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Cost Benefit Analysis of Series Systems with Mixed Standby Components and *k*-stage Erlang Repair Time

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Abstract In this paper, we compare the availability characteristics between three different series system configurations with mixed (cold and warm) standby components and *k*-stage Erlang repair times. Three configurations are studied under the assumption that the time-to-failure for each of the operative and warm standby components are assumed to be exponentially distributed with parameters λ and α, respectively. We present a recursive method, using the supplementary variable technique; we develop the explicit expressions for the steady-state availability. Under the cost/benefit criterion, comparisons are made based on assumed numerical values given to the distribution parameters, and to the cost of the operative and standby units.

Keywords Availability, Cost-Benefit, Erlang Distribution, Supplementary Variable, Series System

1. Introduction

This type of problems is discussed by[2, 3] have studied a two similar or dissimilar unit cold standby redundant system with preventive maintenance, inspection and two types of repairs.[4] carried out the cost-benefit analysis of a one-server two-unit system subject to different repair policies. They carried out the analysis under the assumption that a unit undergoes only one type of failure with no specific mention about inspection.[5, 6] have studied the stochastic behaviour of some standby redundant systems.[9] considered the reliability and mean time to system failure (*MTTF*) analysis of a two-state complex with repairable system, consisting of two sub-systems A and B arranged in series, incorporating the concept of hardware and human failures.[7] considered reliability, availability and cost/benefit of four different series system configurations with mixed standby components.[11] studied a model representing a two-unit active and one-unit on standby human-machine system with general failed system repair time distribution. A two-unit warm standby system with constant failure rate and two types of repairmen and patience time were investigated by[8].[1] studied the two unit standby system and obtained exact confidence limits for the steady-state availability of the system, when the failure rate of an operative unit is constant and the repair time of the failed unit is a two stage Erlang distribution.[10] studied the optimal system for series systems with mixed standby components.

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The main contribution of this paper is three folds. First, we present a recursive method, using the supplementary variable technique and treating the supplementary variable as the remaining repair time, to develop the $Av_i(\infty)$, for configuration *i,* where $i=1,2,3$. The second purpose is to develop the explicit expressions for the steady-state availability, for the three configurations. Finally, we rank three configurations for the $Av_i(\infty)$ based on assumed numerical values given to the system parameters.

2. Assumptions and Configurations Description

We consider a power plant of 10 MW satisfies the following assumptions:

1. The system comprises of operative components and mixed standby components “cold and warm”.
2. The generators are available in components of both 10 and 5 MW.
3. Standby generators are always necessary in case of failure.
4. When the operative component fails, it is immediately replaced by a warm standby if it is available and the cold standby becomes warm standby component.
5. We assume that the switch is perfect and the switchover time from warm standby component to primary component, from cold standby component to warm standby component, from failure to repair, or from repair to cold standby component (or primary component if the system is short) is instantaneous.
6. When operative and warm standby components are repaired, they become as good as new.

7. The operative and warm standby components fail independently of the others and follow an exponential time to failure distribution with parameters λ and α respectively.

8. The time to repair of the components is independent and identically distributed random variable following k-stage Erlang distribution with probability density function $b(u)$ and mean repair time $\frac{1}{\mu}$.

where

$$b(u)=\frac{(\mu k)^k}{(k-1)!}u^{k-1}e^{-\mu k u},\ 0<u<\infty,\ \mu>0.$$

9. If one operative unit or warm standby unit is in repair, then arriving failed units have to wait in the queue until the server is available.

The above assumptions are common to all of the following three configurations:

Configuration 1 is a serial system of one operative 10 MW component, one warm standby 10 MW component and one cold 10 MW component.

Configuration 2 is a serial system of two operative 5 MW components, one warm standby 5 MW component and one cold 5 MW component.

Configuration 3 is a serial system of one operative 10 MW component, two warm standby 10 MW components and one cold 10 MW component.

Cost-benefit factor

In this paper, we assume that the size-proportional costs for the operative components and warm standby components are given in Table 1. With this, we calculate the costs for each configuration $i(i=1,2,3)$ which are shown in Table 2. Throughout this paper, we assume that C_i be the cost of configuration i and F_i the benefit of configuration i, where F_i is $Av_i(\infty)$.

Table 1. The size-proportional cost for the operative, warm and cold standby components

Component	Cost(in $)
Operative 10MW	10×10^6
Operative 5MW	5×10^6
Warm standby 10MW	6×10^6
Warm standby 5MW	3×10^6
Cold standby 10MW	4×10^6
Cold standby 5MW	2×10^6

Table 2. The costs for each configuration $i(i=1,2,3)$

Configuration	Cost(in $)
Configuration 1	20×10^6
Configuration 2	15×10^6
Configuration 3	24×10^6

3. Availability Analysis of the System

We use the following supplementary variable: $U\equiv$ remaining repair time for the component under repair. The state of the system at time t is given by

$N(t)\equiv$ number of working units in the system; and

$U(t)\equiv$ remaining repair time for the component being repaired.

and define

$$P_n(u,t)du=P\{N(t)=n,u<U(t)\le u+du\},\quad u\ge 0,$$

and $P_n(t)=\int_0^\infty P_n(u,t)d$.

