Shear and Longitudinal Modulus of Elasticity in Structural Lumber Beams

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Abstract   This study aimed to present, with the aid of the four points static bending, conducted nondestructively, analytical methodology to determine the longitudinal (E) and shear modulus (G) of elasticity in lumber beams. The woods used were Manilkara spp and Pinus elliottii. Were used three different values to the form factor coefficient of the rectangular cross section, allowing evaluating the differences between the shear stiffness values obtained by wood species. The results of the analysis of variance indicated statistical equivalence between shear modulus of elasticity for the two wood species investigated, unless revealing significant the influence of the form factors used to determine the shear modulus of elasticity. The coefficients of the relationship between the modulus of elasticity ($E=\lambda \cdot G$) obtained from the least squares method were equal to 37 and 33 to the woods Manilkara spp and Pinus elliottii, be revealing 85% and 65% higher respectively then the relationship ($E=20 \cdot G$) presented in the Brazilian standard ABNT NBR 7190. It is emphasized that these results may be different for the same or different wood species, justifying the use of this methodology in each work developed.

Keywords   Lumber, Beams theory, Shear and longitudinal modulus of elasticity

1. Introduction

Among the fundamentals properties required for the design of a structure highlights the modulus of elasticity. In the case of wood, because it is a natural material of great complexity and anatomical variability, with three axes of symmetry in the radial, tangential and longitudinal, obtaining all their elastic parameters is compromised[1]. However, structural designs are made with possess of their equivalents mechanical properties, resulting from experimental tests standardized by regulatory codes aimed at quantifying these variables due to mechanical stress conditions.

In a project of beams, as well as other structural elements, knowledge of the longitudinal (E) and shear (G) modulus of elasticity is of fundamental importance[2]. In Brazil, the characterization of wood in bending is done according to the Brazilian standard ABNT NBR 7190[3] (Design of Wooden Structures), which use of the three points static bending, restricted to small samples and free from defects, having an empirical relationship for obtaining the shear modulus

known the value of the longitudinal modulus, being $G=E/20$.

In Brazil, researches involving the characterization of pieces of wood with structural dimensions follow the assumptions and calculation methods contained in international standards, which may be mentioned the works of Pigozzo et al.[4], Fiorelli et al.[5] and Miotto and Dias[6].

Nondestructive testing methods have been widely used to obtain the longitudinal elastic modulus in wooden beams, emphasizing the use of ultrasound and transverse vibration, emphasizing simplicity and efficiency of the use of such methodologies, justified by the possibility of use the peace after tested[7-14].

With respect to the determination of the shear modulus of elasticity in wooden beams as in specimens of small dimensions is little research developed among them, highlighting the work of Rocco Lahr[15], Burdzik and Nkwera[16], Zangiácomo and Rocco Lahr[17] and Christoforo et al.[18].

Rocco Lahr[15] evaluated, among others[16, 17], the influence of the dimensions of the specimens of lumber for which the effect of shear forces becomes negligible in the calculation of displacements, reaching the ratio $L/h \geq 21$, where L is the useful length and h is the height of the cross section of the specimen.

Christoforo et al.[18] developed, with the aid of the three-points static bending, conducted nondestructively,

analytical methodology to determine the longitudinal (E) and the shear modulus (G) of elasticity in round timber beams, using Eucalyptus wood clone. They used three different values to the form factor coefficient of the circular cross section, allowing evaluating the differences between the shear stiffness values obtained. The results indicated no statistical equivalence between the shear modulus of elasticity, revealing be significant the influence of the form factors used to determine the shear modulus of elasticity. The coefficient (λ) of the relationship between the modulus of elasticity (E=$\lambda \cdot$G) obtained from the least squares method were equal to 118, revealing 5.9 higher than the relationship (E=20$\cdot$G) presented in the Brazilian standard ABNT NBR 7190:1997. It is emphasized that these results may be different for the same or different wood species, justifying the use of this methodology in each research developed.

