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# Estimation of Complex and Linear Uncertainities in S-Parameter Measurements for Metrology Applications

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**Abstract** *The present paper aims to develop a uniform procedure of estimating uncertainty components in VNA measurements whether in complex or linear units. The individual response of each uncertainty components have been studied in the frequency range 1 to 18 GHz, which are applicable for one-port and two-port measurements.* The Vector network analyser (VNA) measurements are performed to assign an overall uncertainty for the respective measuring parameter in terms of complex and linear units for coaxial step attenuator, fixed attenuator and mismatch. These measurements are then verified through the primary and transfer standards of the attenuation and impedance parameters and thus the traceability of the VNA measurements is established. Finally, the outcome of complete study has been presented as VNA measurements based new calibration and measurement capabilities (CMCs) for NPL, India. *It has shown that the final combined uncertainty is found same or nearby by obtaining from uncertainty components either in complex or in linear units. Thus, this paper reports the estimation of VNA measurement uncertainties for various parameters as per the requirements of ISO/IEC 17025:2005 standard.*

**Keywords** Complex S-Parameter, Attenuation, Impedance, Traceability, Calibration Standards, VNA

## 1. Introduction

Today, the broadband measurement of microwave parameters is carried out in terms of complex S-parameters using a vector network analyzer (VNA) at radio and microwave frequencies. A VNA characterizes the behaviour of linear networks quickly, accurately, and completely over broad frequency ranges by measuring its transmission and reflection coefficients in terms of scattering parameters or S-parameters of the device-under-test (DUT). From the measured S-parameters which can be represented by a number of different measurement parameters and units, one can easily deduce a number of microwave parameters in any of form as given in Table1. However, the magnitude and phase components are required for the complete characterization of a linear network and thus one can able to ensure a distortion-free transmission through the network at RF and microwave range. The complex number or vector format is most accurate as it deals with the phases, however a linear format for reflection coefficient or VSWR is required in the reflection measurements. As a range of measuring parameters are expressed in various units, the associated uncertainty should also be evaluate and expressed in the same unit to estimate more reliable uncertainty.

A complete VNA system consists of VNA, calibration and verification kits along with cables and adaptors. The VNA is calibrated against a set of known standards to minimize the measurement uncertainties, which also called the “vector error correction” process[1-4]. In the last few decades, a number of calibration techniques have been realized and implemented to calibrate VNA namely Short-Open-Load-Thru (SOLT), Thru- Reflect- Line (TRL), Thru- Short- Delay (TSD), Line- Reflect- Line (LRL), OSLT (Offset Short -Load -Thru) etc.[5-10]. The impact of each standard’s uncertainty depends on the calibration technique used, stability and repeatability of the system and residual post calibration errors. The quality of calibration also depends on operator experience and random effects such as system sensitivity limits, noise, connector repeatability, etc. The influences of the non-ideal calibration standards on the complex S-parameters measurement in the real/imaginary and magnitude/phase formats and sensitivity coefficients for various calibration techniques have been analyzed and on their uncertainties have been studied[11-14]. In the previous studies, the SOLT and TRL calibration techniques are emerged the best for accurate, most reliable and traceable measurements in general.

***For calibration laboratories, the use of this versatile system for metrology purpose needs special attention as per requirements of ISO/IEC 17025:2005. The international***

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***and national calibration labs have established and still improving the methods of VNA uncertainty estimation and establishing traceability. However, a uniform and widely acceptable realization of such methods has to be implemented in the national laboratories to compare the measurement compatibility more perfectly. In India, VNAs are being utilized as a calibration set up in many government and industrial laboratories for the range of 10 MHz to 20 GHz and installation of 40 GHz VNA is under process at many places. Being a National metrology Institute (NMI) of India, NPL has started the work to establish the traceability of VNA measurements as well as to traceable calibration facility for VNA system using the primary and transfer standards[15-16]. There is still a large gap between NMI and other level-II laboratories in evaluation of uncertainty in VNA measurements and establishing its traceability in accordance with ISO/IEC 17025:2005. For bridging this gap, this paper presents the steps and methods for the utilization of VNA system to calibrate of one-port and two-port components and in the metrology applications. The individual uncertainty components are estimated and evaluated in complex and linear formats as that of measurand. The possibility of getting the same uncertainty value will also be explored, while estimating the uncertainties using various forms for the single measurement value.***

## 2. Uncertainty Contributors in VNA Measurements

The basic details and importance of major uncertainty components applicable for VNA measurements have been studied earlier[17-22]. A number of uncertainty models have been developed to estimate the VNA measurement uncertainty and to establish its traceability[23-27]. Generally, the uncertainty contributors can be categorized in three types for VNA measurements, namely systematic, random and drift errors. The directivity, test port match (or source match for two-port), load match, isolation (or RF leakage), frequency tracking are the systematic error contributors. Such contributors can be effectively removed by perfect calibration of VNA to obtain a correct value. However, due to imperfections of calibration standards, these error contributors are considered as residual systematic errors for Type-B. In the second category, random contributors consist of system repeatability (resolution and noise), connector repeatability (Type A to cover gaps at the connector interface, slots in female connector) etc. Drifts due to signal source (frequency and power stability), instrument (any physical changes between calibration and measurement states), temperature, cable flexure etc. are belong to the drift error contributors. Some of these terms can be minimized by careful control and use of system and calibration kits. Error due to temperature drift can be reduced significantly in a stable and controlled environment.

We have adopted the uncertainty expressions and methodology given in the references[27-28] have been adopted due to their easier implementation and well suited according to ISO standards and guidelines[29-32]. In the present study, frequency range 1 to 18 GHz is divided in three sub-ranges 1-8 GHz, 8-12 GHz and 12-18 GHz for to evaluate contributors. The measurements are performed for Type N connectors by making Port 1 male and Port 2 female for VNA Wiltron 37247B using full-port SOLT technique.

For uncertainty conversion form logarithmic value to linear value and vice versa for any measured S-parameter, the following expressions were used,

$$unc(lin) = 1 - 10^{(-unc(dB)/20)} \tag{1}$$

$$unc(dB) = 20\log_{10}(1 - unc(lin)) \tag{2}$$

**Table 1.** Transmission and Reflection parameters as Measurand on VNA system

| Type of measurements | Log magnitude and phase | Lin magnitude and phase | Real and imaginary |
|---|---|---|---|
| Reflection measurement | Return loss<br>RL= -20 LOG (Snn) | Reflection coefficient, \|Snn\| | Separation of complex reflection component<br>Snn= Xnn+ j Ynn |
| | VSWR<br>=(1+Snn)/(1-Snn) | Reflection phase, φ | |
| Transmission measurement | Insertion loss (/Gain)<br>= -10 LOG ($Smn^2$/(1-$Snn^2$)) | Transmission coefficient, \|Smn\| | Separation of complex transmission Component |
| | Attenuation<br>A= -20 LOG (Smn)<br>where m=1,2., n=1,2.. | Transmission phase, θ | Smn= Xmn+ j Ymn |

### 2.1. Evaluation of Systematic Error Contributors

To evaluate the effective directivity, test port match (or source match) and load match, the methods given in the references[27-28]. The evaluation of these three quantities utilizes the airline and calibration kit components and thus propagates the uncertainties of these standards to the measurements in the uncertainty budgets. The formula for evaluating the effective directivity in linear magnitude is given below,

$$D = r_1/2 \tag{3}$$

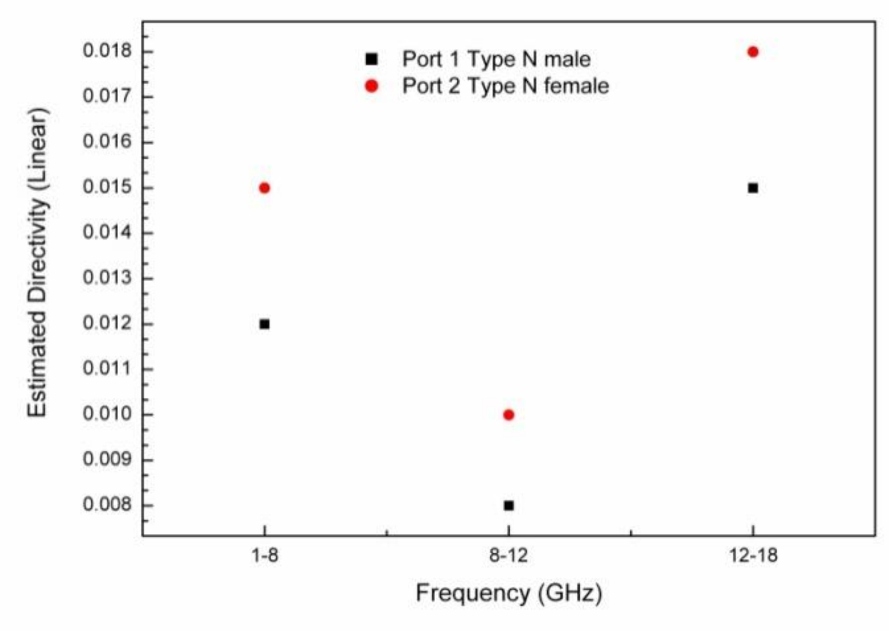
where $r_1$ = maximum ripple amplitude when airline terminated with a fixed load is connected to test port.