3.1. Availability for Configuration 1

Relating the state of the system at time t and $t+dt$, we obtain:

$$\frac{d}{dt}P_3(t)=-(\lambda+\alpha)P_3(t)+P_2(0,t), \tag{1}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_2(u,t)=-(\lambda+\alpha)P_2(u,t)+(\lambda+\alpha)P_3(u,t)+b(u)P_1(0,t), \tag{2}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_1(u,t)=-\lambda P_1(u,t)+(\lambda+\alpha)P_2(u,t)+b(u)P_0(0,t), \tag{3}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_0(u,t)=\lambda P_1(u,t). \tag{4}$$

In steady-state, let us define

$$P_n=\lim_{t\to\infty}P_n(t),\quad n=3,2,1,0.$$

$$P_n(u)=\lim_{t\to\infty}P_n(u,t),\quad n=3,2,1,0.$$

and further define

$$P_3(u)=b(u)P_3, \tag{5}$$

From (1)–(5), we obtain the following steady-state equations as follow:

$$0=-(\lambda+\alpha)P_3+P_2(0), \tag{6}$$

$$-\frac{d}{du}P_2(u)=-(\lambda+\alpha)P_2(u)+(\lambda+\alpha)b(u)P_3+b(u)P_1(0), \tag{7}$$

$$-\frac{d}{du}P_1(u)=-\lambda P_1(u)+(\lambda+\alpha)P_2(u)+b(u)P_0(0), \tag{8}$$

$$-\frac{d}{du}P_0(u)=\lambda P_1(u). \tag{9}$$

Now, from (6), we obtain:

$$P_2(0)=(\lambda+\alpha)P_3, \tag{10}$$

Further, we define

$$B^*(s)=\int_0^\infty e^{-su}b(u)du=\left(\frac{k\mu}{k\mu+s}\right)^k,$$

$$P_n^*(s)=\int_0^\infty e^{-su}P_n(u)du,\quad P_n=P_n^*(0)=\int_0^\infty P_n(u)du,$$

and $\int_0^\infty e^{-su}\frac{d}{du}P_n(u)du=sP_n^*(s)-P_n(0)$.

Now, taking the Laplace- Stieltjes Transform (LST) on both sides of (7)-(9) and using (10) yield:

$$(\lambda+\alpha-s)P_2^*(s)=(B^*(s)-1)P_2(0)+B^*(s)P_1(0), \tag{11}$$

$$(\lambda-s)P_1^*(s)=(\lambda+\alpha)P_2^*(s)+B^*(s)P_0(0)-P_1(0), \tag{12}$$

$$sP_0^*(s)=P_0(0)-\lambda P_1^*(s). \tag{13}$$

We develop a recursive method to get the explicit expressions $P_n^*(0)(n=2,1,0)$.

Setting $s=\lambda+\alpha$ and $s=0$ in (11), respectively yields

$$P_1(0)=\frac{1-B^*(\lambda+\alpha)}{B^*(\lambda+\alpha)}P_2(0)=\frac{(\lambda+\alpha)\left(1-B^*(\lambda+\alpha)\right)}{B^*(\lambda+\alpha)}P_3, \quad (14)$$

and,

$$P_2^*(0)=\frac{1}{(\lambda+\alpha)}P_1(0)=\frac{\left(1-B^*(\lambda+\alpha)\right)}{B^*(\lambda+\alpha)}P_3. \quad (15)$$

Again, setting $s=\lambda$ in (11), it follows that:

$$P_2^*(\lambda)=\frac{(\lambda+\alpha)\left(B^*(\lambda)-B^*(\lambda+\alpha)\right)}{\alpha B^*(\lambda+\alpha)}P_3. \quad (16)$$

Setting $s=\lambda$ in (12) yields:

$$P_0(0)=\frac{P_1(0)-(\lambda+\alpha)P_2^*(\lambda)}{B^*(\lambda)}. \quad (17)$$

Substituting (14) and (16) in (17), we have:

$$P_0(0)=\left\{\frac{(\lambda+\alpha)\left[\alpha\left(1-B^*(\lambda+\alpha)\right)-(\lambda+\alpha)\left(B^*(\lambda)-B^*(\lambda+\alpha)\right)\right]}{\alpha B^*(\lambda)B^*(\lambda+\alpha)}\right\}P_3. \quad (18)$$

Similarly, setting $s=0$ in equation (12), we obtain:

$$\lambda P_1^*(0)=(\lambda+\alpha)P_2^*(0)+P_0(0)-P_1(0). \quad (19)$$

From (19) and (15), we have:

$$P_1^*(0)=\frac{1}{\lambda}P_0(0).$$

Thus

$$P_1^*(0)=\frac{(\lambda+\alpha)}{\lambda}\left\{\frac{\left[\alpha\left(1-B^*(\lambda+\alpha)\right)\right]-(\lambda+\alpha)\left(B^*(\lambda)-B^*(\lambda+\alpha)\right)}{\alpha B^*(\lambda)B^*(\lambda+\alpha)}\right\}P_3. \quad (20)$$

Differentiating (13) with respect to s and setting s = 0 in the result, we obtain:

$$P_0^*(0)=-\lambda P_1^{*(1)}(0). \quad (21)$$

Differentiating (11) with respect to s and then setting s = 0 in the result we have:

$$(\lambda+\alpha)P_2^{*(1)}(0)=P_2^*(0)-\frac{\left[P_2(0)+P_1(0)\right]}{\mu}. \quad (22)$$

Similarly, differentiating (12) with respect to s and setting s = 0 in the result, we find that:

$$\lambda P_1^{*(1)}(0)=P_1^*(0)+P_2^*(0)-\frac{\left[P_2(0)+P_1(0)+P_0(0)\right]}{\mu}. \quad (23)$$

Substituting (23) in (21), we have:

$$P_0^*(0)=\frac{\left[P_2(0)+P_1(0)+P_0(0)\right]}{\mu}-\left(P_2^*(0)+P_1^*(0)\right). \quad (24)$$

where $P_2^*(0), P_2(0), P_1(0)$ and $P_0(0)$ are given in (15), (10), (14) and (18), respectively.