Christoforo et al.[19] presented an analytical method to calculate the longitudinal and shear modulus of structural lumber, using the three points static bending. The wood used in the experiments were *Pinus elliottii* and *Corymbia citriodora*. The equations for the calculation of the elastic moduli were developed by the least squares method, and the shape of the shear coefficient for rectangular cross section was adopted as 0.666. The results of the coefficients between the longitudinal and shear modulus of elasticity for the woods *Pinus elliottii* and *Corymbia citriodora* were equal to 18.70 and 21.20, being proximate to the coefficient (20) established by the Brazilian standard ABNT NBR 7190[3].

This paper, based on the work of Christoforo et al.[18] and Christoforo et al.[19], aimed to present, with the aid of the theory of Timoshenko beams, an analytical methodology for obtaining the longitudinal and shear modulus of elasticity in structural lumber beams, with the aid of the least squares method and the four points static bending, also investigating the influence of the form factor in the calculation of the shear modulus of elasticity.

2. Materials and Methods

The experimental methodology developed for calculate the modulus of elasticity E and G of structural lumber was based on work from Rocco Lahr[15], as also done in the work of Christoforo et al.[18]. The modulus of elasticity was obtained from geometric and linearity condition, with the largest displacement in the experiments was limited to L/200, measurement of small displacements defined by Brazilian standard[3].
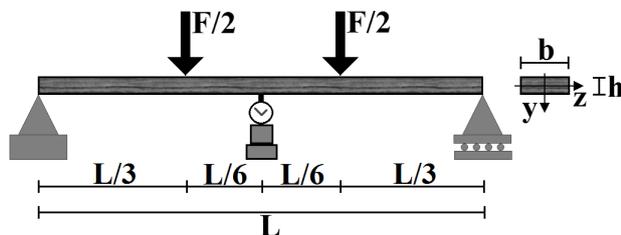

Figure 1. Four points static bending

The Virtual Force Method (VFM) was employed on the structural model of the four-point bending (Figure 1), aiming to find an expression for the calculation of the displacement at the midpoint of the beam (δ), considering the bending moment and shear efforts. Note that the four points static bending static bending is adopted by American Standard ASTM D-198[20] to determine the modulus of elasticity in bending.

Generally, when considering only bending and shearing efforts, the displacement on a point of interest to a structure consisting of bar elements is obtained by Equation 1 (MFV), wherein:

$$1 \cdot \delta = \sum_{i=1}^{n} \left(\int_{\Omega_i} \frac{M(x) \cdot m(x)}{E \cdot I} dx + \int_{\Omega_i} \frac{f_s \cdot Q(x) \cdot q(x)}{G \cdot A} dx \right) \quad (1)$$

δ- Linear displacement or rotation to be calculated by the use of force or virtual moment;

$M(x)$ - variation of bending moment for a slice of the structure according to the actual load history;

$m(x)$ - variation of bending moment for a slice of the structure according to the employment of a force or unit moment applied at a point of interest;

$Q(x)$ - variation of shear for a slice of the structure according to the actual load history;

$q(x)$ - variation of shear for a slice of the structure according to the employment of a force or unit moment applied at a point of interest;

f_s - form factor of the cross section (depending on the geometry of the cross section);

Ω - domain of integration;

E - longitudinal modulus of elasticity or Young's modulus;

I - moment of inertia of the cross section;

G - shear modulus of elasticity;

A - cross-sectional area;

L - length of the beam.

Using Equation 1 in the structural model adapted to the four points static bending (Figure 1), the displacement at the midpoint is expressed by Equation 2, where b and h are respectively the width and height of rectangular cross-section of the beam.

$$\delta = \frac{23 \cdot F \cdot L^3}{108 \cdot E \cdot b \cdot h^3} + \frac{F \cdot L \cdot f_s}{6 \cdot b \cdot h \cdot G} \quad (2)$$

According to the present methodology for calculating the moduli of elasticity are necessary the execution of two successive experimental tests on the same element. At first, assuming the L_1 length of the element, with $L_1/h \geq 21$[15], determines the value of the F_1 force responsible for causing a shift of $\delta_1 = L_1/200$. In the second trial bending the supports were approximate, giving a new useful length (L_2), and must respect the inequality $L_2/h \geq 5/4$[21], ensuring that the sections remain flat after deformed, obtaining a value of force (F_2) responsible for causing a displacement equal to $\delta_2 = L_2/200$.