The impact of effective directivity is same and is independent of type of representations of measurement quantity as shown in Figure 1(a) and (b). However, at the ends of operational frequency range, the values are higher. This component is dominant and directly governs the combined uncertainty in the reflection measurement.

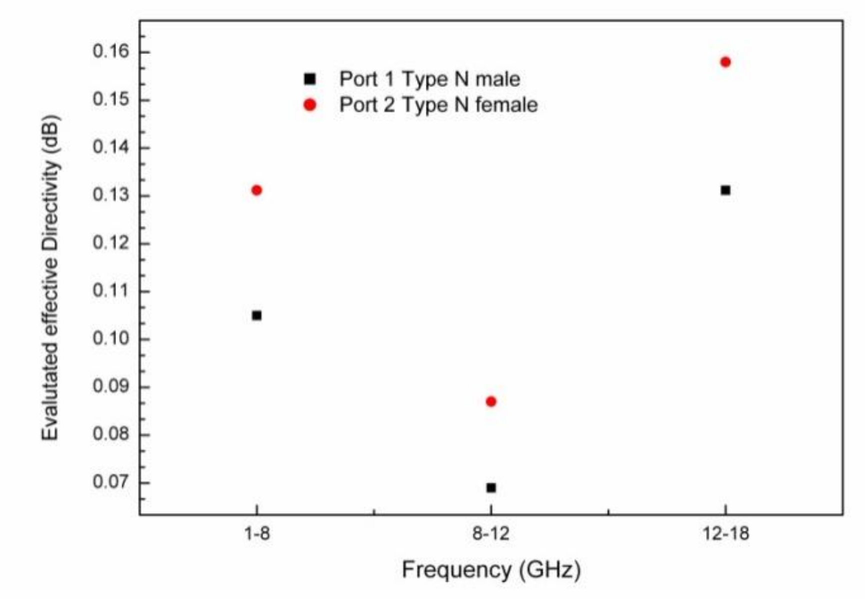
The effective test port match is evaluated from the equation (4).

$$M = r_2/2 \tag{4}$$

where $r_2$ = half of the maximum ripple amplitude when airline terminated with a short and load respectively is connected to test port.

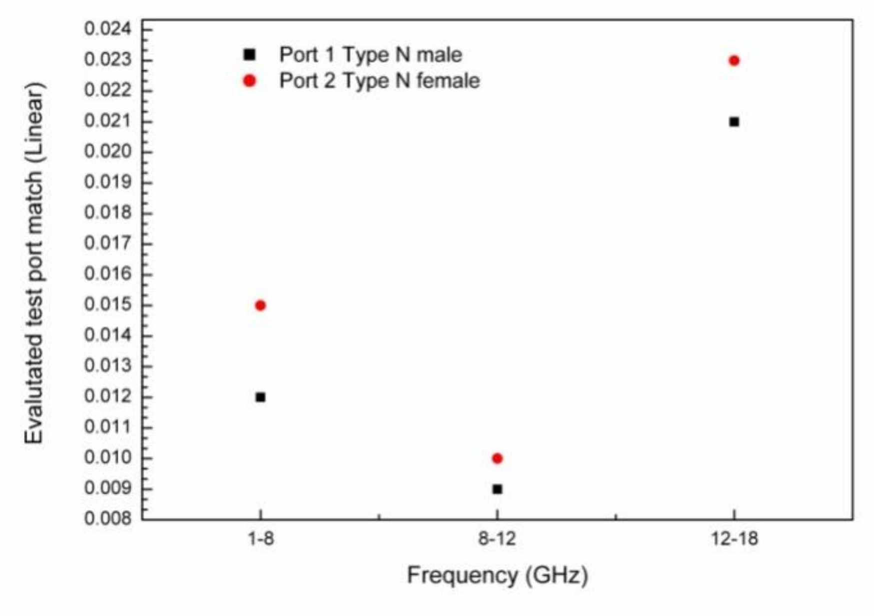



(a)

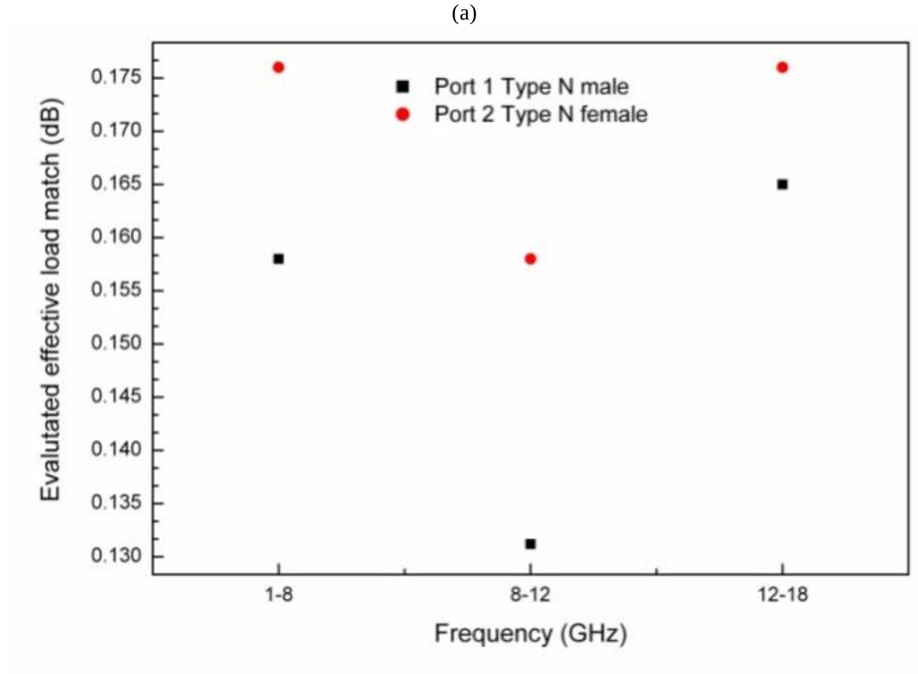



(b)

**Figure 1.** Evaluated effective directivity (a) in linear format, (b) in log format




(a)




(b)

**Figure 2.** Evaluated effective test port match (or source match) (a) in linear format, (b) in log format

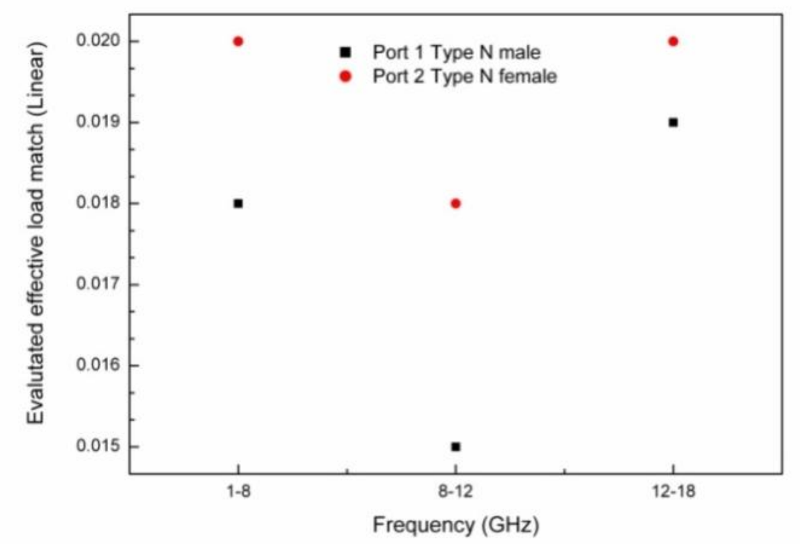
From Figure 2(a) and (b), we found that the evaluated effective test port is high at higher frequency band and minimum at mid-band frequencies. For effective load match, the reflection coefficient of the other test port is determined after full 12- term calibration by measuring through the calibrated port 1. Effective load match can be represented by the uncertainty estimate of this reflection coefficient at port 2. The same technique can be applied for estimation of the source match while the port 2 is calibrated. The values of effective load match are given in Figure 3 (a) and (b).

The reflection and transmission tracking are evaluated and presented in Figure 4 (a) and (b)[27-28]. Reflection tracking response is random and it is dependent on connector type, type of measurement unit and frequency range. However for port 2, the response is different irrespective of unit used. The transmission tracking is comparatively constant with respect to frequency of operation, measurement unit and showed a little variation for change in the signal direction.