Now, using the normalizing condition

$$P_3+P_2^*(0)+P_1^*(0)+P_0^*(0)=1,$$

We can compute P_3 as follow:

$$P_3=\frac{\alpha\mu B^*(\lambda)B^*(\lambda+\alpha)}{\left[(\lambda+\alpha)\left(\alpha-\lambda B^*(\lambda)\right)+\left(\lambda(\lambda+\alpha)+\alpha\mu B^*(\lambda)\right)B^*(\lambda+\alpha)\right]}. \quad (25)$$

The explicit expression for the availability of configuration 1 $\left(Av_1(\infty)\right)$ is given by:

$$Av_1(\infty)=P_3+P_2^*(0)+P_1^*(0). \quad (26)$$

Using (15), (20), (25) and (26), we obtain the explicit expression for the $Av_1(\infty)$

$$Av_1(\infty)=\frac{\gamma_1}{\varpi_1}. \quad (27)$$

where

$$\gamma_1=\mu\left[-\left(\alpha^2+\alpha\lambda+\lambda^2\right)B^*(\lambda)+(\alpha+\lambda)\left(\alpha+\lambda B^*(\lambda+\alpha)\right)\right],$$

and

$$\varpi_1=\lambda(\alpha+\lambda)\left(\alpha+\lambda B^*(\lambda+\alpha)\right)-\lambda B^*(\lambda)\left(\lambda(\lambda+\alpha)-\alpha\mu B^*(\lambda+\alpha)\right).$$

3.2. Availability for Configuration 2

Following the same procedures in the subsection 3.1, we can get the steady-state equations as follow:

$$0=-(2\lambda+\alpha)P_4+P_3(0), \quad (28)$$

$$-\frac{d}{du}P_3(u)=-(2\lambda+\alpha)P_3(u)+(2\lambda+\alpha)b(u)P_4+b(u)P_2(0), \quad (29)$$

$$-\frac{d}{du}P_2(u)=-2\lambda P_2(u)+(2\lambda+\alpha)P_3(u)+b(u)P_1(t), \quad (30)$$

$$-\frac{d}{du}P_1(u)=2\lambda P_2(u). \quad (31)$$

Now, from (28), we obtain:

$$P_3(0)=(2\lambda+\alpha)P_4, \quad (32)$$

Taking the LST on both sides of (29)-(31) and using (32), we get that:

$$(2\lambda+\alpha-s)P_3^*(s)=\left(B^*(s)-1\right)P_3(0)+B^*(s)P_2(0), \quad (33)$$

$$(2\lambda-s)P_2^*(s)=(2\lambda+\alpha)P_3^*(s)+B^*(s)P_1(0)-P_2(0), \quad (34)$$

$$sP_1^*(s)=P_1(0)-2\lambda P_2^*(s). \quad (35)$$

Setting $s=2\lambda+\alpha$ and $s=0$ in (33), respectively yields:

$$P_2(0)=\frac{1-B^*(2\lambda+\alpha)}{B^*(2\lambda+\alpha)}P_3(0)=\frac{(2\lambda+\alpha)\left(1-B^*(2\lambda+\alpha)\right)}{B^*(2\lambda+\alpha)}P_4, \quad (36)$$

and

$$P_3^*(0)=\frac{1}{(2\lambda+\alpha)}P_2(0)=\frac{\left(1-B^*(2\lambda+\alpha)\right)}{B^*(2\lambda+\alpha)}P_4. \quad (37)$$

Again, setting $s=2\lambda$ in (33), it follows that:

$$P_3^*(2\lambda)=\frac{\left(B^*(2\lambda)-1\right)P_3(0)+B^*(2\lambda)P_2(0)}{\alpha} \quad (38)$$

Setting $s=2\lambda$ in (34) yields:

$$P_1(0)=\frac{P_2(0)-(2\lambda+\alpha)P_3^*(2\lambda)}{B^*(2\lambda)}. \quad (39)$$

Substituting (36) and (38) in (39), we have:

$$P_1(0)=(2\lambda+\alpha)\left\{\frac{\left[\alpha\left(1-B^*(2\lambda+\alpha)\right)\right]-(2\lambda+\alpha)\left(B^*(2\lambda)-B^*(2\lambda+\alpha)\right)}{\alpha B^*(2\lambda)B^*(2\lambda+\alpha)}\right\}P_4. \quad (40)$$

Similarly, setting $s=0$ in (34), we obtain:

$$2\lambda P_2^*(0)=(2\lambda+\alpha)P_3^*(0)+P_1(0)-P_2(0).$$

From above equation and (37), we have:

$$P_2^*(0)=\frac{(2\lambda+\alpha)P_3^*(0)+P_1(0)-P_2(0)}{2\lambda}.$$

Now, using (40), we get:

$$P_2^*(0)=\frac{(2\lambda+\alpha)}{2\lambda}\left\{\frac{\left[\alpha\left(1-B^*(2\lambda+\alpha)\right)\right]-(2\lambda+\alpha)\left(B^*(2\lambda)-B^*(2\lambda+\alpha)\right)}{\alpha B^*(2\lambda)B^*(2\lambda+\alpha)}\right\}P_4. \quad (41)$$

Differentiating (35) with respect to s and setting s = 0 in the result, we obtain:

$$P_1^*(0)=-2\lambda P_2^{*(1)}(0). \quad (42)$$

Differentiating (33) with respect to s and then setting s = 0 in the result yields:

$$(2\lambda+\alpha)P_3^{*(1)}(0)=P_3^*(0)-\frac{\left[P_3(0)+P_2(0)\right]}{\mu}. \quad (43)$$

Likewise, differentiating (34) with respect to s and setting s = 0 in the result, we find that:

$$2\lambda P_2^{*(1)}(0)=P_2^*(0)+P_3^*(0)-\frac{\left[P_3(0)+P_2(0)+P_1(0)\right]}{\mu}. \quad (44)$$

Substituting (44) in (42), we have:

$$P_1^*(0)=\frac{\left[P_3(0)+P_2(0)+P_1(0)\right]}{\mu}-\left(P_2^*(0)+P_3^*(0)\right), \quad (45)$$

where $P_3^*(0), P_3(0), P_2(0)$ and $P_1(0)$ are given in (37),(32),(36) and (40),respectively.

