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# On Fuzzy $\delta$-Semi Connectedness

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**Abstract** A.Mukherjee & S.Debnath[10] introduced the concept of $\delta$-semi open sets in fuzzy setting and investigate their basic properties. Also S.Debnath[4] introduced the concept of fuzzy $\delta$-semi continuous functions in fuzzy topological spaces. The purpose of the present paper is to introduce and study the concept of $\delta$-semi connectedness in fuzzy topological spaces. It is seen that every fuzzy $\delta$-semi separated sets is fuzzy semi separated but the converse is not true. Also every fuzzy connected set is fuzzy $\delta$-semi connected but the converse may not be true.

**Keywords** Fuzzy $\delta$-semi open, fuzzy semi separated, fuzzy $\delta$-semi separated, fuzzy $\delta$-semi continuous, fuzzy semi connected, fuzzy $\delta$-semi connected

## 1. Introduction and Preliminaries

According to Zadeh[17], a fuzzy subset $\mu$ in a set $X(\neq \phi)$ is a function from $X$ to the unit closed interval $I=[0,1]$ i.e., $\mu \in I^x$. In what follows, by $(X,\tau)$ or simply by $X$ we mean a fuzzy topological space (fts, for short) in the sense of Chang[3]. The notations $cl\mu$, $\text{int}\,\mu$ and $1-\mu$ will stand for the fuzzy closure, interior and complement of a fuzzy set $\mu$ in a fts. $X$.

A fuzzy point $x_p$ in a fts. $X$ is quasi-coincident with a fuzzy set $\mu$ denoted by $x_p q\mu$ if $p+\mu(x)>1$ [12].The negation of this statement written as $x_p \not{q} \mu$. A fuzzy set $\mu$ of a fts. $X$ is called a $Q$-neighbourhood ($Q$-nbd, for short) of a fuzzy point $x_p$ if there exists a fuzzy open set $\lambda$ in $X$ such that $x_p q\lambda \leq \mu$ [12]. A fuzzy set $\mu$ of $X$ is called regular open if $\mu = \text{int}\,cl\mu$ [1]. A fuzzy point $x_p$ is called a $\delta$-cluster point of a fuzzy set $\mu$ of $X$ if every fuzzy regular open $Q$-nbd. of $x_\alpha$ is quasi-coincident with $\mu$. The union of all $\delta$-cluster of $\mu$ is called $\delta$-closure of $\mu$ and is denoted by $\delta cl\mu$. A fuzzy set $\mu$ is called $\delta$-closed if $\mu=\delta cl\mu$ and the complement of such a fuzzy set is called fuzzy $\delta$-open. Fuzzy $\delta$-interior of a fuzzy set $\mu$ in a fts $X$, denoted by $\delta\,\text{int}\,\mu$, is defined by $\delta\,\text{int}\,\mu = 1-\delta cl(1-\mu)$[11]. It is known[14] that a fuzzy set $\mu$ in a fts $X$ is fuzzy $\delta$-open iff $\mu=\delta\,\text{int}\,\mu$, and the set of all fuzzy $\delta$-open sets in $(X,\tau)$ forms a fuzzy topology $\tau_\delta$(say) on $X$, called the fuzzy semi-regularization topology of $(X,\tau)$, such that $\tau_\delta \subseteq \tau$, and for which the fuzzy regular open sets form a fuzzy open base.

**Definition 1.1:** A fuzzy set $\lambda$ in a fuzzy topplogical space $X$ is called:

(a) fuzzy semi open[1] if $\lambda \leq cl(\text{int}\,\lambda)$

(b) fuzzy $\delta$-semi open[10] if $\exists$ a fuzzy $\delta$-open set $\mu$ in $X$ such that $\mu \leq \lambda \leq cl(\mu)$.

**Remark 1.2:** Every fuzzy $\delta$-semi open set is fuzzy semi open but the converse may not be true[10].

**Definition 1.3:** The complement of a fuzzy semi open (resp. fuzzy $\delta$-semi open) set is called fuzzy semi closed (resp. fuzzy $\delta$-semi closed).

**Definition 1.4:** The intersection of all fuzzy semi closed (resp. fuzzy $\delta$-semi closed) sets containing a fuzzy set $\lambda$ of a fuzzy topological space $X$ is called fuzzy semiclosure[16] (resp. fuzzy $\delta$-semi closure[13]) of $\lambda$ and is denoted by $scl(\lambda)$(resp. $\delta-scl(\lambda)$).

