International Journal of Control Science and Engineering 2019, 9(2): 27-36
DOI: 10.5923/j.control.20190902.01

Observer-Based Adaptive Fuzzy Sliding Mode Control for Switched Uncertain Nonlinear Systems with Dead-Zone Input

Chiang-Cheng Chiang*, Chin-Cheng Kuo

Department of Electrical Engineering, Tatung University, Taipei, Taiwan, Republic of China

Abstract This paper considers the problem of observer-based adaptive fuzzy sliding mode control for switched uncertain nonlinear systems with dead-zone input in strict-feedback form. The explored switched systems include unknown nonlinearities, dead-zone and immeasurable states. Fuzzy logic systems are used to approximate unknown nonlinear functions of the dynamic system and unknown upper bounds of uncertainties, respectively. A state observer based on state variable filters is developed to estimate the immeasurable states. Adaptive technique and sliding mode control method are utilized to construct a controller. By choosing an appropriate Lyapunov function, the proposed controller is designed to demonstrate that all the signals in the closed-loop system can not only guarantee uniformly ultimately bounded, but also achieve good tracking performance. Finally, the simulation results are provided to demonstrate the effectiveness of the proposed approach.

Keywords Switched system, Strict-feedback form, Lyapunov function, Sliding mode control, Dead-zone, Fuzzy logic system

1. Introduction

In recent years, switched systems have drawn much attention and some significant results have been obtained in the literature. As an important class of hybrid systems, switched systems can be described by a family of subsystems and a switching law between them. Actually, many practical systems, such as power electronic, automotive industry, mechanical systems, air traffic control, etc., can be expressed as switched systems [1-4]. Motivated by the above reasons, the control design and stability of the switched systems have received many researchers a great interest. In addition, dead-zone input nonlinearity is a nonsmooth function that features certain insensitivity for small control inputs which is often encountered in a variety of practical systems such as the single-link flexible joint manipulator, and so on [5, 17-18]. In this paper, the switched system in strict-feedback form is investigated and a feasible and systematic methodology is proposed to solve the tracking control problem.

Due to uncertainties inherent in practical systems, the design control capable of handling uncertainties is of practical interest and is challenging. To achieve the desired system performance, adaptive control is a valid methodology, which supplies adaptation mechanisms to regulate controllers for systems with some uncertainties, such as parametric, structural, and environmental uncertainties [6-7]. For non-switched nonlinear systems using fuzzy logic systems or neural networks to parameterise the unknown non-linearities, adaptive control of uncertain nonlinear systems has attracted much attention [8-9]. In recent years, adaptive fuzzy or neural backstepping approaches for strict-feedback form systems [10-11] provide some systematic methods to achieve good tracking performance.

Fuzzy logic systems (FLS) with appropriate adaptive laws algorithms are used to approximate the unknown nonlinear functions appearing in the structure of the switched system. A FLS consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The problem of adaptive fuzzy output-feedback control for switched uncertain non-linear systems is investigated in [12, 19]. In [13], Li *et al.* introduced an adaptive fuzzy output-feedback stabilization controller to treat the problem for a class of switched nonstrict-feedback nonlinear systems.

Recently, sliding mode control (SMC) has been adopted as a powerful approach for the control of systems with external disturbances and parameter variations [14, 20].

* Corresponding author:
ccchiang@ttu.edu.tw (Chiang-Cheng Chiang)
Published online at http://journal.sapub.org/control

Since SMC technique has some advantages, such as insensitivity to system parameter variations, invariance to external disturbances, good transient and fast response. Basically, SMC laws consist of two parts: switching controller design and equivalent controller design. The switching control law is employed to lead the system's states to a given sliding surface and the equivalent control law guarantees the system's states to stay on the aforementioned sliding surface and converge to zero along the sliding surface.

The main contribution of this paper is that the proposed SMC method deals with the problem of tracking performance and stability for a class of switched nonlinear systems in strict-feedback form with dead-zone input. Based on SMC technique, an adaptive fuzzy sliding mode controller is designed for the switched system by using fuzzy logic systems with some adaptive laws to approximate uncertain functions. The fuzzy state observer is employed to estimate the unavailable state for measurement. By choosing an appropriate Lyapunov function, it is theoretically ensured that all the signals in the closed-loop system are uniformly ultimately bounded and receive good tracking performance under our designed controller.

This paper organized as follows. Section II contains system description, dead-zone characterization, a detailed description systems, and fuzzy basis functions. Section III, the observer-based fuzzy sliding mode controller is utilized to treat the nonlinear switched system with dead-zone input. Simulation results are provided in Section IV to demonstrate the advantages and effectiveness of the proposed approaches Finally, the concluding remarks are gathered in Section V.

2. Problem Statement and Preliminaries

2.1. System Description

Consider a class of single-input-single-output switched uncertain nonlinear systems in strict-feedback form with dead-zone input expressed as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \quad\vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f_{\sigma(t)}(\mathbf{x}) + D(u_{\sigma(t)}(t)) + d_{\sigma(t)}(\mathbf{x},t) \\ y = x_1 \end{cases} , \quad (1)$$

where $\mathbf{x} = [x_1, x_{2,} \cdots, x_n]^{\mathrm{T}} \in R^n$ is the system state vector which is assumed to be available for measurement, $u_{\sigma(t)} \in R$ and $y \in R$ are the input and output of the system output, respectively. The function $\sigma(\mathrm{t}) : [0,\infty) \to \mathrm{M} = \{1, 2, ..., \mathrm{m}\}$, is a switching signal which is assumed to be a piecewise continuous (from the right) function of time. If $\sigma(t) = k$, then we say the kth switched subsystem is active and the remaining switched subsystems are inactive. $f_{\sigma(t)}(\mathbf{x})$ is the unknown smooth nonlinear function, $d_{\sigma(t)}(\mathbf{x},t)$ is unknown external bound disturbance. $D(u_{\sigma(\mathrm{t})}(\mathrm{t}))$ denotes the input function containing a dead-zone.