Now, using the normalizing condition:

$$P_4+P_3^*(0)+P_2^*(0)+P_1^*(0)=1, \quad (46)$$

We can compute P_4 as follow.

$$P_4=\frac{\alpha\,\mu B^*(2\lambda)B^*(2\lambda+\alpha)}{\left[\begin{array}{l}(2\lambda+\alpha)\left(\alpha-2\lambda B^*(2\lambda)\right)\\+\left(2\lambda(2\lambda+\alpha)+\alpha\,\mu B^*(2\lambda)\right)B^*(2\lambda+\alpha)\end{array}\right]}. \quad (47)$$

The availability of configuration 2 $(Av_2(\infty))$ is given by:

$$Av_2(\infty)=P_4+P_3^*(0)+P_2^*(0). \quad (48)$$

Subsisting (37), (41) and (47) into (48), we obtain the $Av_2(\infty)$:

$$Av_2(\infty)=\frac{\gamma_2}{\varpi_2}. \quad (49)$$

where

$$\gamma_2=\mu\left[-\left(\alpha^2+2\alpha\lambda+4\lambda^2\right)B^*(2\lambda)+(\alpha+2\lambda)\left(\alpha+2\lambda B^*(2\lambda+\alpha)\right)\right],$$

and

$$\varpi_2=2\lambda\left[(\alpha+2\lambda)\left(\alpha-2\lambda B^*(2\lambda)\right)+\left(2\lambda(2\lambda+\alpha)+\alpha\,\mu B^*(2\lambda)\right)B^*(2\lambda+\alpha)\right].$$

3.3. Availability for Configuration 3

We use the same procedure as above to obtain the steady-state equations as follow:

$$0=-(\lambda+2\alpha)P_4+P_3(0), \quad (50)$$

$$-\frac{d}{du}P_3(u)=-(\lambda+2\alpha)P_3(u)+(\lambda+2\alpha)b(u)P_4+b(u)P_2(0), \quad (51)$$

$$-\frac{d}{du}P_2(u)=-(\lambda+\alpha)P_2(u)+(\lambda+2\alpha)P_3(u)+b(u)P_1(0), \quad (52)$$

$$-\frac{d}{du}P_1(u)=-\lambda P_1(u)+(\lambda+\alpha)P_2(u)+b(u)P_0(0), \quad (53)$$

$$-\frac{d}{du}P_0(u)=\lambda P_1(u). \quad (54)$$

Now, from (50), we obtain:

$$P_3(0)=(\lambda+2\alpha)P_4. \quad (55)$$

Taking the LST on both sides of (51)-(54) and using (55), we obtain:

$$(\lambda+2\alpha-s)P_3^*(s)=\left(B^*(s)-1\right)P_3(0)+B^*(s)P_2(0), \quad (56)$$

$$(\lambda+\alpha-s)P_2^*(s)=(\lambda+2\alpha)P_3^*(s)+B^*(s)P_1(0)-P_2(0), \quad (57)$$

$$(\lambda-s)P_1^*(s)=(\lambda+\alpha)P_2^*(s)+B^*(s)P_0(0)-P_1(0), \quad (58)$$

$$sP_0^*(s)=P_0(0)-\lambda P_1^*(s). \quad (59)$$

Setting $s=\lambda+2\alpha$, $s=0$, $s=\lambda+\alpha$ and $s=\lambda$ in (56), respectively yields:

$$P_2(0)=\frac{1-B^*(\lambda+2\alpha)}{B^*(\lambda+2\alpha)}P_3(0)=\frac{(\lambda+2\alpha)\left(1-B^*(\lambda+2\alpha)\right)}{B^*(\lambda+2\alpha)}P_4, \quad (60)$$

$$P_3^*(0)=\frac{P_2(0)}{(\lambda+2\alpha)}=\frac{\left(1-B^*(\lambda+2\alpha)\right)}{B^*(\lambda+2\alpha)}P_4, \quad (61)$$

$$P_3^*(\lambda+\alpha)=\frac{(\lambda+2\alpha)\left(B^*(\lambda+\alpha)-B^*(\lambda+2\alpha)\right)}{\alpha B^*(\lambda+2\alpha)}P_4, \quad (62)$$

$$P_3^*(\lambda)=\frac{\left(B^*(\lambda)-1\right)P_3(0)+B^*(\lambda)P_2(0)}{2\alpha}. \quad (63)$$

Again, setting $s=\lambda+\alpha$, $s=0$ and $s=\lambda$ in (57), respectively yields:

$$P_1(0)=\frac{P_2(0)-(\lambda+2\alpha)P_3^*(\lambda+\alpha)}{B^*(\lambda+\alpha)}, \quad (64)$$

$$P_2^*(0)=\frac{(\lambda+2\alpha)P_3^*(0)+P_1(0)-P_2(0)}{(\lambda+\alpha)}, \quad (65)$$

$$P_2^*(\lambda)=\frac{(\lambda+2\alpha)P_3^*(\lambda)+B^*(\lambda)P_1(0)-P_2(0)}{\alpha}. \quad (66)$$