**Definition 1.5:** A mapping $f:X\to Y$ is said to be fuzzy $\delta$-semi continuous[4] (resp. fuzzy $\delta$-semi irresolute[4]) if $f^{-1}(\lambda)$ is fuzzy $\delta$-semi open in $X$ for every fuzzy open (resp. fuzzy $\delta$-semi open) set $\lambda$ in $Y$.

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**Definition 1.6:** Two nonempty fuzzy sets $\lambda$ and $\mu$ of a fuzzy topological space $X$ are said to be fuzzy separated[5] (resp. fuzzy semi separated[7]) if $cl(\lambda)\not q\mu$ and $cl(\mu)\not q\lambda$ (resp. $scl(\lambda)\not q\mu$ and $scl(\mu)\not q\lambda$).

**Definition 1.7:** A fuzzy set $\eta$ in a fuzzy topological space $X$ is said to be fuzzy connected[6] (resp. fuzzy semi connected[7]) if $\eta$ can not be expressed as the union of two fuzzy separated (resp. fuzzy semi separated) sets.

**Remark 1.8:** Every fuzzy semi connected set is fuzzy connected but the converse may not be true[7].

Throughout this paper fts $X$ denotes a fuzzy topological space $X$ and $F_\delta SO(X)$(resp. $F_\delta SC(X)$) denotes the family of all fuzzy $\delta$-semi open (resp. fuzzy $\delta$-semi closed) sets of $X$.

## 2. Fuzzy $\delta$-semi Separated Sets

**Definition 2.1:** Two nonempty fuzzy sets $\lambda$ and $\mu$ in a fts $X$ are said to be fuzzy $\delta$-semi separated if $\delta-scl(\lambda)\not q\mu$ and $\delta-scl(\mu)\not q\lambda$.

**Remark 2.2:** Any two fuzzy $\delta$-semi separated sets in a fts $X$ are fuzzy semi separated but the converse is not true.

**Example 2.3:** Suppose $X=\{a,b\}$ and $\tau=\{0,1,\mu_1,\mu_2\}$, where

$\mu_1(a)=0.5 \quad \mu_1(b)=0.3$

$\mu_2(a)=0.5 \quad \mu_2(b)=0.1$

Then $(X,\tau)$ is a fts. Let $\mu_3$ be a fuzzy set in $X$ such that

$\mu_3(a)=0.5 \quad \mu_3(b)=0.2$

Then the fuzzy set $\mu_1$ and $\mu_3$ are fuzzy semi separated but not fuzzy $\delta$-semi separated in $X$.

**Theorem 2.4:** Let $\lambda$ and $\mu$ are fuzzy $\delta$-semi separated sets in a fts $X$ and $\lambda_1$ and $\mu_1$ are two nonempty fuzzy sets such that $\lambda_1\le\lambda$ and $\mu_1\le\mu$, then $\lambda_1$ and $\mu_1$ are fuzzy $\delta$-semi separated sets in $X$

**Proof:** Since $\lambda_1\le\lambda$ and $\mu_1\le\mu$ ,we have $\delta-scl(\lambda_1)\le\delta-scl(\lambda)$ and $\delta-scl(\mu_1)\le\delta-scl(\mu)$.

Therefore $\delta-scl(\lambda)\not q\mu\Rightarrow\delta-scl(\lambda_1)\not q\mu_1$ and $\lambda\not q\delta-scl(\mu)\Rightarrow\lambda_1\not q\delta-scl(\mu_1)$. Hence the theorem.

**Lemma 2.5[12]:** Let $\lambda$ and $\mu$ be two fuzzy sets in fts $X$, then $\lambda\le\mu$ if and only if $\lambda\not q(1-\mu)$.

**Theorem 2.6:** Let $\lambda,\mu\in F_\delta SO(X)$ . Then $\lambda$ and $\mu$ are fuzzy $\delta$-semi separated in $X$ if and only if $\lambda\not q\mu$.

**Proof:**

***Necessary:*** If $\lambda q\mu$ then there exists a point $x\in X$ such that $\lambda(x)+\mu(x)>1$ . This implies that $\delta-scl(\lambda(x))+\mu(x)>1$ and $\lambda(x)+\delta-scl(\mu(x))>1$ . Hence $\delta-scl(\lambda)q\mu$ and $\lambda q\delta-scl(\mu)$, which is contradiction . As $\lambda$ and $\mu$ are fuzzy $\delta$-semi separated sets in $X$ . Hence $\lambda\not q\mu$.

***Sufficient:*** Suppose that $\lambda\not q\mu$, then $\lambda\le(1-\mu)$.

Therefore, $\delta-scl(\lambda)\le\delta-scl(1-\mu)\le1-\mu$ , because $1-\mu\in F_\delta SC(X)$.