The employment of F_1 and F_2 forces, L_1 and L_2 measures and displacements δ_1 and δ_2 obtained in trials in Equation 2 leads to a system with two equations in two unknowns, whose solution provides the longitudinal and shear modulus of elasticity of structural lumber beams, expressed respectively by Equations 3 and 4.

$$E = \frac{23 \cdot L_1 \cdot L_2 \cdot F_1 \cdot F_2 \cdot (L_2^2 - L_1^2)}{108 \cdot b \cdot h^3 \cdot (F_1 \cdot L_1 \cdot \delta_2 - F_2 \cdot L_2 \cdot \delta_1)} \qquad (3)$$

$$G = \frac{f_s \cdot L_1 \cdot L_2 \cdot F_1 \cdot F_2 \cdot (L_1^2 - L_2^2)}{6 \cdot b \cdot h \cdot (F_1 \cdot L_1^3 \cdot \delta_2 - F_2 \cdot L_2^3 \cdot \delta_1)} \qquad (4)$$

Equations 3 and 4 can be adapted for measurements of displacements $\delta_1 = L_1/200$ and $\delta_2 = L_2/200$, as well as expressing equations 5 and 6.

$$E = \frac{1150 \cdot F_1 \cdot F_2 \cdot (L_2^2 - L_1^2)}{27 \cdot b \cdot h^3 \cdot (F_1 - F_2)} \qquad (5)$$

$$G = \frac{100 \cdot F_1 \cdot F_2 \cdot f_s \cdot (L_1^2 - L_2^2)}{3 \cdot b \cdot h \cdot (F_1 \cdot L_1^2 - F_2 \cdot L_2^2)} \qquad (6)$$

The equations 5 and 6 used in obtaining the modulus of elasticity not take into consideration the own weight of the piece. However, Christoforo et al.[18] proved to be negligible the influence of own weight in the calculation of displacements, validating the methodology here presented.

The proposed methodology was used in wooden beams of the *Pinus elliottii* and *Manilkara* spp. (Maçaranduba) species, provided by MJ Wood Company of São Carlos (SP-Brazil), tested in the position of least inertia (flatwise) and positioned in the Longitudinal and Radial plane (LR). For each species of wood were manufactured twelve specimens of dimensions 140×5×3cm.

The lengths used in bending tests were L_1=130cm and L_2=80cm, obtained from each specimen for each species the values of the forces responsible for causing the displacements of 0.65cm ($L_1/200$) and 0.40cm ($L_2/200$).

The form factor (f_s) in the present calculation of shear modulus of elasticity is a constant that depends on the geometry of the cross section of the peace. In literature, for square or rectangular cross section, some authors have different values of form factors. In order to evaluate the influence of employment of the coefficient in order to calculate the shear modulus of elasticity, these were varied, assuming values: 0.667, 0.822 and 0.833, respectively obtained from the works of Timoshenko[22], Mindlin and Deresiewicz[23] and Roark[24].

To check the influence of the shape coefficient for calculating the shear modulus was used the analysis of variance (ANOVA), with proven assumptions of residuals normality, homogeneity and independence with the normality test of Anderson-Darling, with the residual plots versus fitted values and residuals versus order respectively by response investigated, both with the aid of the software Minitab® version 14.

In order to relate the values of the elastic moduli E and G for the species evaluated and later compared the relationship defined by the Brazilian standard ABNT NBR 7190[3], has been used the least squares method[18, 25], expressed by Equation 7, λ is the coefficient to be adjusted to the lesser residue ($E=\lambda \cdot G$).

$$f(\alpha) = \frac{1}{2} \cdot \sum_{i=1}^{n} \left(E_i - \lambda \cdot G_i \right)^2 \qquad (7)$$

3. Results

Tables 1 and 2 shows the mean values (X_m), standard deviations (SD) and coefficients of variation (CV) of the longitudinal and shear modulus of elasticity for the woods *Manilkara* spp and *Pinus elliottii* respectively.