Likewise, setting $s=\lambda$ and $s=0$ in (58), we obtain:

$$P_0(0)=\frac{P_1(0)-(\lambda+\alpha)P_2^*(\lambda)}{B^*(\lambda)}, \quad (67)$$

$$P_1^*(0)=\frac{(\lambda+\alpha)P_2^*(0)+P_0(0)-P_1(0)}{\lambda}. \quad (68)$$

Differentiating (59) with respect to s and setting s = 0 in the result, we obtain:

$$P_0^*(0)=-\lambda P_1^{*(1)}(0). \quad (69)$$

Differentiating (56) with respect to s and then setting s = 0 in the result yields:

$$(\lambda+2\alpha)P_3^{*(1)}(0)=P_3^*(0)-\frac{\left[P_3(0)+P_2(0)\right]}{\mu}. \quad (70)$$

Differentiating (57) with respect to s and then setting s = 0 in the result, it follows that:

$$(\lambda+\alpha)P_2^{*(1)}(0)=P_2^*(0)+P_3^*(0)-\frac{\left[P_3(0)+P_2(0)+P_1(0)\right]}{\mu}. \quad (71)$$

Likewise, differentiating (58) with respect to s and setting s = 0 in the result, we find that:

$$\lambda P_1^{*(1)}(0)=P_1^*(0)+P_2^*(0)+P_3^*(0)-\frac{\left[P_3(0)+P_2(0)+P_1(0)+P_0(0)\right]}{\mu}. \quad (72)$$

Substituting (72) into (69), we have:

$$P_0^*(0)=\frac{\left[P_3(0)+P_2(0)+P_1(0)+P_0(0)\right]}{\mu}-\left(P_3^*(0)+P_1^*(0)+P_2^*(0)\right). \quad (73)$$

Now, using the normalizing condition:

$$P_4+P_3^*(0)+P_2^*(0)+P_1^*(0)+P_0^*(0)=1, \quad (74)$$

We can compute P_4 as follow:

$$P_4=\frac{-2\alpha^2\,\mu\,B^*(\lambda)B^*(\lambda+\alpha)B^*(\lambda+2\alpha)}{\psi}. \quad (75)$$

where

$$\psi=\left[(\lambda+2\alpha)\begin{pmatrix}2\alpha^2\left(-1+B^*(\alpha+\lambda)\right)\\ +\lambda B^*(\lambda)\left(2\alpha-(\alpha+\lambda)B^*(\alpha+\lambda)\right)\end{pmatrix}-\left(2(\lambda+\alpha)(\lambda+2\alpha)\right.\right.$$
$$\left.\left.\left(\alpha-\lambda B^*(\lambda)\right)+\begin{pmatrix}\lambda(\lambda+\alpha)(\lambda+2\alpha)\\ +2\alpha^2\mu B^*(\lambda)\end{pmatrix}B^*(\lambda+\alpha)\right)B^*(\lambda+2\alpha)\right].$$

The availability of configuration 3 $(Av_3(\infty))$ is given by:

$$Av_3(\infty)=P_4+P_3^*(0)+P_2^*(0)+P_1^*(0). \quad (76)$$

Using (61), (65), (68), (75) and (76), we obtain the $Av_3(\infty)$:

$$Av_3(\infty)=\frac{\gamma_3}{\varpi_3}. \quad (77)$$

where

$$\gamma_3=\mu\left[(2\alpha+\lambda)(\alpha+\lambda)\left(-2\alpha^2\begin{pmatrix}-1+B^*(\alpha+\lambda)\\ +(\alpha+\lambda)\left(2\alpha+\lambda B^*(\alpha+\lambda)\right)B^*(2\alpha+\lambda)\end{pmatrix}\right.\right.$$
$$+B^*(\lambda)\left(\begin{pmatrix}4\alpha^4+6\alpha^3\lambda\\ +7\alpha^2\lambda^2+4\alpha\lambda^3+\lambda^4\end{pmatrix}B^*(\alpha+\lambda)-2(2\alpha+\lambda)\left(\alpha^2+\alpha\lambda+\lambda^2\right)\right.$$
$$\left.\left.\left.\left(\alpha+(\alpha+\lambda)B^*(2\alpha+\lambda)\right)\right)\right)\right],$$

and

$$\varpi_3=\lambda(\alpha+\lambda)\left[(2\alpha+\lambda)\begin{pmatrix}-2\alpha^2\left(-1+B^*(\alpha+\lambda)\right)\\ +(\alpha+\lambda)\left(2\alpha+\lambda B^*(\alpha+\lambda)\right)B^*(2\alpha+\lambda)\end{pmatrix}\right.$$
$$+B^*(\lambda)\left(-2\lambda(2\alpha+\lambda)\left(\alpha+(\alpha+\lambda)B^*(2\alpha+\lambda)\right)\right.$$
$$\left.\left.+B^*(\alpha+\lambda)\left(\lambda(\alpha+\lambda)(2\alpha+\lambda)+2\alpha^2\mu B^*(2\alpha+\lambda)\right)\right)\right].$$

4. Comparison between the three Configurations

The purpose of this section is to compare $Av_i(\infty)$ for $i=1,2,3$ when $\lambda=0.001$, $\mu=0.1$, $\alpha=0.2\lambda$ and $k=1,2,3$.

Comparisons for the $Av_i(\infty)$

Three cases are illustrated in Tables 3-10.