Hence by lemma 2.5 $\delta-scl(\lambda)\not q\mu$ . Similarly, $\lambda\not q\delta-scl(\mu)$.

Hence the theorem.

**Theorem 2.7:** Let $\lambda,\mu\in F_\delta SC(X)$. Then $\lambda$ and $\mu$ are fuzzy $\delta$-semi separated in $X$ if and only if $\lambda\not q\mu$.

**Proof:** Obvious.

**Theorem 2.8:** Let $\lambda,\mu\in F_\delta SO(X)$. Then the fuzzy sets $A_\lambda\mu=\lambda\cap(1-\mu)$ and $A_\mu\lambda=\mu\cap(1-\lambda)$ are fuzzy $\delta$-semi separated in $X$.

**Proof:** Since $A_\lambda\mu\le(1-\mu)$ ,So $\delta-scl(A_\lambda\mu)\le\delta-scl(1-\mu)=(1-\mu)$ because $\mu\in F_\delta SO(X)$.

And so by lemma 2.5, $\delta-scl(A_\lambda\mu)\not q\mu$ . Thus, $\delta-scl(A_\lambda\mu)\not q(A_\mu\lambda)$ . Similarly, $\delta-scl(A_\mu\lambda)\not q(A_\lambda\mu)$ . Hence $A_\lambda\mu$ and $A_\mu\lambda$ are fuzzy $\delta$-semi separated sets.

**Theorem 2.9:** Let $\lambda,\mu\in F_\delta SC(X)$. Then the fuzzy sets $A_\lambda\mu=\lambda\cap(1-\mu)$ and $A_\mu\lambda=\mu\cap(1-\lambda)$ are fuzzy $\delta$-semi separated in $X$.

**Proof:** Obvious.

**Theorem 2.10:** Two fuzzy sets $\lambda$ and $\mu$ of a fts X are fuzzy $\delta$-semi separated if and only if there exists fuzzy sets $\alpha,\beta \in F_{\delta}SO(X)$ such that $\lambda \le \alpha, \mu \le \beta, \lambda \not{q} \beta$ and $\mu \not{q} \alpha$.

**Proof:** *Necessary*: Let $\lambda$ and $\mu$ be two fuzzy $\delta$-semi separated sets in $X$. Put $\beta = 1-(\delta - scl(\lambda))$ and $\alpha = 1-(\delta - scl(\mu))$, then $\alpha,\beta \in F_{\delta}SO(X)$ such that $\lambda \le \alpha, \mu \le \beta, \lambda \not{q} \beta$ and $\mu \not{q} \alpha$.

*Sufficient:* Let $\alpha,\beta \in F_{\delta}SO(X)$ such that $\lambda \le \alpha, \mu \le \beta, \lambda \not{q} \beta$ and $\mu \not{q} \alpha$.

Now $(1-\beta),(1-\alpha) \in F_{\delta}SC(X)$, we have $(\delta - scl(\lambda)) \le 1-\beta \le 1-\mu$ and $(\delta - scl(\mu)) \le 1-\alpha \le 1-\lambda$. Therefore by lemma 2.5, $(\delta - scl(\lambda)) \not{q} \mu$ and $(\delta - scl(\mu)) \not{q} \lambda$. Hence $\lambda$ and $\mu$ are fuzzy $\delta$-semi separated in $X$.

## 3. Fuzzy $\delta$-semi Connectedness

**Definition 3.1:** A fuzzy set $\alpha$ in a fts $X$ is said to be fuzzy $\delta$-semi connected if it cannot be expressed as the union of two fuzzy $\delta$-semi separated sets.

**Remark 3.2:** Every fuzzy connected set is fuzzy $\delta$-semi connected but the converse may not be true. For the fuzzy set $\mu_3$ considered in example 2.3 is fuzzy $\delta$-semi connected but not fuzzy semi connected.