**Table 1.** Modulus of elasticity of *Manilkara* spp wood species

		f_s (0,667)	f_s (0,822)	f_s (0,833)
Sp.	E (MPa)	G (MPa)	G (MPa)	G (MPa)
1	19922	274	337	342
2	15524	428	528	535
3	19121	187	231	234
4	20551	294	363	368
5	14424	513	632	641
6	16767	509	627	636
7	15212	301	371	376
8	15824	293	361	366
9	20356	351	433	439
10	17463	486	599	607
11	14578	419	517	524
12	17504	390	480	487
X_m	17271	370	457	463
SD	2253	104	128	130
CV (%)	13	28	28	28

Table 2. Modulus of elasticity of *Pinus elliottii* wood species

		f_s (0,667)	f_s (0,822)	f_s (0,833)
Sp.	E (MPa)	G (MPa)	G (MPa)	G (MPa)
1	9099	300	370	375
2	11330	261	321	326
3	11033	131	161	163
4	10680	348	428	434
5	13191	156	192	195
6	11205	351	432	438
7	10940	269	331	336
8	9321	219	270	274
9	10224	138	170	172
10	10429	392	483	490
11	9571	223	275	279
12	10441	263	323	328
X_m	10622	254	313	318
SD	1090	85	105	107
CV (%)	10	34	34	34

Figures 2 and 3 illustrate the normality plot of Anderson-Darling test for the modulus of elasticity of the

timbers *Manilkara* spp and *Pinus elliottii* respectively, showing normal distributions for all responses by the P-values found are greater than 0.05 (5%).


(a)

(b)
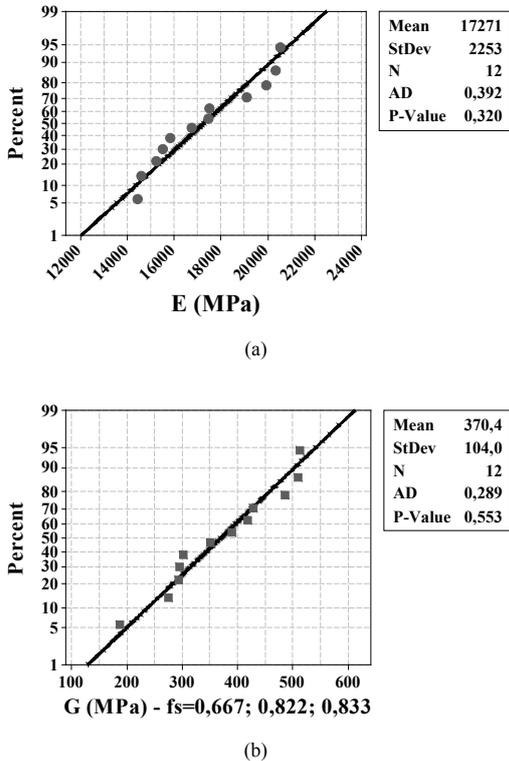
Figure 2. Normality plot of modulus of elasticity of *Manilkara* spp wood species

(a)