Then, the system (1) can be rewritten as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}[f_k(\mathbf{x}) + D(u_k(\mathrm{t})) + d_k(\mathbf{x},\mathrm{t})] \\ y = \mathbf{C}\mathbf{x} \end{cases} \quad (2)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix}^T \in R^{n\times 1}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \in R^{1\times n}$$

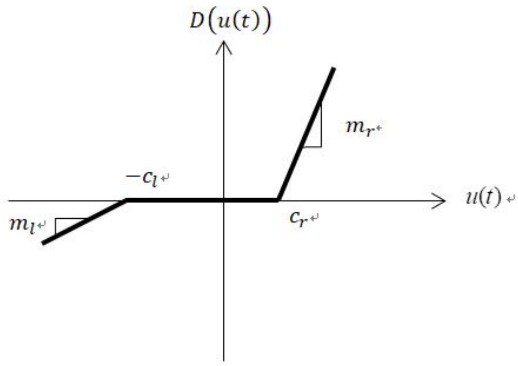

Figure 1. Dead-zone model

The non-symmetric dead-zone with input $u(t)$ and output as shown in the above Fig. 1 is described by

$$D(u(t)) = \begin{cases} m_r(u(t) - c_r) & \text{for } u(t) \ge c_r \\ 0 & \text{for } -c_l \le u(t) \le c_r \\ m_l(u(t) + c_l) & \text{for } u(t) \le -c_l \end{cases} \quad (3)$$

where, c_l, c_r, m_r, and m_l are parameters and slopes of the dead-zone, respectively. In order to investigate the key features of the dead-zone in the control problems, the following assumptions should be made:

Assumption 1: The dead-zone output $D(u(t))$ is not available to obtain.

Assumption 2: The coefficients c_r, c_l, and m_r, m_l are unknown.

Assumption 3: The maximum and minimum values of the characteristic slopes are known. $\max\{m_r, m_l\} = m_{\max}$, $\min\{m_r, m_l\} = m_{\min}$

Based on the above assumptions the expression (3) can be represented as

$$D(u(t)) = mu(t) + d(u(t)) \tag{4}$$

where $d(u(t))$ can be calculated form (3) and (4) as

$$d(u(t)) = \begin{cases} -m_r c_r & \text{for } u(t) \geq c_r \\ -mu(t) & \text{for } -c_l \leq u(t) \leq c_r \\ m_l c_l & \text{for } u(t) \leq -c_l \end{cases} \tag{5}$$

From *Assumption 1* and *Assumption 2*, we can conclude that $d(u(t))$ is bounded, and satisfies:

$$\left| d(u(t)) \right| \leq \rho \tag{6}$$

where ρ is an upper bound, which can be chosen as

$$\rho = \max\left\{ m_r c_r,\ m_l c_l \right\} \tag{7}$$

Assumption 4 : $0 < \left| d_k(\mathbf{x}, t) \right| \leq h_k(\mathbf{x}, t) < \infty$

where $h_k(\mathbf{x}, t)$ is an unknown function.

Control objective: Design a controller for (1) such that the system output y(t) would track the desired output vector $y_d(t)$, where $y_d(t)$ be a given bounded desired signal and contain finite derivative up to the n order. Define the vector of the output tracking error as $e_i = y^{(i-1)} - y_d^{(i-1)}$, $i = 1.2, \ldots, n$. Thus,

$$\begin{aligned} \dot{e}_1 &= \dot{y} - \dot{y}_d = e_2 \\ \dot{e}_2 &= \ddot{y} - \ddot{y}_d = e_3 \\ &\vdots \\ \dot{e}_{n-1} &= y^{(n-1)} - y_d^{(n-1)} = e_n \\ \dot{e}_n &= y^{(n)} - y_d^{(n)} \end{aligned} \tag{8}$$

2.2. Description of Fuzzy Logic Systems

The fuzzy logic system performs a mapping from $U \subset R^n$ to $V \subset R$. Let $U = U_1 \times \cdots \times U_n$ where $U_i \subset R$, $i = 1, 2, \cdots, n$. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules:

$$\begin{aligned} R^{(l)} :\ & \text{IF } x_1 \text{ is } F_1^l, \text{ and } x_2 \text{ is } F_2^l, \text{ and } \cdots \text{ and, } x_n \text{ is } F_n^l \\ & \text{THEN } y \text{ is } G^l, \quad \text{for } l = 1, 2, \cdots, B. \end{aligned} \tag{9}$$

in which $\mathbf{x} = \left[x_1, x_2, \cdots, x_n \right]^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy logic system, F_i^l and G^l are fuzzy sets in U_i and V, respectively. The fuzzifier maps a crisp point $\mathbf{x} = \left[x_1, x_2, \cdots, x_n \right]^T$ into a fuzzy set in U. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V.

The fuzzy systems with center-average defuzzifier, product inference and singleton fuzzifier are of the following form:

$$y = \boldsymbol{\theta}^T \boldsymbol{\xi}(\mathbf{x}) \tag{10}$$

where $\boldsymbol{\theta}^T = \left[\theta^1, \ldots, \theta^B \right]$ with each variable θ^l as the point at which the fuzzy membership function of G^l achieves the maximum value and $\boldsymbol{\xi}(\mathbf{x}) = \left[\xi^1(\mathbf{x}), \ldots, \xi^B(\mathbf{x}) \right]^T$ with each variable $\xi^l(\mathbf{x})$ as the fuzzy basis function defined as

$$\xi^l(\mathbf{x}) = \frac{\prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{F_i^l}(x_i) \right)} \tag{11}$$

where $\mu_{F_i^l}(x_i)$ is the membership function of the fuzzy set.

3. Controller Design and Stability Analysis

3.1. Observer Design

According to the description of the fuzzy logic system presented in Section 2.2, we can construct the following fuzzy logic systems, over a compact set, the unknown nonlinear functions $f_k(\mathbf{x})$ and the uncertainty $h_k(\mathbf{x}, t)$ can be approximated as

$$\hat{f}_{\mathrm{k}}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_{f,k}) = \hat{\boldsymbol{\theta}}_{f,k}^T \boldsymbol{\xi}(\mathbf{x}) \tag{12}$$

$$\hat{h}(\mathbf{x} \mid \hat{\boldsymbol{\theta}}_{h,k}) = \hat{\boldsymbol{\theta}}_{h,k}^T \boldsymbol{\xi}(\mathbf{x}) \tag{13}$$

where $\boldsymbol{\xi}(\mathbf{x})$ is the fuzzy basic vector, $\hat{\boldsymbol{\theta}}_{f,k}$ and $\hat{\boldsymbol{\theta}}_{\mathrm{h},k}$ are the corresponding adjustable parameter vectors of each fuzzy logic systems.

Owing to the unavailable states of the system and the unavailable elements of the output error vector in many practical systems, the fuzzy logic systems (12) and (13) are not used to control nonlinear systems whose states are not obtained for measurement. Therefore, we must employ an observer to estimate. Let $\hat{\mathbf{x}}$ be the estimate of $\mathbf{x}$ at first. Then, we can obtain the following fuzzy logic systems as

$$\hat{f}_{\mathrm{k}}(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) = \hat{\boldsymbol{\theta}}_{f,k}^T \boldsymbol{\xi}(\hat{\mathbf{x}}) \tag{14}$$

$$\hat{h}(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{h,k}) = \hat{\boldsymbol{\theta}}_{h,k}^T \boldsymbol{\xi}(\hat{\mathbf{x}}) \tag{15}$$

In order to estimate the state, the observer can be chosen as follows:

$$\begin{cases} \dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\left[\hat{f}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) + D(\mathrm{u}_k(t)) + v_k \right] + \mathbf{L}_k(y - \hat{y}) \\ y = \mathbf{C}\hat{\mathbf{x}} \end{cases} \tag{16}$$

where $\mathbf{L}_k = [l_{1k}\ l_{2k}\ \cdots\ l_{nk}]^T \in R^{n\times 1}$ is the observer gain vector, and $l_{ik} > 0\ (i = 1,2,\ldots,n)$ are coefficients of the Hurwitz polynomial $p(s) = s^n + l_1 s^{n-1} + \cdots + l_{n-1}s + l_n$. Define the estimation error vector as $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ and $\tilde{y} = y - \hat{y}$.