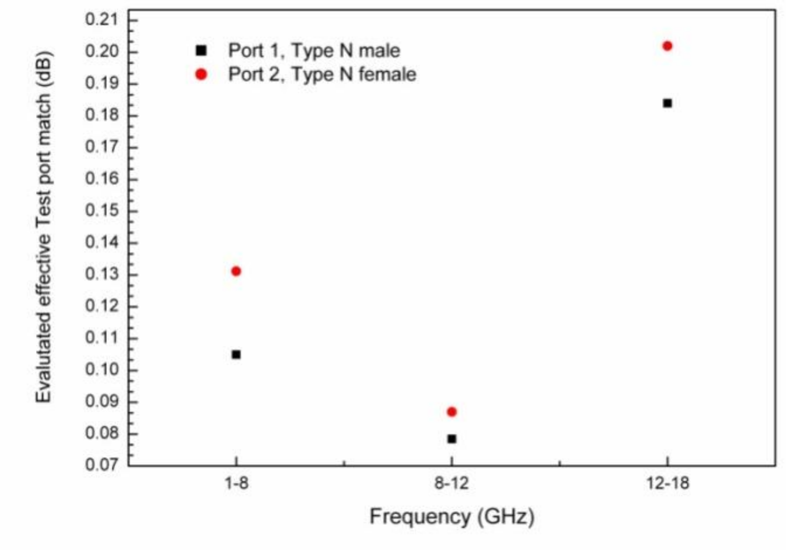
Isolation is the direct measurement by connecting the matched loads to both test ports i.e. thru measurement, otherwise the manufacturer's specification in terms of I (dB) or I (Lin) can be used for the first time users[27-28].

$$dI(dB) = \pm 20\log_{10}\left[1+10^{-\frac{(I-A)}{20}}\right] \quad (5)$$

where A: Measured attenuation level (in dB)

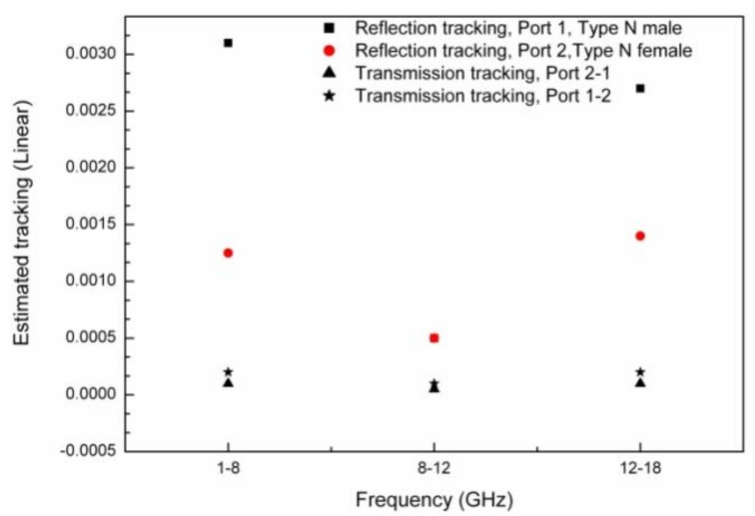



(a)

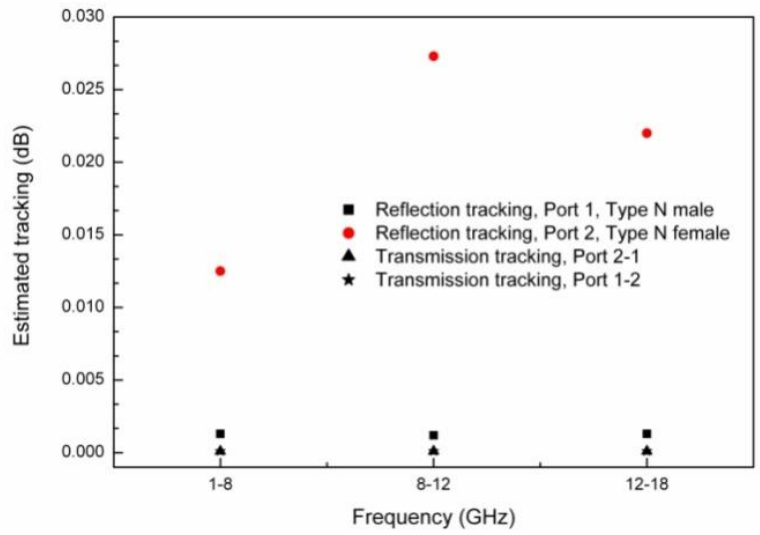



(b)

**Figure 3.** Evaluated effective load match (a) in linear format, (b) in log format




(a)




(b)

**Figure 4.** Estimated tracking (a) in linear format, (b) in log format

It is clear from the equation (5), this component will increase for higher attenuation levels and dependent on the port direction. The isolation is calculated for various attenuation levels from 50 dB to 70 dB and for the direction of signal and presented in Figure 5. The response is decreasing with increase in the frequency and almost same irrespective of unit of measurand.

## 2.2. Mismatch

The expressions given in this section are briefly describe and evaluated earlier for the attenuation measurement systems in accordance to the fixed and variable attenuators [32-35]. Here the expressions are written again in context of VNA.

### 2.2.1. Mismatch Uncertainty Calculation for a Fixed Attenuator

Now if we consider,

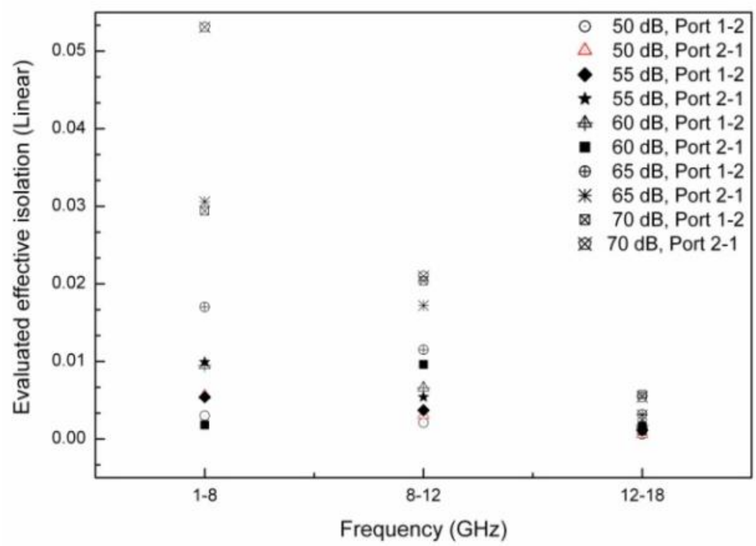
M: Effective Test port match or Effective Source match

$\Gamma_L$: Effective load match

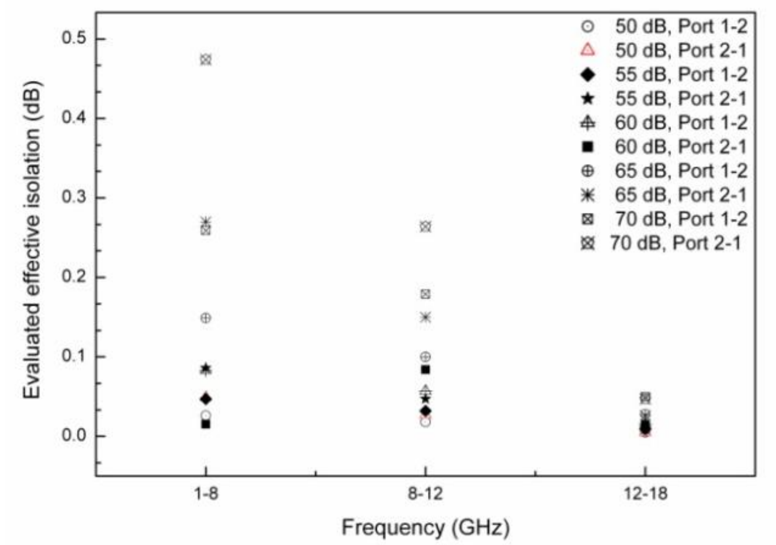
$S_{11}$, $S_{12}$, $S_{21}$, $S_{22}$: Scattering coefficients of the attenuator (at the attenuation level)

$S'_{11}$, $S'_{12}$, $S'_{21}$, and $S'_{22}$: Scattering coefficients of the two-port device at the initial state '0' dB (applicable for step attenuator)

If $S_{11}$, $S_{22}$, M and $\Gamma_L$ are extremely small i.e. <<1, in dB, mismatch uncertainty can be estimated using (6) in dB,




(a)




(b)

**Figure 5.** Estimated effective isolation (a) in linear format, (b) in log format

$$Umismfa_{Max} = 8.686\left[|M||S_{11}| + |\Gamma_{L}||S_{22}| + |M\Gamma_{L}|\left(1+|S_{21}S_{12}|\right)\right] \tag{6}$$