(b)
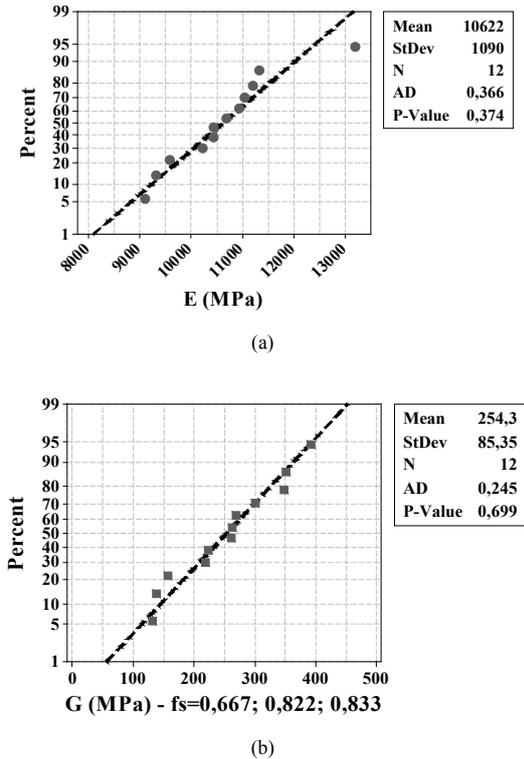
Figure 3. Normality plot of modulus of elasticity of *Pinus elliottii* wood species

The P-value of the ANOVA on the influence of form factors in calculating the shear modulus of elasticity was equal to 0.129, an adjusted coefficient of determination (R^2(Adj.)) equal to 6.32%. By P-value is greater than 0.05, it appears to be equivalent the shear modulus of elasticity for *Manilkara* spp wood species. Figure 4 illustrates the normality plot of the residuals (4a), homogeneity (4b) and independence (4c).

(a)

(b)

(c)
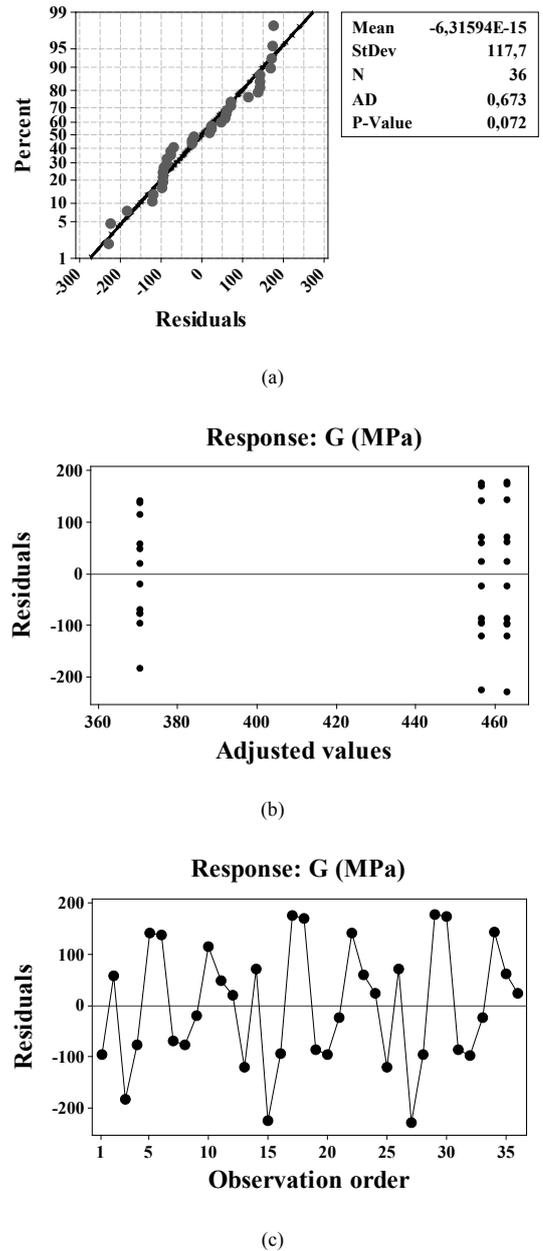
Figure 4. ANOVA validation of the form factor on the shear modulus of elasticity of *Manilkara* spp wood species

