6.55256	9.84196	8.16357	5.97884	6.61848
6.12873	8.92951	9.34479	5.07719	6.94334	5.1727
7.07208	9.06227	7.6745	9.3218	5.45887	6.86871
8.8666	7.43472	8.8224	9.27425	9.21156	6.6648
5.14106	7.59071	5.66978	6.69994	6.65606	7.02232
5.92786	9.75104	8.55321	8.47856	8.93543	6.29558
7.92396	5.0605	8.51211	6.12383	8.32792	6.86106
5.46176	5.96106	5.46517	5.55743	5.25943	6.62341
<b>7.5018816</b>	<b>7.5296848</b>	<b>7.6000715</b>	<b>7.488132</b>	<b>7.3283083</b>	<b>7.5491417</b>
<b>5.07027</b>	<b>5.0605</b>	<b>5.05615</b>	<b>5.06334</b>	<b>5.09864</b>	<b>5.00843</b>
<b>9.91705</b>	<b>9.96533</b>	<b>9.97077</b>	<b>9.95609</b>	<b>9.92731</b>	<b>9.85544</b>
<b>7.538805</b>	<b>7.54978</b>	<b>7.77574</b>	<b>7.543245</b>	<b>7.277385</b>	<b>7.846795</b>
<b>7.58545</b>	<b>7.54978</b>	<b>7.77574</b>	<b>7.543245</b>	<b>7.277385</b>	<b>7.846795</b>
<b>5.0209727</b>	<b>5.011105</b>	<b>5.0067115</b>	<b>5.0139734</b>	<b>5.0496264</b>	<b>4.9585143</b>
1.62205	6.49833	7.04777	5.56509	2.65831	-4.60056
<b>5.0037632</b>	<b>5.0593696</b>	<b>5.200143</b>	<b>4.976264</b>	<b>4.6566166</b>	<b>5.0982834</b>
5.1709	5.09956	5.55148	5.08649	4.55477	5.69359
5.07761	5.09956	5.55148	5.08649	4.55477	5.69359
9.9662205	10.0149833	10.0204777	10.0056509	9.9765831	9.9039944
12.09727	11.1105	10.67115	11.39734	14.96264	5.85143
<b>10.0037632</b>	<b>10.0593696</b>	<b>10.200143</b>	<b>9.976264</b>	<b>9.6566166</b>	<b>10.0982834</b>
10.1709	10.09956	10.55148	10.08649	9.55477	10.69359
10.07761	10.09956	10.55148	10.08649	9.55477	10.69359
4.885022428	4.874885294	4.870371831	4.877832015	4.914458505	4.820858627
5.069021743	5.059249269	5.054898167	5.062089988	5.097398926	5.007166084
9.918295047	9.966587272	9.972028649	9.957344932	9.928557645	9.856669446
10.10182109	10.15191534	10.15755976	10.14232812	10.11246663	10.03789593

A Simulation Study on A Uniform Distribution Over the Interval (5,10) (continued)

Sample 97	Sample 98	Sample 99	Sample 100
6.06981	7.196	6.26284	7.8831
9.32288	6.3471	9.21827	8.70313
7.48206	5.5979	5.66504	9.28785
5.55559	8.9968	9.67986	5.68158
6.947	7.4866	6.61298	5.95971
5.91696	6.8486	6.75368	7.04876
9.33183	5.4834	7.73034	9.27037
6.13787	8.7877	8.30374	6.8351
7.91341	7.3681	5.10907	8.98629
5.61384	5.5579	7.51722	9.66273
9.87432	5.1294	8.15625	8.05645
9.92302	9.1291	8.57707	8.20657
6.03787	7.27	9.22589	7.6988
9.72749	6.2725	5.20106	6.38067
8.09217	8.2146	7.54133	9.60089
6.23279	7.2335	5.3949	7.06617