If values are in terms of magnitude and phase, and applicable for high attenuation values > 10 dB,

$$Umismfa_{mp} = 8.686\begin{bmatrix} |M||\Gamma_{L}|\left\langle \begin{matrix} \cos(\phi M + \phi\Gamma_{L}) - \\ |S_{21}||S_{12}|\cos(\phi M + \phi\Gamma_{L} + \phi 12 + \phi 21) \end{matrix} \right\rangle - \\ |M||S_{11}|\cos(\phi M + \phi 11) - |\Gamma_{L}||S_{22}|\cos(\phi\Gamma_{L} + \phi 22) \end{bmatrix} \tag{7}$$

When values are in terms of real and imaginary, the mismatch uncertainty in dB will be,

$$Umismfa_{ri} = 8.686\begin{bmatrix} |M|\sqrt{a^2+b^2} + |\Gamma_{L}|\sqrt{e^2+f^2} + \\ |M||\Gamma_{L}|\left(\sqrt{(pk-ql)^2+(pl+qk)^2}\right) \end{bmatrix} \tag{8}$$

2.2.2. Mismatch Uncertainty Calculation for Incremental Attenuation I.E. A Step Attenuator

If $S_{11}$, $S_{22}$, M and $\Gamma_L$ are extremely small i.e. <<1, in dB

$$Umismsa_{Max} = 8.686\begin{bmatrix} |M|^2\left(|S_{11}|^2+|S'_{11}|^2\right)+|\Gamma_{L}|^2\left(|S_{22}|^2+|S'_{22}|^2\right) \\ +|M|^2|\Gamma_{L}|^2\left(|S_{21}|^2|S_{12}|^2+|S'_{21}|^2|S'_{12}|^2\right) \end{bmatrix}^{1/2} \tag{9}$$

If values are in terms of magnitude and phase, and applicable for high attenuation values> 10 dB,

$$Umismsa_{mp} = 8.686\begin{bmatrix} |M|\left(|S'_{11}|\cos(\phi M+\phi' 11) - |S_{11}|\cos(\phi\Gamma_{G}+\phi 11)\right) + \\ |M||\Gamma_{L}|\left\langle \begin{matrix} |S'_{21}||S'_{12}|\cos(\phi M+\phi\Gamma_{L}+\phi'_{12}+\phi'_{21}) - \\ |S_{21}||S_{12}|\cos(\phi M+\phi\Gamma_{L}+\phi 12+\phi 21) \end{matrix} \right\rangle \\ + |\Gamma_{L}|\left(|S'_{22}|\cos(\phi\Gamma_{L}+\phi' 22) - |S_{22}|\cos(\phi\Gamma_{L}+\phi 22)\right) \end{bmatrix} \tag{10}$$

When values are in terms of real and imaginary, the mismatch uncertainty in dB will be,

$$Umismsa_{ri} = 8.686\begin{bmatrix} |M|\sqrt{(a-c)^2+(\mathrm{b}-\mathrm{d})^2} + \\ |\Gamma_{L}|\sqrt{(e-g)^2+(\mathrm{f}-\mathrm{h})^2} + \\ |M||\Gamma_{L}|\left(\sqrt{\begin{matrix}(pk-ql-mr+ns)^2 + \\ (pl+qk-ms-nr)^2\end{matrix}}\right) \end{bmatrix} \tag{11}$$

The mismatch uncertainties for a 50 dB attenuator is calculated using different input formats with the existing VNA system in the frequency range 1 to 18 GHz and presented in Figure 6.

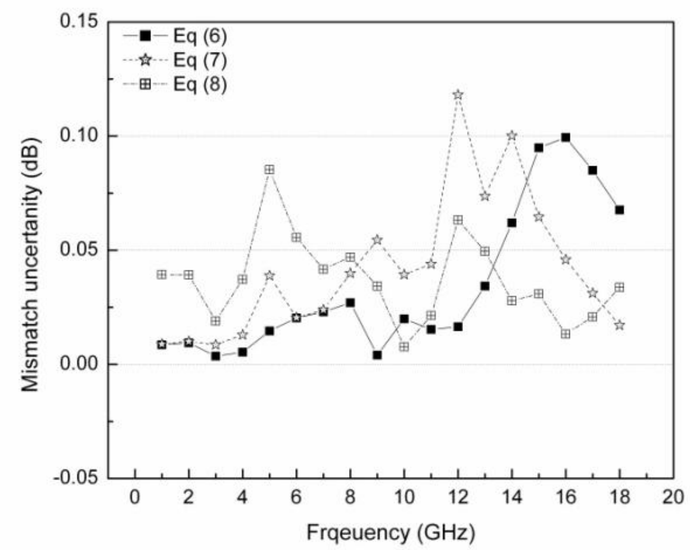
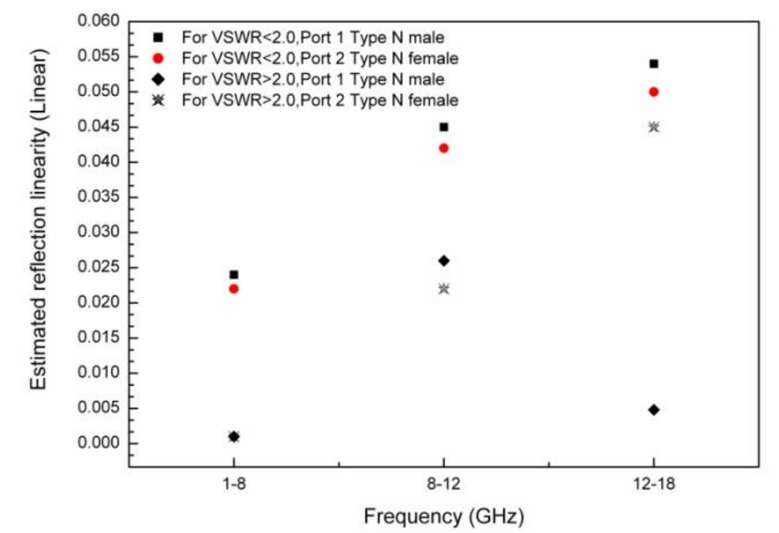
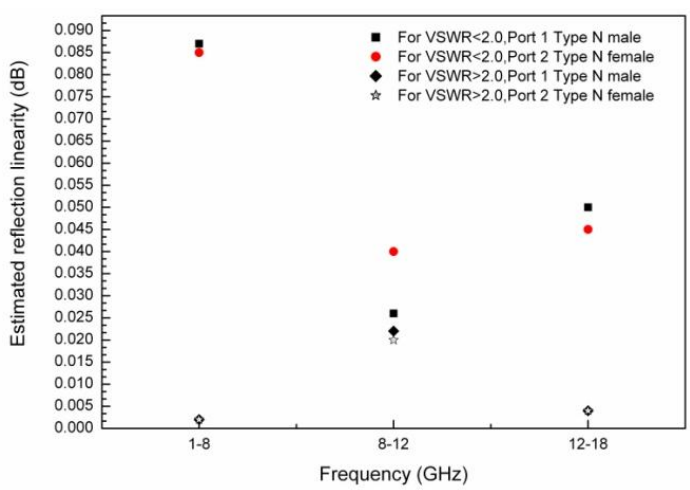



**Figure 6.** Calculated mismatch uncertainty of a 50 dB attenuator

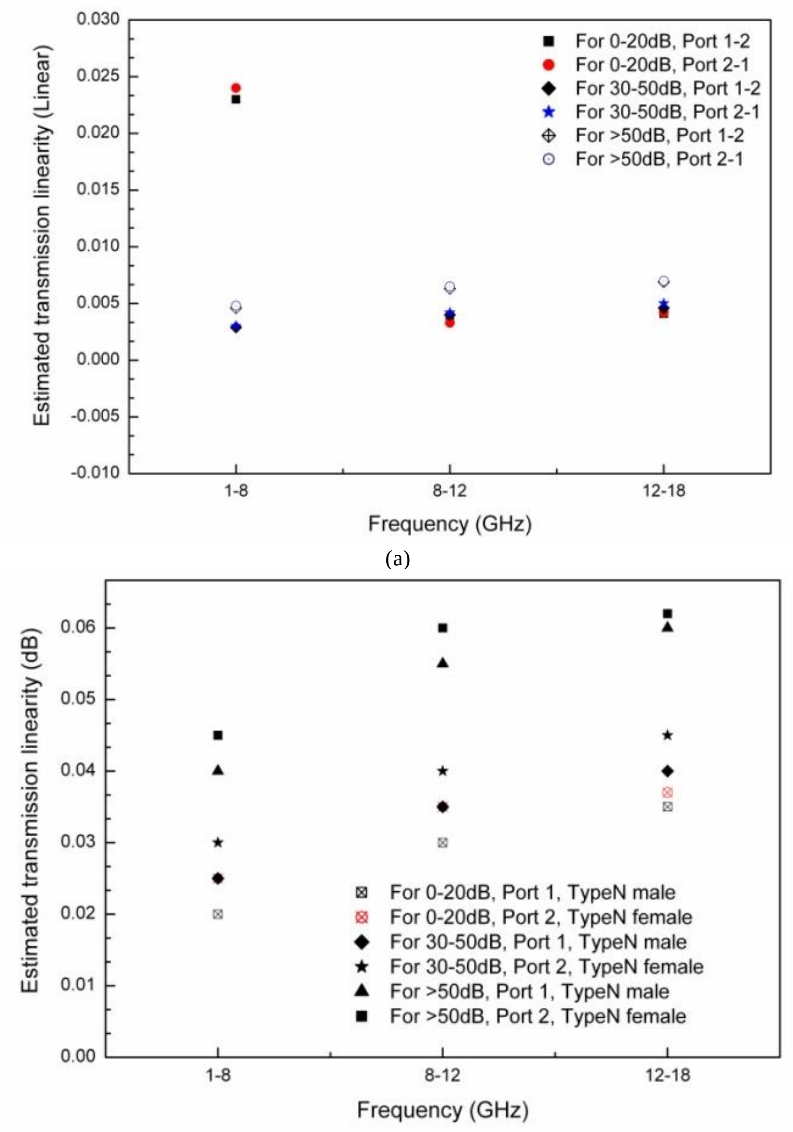



(a)