The P-value of the ANOVA on the influence of form factors in calculating the shear modulus of elasticity was equal to 0.236, an adjusted coefficient of determination (R^2(Adj.)) equal to 2.83%. By P-value is greater than 0.05, it appears to be equivalent the shear modulus of elasticity for *Pinus elliottii* wood species. Figure 5 illustrates the

normality plot of the residuals (5a), homogeneity (5b) and independence (5c).
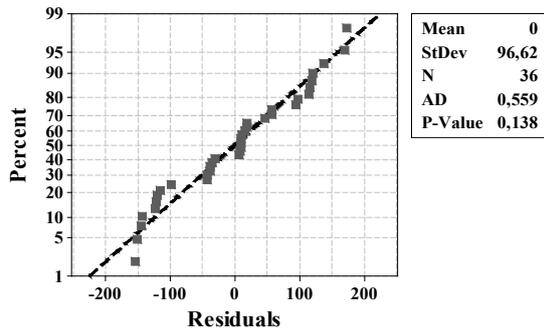
(a)
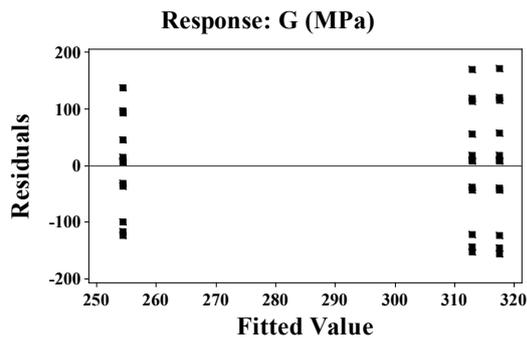
(b)
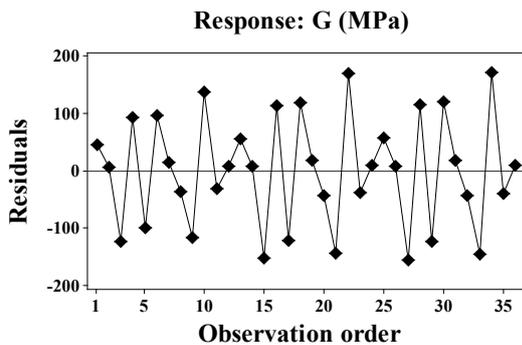
(c)

Figure 5. ANOVA validation of the form factor on the shear modulus of elasticity of *Pinus elliottii* wood species

Table 3. Ratios between the longitudinal and shear modulus of elasticity for the wood species

	Manilkara spp		
	$E=\lambda \cdot G$ ($f_s = 0{,}667$)	$E=\lambda \cdot G$ ($f_s = 0{,}822$)	$E=\lambda \cdot G$ ($f_s = 0{,}833$)
λ	43	35	34
	Pinus elliottii		
	$E=\lambda \cdot G$ ($f_s = 0{,}667$)	$E=\lambda \cdot G$ ($f_s = 0{,}822$)	$E=\lambda \cdot G$ ($f_s = 0{,}833$)
λ	38	31	30

Table 3 shows the results of the λ coefficients obtained by the least square method of the relationship between the shear and longitudinal modulus of elasticity of the wood species investigated.

The results of the average values of the coefficients between the longitudinal and shear modulus of elasticity for *Manilkara* spp and *Pinus elliottii* timber beams were respectively equal to 37 and 33, showing be 85% and 65% greater than the coefficient established between the modulus of elasticity by Brazilian standard ABNT 7190[3], implying shear modulus of elasticity 46 and 39% lower than shown by the this standard.

The ratio between the longitudinal and shear modulus of elasticity found in this work for the *Pinus elliottii* (33) wood were 76.47% higher than the correlation coefficient between modules of *Pinus elliottii* (18.70) wood obtained from the work of Christoforo et al.[19], justified by the anisotropy of wood, local extraction of woods among others.

4. Conclusions

The results of the longitudinal and shear modulus of elasticity to the wood species investigated proved to be independent the choice of the form coefficients of cross section, being equivalent in both cases.

The average values of the coefficients of the relationship between the modulus of elasticity for the wood species were significantly different (higher) than the value set by the Brazilian standard[3], being the least of them, arising from Pinus elliottii wood, but still 65% higher, providing values of shear modules below the Brazilian standard.

For the anisotropy of wood combined with physical and chemical factors, the obtained results should not be extrapolated to the same or different woods species, thereby justifying the use of this calculation method developed in each study.

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