Then from (2) and (16), we obtain

$$\begin{cases}\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{L}_k\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}\left[f_k(\mathbf{x}) - \hat{f}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{f,k}) + d_k(\mathbf{x},t) - \nu_k\right] \\ \tilde{y} = \mathbf{C}\tilde{\mathbf{x}}\end{cases} \tag{17}$$

It is assumed that $\mathbf{x}$, $\hat{\mathbf{x}}$, $\hat{\boldsymbol{\theta}}_{f,k}$ and $\hat{\boldsymbol{\theta}}_{h,k}$ belong to compact sets $\Omega_{\mathbf{x}}$, $\Omega_{\hat{\mathbf{x}}}$, $\Omega_{\hat{\boldsymbol{\theta}}_{f,k}}$ and $\boldsymbol{\Omega}_{\hat{\boldsymbol{\theta}}_{h,k}}$ respectively, which is defined as

$$\Omega_{\mathbf{x}} = \left\{\mathbf{x} \in R^{n\times 1} : \|\mathbf{x}\| \le N_{\mathbf{x}} < \infty\right\} \tag{18}$$

$$\Omega_{\hat{\mathbf{x}}} = \left\{\hat{\mathbf{x}} \in R^{n\times 1} : \|\hat{\mathbf{x}}\| \le N_{\hat{\mathbf{x}}} < \infty\right\} \tag{19}$$

$$\Omega_{\hat{\boldsymbol{\theta}}_{f,k}} = \left\{\hat{\boldsymbol{\theta}}_{f,k} \in R^{n\times 1} : \left\|\boldsymbol{\theta}_{f,k}\right\| \le N_f < \infty\right\} \tag{20}$$

$$\Omega_{\hat{\boldsymbol{\theta}}_{h,k}} = \left\{\hat{\boldsymbol{\theta}}_{h,k} \in R^{n\times 1} : \left\|\boldsymbol{\theta}_{h,k}\right\| \le N_h < \infty\right\} \tag{21}$$

where $N_{\mathbf{x}}, N_{\hat{\mathbf{x}}}, N_f$ and N_h are the designed parameters, and N is the number of fuzzy inference rules. Let us define the optimal parameter vector $\boldsymbol{\theta}^*_{f,k}$ and $\boldsymbol{\theta}^*_{h,k}$ as follows:

$$\boldsymbol{\theta}^*_{f,k} = \arg\min_{\hat{\boldsymbol{\theta}}_{f,k}\in\Omega_{\hat{\boldsymbol{\theta}}_{f,k}}}\left\{\sup_{\mathbf{x}\in\Gamma}\left|f_k(\mathbf{x}) - \hat{f}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{f,k})\right|\right\} \tag{22}$$

$$\boldsymbol{\theta}^*_{\mathrm{h},k} = \arg\min_{\hat{\boldsymbol{\theta}}_{\mathrm{h},k}\in\Omega_{\hat{\boldsymbol{\theta}}_{\mathrm{h},k}}}\left\{\sup_{\mathbf{x}\in\Gamma}\left|h_k(\mathbf{x}) - \hat{h}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{h,k})\right|\right\} \tag{23}$$

where $\boldsymbol{\theta}^*_{f,k}$ and $\boldsymbol{\theta}^*_{\mathrm{h},k}$ are bounded in the suitable closed set $\Omega_{\hat{\boldsymbol{\theta}}_{f,k}}$ and $\Omega_{\hat{\boldsymbol{\theta}}_{h,k}}$. The parameter estimation errors can be defined as

$$\tilde{\boldsymbol{\theta}}_{f,k} = \boldsymbol{\theta}^*_{f,k} - \hat{\boldsymbol{\theta}}_{f,k} \tag{24}$$

$$\tilde{\boldsymbol{\theta}}_{h,k} = \boldsymbol{\theta}^*_{h,k} - \hat{\boldsymbol{\theta}}_{h,k} \tag{25}$$

Then,

$$w_{1k} = f_k(\mathbf{x}) - \hat{f}_k(\hat{\mathbf{x}}\mid\boldsymbol{\theta}^*_{f,k}) \tag{26}$$

$$w_{2k} = h_k(\mathbf{x}) - h_k(\hat{\mathbf{x}}\mid\boldsymbol{\theta}^*_{\mathrm{h},k}) \tag{27}$$

are the minimum approximation errors, which correspond to approximation errors obtained when optimal parameters are used.

Applying (24) and (26) to (17), it yields

$$\begin{aligned}&\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{L}_k\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}\left[\tilde{\boldsymbol{\theta}}^T_{f,k}\xi(x) + w_{1k} + d_k(\mathbf{x},t) - \nu_k\right] \\ &\tilde{y} = \mathbf{C}\tilde{\mathbf{x}}\end{aligned} \tag{28}$$

The output error dynamic of (28) can be expressed as follows:

$$\tilde{y} = H(s)\left[\tilde{\boldsymbol{\theta}}^T_{f,k}\xi(x) + w_{1k} + d_k(\mathbf{x},t) - \nu_k\right] \tag{29}$$

where $H(s) = \mathbf{C}(s\mathbf{I} - (\mathbf{A} - \mathbf{L}_k\mathbf{C}))^{-1}\mathbf{B}$ and s denotes the complex Laplace transform variable. As has been discussed, we could not obtain all elements of $\mathbf{x}$, because not all states of the system are available or measurement. Hence, we could not obtain all elements of $\mathbf{x}$. We will employ the state variable filters [15] to cope with this problem. First, we choose a stable filter $G(\mathrm{s})$ as the following form:

$$G(s) = \frac{1}{s^n + g_1 s^{n-1} + g_2 s^{n-2} + \cdots + g_{n-1}} \tag{30}$$

Where $g_i>0$ $i=1,2,\cdots,n-1$ are coefficients of the Hurwitz polynomial $p(s) = s^n + g_1 s^{n-1} + g_2 s^{n-2} + \cdots + g_{n-1}$.