(b)

**Figure 7.** Estimated reflection linearity (a) in linear format, (b) in log format




(b)

**Figure 8.** Estimated transmission linearity (a) in linear format, (b) in log format

### 2.3. Effective Linearity

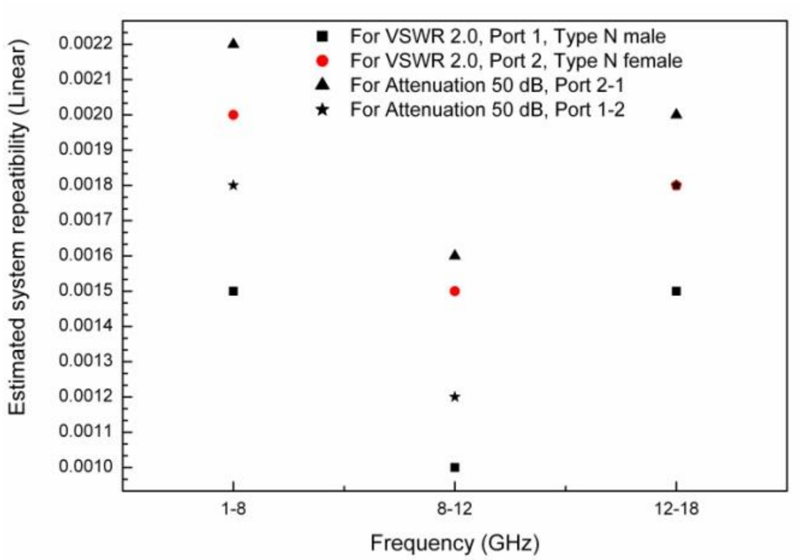
The evaluation and inclusion of linearity in uncertainty budgets for one-port and two-port measurements, is the link to establish traceability to national standards of measuring parameters. Thus for evaluating linearity in transmission and reflection measurements, a step attenuator Agilent 8496B calibrated against the signal and attenuation calibrator model VM-7 and a mismatch set 2562L of Maury Microwave in Type-N connector calibrated against coaxial airline standard Anritsu 18N50-10 have been used, respectively. These linear contributions are shown in Figure 7(a-b) and Figure 8(a-b) for various ranges of reflection and transmission values separately.

In Figure 7 (a-b), it has been noticed that the reflection linearity depends of type of connector used and the value of VSWR. Except for VSWR>2.0 at Type N female, the uncertainty contribution is increasing with frequency.

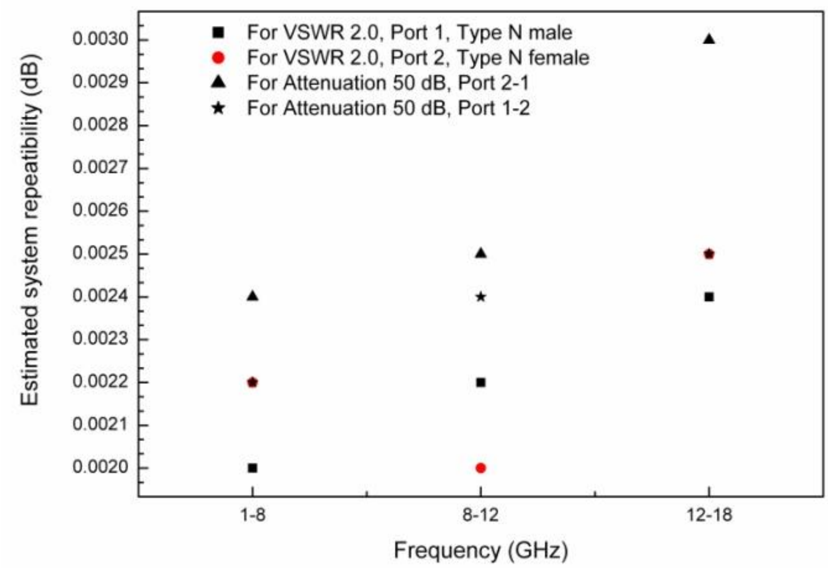
Transmission linearity in linear values is having almost constant values except for lower attenuation ranges and independent of the direction of signal as shown in Figure 8 (a), whereas this uncertainty contribution is increasing with the applied frequency and attenuation range in dB given in Figure 8 (b).

### 2.4. Evaluation of Random Error Contributors

The system repeatability has been estimated by the root square sum of two standard deviations for repeatability measurements, the standard deviation of 5 times measurements on the same calibration and the standard deviation of 5 times measurements after recalibrations and shown in Figure 9 (a) and (b). Separate assessment of connector repeatability is not performed in our case considering the same approach would be applied during the assessment of repeatability (type A contribution) using DUT for an individual parameter[27-28].

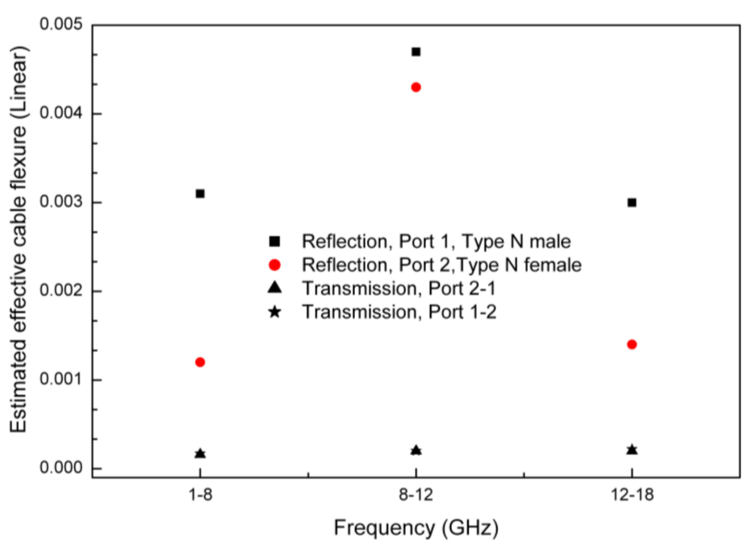



(a)

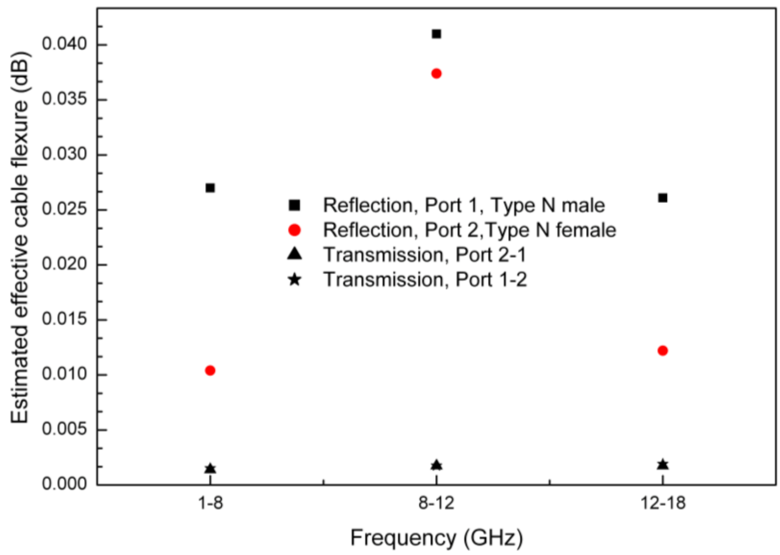



(b)