**Theorem 3.3:** Let $\lambda$ and $\mu$ be two fuzzy $\delta$-semi separated sets in a fts $X$ and $\alpha$ be a fuzzy $\delta$-semi connected set in $X$ such that $\alpha \le \lambda \cup \mu$. Then exactly one of the following conditions (a) and (b) holds:

(a) $\alpha \le \lambda$ and $\alpha \cap \mu = 0$

(b) $\alpha \le \mu$ and $\alpha \cap \lambda = 0$

**Proof:** We first note that when $\alpha \cap \mu = 0$, then $\alpha \le \lambda$, since $\alpha \le \lambda \cup \mu$. Similarly, since $\alpha \cap \lambda = 0$, we have $\alpha \le \mu$. Now, since $\alpha \le \lambda \cup \mu$, both $\alpha \cap \lambda = 0$ and $\alpha \cap \mu = 0$ cannot hold simultaneously. Again if $\alpha \cap \mu \ne 0$ and $\alpha \cap \lambda \ne 0$, then $\alpha \cap \lambda$ and $\alpha \cap \mu$ are fuzzy $\delta$-semi separated in $X$ such that $\alpha = (\alpha \cap \lambda) \cup (\alpha \cap \mu)$, contradicting the fuzzy $\delta$-semi connectedness of $\alpha$. Hence exactly one of the conditions (a) and (b) must holds.

**Theorem 3.4:** Let $\alpha$ and $\beta$ be two fuzzy sets of a fts $X$. If $\alpha$ is fuzzy $\delta$-semi connected and $\alpha \le \beta \le \delta - scl(\alpha)$, then $\beta$ is fuzzy $\delta$-semi connected.

**Proof:** If $\alpha = 0$ then the result is true. Let $\alpha \ne 0$, suppose $\beta$ is not fuzzy $\delta$-semi connected then $\exists$ two fuzzy $\delta$-semi separated sets $\lambda$ and $\mu$ in $X$ such that $\beta = \lambda \cup \mu$. Since $\alpha$ is fuzzy $\delta$-semi connected and $\alpha \le \beta \le \lambda \cup \mu$, by theorem 3.3 $\alpha \le \lambda$ and $\alpha \cap \mu = 0$ or $\alpha \le \mu$ and $\alpha \cap \lambda = 0$. Let $\alpha \le \lambda$ and $\alpha \cap \mu = 0$, then

$$\mu = \mu \cap \beta \le \mu \cap \delta - scl(\alpha)$$
$$\le \mu \cap \delta - scl(\lambda) \le \mu \cap (1-\mu) \le \mu.$$

It follows that $\mu = \mu \cap (1-\mu)$ and since $\mu \ne 0$, $\mu(x) = 1/2$ for all $x \in X$. Thus $\mu_0 = X$, where $\mu_0$ denotes the support of $\mu$. Now $\alpha \cap \mu = 0 \Rightarrow \alpha_0 \cap \mu_0 = \phi \Rightarrow \alpha_0 = \phi$. Hence $\alpha = 0$, which is a contradiction.

**Theorem 3.5:** Let $f: X \to Y$ be a fuzzy $\delta$-semi irresolute surjective mapping. If $\eta$ is a fuzzy $\delta$-semi connected subset in $X$, then $f(\eta)$ is fuzzy $\delta$-semi connected subset in $Y$.

**Proof:** Suppose that $f(\eta)$ is not fuzzy $\delta$-semi connected in $Y$. Then there exists fuzzy $\delta$-semi separated subset $\lambda$ and $\mu$ in $Y$ such that $f(\eta) = \lambda \cup \mu$. By theorem 2.10, there exists fuzzy $\delta$-semi open subsets $\alpha$ and $\beta$ such that $\lambda \le \alpha, \mu \le \beta, \lambda \not{q} \beta$ and $\mu \not{q} \alpha$. Since $f$ is fuzzy $\delta$-semi irresolute,

$$\eta = f^{-1}(f(\eta)) = f^{-1}(\lambda) \cup f^{-1}(\mu)$$
$$= f^{-1}(\lambda \cup \mu).$$

Also it can be easily seen that $f^{-1}(\lambda) \le f^{-1}(\alpha)$, $f^{-1}(\mu) \le f^{-1}(\beta)$, $f^{-1}(\lambda) \not{q} f^{-1}(\beta)$ and $f^{-1}(\mu) \not{q} f^{-1}(\alpha)$. Thus $f^{-1}(\lambda)$ and $f^{-1}(\mu)$ are fuzzy $\delta$-semi separated in $X$. Hence $\eta$ is not fuzzy $\delta$-semi connected, which is a contradiction.

**Theorem 3.6:** Let $f: X \to Y$ be a fuzzy $\delta$-semi continuous surjective mapping. If $\eta$ is a fuzzy $\delta$-semi connected subset in $X$, then $f(\eta)$ is fuzzy connected subset in $Y$.

**Proof:** Analogues to that theorem above.

# 4. Conclusions

Based on $\delta$ -semi open sets and $\delta$ -Semi continuous functions, $\delta$ -semi connectedness was introduced, which is the weaker form of different existing connectedness in fuzzy topological spaces.

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