Introducing (30) into (29), we can obtain the steady-state equation

$$G(s)H(\mathrm{s})^{-1}\{\tilde{y}\} = G(\mathrm{s})\left[\tilde{\boldsymbol{\theta}}^T_{f,k}\xi(x) + w_{1k} + d_k(\mathbf{x},t) - \nu_k\right] \tag{31}$$

Define a set of state variable filters $T_i(\mathrm{s})$ as

$$T_i(s) = G(s)s^i\ ,i = 0,1,2,\ldots,n-1 \tag{32}$$

The corresponding filtered signals are defined as follows:

$$\tilde{x}_{if} = T_i(s)\{\tilde{x}_1\},\ i = 0,1,\ldots,n-1 \tag{33}$$

$$w_{1kf} = T_0(s)\{w_{1k}\} \tag{34}$$

$$\xi_f = T_0(s)\{\xi\} \tag{35}$$

$$\nu_{kf} = T_0(s)\{\nu_k\} \tag{36}$$

$$d_{kf}(\mathbf{x},t) = T_0(s)\{d_k(\mathbf{x},t)\} \tag{37}$$

Then, Eq. (28) can be rewritten as follows:

$$\begin{cases}\dot{\tilde{\mathbf{x}}}_f = (\mathbf{A} - \mathbf{L}_k\mathbf{C})\tilde{\mathbf{x}}_f + \mathbf{B}\left[\tilde{\boldsymbol{\theta}}^T_{f,k}\xi_f(\hat{\mathbf{x}}) + w_{1kf} + d_{kf}(\mathbf{x},t) - \nu_{kf}\right] \\ \tilde{y}_f = \mathbf{C}\tilde{\mathbf{x}}_f\end{cases} \tag{38}$$

where

$$\tilde{\mathbf{x}}_f = \left[\tilde{x}_{1f}\ \tilde{x}_{2f}\ \tilde{x}_{3f}\ \cdots\ \tilde{x}_{nf}\right] \in R^n$$

$$\mathbf{A} - \mathbf{L}_k\mathbf{C} = \begin{bmatrix} -l_{1,k} & 1 & 0 & \cdots & 0 \\ -l_{2,k} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -l_{\mathrm{n}-1,k} & 0 & 0 & \cdots & 1 \\ -l_{\mathrm{n},k} & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n}$$

Define

$$\tilde{w} = w - \hat{w} \tag{39}$$

where $\hat{w}$ is the estimated of w, and

$$|w_{1kf}| + |w_{2k}| \le w \tag{40}$$

Based on the Lyapunov stable theorem, we can obtain the robust compensation term v_{kf} and the parameter update laws as follows:

$$v_{kf} = \frac{\mathbf{B}^T \mathbf{P}_k \tilde{\mathbf{x}}_f}{\left\|\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\right\|}\hat{w} + \frac{\mathbf{B}^T \mathbf{P}_k \tilde{\mathbf{x}}_f}{\left\|\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\right\|}\hat{h}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{h,k}) \tag{41}$$

$$\dot{\hat{\boldsymbol{\theta}}}_{f,k} = \gamma_f \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \tag{42}$$

$$\dot{\hat{\boldsymbol{\theta}}}_{h,k} = \gamma_h \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \tag{43}$$

$$\dot{\hat{w}} = \gamma_w \left\|\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\right\| \tag{44}$$

where γ_f, γ_h and γ_w are positive constants

Remark 1: Without loss of generality, the adaptive laws used in this paper are assumed that the parameter vectors are within the constraint sets or on the boundaries of the constraint sets but moving toward the inside of the constraint sets. If the parameter vectors are on the boundaries of the constraint sets but moving toward the outside of the constraint sets, we have to use the projection algorithm to modify the adaptive laws such that the parameter vectors will remain inside of the constraint sets. Readers can refer to reference [16]. The proposed adaptive law (41)-(44) can be modified as the following form:

$$\dot{\hat{\boldsymbol{\theta}}}_{f,k} = \begin{cases} \gamma_f \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}, & \text{if } (\left\|\hat{\boldsymbol{\theta}}_{f,k}\right\| < N_f) \\ & \text{or } (\left\|\hat{\boldsymbol{\theta}}_{f,k}\right\| = N_f \\ & \text{and } \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \cdot \hat{\boldsymbol{\theta}}_{f,k}^{\mathrm{T}} \boldsymbol{\xi}_f(\hat{\mathbf{x}}) \ge 0) \\ P\left\{\gamma_f \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k\right\}\mathbf{B}, & \text{if } (\left\|\hat{\boldsymbol{\theta}}_{f,k}\right\| = N_f \\ & \text{and } \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \cdot \hat{\boldsymbol{\theta}}_{f,k}^{\mathrm{T}} \boldsymbol{\xi}_f(\hat{\mathbf{x}}) < 0) \end{cases} \tag{45}$$

where $P\left\{\gamma_f \tilde{\mathbf{x}}_f^T \mathbf{P}_k \boldsymbol{\xi}_f(\hat{\mathbf{x}})\right\}$ is defined as

$$\begin{aligned} &P\left\{\gamma_f \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \boldsymbol{\xi}_f(\hat{\mathbf{x}})\right\} \\ &= \gamma_f \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} + \gamma_f \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \frac{\hat{\boldsymbol{\theta}}_{f,k}\hat{\boldsymbol{\theta}}_{f,k}^{\mathrm{T}}}{\left\|\hat{\boldsymbol{\theta}}_{f,k}\right\|^2}\boldsymbol{\xi}_f(\hat{\mathbf{x}}) \end{aligned} \tag{46}$$

$$\dot{\hat{\boldsymbol{\theta}}}_{\mathrm{h},k} = \begin{cases} \gamma_h \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}, & \text{if } (\left\|\hat{\boldsymbol{\theta}}_{\mathrm{h},k}\right\| < N_h) \\ & \text{or } (\left\|\hat{\boldsymbol{\theta}}_{\mathrm{h},k}\right\| = N_h \\ & \text{and } \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \cdot \hat{\boldsymbol{\theta}}_{\mathrm{h},k}^{\mathrm{T}} \boldsymbol{\xi}_f(\hat{\mathbf{x}}) \ge 0) \\ P\left\{\gamma_h \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k\right\}\mathbf{B}, & \text{if } (\left\|\hat{\boldsymbol{\theta}}_{\mathrm{h},k}\right\| = N_h \\ & \text{and } \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \cdot \hat{\boldsymbol{\theta}}_{\mathrm{h},k}^{\mathrm{T}} \boldsymbol{\xi}_f(\hat{\mathbf{x}}) < 0) \end{cases} \tag{47}$$

where $P\left\{\gamma_f \tilde{\mathbf{x}}_f^T \mathbf{P}_k \boldsymbol{\xi}_f(\hat{\mathbf{x}})\right\}$ is defined as

$$\begin{aligned} &P\left\{\gamma_h \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \boldsymbol{\xi}_f(\hat{\mathbf{x}})\right\} \\ &= \gamma_h \boldsymbol{\xi}_f(\hat{\mathbf{x}})\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} + \gamma_h \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} \frac{\hat{\boldsymbol{\theta}}_{\mathrm{h},k}\hat{\boldsymbol{\theta}}_{\mathrm{h},k}^{\mathrm{T}}}{\left\|\hat{\boldsymbol{\theta}}_{\mathrm{h},k}\right\|^2}\boldsymbol{\xi}_f(\hat{\mathbf{x}}) \end{aligned} \tag{48}$$

The main result of the robust adaptive fuzzy observer scheme is summarized on the following theorem.