8.43305	7.0493	7.31186	5.0671
5.49295	8.5185	6.88068	7.80063
8.56085	6.4932	9.53571	5.31537
7.27365	6.4598	6.97175	5.88511
6.90708	5.8214	7.61343	6.67453
7.18443	7.6224	5.0642	9.90066
9.4264	6.9188	9.54913	5.46897
6.0879	6.3237	6.29712	7.24598
5.66977	9.6747	9.38268	5.17785
7.12944	6.8755	9.1996	8.46317
5.70038	7.1885	7.23291	9.3592
7.82778	5.7789	8.92291	7.85412
7.29484	5.0599	9.63473	6.73056
7.62466	6.9333	6.36501	9.20233
5.83423	5.7463	5.39357	5.70454
8.25272	5.2885	5.99405	5.09388
5.51224	5.6281	9.57646	9.42176
7.39638	5.9825	9.99017	7.65081
8.83294	6.6391	7.7901	5.14826
5.28796	8.035	9.48686	5.03304
8.17813	9.0869	7.56432	8.14471
7.64573	8.536	6.88738	8.72843
7.70506	9.42	7.02482	6.23009
6.16992	6.6216	8.55453	9.32639
7.57258	8.2968	5.28664	7.84153
8.7078	5.9088	5.65771	7.28903
8.33025	9.3744	5.87207	5.27575
5.72844	6.2502	6.4284	7.56508
6.32623	7.8262	6.0977	8.174
8.79575	9.3025	6.42639	7.77993
7.9307	9.7544	9.99122	8.17368
8.35479	7.9114	5.19999	6.1531
5.90778	8.7225	7.94356	7.13526
8.04123	8.9966	9.27531	5.6579
7.37065	9.0767	8.66941	9.92382
5.54969	9.6716	7.19476	7.44799
7.85046	8.0562	6.08785	5.15208
9.62641	7.4477	9.78405	8.02122
7.68202	7.1621	9.04268	8.50513
7.51337	7.3528	5.80391	6.21122
5.69447	8.8631	6.96021	9.57822
5.86206	9.6208	8.87941	6.69874
6.61621	5.9569	5.5584	9.50193
6.65144	8.4801	8.25181	6.95405
8.36578	9.7359	8.23355	6.22647
8.09951	5.133	7.71347	9.06748
6.13185	7.0257	6.93728	7.48149
5.87743	8.9391	8.12423	5.65091
7.30891	9.3774	5.58766	6.79316
7.10725	7.654	6.47934	9.08146
9.83079	9.5668	6.35068	5.83882