Case 1: fix $\alpha=0.2\lambda$ and $\mu=0.1$, vary the values of $\lambda\in\{0.001,...,0.1\}$ at $k=1,2,3$.

Case 2: fix $\alpha=0.2\lambda$ and $\lambda=0.001$, vary the values of $\mu\in\{0.06,...,0.16\}$ at $k=1,2,3$.

Case 3: fix $\mu=0.1$ and $\lambda=0.001$, vary the values of $k\in\{1,...,8\}$.

Table 3. Comparison of $Av_i(\infty)$ by using three configurations when $\alpha=0.2\lambda$, $\mu=0.1$ and $k=1$

λ	$Av_1(\infty)$	$Av_2(\infty)$	$Av_3(\infty)$
0.001	0.999999	0.999991	1.000
0.002	0.999991	0.999932	1.000
0.003	0.999966	0.99977	0.999999
0.004	0.999919	0.999459	0.999995
0.005	0.999842	0.998959	0.999988
0.006	0.999727	0.998235	0.999975
0.007	0.999568	0.997255	0.999954
0.008	0.999359	0.995994	0.999922
0.009	0.999096	0.994429	0.999875
0.01	0.998771	0.992545	0.999811
0.02	0.992426	0.957671	0.998069
0.03	0.976729	0.892932	0.990726
0.04	0.951806	0.814595	0.974674
0.05	0.919313	0.735678	0.948951
0.06	0.881643	0.663011	0.91485
0.07	0.841152	0.598937	0.874934
0.08	0.799778	0.54351	0.831965
0.09	0.758932	0.495871	0.78826
0.1	0.719546	0.454912	0.745468

Table 4. Comparison of $Av_i(\infty)$ by using three configurations when $\alpha=0.2\lambda$, $\mu=0.1$ and $k=2$

λ	$Av_1(\infty)$	$Av_2(\infty)$	$Av_3(\infty)$
0.001	0.999999	0.999995	1.000
0.002	0.999995	0.999965	1.000
0.003	0.999983	0.999879	1.000
0.004	0.999958	0.99971	0.999998
0.005	0.999917	0.999432	0.999996
0.006	0.999856	0.99902	0.999991
0.007	0.99977	0.99845	0.999984
0.008	0.999655	0.9977	0.999972
0.009	0.999509	0.996749	0.999954
0.01	0.999327	0.995579	0.99993
0.02	0.995506	0.971104	0.999171
0.03	0.985057	0.918064	0.995381
0.04	0.966795	0.845963	0.985617
0.05	0.940898	0.767694	0.967553
0.06	0.908673	0.692451	0.940579
0.07	0.871984	0.624608	0.905915
0.08	0.832751	0.565343	0.865905
0.09	0.792637	0.514266	0.823133
0.1	0.75292	0.4704	0.779798

Table 5. Comparison of $Av_i(\infty)$ by using three configurations when $\alpha = 0.2\lambda$, $\mu = 0.1$ and $k = 3$

λ	$Av_1(\infty)$	$Av_2(\infty)$	$Av_3(\infty)$
0.001	0.999999	0.999996	1.000
0.002	0.999996	0.999974	1.000
0.003	0.999987	0.999908	1.000
0.004	0.999969	0.999779	0.999999
0.005	0.999937	0.999565	0.999998
0.006	0.999891	0.999245	0.999995
0.007	0.999825	0.998797	0.99999
0.008	0.999738	0.998204	0.999982
0.009	0.999625	0.997445	0.999971
0.01	0.999484	0.996503	0.999955
0.02	0.996443	0.975751	0.999439
0.03	0.987786	0.927628	0.996664
0.04	0.972029	0.858568	0.988983
0.05	0.948842	0.780833	0.973811
0.06	0.91904	0.704505	0.949846
0.07	0.884176	0.634948	0.917614
0.08	0.846067	0.573927	0.879105
0.09	0.806427	0.521301	0.836902
0.1	0.766664	0.476157	0.793417

Table 6. Comparison of $Av_i(\infty)$ by using three configurations when $\alpha = 0.2\lambda$, $\lambda = 0.001$ and $k = 1$

μ	$Av_1(\infty)$	$Av_2(\infty)$	$Av_3(\infty)$
0.06	0.999993	0.999957	1.000
0.07	0.999996	0.999973	1.000
0.08	0.999997	0.999982	1.000
0.09	0.999998	0.999987	1.000
0.1	0.999999	0.999991	1.000
0.11	0.999999	0.999993	1.000
0.12	0.999999	0.999995	1.000
0.13	0.999999	0.999996	1.000
0.14	0.999999	0.999997	1.000
0.15	1.000	0.999997	1.000
0.16	1.000	0.999998	1.000

Table 7. Comparison of $Av_i(\infty)$ by using three configurations when $\alpha = 0.2\lambda$, $\lambda = 0.001$ and $k = 2$

μ	$Av_1(\infty)$	$Av_2(\infty)$	$Av_3(\infty)$
0.06	0.999997	0.999978	1.000
0.07	0.999998	0.999986	1.000
0.08	0.999999	0.999991	1.000
0.09	0.999999	0.999993	1.000
0.1	0.999999	0.999995	1.000
0.11	0.999999	0.999996	1.000
0.12	1.000	0.999997	1.000
0.13	1.000	0.999998	1.000
0.14	1.000	0.999998	1.000
0.15	1.000	0.999999	1.000
0.16	1.000	0.999999	1.000

Table 8. Comparison of $Av_i(\infty)$ by using three configurations when $\alpha = 0.2\lambda$, $\lambda = 0.001$ and $k = 3$