Theorem 1: Consider the single-input-single-output uncertain switched system in strict-feedback form (1). The robust adaptive fuzzy observer is defined by (16) with adaptation laws given by (41)-(44). For the given positive definite matrix $\mathbf{Q}_k$, if there exist symmetric positive definite matrix $\mathbf{P}_k$ such that the following Lyapunov equation

$$(\mathrm{A} - \mathrm{L}_k \mathrm{C})^T \mathrm{P}_k + \mathrm{P}_k(\mathrm{A} - \mathrm{L}_k \mathrm{C}) = -\mathrm{Q}_k \tag{49}$$

is satisfied, then all signals of the closed-loop system are bounded, and the estimation errors converge to a neighborhood of zero.

Proof. Consider the Lyapunov function candidate

$$V_{1k} = \frac{1}{2}\left(\tilde{\mathbf{x}}_f^T \mathbf{P}_k \tilde{\mathbf{x}}_f + \frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{f,k}^T \tilde{\boldsymbol{\theta}}_{f,k} + \frac{1}{\gamma_h}\tilde{\boldsymbol{\theta}}_{\mathrm{h},k}^T \tilde{\boldsymbol{\theta}}_{\mathrm{h},k} + \frac{1}{\gamma_w}\tilde{w}^2\right) \tag{50}$$

By the time derivate of V_1 and the facts $\dot{\tilde{\theta}}_{f,k} = -\dot{\hat{\theta}}_{f,k}$, $\dot{\tilde{\theta}}_{h,k} = -\dot{\hat{\theta}}_{h,k}$, $\dot{\tilde{w}} = -\dot{\hat{w}}$ we obtain

$$\begin{aligned} \dot{V}_{1k} &= \frac{1}{2}\tilde{\mathbf{x}}_f^T\left[\left(\mathbf{A} - \mathbf{L}_k \mathbf{C}\right)^T \mathbf{P}_k + \mathbf{P}_k\left(\mathbf{A} - \mathbf{L}_k \mathbf{C}\right)\right]\tilde{\mathbf{x}}_f \\ &\quad + \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\tilde{\boldsymbol{\theta}}_{f,k}^T \boldsymbol{\xi}_f(\hat{\mathbf{x}}) + \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} w_{1kf} + \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} d_{kf}(\mathbf{x}, t) \\ &\quad - \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} v_{kf} - \frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{f,k}^T \dot{\hat{\boldsymbol{\theta}}}_{f,k} - \frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{h,k}^T \dot{\hat{\boldsymbol{\theta}}}_{h,k} - \frac{1}{\gamma_w}\tilde{w}\dot{\hat{w}} \\ &\le \frac{1}{2}\tilde{\mathbf{x}}_f^T\left[\left(\mathbf{A} - \mathbf{L}_k \mathbf{C}\right)^T \mathbf{P}_k + \mathbf{P}_k\left(\mathbf{A} - \mathbf{L}_k \mathbf{C}\right)\right]\tilde{\mathbf{x}}_f \\ &\quad + \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\tilde{\boldsymbol{\theta}}_{f,k}^T \boldsymbol{\xi}_f(\hat{\mathbf{x}}) + \left\|\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\right\|\left|w_{1kf}\right| + \left\|\tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B}\right\|\left|d_{kf}(\mathbf{x}, t)\right| \\ &\quad - \tilde{\mathbf{x}}_f^T \mathbf{P}_k \mathbf{B} v_{kf} - \frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{f,k}^T \dot{\hat{\boldsymbol{\theta}}}_{f,k} - \frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{h,k}^T \dot{\hat{\boldsymbol{\theta}}}_{h,k} - \frac{1}{\gamma_w}\tilde{w}\dot{\hat{w}} \end{aligned} \tag{51}$$

Applying *Assumption 4* and (27) and (40), it yields

$$\begin{aligned}\dot{V}_{1k} \le & \frac{1}{2}\tilde{\mathbf{x}}_f^T\left[\left(\mathbf{A}-\mathbf{L}_k\mathbf{C}\right)^T\mathbf{P}_k+\mathbf{P}_k\left(\mathbf{A}-\mathbf{L}_k\mathbf{C}\right)\right]\tilde{\mathbf{x}}_f \\ & +\left[\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}\tilde{\boldsymbol{\theta}}_{f,k}^T\xi_f(\hat{\mathbf{x}})-\frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{f,k}^T\dot{\hat{\boldsymbol{\theta}}}_{f,k}\right] \\ & +\left[\left\|\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}\right\|\tilde{\boldsymbol{\theta}}_{h,k}^T\xi_f(\hat{\mathbf{x}})-\frac{1}{\gamma_f}\tilde{\boldsymbol{\theta}}_{h,k}^T\dot{\hat{\boldsymbol{\theta}}}_{h,k}\right] \\ & +\left[\left\|\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}\right\|w-\frac{1}{\gamma_w}\tilde{w}\dot{\hat{w}}\right]+\left\|\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}\right\|\hat{h}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{h,k})-\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}v_{kf}\end{aligned} \tag{52}$$

By employing (42)-(44), we have

$$\begin{aligned}\dot{V}_{1k} \le & \frac{1}{2}\tilde{\mathbf{x}}_f^T\left[\left(\mathbf{A}-\mathbf{L}_k\mathbf{C}\right)^T\mathbf{P}_k+\mathbf{P}_k\left(\mathbf{A}-\mathbf{L}_k\mathbf{C}\right)\right]\tilde{\mathbf{x}}_f \\ & +\left\|\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}\right\|\hat{w}+\left\|\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}\right\|\hat{h}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{h,k})-\tilde{\mathbf{x}}_f^T\mathbf{P}_k\mathbf{B}v_{kf}\end{aligned} \tag{53}$$

Then using the robust compensation term v_{kf} (41), the above equation can be rewritten as

$$\dot{V}_{1k} \le \frac{1}{2}\tilde{\mathbf{x}}_f^T\left[\left(\mathbf{A}-\mathbf{L}_k\mathbf{C}\right)^T\mathbf{P}_k+\mathbf{P}_k\left(\mathbf{A}-\mathbf{L}_k\mathbf{C}\right)\right]\tilde{\mathbf{x}}_f \tag{54}$$

According to (49), it can be easily shown that

$$\dot{V}_{1k} \le -\frac{1}{2}\tilde{\mathbf{x}}_f^T\mathbf{Q}_k\tilde{\mathbf{x}}_f \tag{55}$$

Therefore, it can be concluded that $\dot{V}_{1k} \le 0$ from (55), and the estimation errors of the closed-loop system converge to a neighborhood of zero based on Lyapunov synpaper approach. This completes the proof.