6.92512	6.6621	5.00661	9.82196		
9.11534	8.4111	8.17905	6.12704		
8.18818	9.0767	8.79637	8.13823		
6.48844	5.6448	7.49101	9.54837		
9.70725	5.3183	7.1652	7.82948		
9.6157	7.207	7.50207	9.77303		
6.259	8.6083	6.00227	7.87994		
8.22683	6.7875	6.34944	6.07921		
6.72646	6.0566	9.87433	7.84758		
5.47933	7.7005	6.65184	9.38024		
5.82503	8.0103	6.45383	5.42378		
9.36158	8.145	5.14453	9.9326		
9.37292	8.0809	6.31914	6.50298		
9.81604	7.4179	5.37785	5.25003		
5.98888	7.0782	7.88375	6.43828		
8.08137	5.4684	7.20134	7.69912		
7.16463	5.9356	6.76466	5.2247		
7.40421	5.7293	6.96203	6.99122		
8.51	8.3009	6.53925	9.4795		
6.22441	6.515	9.75635	8.72266		
8.6553	5.2525	6.88367	8.73144		
5.08693	5.9486	8.11424	5.93893		
6.74125	9.9999	7.10754	7.14105		
5.42597	6.3915	8.75378	5.44901		
7.10294	5.6972	8.27718	7.25408		
5.74783	8.5311	5.83878	8.9644		
6.35642	5.0193	9.85545	7.90506		
9.47188	5.0379	6.45491	6.85422		
5.25334	8.4727	7.26275	8.52135		
8.91317	5.0021	8.39853	5.36629		
7.91003	5.0652	9.01336	9.01209		
7.77594	6.9555	8.0859	9.74762		
9.78677	9.9591	7.20513	9.88968	Mean	Var
<b>7.3914974</b>	<b>7.315718</b>	<b>7.4168335</b>	<b>7.5100028</b>	<b>7.502282583</b>	<b>0.021270717</b>
<b>5.08693</b>	<b>5.0021</b>	<b>5.00661</b>	<b>5.03304</b>	<b>5.0536892</b>	<b>0.003032216</b>
<b>9.92302</b>	<b>9.9999</b>	<b>9.99122</b>	<b>9.9326</b>	<b>9.9556945</b>	<b>0.001421328</b>
<b>7.383515</b>	<b>7.2015</b>	<b>7.21902</b>	<b>7.674805</b>	<b>7.5061214</b>	<b>0.06102876</b>
<b>7.383515</b>	<b>7.19775</b>	<b>7.24783</b>	<b>7.607945</b>	<b>7.50175925</b>	<b>0.063832164</b>
<b>5.0377993</b>	<b>4.952121</b>	<b>4.9566761</b>	<b>4.9833704</b>	<b>5.004226092</b>	<b>0.003093163</b>
<b>2.22502</b>	<b>9.9899</b>	<b>9.11322</b>	<b>3.1926</b>	<b>5.5251445</b>	<b>14.49896702</b>
<b>4.7829948</b>	<b>4.631436</b>	<b>4.833667</b>	<b>5.0200056</b>	<b>5.004565166</b>	<b>0.085082868</b>
<b>4.76703</b>	<b>4.3955</b>	<b>4.49566</b>	<b>5.21589</b>	<b>5.0035185</b>	<b>0.255328655</b>
<b>4.76703</b>	<b>4.403</b>	<b>4.43804</b>	<b>5.34961</b>	<b>5.0122428</b>	<b>0.244115039</b>
<b>9.9722502</b>	<b>10.049899</b>	<b>10.0411322</b>	<b>9.981926</b>	<b>10.00525145</b>	<b>0.001449897</b>
<b>13.77993</b>	<b>5.2121</b>	<b>5.66761</b>	<b>8.33704</b>	<b>10.4226092</b>	<b>30.93163269</b>
<b>9.7829948</b>	<b>9.631436</b>	<b>9.833667</b>	<b>10.0200056</b>	<b>10.00456517</b>	<b>0.085082868</b>
<b>9.76703</b>	<b>9.3955</b>	<b>9.49566</b>	<b>10.21589</b>	<b>10.0035185</b>	<b>0.255328655</b>
<b>9.76703</b>	<b>9.403</b>	<b>9.43804</b>	<b>10.34961</b>	<b>10.0122428</b>	<b>0.244115039</b>
<b>4.902308471</b>	<b>4.81429076</b>	<b>4.818970236</b>	<b>4.846393412</b>		
<b>5.085685961</b>	<b>5.000834481</b>	<b>5.005345623</b>	<b>5.031782316</b>		
<b>9.924266558</b>	<b>10.00116603</b>	<b>9.992483827</b>	<b>9.933848984</b>		

10.10801543	10.18778439	10.17877822	10.11795542		
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LCL=Lower Confidence Limit; UCL=Upper Confidence Limit; LB=Lower Bound ; UB= Upper Bound

	Mean	Var
Mean	7.502282583	0.021270717
Min	5.0536892	0.003032216
Max	9.9556945	0.001421328
M(even)	7.5061214	0.06102876
M( odd)	7.50175925	0.063832164
T1	5.004226092	0.003093163
T2	5.5251445	14.49896702
T3	5.004565166	0.085082868
T4	5.0035185	0.255328655
T5	5.0122428	0.244115039
W1	10.4226092	30.93163269
W2	10.00525145	0.001449897
W3	10.00456517	0.085082868
W4	10.0035185	0.255328655
W5	10.0122428	0.244115039

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