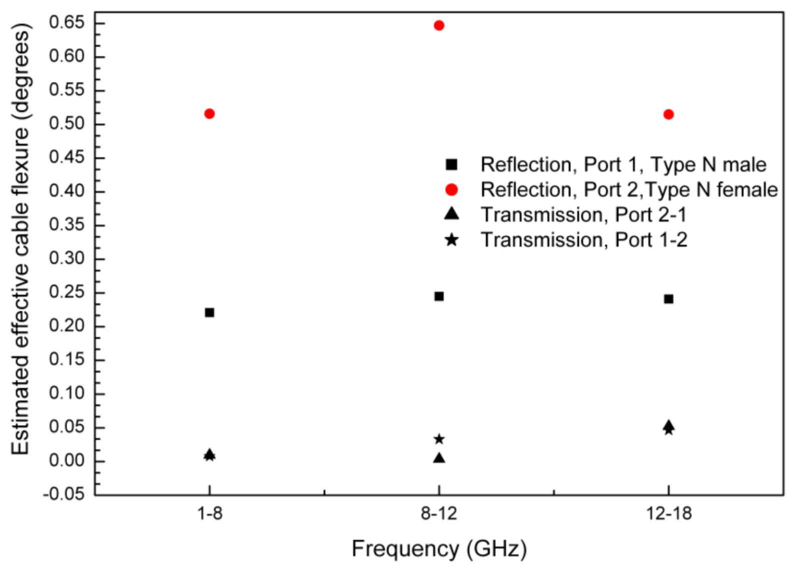
**Figure 9.** Estimated system repeatability (a) in linear format, (b) in log format




(a)




(b)



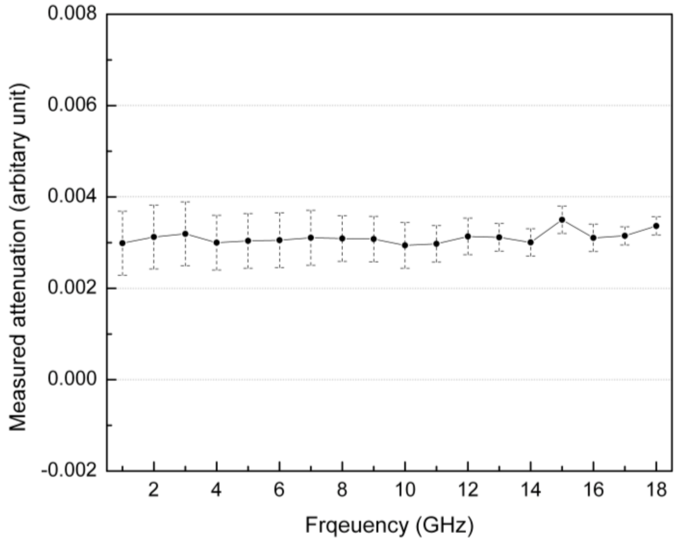

(c)

**Figure 10.** Estimated effective cable flexure (a) in linear format, (b) in log format, and (c) in degrees

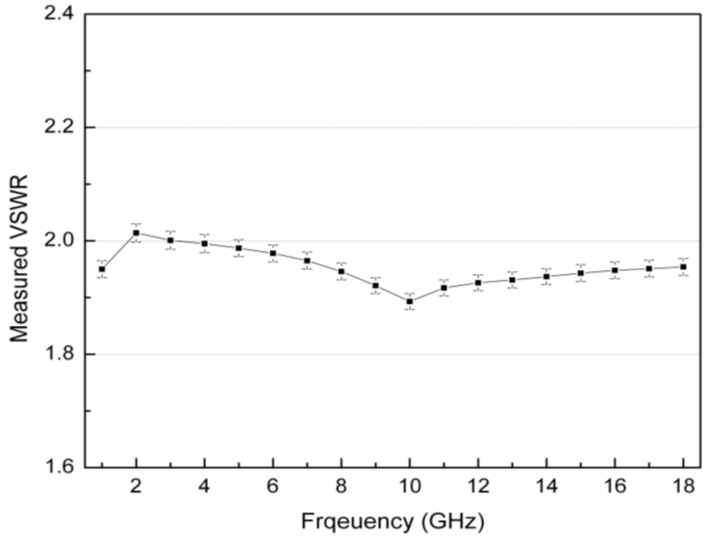
The cable flexure contribution has a greater importance for phase measurements as signal phase changes along the cable for different frequencies i.e. two times for reflection measurements, and longer time for transmission measurements. For transmission phase, the root sums of squares for cable flexure uncertainties of reflection phases at both ports have taken into account for an estimate. As shown in Figure 10 (a), (b) and (c), the uncertainty due to cable flexure has similar response for reflection and transmission measurements, respectively, irespective of units.

## 3. Complex S-Parameters Measurement and Its Verification

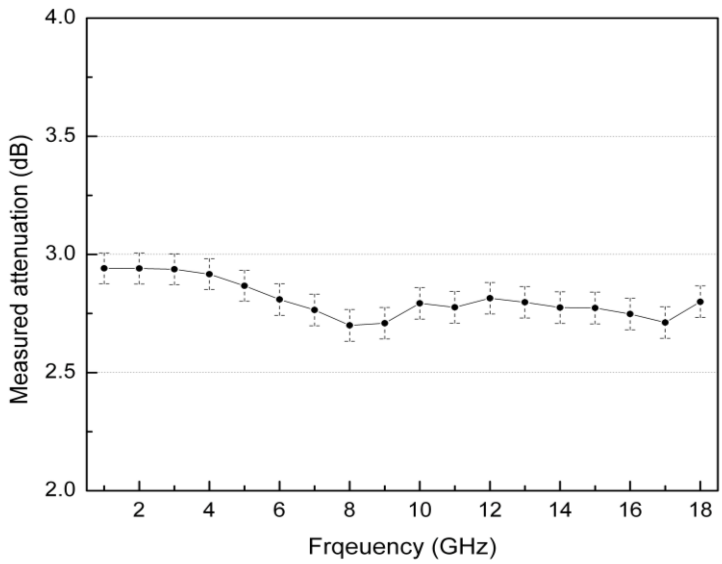
After the evaluation of the uncertainty components of uncertainty as described in the previous sections, the associated uncertainties of various parameters are estimated as per GUM document. The uncertainty estimation of reflection and transmission magnitude in linear and logarithmic formats is performed using the model equations given in the reference[27], whereas the uncertainty in the reflection and transmission phases are estimated based on the references[16] and [25]. We calibrated two-port VNA Wiltron 37247B for SOLT calibration using Anritsu coaxial calibration kit 3653 in Type- N connector. For verification purpose, the DUTs were chosen Maury Microwave mismatch of VSWR 2.0, a Weinschel fixed attenuator 3 dB and Agilent step attenuator for 50 dB attenuation in 1 to 18 GHz range. Some of the results along with the uncertainties are presented in Figure 11(a-d). Model uncertainty budgets for few parameters are given in Appendix A for a frequency 2 GHz.


Measured attenuation (arbitary unit)
0.008
0.006
0.004
0.002
0.000
-0.002
2 4 6 8 10 12 14 16 18
Frqeuency (GHz)

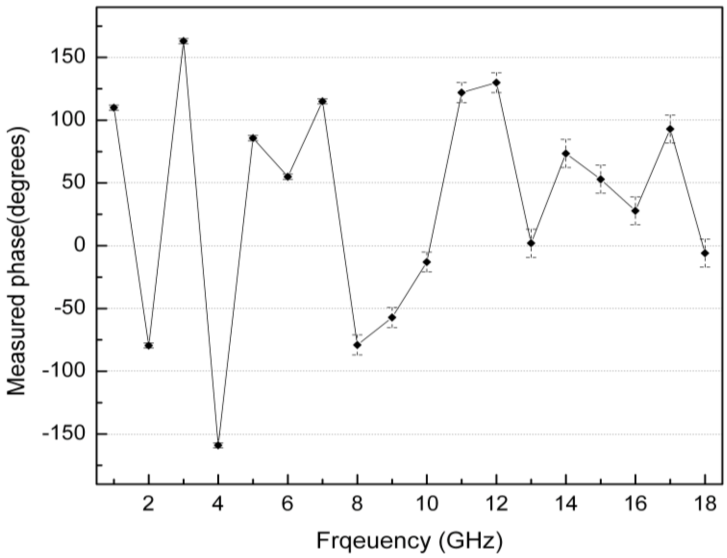
(a)


Measured VSWR
2.4
2.2
2.0
1.8
1.6
2 4 6 8 10 12 14 16 18
Frqeuency (GHz)

(b)


Measured attenuation (dB)
4.0
3.5
3.0
2.5
2.0
2 4 6 8 10 12 14 16 18
Frqeuency (GHz)

(c)




(d)

**Figure 11.** Measurement results with estimated uncertainties (a) VSWR of coaxial mismatch, (b) Measured attenuation in linear magnitudes of 50 dB attenuator, (c) Measured attenuation in log magnitudes of 3 dB attenuator, and (d) Measured $S_{21}$ phase values of 3 dB attenuator

The verification kit has been used to experimentally validate the VNA measurements and establishing the traceability of these measurements. The $S_{11}$ and $S_{22}$ of the airline were measured at port 1 and 2 of the calibrated VNA compared to the theoretically calculated values along with skin depth correction to check their closeness against the estimated uncertainty of the reflection coefficient (or $Z_0$)[36]. The phase shift (in degree) introduced by a coaxial airline is calculated to estimate the standard contribution in phase using the equation (17)[27],

$$\Phi(f) = 360Lf\sqrt{\varepsilon_r\mu_r}/c = 0.012\times Lf\frac{1}{VF} \tag{12}$$

where L is the geometrical length of the airline (cm) and f is the frequency (MHz). Here c is the velocity of light × velocity factor (VF). (For air-filled airline, velocity factor is approximately 1).