3.2. Controller Design

Next, we design the observer-based sliding mode controller. By employing (8) and (16), we get

$$\begin{aligned}\dot{\hat{e}}_1 &= \hat{x}_2+l_{1k}\tilde{y}-\dot{y}_d \\ \dot{\hat{e}}_2 &= \hat{x}_3+l_{2k}\tilde{y}+l_{1k}\dot{\tilde{y}}-\ddot{y}_d \\ \dot{\hat{e}}_3 &= \hat{x}_4+\sum_{i=1}^{3}l_{ik}\tilde{y}^{(3-i)}-y_d^{(3)} \\ &\vdots \\ \dot{\hat{e}}_{n-1} &= \hat{x}_n+\sum_{i=1}^{n-1}l_{ik}\tilde{y}^{(n-1-i)}-y_d^{(\mathrm{n}-1)} \\ \dot{\hat{e}}_n &= \left[f_k\left(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{f,k}\right)+D\left(u_k(t)\right)+v_k\right]+\sum_{i=1}^{n}l_{ik}\tilde{y}^{(n-i)}-y_d^{(\mathrm{n})}\end{aligned}$$

Define the sliding surfaces as follows:

$$S_k = c_{1k}\hat{e}_1+c_{2k}\hat{e}_2+c_{3k}\hat{e}_3+\cdots+c_{n-1,k}\hat{e}_{n-1}+\hat{e}_n \tag{56}$$

where $c_{ik}>0,\ i=1,2,\cdots,n-1$ is designed parameters.

Differentiating S_k with respect to time, we have

$$\begin{aligned}\dot{S}_k &= c_{1k}\dot{\hat{e}}_1+c_{2k}\dot{\hat{e}}_2+c_{3k}\dot{\hat{e}}_3+\cdots c_{n-1,k}\dot{\hat{e}}_{n-1}+\dot{\hat{e}}_n \\ &= \sum_{i=1}^{n-1}c_{ik}\dot{\hat{e}}_i+\dot{\hat{e}}_n \\ &= \sum_{i=1}^{n-1}c_{ik}(\hat{x}_{i+1}+\sum_{j=1}^{i}l_{ik}\tilde{y}^{(i-j)}-y_d^{(i)})+\hat{f}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{f,k}) \\ &\quad +D(u_k(t))+v_k+\sum_{i=1}^{n}l_{ik}\tilde{y}^{(n-i)}-y_d^{(n)}\end{aligned} \tag{57}$$

Consider the following controller

$$\begin{aligned}u_k = & -\hat{\phi}\left[\sum_{i=1}^{n-1}c_{ik}(\hat{x}_{i+1}+\sum_{j=1}^{i}l_{jk}\tilde{y}^{(i-j)}-y_d^{(i)})\right. \\ & \left.+\hat{f}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{f,k})+v_k+\sum_{i=1}^{n}l_{ik}\tilde{y}^{(n-i)}-y_d^{(n)}\right] \\ & -\frac{1}{m_{\min}}\operatorname{sgn}(S_k)\hat{\rho}-KS_k\end{aligned} \tag{58}$$

in which K is a positive constant, and ρ is defined in (7).

We defined

$$\tilde{\phi} = \phi-\hat{\phi} \tag{59}$$

$$\tilde{\rho} = \rho-\hat{\rho} \tag{60}$$

where $\hat{\phi}$ is the estimate of ϕ, which is defined as $\phi=\frac{1}{m}$. $\hat{\rho}$ is the estimate of ρ. The parameter update laws are as follows:

$$\begin{aligned}\dot{\hat{\phi}} = & \eta S_k\left[\sum_{i=1}^{n-1}c_{ik}(\hat{x}_{i+1}+\sum_{j=1}^{i}l_{jk}\tilde{y}^{(i-j)}-y_d^{(i)})\right. \\ & \left.+\hat{f}_k(\hat{\mathbf{x}}\mid\hat{\boldsymbol{\theta}}_{f,k})+v_k+\sum_{i=1}^{n}l_{ik}\tilde{y}^{(n-i)}-y_d^{(n)}\right]\end{aligned} \tag{61}$$

$$\dot{\hat{\rho}} = \gamma_\rho\left|S_k\right| \tag{62}$$

where the scalar γ_ρ and η are positive constants, and v_k can be obtained by backward from v_{kf}.

Theorem 2: Consider the nonlinear switched system (1) with an unknown dead-zone input (4). The proposed observed-based fuzzy sliding mode controller defined by (58) guarantees that all signals of the closed-loop system are bounded and converge to a neighborhood of zero.

Proof. Consider the Lyapunov function candidate

$$V_{2k} = \frac{1}{2}\left(\frac{S_k{}^2}{m}+\frac{1}{\eta}\tilde{\phi}^2+\frac{1}{m_{\min}\gamma_\rho}\tilde{\rho}^2\right) \tag{63}$$

Differentiating the Lyapunov function V_{2k} with the respect to time, we obtain

$$\dot{V}_{2k} = \frac{1}{m} S_k \dot{S}_k + \frac{1}{\eta} \tilde{\phi}\dot{\tilde{\phi}} + \frac{1}{m_{\min}\gamma_\rho} \tilde{\rho}\dot{\tilde{\rho}} \tag{64}$$

By the fact $\dot{\tilde{\phi}} = -\dot{\hat{\phi}}$ and $\dot{\tilde{\rho}} = -\dot{\hat{\rho}}$, the above equation becomes

$$\dot{V}_{2k} = \frac{1}{m} S_k \dot{S}_k - \frac{1}{\eta} \tilde{\phi}\dot{\hat{\phi}} - \frac{1}{m_{\min}\gamma_\rho} \tilde{\rho}\dot{\hat{\rho}} \tag{65}$$