μ	$Av_1(\infty)$	$Av_2(\infty)$	$Av_3(\infty)$
0.06	0.999998	0.999983	1.000
0.07	0.999998	0.99999	1.000
0.08	0.999999	0.999993	1.000
0.09	0.999999	0.999995	1.000
0.1	0.999999	0.999996	1.000
0.11	1.000	0.999997	1.000
0.12	1.000	0.999998	1.000
0.13	1.000	0.999998	1.000
0.14	1.000	0.999999	1.000
0.15	1.000	0.999999	1.000
0.16	1.000	0.999999	1.000

Table 9. Comparison of configurations 1,2,3, for $Av_i(\infty)$

Result	
$\lambda \in \{0.001,...,0.1\}, k \in \{1,2,3\}$	$Av_3(\infty) > Av_1(\infty) > Av_2(\infty)$
$\mu \in \{0.06,...,0.14\}$, at $k = 1$	$Av_3(\infty) > Av_1(\infty) > Av_2(\infty)$
$\mu \in \{0.15, 0.16\}$, at $k = 1$	$Av_3(\infty) = Av_1(\infty) > Av_2(\infty)$
$\mu \in \{0.06,...,0.11\}$, at $k = 2$	$Av_3(\infty) > Av_1(\infty) > Av_2(\infty)$
$\mu \in \{0.12,...,0.16\}$, at $k = 2$	$Av_3(\infty) = Av_1(\infty) > Av_2(\infty)$
$\mu \in \{0.06,...,0.1\}$, at $k = 3$	$Av_3(\infty) > Av_1(\infty) > Av_2(\infty)$
$\mu \in \{0.11,...,0.16\}$, at $k = 3$	$Av_3(\infty) = Av_1(\infty) > Av_2(\infty)$
$k \in \{1, 2, 3\}$	$Av_3(\infty) > Av_1(\infty) > Av_2(\infty)$
$k \in \{4, ..., 8\}$	$Av_3(\infty) = Av_1(\infty) > Av_2(\infty)$

Cost/ benefit ratio comparisons

Let $b_i = C_i / Av_i(\infty)$, where C_i be the cost of configuration i for $i = 1,2,3$ which are listed in Table 2. We compare b_i, $i = 1,2,3$, in three cases as follow:

Table 10. Comparison of b_i by using three configurations when $\alpha = 0.2\lambda$, $\mu = 0.1$ and $k = 1$

λ	b_1	b_2	b_3
0.001	2.0×10^7	1.50001×10^7	2.4×10^7
0.002	2.00002×10^7	1.50001×10^7	2.4×10^7
0.003	2.00007×10^7	1.50035×10^7	2.4×10^7
0.004	2.00016×10^7	1.50081×10^7	2.40001×10^7
0.005	2.00032×10^7	1.50156×10^7	2.40003×10^7
0.006	2.00055×10^7	1.50265×10^7	2.40006×10^7
0.007	2.00086×10^7	1.50413×10^7	2.40011×10^7
0.008	2.00128×10^7	1.50603×10^7	2.40019×10^7
0.009	2.00181×10^7	1.5084×10^7	2.4003×10^7
0.01	2.00246×10^7	1.51127×10^7	2.40045×10^7
0.02	2.01526×10^7	1.5663×10^7	2.40464×10^7
0.03	2.04765×10^7	1.67986×10^7	2.42247×10^7
0.04	2.10127×10^7	1.84141×10^7	2.46236×10^7
0.05	2.17554×10^7	2.03894×10^7	2.52911×10^7
0.06	2.26849×10^7	2.26241×10^7	2.62338×10^7
0.07	2.37769×10^7	2.50444×10^7	2.74307×10^7
0.08	2.50069×10^7	2.75984×10^7	2.88474×10^7
0.09	2.63528×10^7	3.02498×10^7	3.04468×10^7
0.1	2.77953×10^7	3.29734×10^7	3.21946×10^7

Table 11. Comparison of b_i by using three configurations when $\alpha = 0.2\lambda$, $\mu = 0.1$ and $k = 2$

λ	b_1	b_2	b_3
0.001	2.0×10^7	1.50001×10^7	2.4×10^7
0.002	2.00001×10^7	1.50005×10^7	2.4×10^7
0.003	2.00003×10^7	1.50018×10^7	2.4×10^7
0.004	2.00008×10^7	1.50044×10^7	2.4×10^7
0.005	2.00017×10^7	1.50085×10^7	2.40001×10^7
0.006	2.00029×10^7	1.50147×10^7	2.40002×10^7
0.007	2.00046×10^7	1.50233×10^7	2.40004×10^7
0.008	2.00069×10^7	1.50346×10^7	2.40007×10^7
0.009	2.00098×10^7	1.50489×10^7	2.40011×10^7
0.01	2.00135×10^7	1.50666×10^7	2.40017×10^7
0.02	2.00903×10^7	1.54463×10^7	2.40199×10^7
0.03	2.03034×10^7	1.63387×10^7	2.41114×10^7
0.04	2.06869×10^7	1.77313×10^7	2.43502×10^7
0.05	2.12563×10^7	1.9539×10^7	2.48048×10^7
0.06	2.20101×10^7	2.16622×10^7	2.55162×10^7
0.07	2.29362×10^7	2.40151×10^7	2.64926×10^7
0.08	2.40168×10^7	2.65326×10^7	2.77167×10^7
0.09	2.52322×10^7	2.91678×10^7	2.91569×10^7
0.1	2.65633×10^7	3.18877×10^7	3.07772×10^7

Table 12. Comparison of b_i by using three configurations when $\alpha = 0.2\lambda$, $\mu = 0.1$ and $k = 3$.