So the measured phase shifts introduced by the airline along with the estimated uncertainty are compared to the values calculated by the equation (17). The electrical lengths of the airlines decide the usable frequency of operation for the insertion phase and reflection coefficients on VNA system. We have used a coaxial airline of length 10 cm and thru connection (zero length line) to cover 0.5 to 17 GHz frequency range to claim traceability. From 17 to 18 GHz, extrapolation has been used to estimate the airline uncertainty contribution in the reflection coefficient and phase in the presented results. Similarly, the suitable lengths of airline can be selected accordingly to cover the desired frequency ranges in respect to establish traceability. In the similar fashion, it was found that the difference between the measured values of a calibrated attenuator from IF substitution technique and VNA technique, is less than the root sum square of their uncertainties obtained for both techniques.

# 5. Conclusions

New calibration and measurement capabilities (CMC) in the frequency range 1 to 18 GHz by VNA systemare realized and summarized in Table 2.

The present work reports the establishment of the following calibration expertise at NPL, India in accordance to ISO/IEC 17025:

a) Traceable calibration and uncertainty estimation of the lab's VNA system (WILTRON 37247B) and its transmission and reflection measurements.

b) Traceable calibration and uncertainty estimation of the user's VNA system along with its calibration kits for regional and other calibration laboratories.

c) Traceable calibration and uncertainty estimation of the user's individual calibration kits and their components against the national standards of attenuation and impedance through VNA measurements.

It has also verified that the final combined uncertainty is estimated the same or almost close by obtaining two practices for same measurement parameter, set-up and constant environment conditions, One practice is to estimate the most valid combined uncertainty from the individual uncertainty components evaluated in same terms as of the measurement parameters, and second, the total combined uncertainty have achieved from one format to another using handy formulas like VSWR and reflection coefficient or linear to logarithmic (dB) conversion formula etc. Thus, the present paper provides the descriptive solutions for estimation of uncertainties and traceable VNA measurements in vector and linear formats at one place for the RF metrologists.

**Table 2.** Established Calibration ranges by vector network analyzer technique

| S.No. | Type of Measurements | Parameter | Measurement Range | Range of Expanded uncertainty (±) |
|---|---|---|---|---|
| 1 | One-port Reflection | $S_{nn}$ (lin) | 0.005 to 1.0 | 0.002 to 0.050 |
| | | RL (dB) | 0.0 to 50 dB | 0.020 to 0.500 |
| | | $S_{nn}$ (ReIm) | 0.005 to 1.0 | 0.001 to 0.025 |
| | | VSWR | 1.04 - 2.0 | 0.010 to 0.025 |
| | | $S_{nn}$(Phase) | (±) 0° to 180° | 0.10° to 5.00° |
| 2 | Two-port Reflection | $S_{nn}$ (lin) | 0.002 to 0.10 | 0.005 to 0.020 |
| | | RL (dB) | 20 to 50 dB | 0.020 to 0.500 |
| | | $S_{nn}$ (ReIm) | 0.002 to 1.0 | 0.005 to 0.020 |
| | | VSWR | 1.01 – 1.05 | 0.005 to 0.020 |
| | | $S_{nn}$(Phase) | (±) 0° to 180° | 0.10° to 10.00° |
| 3 | Two-port Transmission | $S_{mn}$ (lin) | 0.001 to 1.0 | 0.005 to 0.020 |
| | | IL (dB)/ Attenuation | 0.0 to 70 dB | 0.010 to 1.00 |
| | | $S_{mn}$ (ReIm) | 0.001 to 1.0 | 0.005 to 0.010 |
| | | $S_{mn}$(Phase) | (±) 0° to 180° | 0.10° to 5.00° |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# Appendix A: Detail uncertainty budgets

To estimate uncertainty of the S-parameters for their different forms, various uncertainty budgets have been prepared as examples for understanding of the reader.

**A.1. One-port uncertainty budgets**

Device under calibration: Coaxial mismatch

Range: VSWR 2.0, DC to 18 GHz

Measurement frequency: 2 GHz

Uncertainty estimation for one –port component are given in Table–A1.1, Table–A1.2 and Table– A1.3 for VSWR, Reflection coefficient magnitude (linear) and Reflection coefficient phase (degrees) respectively.

**Table A1.1.** Uncertainty estimation for one-port measurement- VSWR

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Effective Directivity, UB1 | 0.012 | 0.012 | U-shaped | - | - | - |
| Effective Test Port match, UB2 | 0.012 | 0.001386 | U-shaped | - | - | - |
| Sum of Correlated quantities | | 0.013386 | U-shaped | 1 | ∞ | 0.009465 |
| Reflection Tracking, UB3 | 0.0031 | 0.001054 | Rectangular | 1 | ∞ | 0.000608 |
| Effective Linearity, UB4 | 0.024 | 0.008156 | Rectangular | 1 | ∞ | 0.004709 |

| | | | | | | |
|---|---|---|---|---|---|---|
| System repeatability, UB5 | 0.0015 | 0.00051 | Gaussian | 1 | ∞ | 0.000255 |
| Cable flexure, UB6 | 0.0031 | 0.001054 | Gaussian | 1 | ∞ | 0.000526 |
| Type B | | | | | | 0.010605 |
| Repeatability,(Type A) | 2.03 | 0.000248 | Gaussian | 1 | 9 | 0.000248 |
| Combined Std unc. | k=1 | | | | | 0.010608 |
| Expanded unc. | k=2 | Eff. DOF | 482244651 | | | 0.021 |

**Table A1.2.** Uncertainty estimation for one-port measurement- Reflection coefficient magnitude (Linear)

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Effective Directivity, UB1 | 0.011 | 0.011 | U-shaped | - | - | - |
| Effective Test Port match, UB2 | 0.012 | 0.0013894 | U-shaped | - | - | - |
| Sum of Correlated quantities | | 0.012389 | U-shaped | 1 | ∞ | 0.008761 |
| Reflection Tracking, UB3 | 0.0031 | 0.001055 | Rectangular | 1 | ∞ | 0.000609 |
| Effective Linearity, UB4 | 0.024 | 0.008166 | Rectangular | 1 | ∞ | 0.004715 |
| System repeatability, UB5 | 0.0015 | 0.000511 | Gaussian | 1 | ∞ | 0.000255 |
| Cable flexure, UB6 | 0.0031 | 0.001055 | Gaussian | 1 | ∞ | 0.000527 |
| Type B | | | | | | 0.009985 |
| Repeatability, (Type A) | 0.340 | 0.000089 | Gaussian | 1 | 9 | 0.000089 |
| Combined Std unc. | k=1 | | | | | 0.009985 |
| Expanded unc. | k=2 | Eff. DOF | 1469010704 | | | 0.020 |

**Table A1.3.** Uncertainty estimation for one-port measurement- Reflection coefficient phase (degrees)

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Magnitude VRC | 0.339 | | | | | - |
| Uncertainty in Measured VRC | 0.021 | | | | | - |
| Arcsine ($U_{VRC}$/VRC)*(180/pi), UB1 | | 3.55157 | Gaussian | 1 | ∞ | 1.775785 |
| Effective cable flexure, UB2 | 0.2211 | - | | | | - |
| Freq (GHz) of measurement | 2 | - | | | | - |
| Uncertainty due to cable phase stability | | 0.4422 | Gaussian | 1 | ∞ | 0.2211 |
| Cable length (cm) | 60 | - | | | | - |
| Temp change TD, UB4 | 0.0012 | 0.0688 | Gaussian | 1 | ∞ | 0.0343775 |
| Type B | | | | | | 1.789827 |
| Repeatability, (Type A) | 2.633 | 0.0168 | Gaussian | 1 | 9 | 0.0168 |
| Combined Std unc. | k=1 | | | | | 1.7899 |
| Expanded unc. | k=2 | Eff. DOF | 1160697569 | | | 3.580 |

### A.2. Two-port uncertainty budgets

Device under calibration: Coaxial mismatch

Range: VSWR 2.0, DC to 18 GHz
Measurement frequency: 2 GHz
Measured S-parameters (linear)
$S_{11}$ =0.023 $S_{12}$ =0.003 $S_{21}$ =0.003 $S_{22}$ =0.006

Uncertainty estimation for two –port component are given in Table–A2.1 to Table–A2.5 for Reflection coefficient magnitude (Logarithmic-dB), Reflection coefficient (Real and Imaginary components), Transmission coefficient magnitude (Linear), Transmission coefficient magnitude (Logarithmic-dB) and Transmission coefficient phase (degrees) respectively.