Applying equation (57), equation (65) can be written as

$$\begin{aligned}
\dot{V}_{2k} &= \frac{S_k}{m}\left\{ \sum_{i=1}^{n-1} c_{ik}(\hat{x}_{i+1} + \sum_{j=1}^{i} l_{jk}\tilde{y}^{(i-j)} - y_d^{(i)}) + \hat{f}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) \right. \\
&\quad \left. + D(u_k(t)) + v_k + \sum_{i=1}^{n} l_{ik}\tilde{y}^{(n-i)} - y_d^{(n)} \right\} \\
&\quad - \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} - \frac{1}{m_{\min}\gamma_\rho}\tilde{\rho}\dot{\hat{\rho}} \\
&\le \frac{S_k}{m}\left\{ \sum_{i=1}^{n-1} c_{ik}(\hat{x}_{i+1} + \sum_{j=1}^{i} l_{jk}\tilde{y}^{(i-j)} - y_d^{(i)}) + \hat{f}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) \right. \\
&\quad \left. + m u_k(t) + v_k + \sum_{i=1}^{n} l_{ik}\tilde{y}^{(n-i)} - y_d^{(n)} \right\} \qquad (66) \\
&\quad + \frac{1}{m}|S_k|\rho - \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} - \frac{1}{m_{\min}\gamma_\rho}\tilde{\rho}\dot{\hat{\rho}} \\
&\le \frac{S_k}{m}\left\{ \sum_{i=1}^{n-1} c_{ik}(\hat{x}_{i+1} + \sum_{j=1}^{i} l_{jk}\tilde{y}^{(i-j)} - y_d^{(i)}) + \hat{f}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) \right. \\
&\quad \left. + v_k + \sum_{i=1}^{n} l_{ik}\tilde{y}^{(n-i)} - y_d^{(n)} \right\} + S_k u_k(t) \\
&\quad + \left[\frac{1}{m_{\min}}|S_k|\rho - \frac{1}{m_{\min}\gamma_\rho}\tilde{\rho}\dot{\hat{\rho}} \right] - \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}
\end{aligned}$$

According to $\phi = \dfrac{1}{m}$, the above equation (66) becomes

$$\begin{aligned}
\dot{V}_{2k} &\le \phi S_k \left\{ \sum_{i=1}^{n-1} c_{ik}(\hat{x}_{i+1} + \sum_{j=1}^{i} l_{ik}\tilde{y}^{(i-j)} - y_d^{(i)}) + \hat{f}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) \right. \\
&\quad \left. + v_k + \sum_{i=1}^{n} l_{ik}\tilde{y}^{(n-i)} - y_d^{(n)} \right\} + S_k u_k(t) \qquad (67) \\
&\quad + \left[\frac{1}{m_{\min}}|S_k|\rho - \frac{1}{m_{\min}\gamma_\rho}\tilde{\rho}\dot{\hat{\rho}} \right] - \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}
\end{aligned}$$

Applying adaptive laws (61) and (62), we obtained

$$\begin{aligned}
V_{2k} &\le \hat{\phi} S_k \left[\sum_{i=1}^{n-1} c_{ik}(\hat{x}_{i+1} + \sum_{j=1}^{i} l_{jk}\tilde{y}^{(i-j)} - y_d^{(i)}) + \hat{f}_k(\hat{\mathbf{x}} \mid \hat{\boldsymbol{\theta}}_{f,k}) \right. \\
&\quad \left. + v_k + \sum_{i=1}^{n} l_{ik}\tilde{y}^{(n-i)} - y_d^{(n)} \right] + S_k u_k(t) + + \frac{1}{m_{\min}}|S_k|\hat{\rho}
\end{aligned} \tag{68}$$

Using the control law (58) the above equation can be rewritten as

$$\dot{V}_{2k} \le -K S_k{}^2 \le 0 \tag{69}$$

Therefore, it can be concluded that from (69), and the all signals of the closed-loop system converge asymptotically to a neighborhood of zero based on the Lyapunov synthesis approach. This completes the proof.

4. An Example and Simulation Results

In this section, a mass-spring-damper system [12] in the presence of uncertain parameter and exogenous disturbances is considered as our simulation example in Fig. 2. The corresponding mathematical model is described as follows:

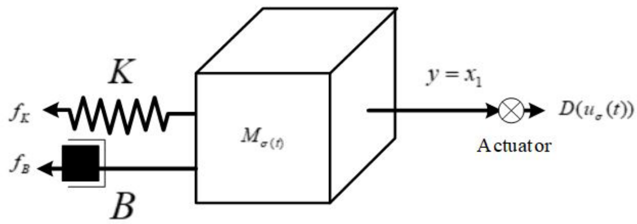

Figure 2. The mass-spring-damper system

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-f_K(\mathbf{x},t) - f_B(\mathbf{x},t) + \mathrm{D}(u_{\sigma(t)}(t))}{M_{\sigma(t)}} + d_{\sigma(t)}(\mathbf{x},t)
\end{aligned}$$

where $y = x_1$ is the displacement of the mass, x_2 is the velocity of the mass, $f_K(\mathbf{x},t) = x_1^2$ are the spring force, $f_B(\mathbf{x},t) = 0.5x_2^3$, are the friction force, $\sigma(t) = k,\ k \in 1,\ 2$ is the switched signal, $M_k = 0.75\text{kg}, k = 1$ and $M_k = 1\text{kg}$, $k = 2$ is the body mass, and u_k is the applied force. The structures of spring force and friction force are assumed to be known. The exogenous disturbance is assumed to be $d_k(\mathbf{x},t) = 0.1x_2\sin(t)$ and $d_k(\mathbf{x},t) = 0.2x_2\sin(t)$, In the implementation, five fuzzy sets are defined over interval [-3,3] for x_1 and x_2 , with labels F_1, F_2, F_3, F_4 and F_5 and their membership functions are