λ	b_1	b_2	b_3
0.001	2.0×10^7	1.50001×10^7	2.4×10^7
0.002	2.00001×10^7	1.50004×10^7	2.4×10^7
0.003	2.00003×10^7	1.50014×10^7	2.4×10^7
0.004	2.00006×10^7	1.50033×10^7	2.4×10^7
0.005	2.00013×10^7	1.50065×10^7	2.40001×10^7
0.006	2.00022×10^7	1.50113×10^7	2.40001×10^7
0.007	2.00035×10^7	1.50181×10^7	2.40002×10^7
0.008	2.00053×10^7	1.5027×10^7	2.40004×10^7
0.009	2.00075×10^7	1.50384×10^7	2.40007×10^7
0.01	2.00103×10^7	1.50526×10^7	2.40011×10^7
0.02	2.00714×10^7	1.53728×10^7	2.40135×10^7
0.03	2.02473×10^7	1.61703×10^7	2.40803×10^7
0.04	2.05755×10^7	1.74709×10^7	2.42674×10^7
0.05	2.10783×10^7	1.92103×10^7	2.46454×10^7
0.06	2.17618×10^7	2.12915×10^7	2.52673×10^7
0.07	2.26199×10^7	2.3624×10^7	2.61548×10^7
0.08	2.36388×10^7	2.61357×10^7	2.73005×10^7
0.09	2.48008×10^7	2.87741×10^7	2.86772×10^7
0.1	2.6087×10^7	3.15022×10^7	3.02489×10^7

Table 13. Comparison of b_i by using three configurations when $\alpha = 0.2\lambda$, $\lambda = 0.001$ and $k = 1$

μ	b_1	b_2	b_3
0.06	2.00001×10^7	1.50006×10^7	2.4×10^7
0.07	2.00001×10^7	1.50004×10^7	2.4×10^7
0.08	2.00001×10^7	1.50003×10^7	2.4×10^7
0.09	2.0×10^7	1.50002×10^7	2.4×10^7
0.1	2.0×10^7	1.50001×10^7	2.4×10^7
0.11	2.0×10^7	1.50001×10^7	2.4×10^7
0.12	2.0×10^7	1.50001×10^7	2.4×10^7
0.13	2.0×10^7	1.50001×10^7	2.4×10^7
0.14	2.0×10^7	1.50001×10^7	2.4×10^7
0.15	2.0×10^7	1.5×10^7	2.4×10^7
0.16	2.0×10^7	1.5×10^7	2.4×10^7

Table 14. Comparison of b_i by using three configurations when $\alpha = 0.2\lambda$, $\lambda = 0.001$ and $k = 2$.

μ	b_1	b_2	b_3
0.06	2.00001×10^7	1.50003×10^7	2.4×10^7
0.07	2.0×10^7	1.50002×10^7	2.4×10^7
0.08	2.0×10^7	1.50001×10^7	2.4×10^7
0.09	2.0×10^7	1.50001×10^7	2.4×10^7
0.1	2.0×10^7	1.50001×10^7	2.4×10^7
0.11	2.0×10^7	1.50001×10^7	2.4×10^7
0.12	2.0×10^7	1.5×10^7	2.4×10^7
0.13	2.0×10^7	1.5×10^7	2.4×10^7
0.14	2.0×10^7	1.5×10^7	2.4×10^7
0.15	2.0×10^7	1.5×10^7	2.4×10^7
0.16	2.0×10^7	1.5×10^7	2.4×10^7

Table 15. Comparison of b_i by using three configurations when $\alpha = 0.2\lambda$, $\lambda = 0.001$ and $k = 3$.

μ	b_1	b_2	b_3
0.06	2.0×10^7	1.50002×10^7	2.4×10^7
0.07	2.0×10^7	1.50002×10^7	2.4×10^7
0.08	2.0×10^7	1.50001×10^7	2.4×10^7
0.09	2.0×10^7	1.50001×10^7	2.4×10^7
0.1	2.0×10^7	1.50001×10^7	2.4×10^7
0.11	2.0×10^7	1.5×10^7	2.4×10^7
0.12	2.0×10^7	1.5×10^7	2.4×10^7
0.13	2.0×10^7	1.5×10^7	2.4×10^7
0.14	2.0×10^7	1.5×10^7	2.4×10^7
0.15	2.0×10^7	1.5×10^7	2.4×10^7
0.16	2.0×10^7	1.5×10^7	2.4×10^7

Numerical results of the b_i for configuration $i(i=1,2,3)$ are calculated in Tables 10-15 for three cases, respectively. Tables 10-12 show that the b_i increases as λ or α increases for any configuration. On the other hand, Tables 13-15 show that the b_i decreases as μ increase for any configuration. One observes from Tables 10-12 that optimal configuration using b_i value depends on the value of λ. When $\lambda \in \{0.001,...,0.06\}$ at $k \in \{1,2,3\}$, the optimal configuration is configuration 2, but when $\lambda \in \{0.07,...,0.09,0.1\}$ at $k \in \{1,2,3\}$, the optimal configuration is configuration 1. It appears from Tables 13-15 that the optimal configuration using the b_i value does not depend on the value of μ. We see from Tables 13-15 that the optimal configuration using the b_i value is configuration 2.

5. Conclusions

In this paper, we constructed three different series system configurations with mixed standby components and *k*-stage Erlang repair times to study the b_i analysis of three configurations. We developed the explicit expressions for the steady-state availability, $Av_i(\infty)$, for three configurations and performed a comparative analysis for them. We rank three configurations based on the $Av_i(\infty)$, and the b_i. Using the b_i measure, the optimal configuration depends on the values of λ and α but does not depend on the value of μ.

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