**Table A2.1.** Uncertainty estimation for two-port measurement- Reflection coefficient magnitude (Logarithmic-dB)

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Effective Directivity, UB1 | 0.105 | 0.105 | U-shaped | - | - | - |
| Effective Test Port match, UB2 | 0.105 | 0.0063486 | U-shaped | - | - | - |
| Sum of Correlated quantities | | 0.1113486 | U-shaped | 1 | ∞ | 0.078735372 |
| Reflection Tracking, UB3 | 0.0031 | 9.372E-05 | Rectangular | 1 | ∞ | 5.4108E-05 |
| Effective Linearity, UB4 | 0.087 | 0.0026301 | Rectangular | 1 | ∞ | 0.001518516 |
| System repeatability, UB5 | 0.002 | 6.046E-05 | Gaussian | 1 | ∞ | 3.02316E-05 |
| Cable flexure, UB6 | 0.027 | 0.0008163 | Gaussian | 1 | ∞ | 0.000408126 |
| Effective Load match, UB7 | 0.158 | 0.0313134 | U-shaped | 1 | ∞ | 0.022141942 |
| Type B | | 0.082 | | | | |
| Repeatability, (Type A) | 33.078 | 0.09824196 | Gaussian | 1 | 9 | 0.098242 |
| Combined Std unc. | k=1 | | 0.1278 | | | |
| Expanded unc. | k=2 | Eff. DOF | 128.15 | | 0.256 | |

**Table A2.2.** Uncertainty estimation for two-port measurement- Reflection coefficient (Real & Imaginary components) Real component of reflection coefficient

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Effective Directivity, UB1 | 0.012 | 0.012 | U-shaped | - | - | - |
| Effective Test Port match, UB2 | 0.012 | 1.08E-09 | U-shaped | - | - | - |
| Sum of Correlated quantities | | 0.012 | U-shaped | 1 | ∞ | 0.008486 |
| Reflection Tracking, UB3 | 0.0031 | 9.3E-07 | Rectangular | 1 | ∞ | 5.369E-07 |
| Effective Linearity, UB4 | 0.024 | 7.2E-06 | Rectangular | 1 | ∞ | 4.157E-06 |
| System repeatability, UB5 | 0.0015 | 4.5E-07 | Gaussian | 1 | ∞ | 2.25E-07 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Cable flexure, UB6 | 0.0031 | 9.3E-07 | Gaussian | 1 | ∞ | 4.65E-07 |
| Effective Load match, UB7 | 0.018 | 1.72E-07 | U-shaped | 1 | ∞ | 1.214E-07 |
| Type B | | 0.008 | | | | |
| Repeatability, (Type A) | 0.0003 | 0.000107 | Gaussian | 1 | 9 | 0.000107 |
| Combined Std unc. | k=1 | | 0.0085 | | | |
| Expanded unc. | k=2 | Eff. DOF | 357738477 | | 0.017 | |

Imaginary component of reflection coefficient:

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Effective Directivity, UB1 | 0.012 | 0.012 | U-shaped | - | - | - |
| Effective Test Port match, UB2 | 0.012 | 1.2E-08 | U-shaped | - | - | - |
| Sum of Correlated quantities | | 0.012 | U-shaped | 1 | ∞ | 0.008486 |
| Reflection Tracking, UB3 | 0.0031 | 3.1E-06 | Rectangular | 1 | ∞ | 1.79E-06 |
| Effective Linearity, UB4 | 0.024 | 0.00003 | Rectangular | 1 | ∞ | 1.39E-05 |
| System repeatability, UB5 | 0.0015 | 1.5E-06 | Gaussian | 1 | ∞ | 0.000001 |
| Cable flexure, UB6 | 0.0031 | 3.1E-06 | Gaussian | 1 | ∞ | 0.000001 |
| Effective Load match, UB7 | 0.018 | 1.7E-07 | U-shaped | 1 | ∞ | 1.22E-07 |
| Type B | | 0.008 | | | | |
| Repeatability, (Type A) | 0.001 | 0.00028 | Gaussian | 1 | 9 | 0.000280 |
| Combined Std unc. | k=1 | | 0.0085 | | | |
| Expanded unc. | k=2 | Eff. DOF | 7607632 | | 0.017 | |

**Table A2.3.** Uncertainty estimation for two-port measurement-Transmis sion coefficient magnitude (Linear)

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Effective Test Port match, UB1 | 0.012 | 3.5E-05 | U-shaped | | | - |
| Effective Load match, UB2 | 0.018 | 0.0001 | U-shaped | | | - |
| Effective Test port match*Load match | 0.001 | 0.0003 | U-shaped | | | - |
| Mismatch calculated | | 0.0004 | U-shaped | 1 | ∞ | 0.0002498 |
| Transmission Tracking, UB3 | 0.064 | 0.0002 | Rectangular | | | - |
| Effective Linearity, UB4 | 0.92 | 0.0029 | Gaussian | | | - |
| Sum of Correlated quantities | | 0.0029 | Gaussian | 1 | | 0.0014351 |
| Isolation, UB5 | 95 dB | 0.0057 | Rectangular | 1 | | 0.00328018 |
| System repeatability, UB6 | 0.002 | 0.0018 | Gaussian | 1 | ∞ | 0.0009 |
| Cable flexure, UB7 | 0.001 | 0.0002 | Gaussian | 1 | ∞ | 0.0001 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Type B | | | | | | 0.003702 |
| Repeatability, (Type A) | 0.003 | 0.0001 | Gaussian | 1 | 9 | 0.0000078 |
| Combined Std unc. | k=1 | | | | | 0.003702 |
| Expanded unc. | k=2 | Eff. DOF | 7.3194E+12 | | | 0.007 |

**Table A2.4.** Uncertainty estimation for two -port measurement- Transmission coefficient magnitude (dB)

| Sources of Uncertainty | Estimate | Uncertainty contribution (dB) | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty (dB) |
|---|---|---|---|---|---|---|
| Effective Test Port match, UB1 | 0.012 | 3.5E-05 | U-shaped | | | - |
| Effective Load match, UB2 | 0.018 | 0.0001 | U-shaped | | | - |
| Effective Test port match*Load match | 0.0002 | 0.0002 | U-shaped | | | - |
| Mismatch calculated | | 0.0031 | U-shaped | 1 | ∞ | 0.00217 |
| Transmission Tracking, UB3 | 0.0001 | 0.0050 | Rectangular | | | |
| Effective Linearity, UB4 | 0.0005 | 0.0251 | Gaussian | | | |
| Sum of Correlated quantities | | 0.0256 | Gaussian | 1 | | 0.01279 |
| Isolation, UB5 | 95 dB | 0.0495 | Rectangular | 1 | | 0.02859 |
| System repeatability, UB6 | 0.0022 | 0.0022 | Gaussian | 1 | ∞ | 0.0011 |
| Cable flexure, UB7 | 0.0015 | 0.0015 | Gaussian | 1 | ∞ | 0.00075 |
| Type B | | | | | | 0.03153 |
| Repeatability, (Type A) | 50.18 | 0.0118 | Gaussian | 1 | 9 | 0.01180 |
| Combined Std unc. | k=1 | | | | | 0.0398 |
| Expanded unc. | k=2 | Eff. DOF | 595.783 | | | 0.067 |

**Table A2.5.** Uncertainty estimation for two-port measurement-Transmission coefficient phase (degrees)

| Sources of Uncertainty | Estimate | Uncertainty contribution | Probability Distribution | Sensitivity Coefficient | DOF | Standard uncertainty |
|---|---|---|---|---|---|---|
| Magnitude S21, dB | 50.18 | | | | | - |
| Uncertainty in Measured S21, dB | 0.067 | | | | | - |
| Arcsine (US21/S21)*(180/pi), UB1 | | 0.4437 | Gaussian | 1 | ∞ | 0.2218368 |
| Uncertainty in the phase standard (airline),(Degs) | 1.2 | | | | | - |
| Uncertainty in airline length, (mm) | 0.015 | | | | | - |
| Uncertainty in phase shift, (Degs), UB2 | 0.5 | 0.018 | | | | 0.009 |
| Effective cable flexure, UB3 | 0.0077 | | | | | - |
| Freq (GHz) of measurement | 2 | | | | | - |
| Unc cable phase stability | | 1.848 | Gaussian | 1 | ∞ | 0.924 |
| Cable length (cm) | 60 | | | | | - |
| Uncertainty Iphi from intercomparison, (degs), UB4 | 0.1076 | 0.1076 | | | | 0.0538213 |
| Temp change TD, UB5 | 0.0012 | 0.0688 | Gaussian | 1 | ∞ | 0.0343775 |
| Type B | | | | | | 0.9524 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Repeatability, (Type A) | 44.589 | 0.3266 | Gaussian | 1 | 9 | 0.3266 |
| Combined Std unc. | k=1 | | | | | 1.00687 |
| Expanded unc. | k=2 | Eff. DOF | 813.303 | | | 2.014 |

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