$$\mu_{F_1}(x_i) = \exp\left[\frac{-(x_i+2)^2}{4}\right],$$

$$\mu_{F_2}(x_i) = \exp\left[\frac{-(x_i+1)^2}{4}\right],$$

$$\mu_{F_3}(x_i) = \exp\left[\frac{-(x_i)^2}{4}\right],$$

$$\mu_{F_4}(x_i) = \exp\left[\frac{-(x_i - 1)^2}{4}\right],$$

$$\mu_{F_5}(x_i) = \exp\left[\frac{-(x_i - 2)^2}{4}\right],\ i = 1,\ 2$$

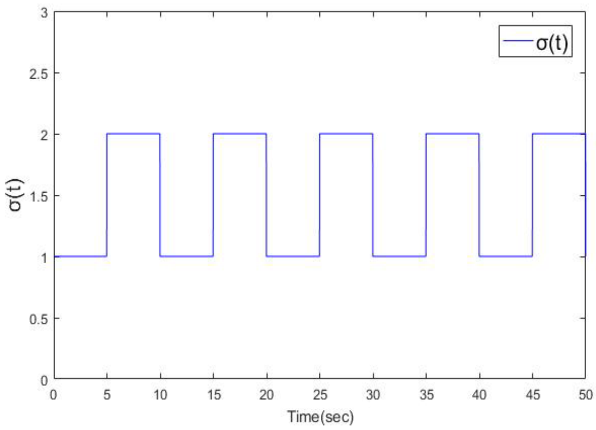

Figure 3. The switched signal $\sigma(t)$ with dwell time is 5secs

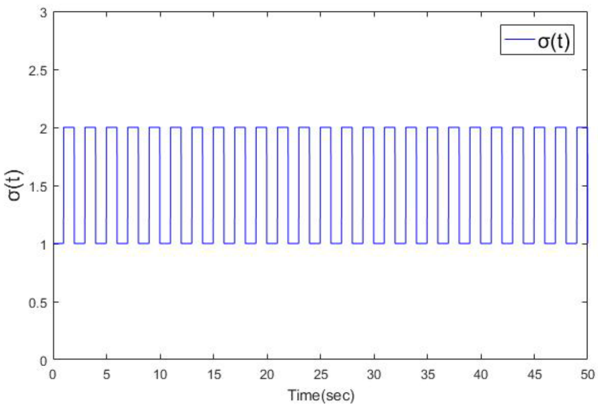

Figure 4. The switched signal $\sigma(t)$ with dwell time is 1sec

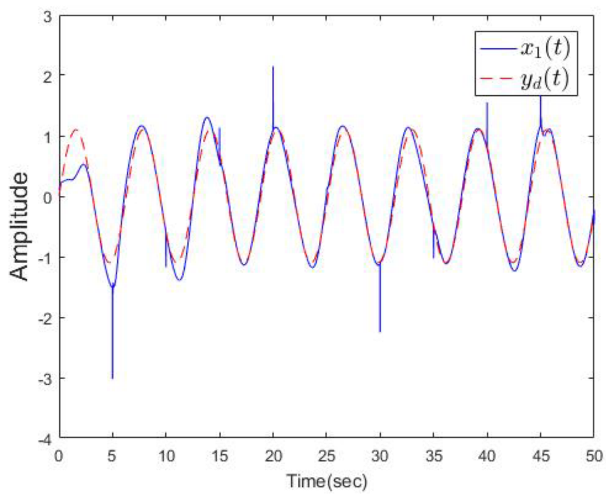

Figure 5. The outputs x_1 and y_d with dwell time is 5secs

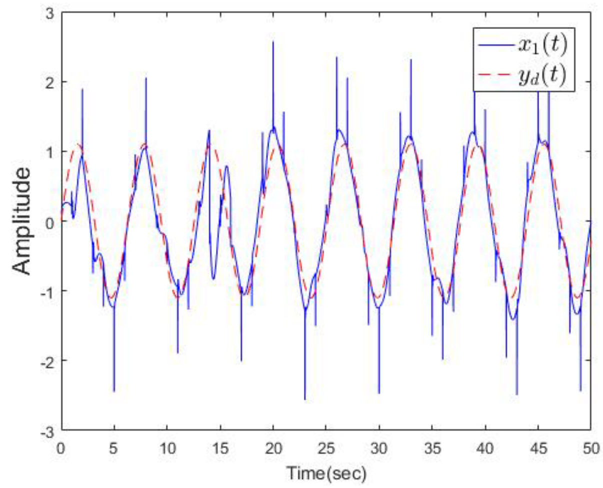

Figure 6. The outputs x_1 and y_d with dwell time is 1sec

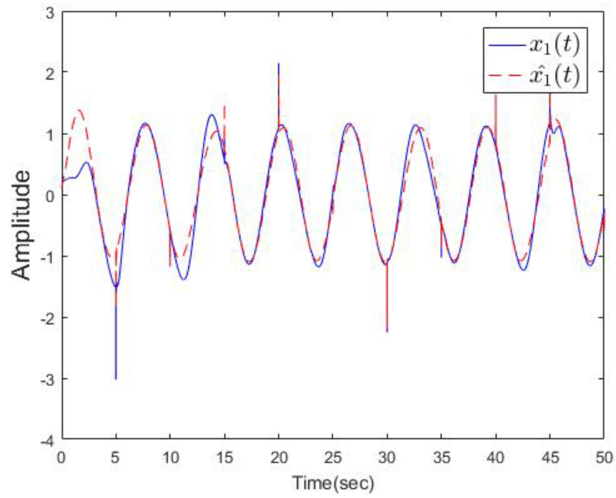

Figure 7. The trajectories of x_1 and $\hat{x}_1$ with dwell time is 5secs

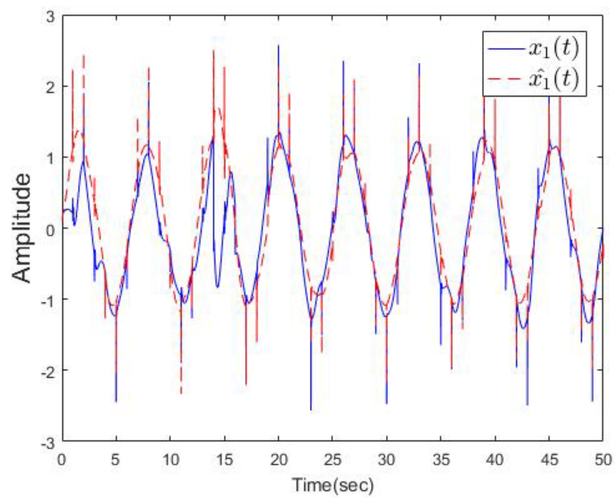

Figure 8. The trajectories of x_1 and $\hat{x}_1$ with dwell time is 1sec

In this section, the control objective is to maintain the system output $y(t)$ to follow the reference signal $y_d = 1.1\sin(t)$. First, we select the observer gain matrix as $\mathbf{L}_k = [2,1]^T$, $k=1$ and $\mathbf{L}_k = [4,12]^T$, $k=2$. In this example, the sampling time is 0.01s. The sliding surface are select as $S_k = c_{1k}\hat{e}_1 + \hat{e}_2$, when k=1 $S_1 = c_{11}\hat{e}_1 + \hat{e}_2$, $c_{11} = 40$ and when k = 2 $S_2 = c_{12}\hat{e}_1 + \hat{e}_2$, $c_{12} = 0.9$. The initial values are chosen as $x_1(0) = x_2(0) = 0.1$, $\theta_{f,k}(0) = -3$, $k = 1,2$, $\theta_{h,k}(0) = -0.55$, $k = 1,2$, $\hat{w}(0) = 1$. The other parameters are selected as $\gamma_{fk} = 5$, $\gamma_{hk} = 1$, $k = 1, 2$ and, $\gamma_\rho = 1$, $\gamma_w = 0.1$. The simulation is divided into two cases, one for the dwell time of 5 seconds and the other for the dwell time of 1 second. Finally, the simulation results are shown in Figs. 3-12.

Remark 2: According to the above simulation results, it can be found that if the dwell time of the system is longer, then the system tracking performance is better, and if the dwell time of the system is shorter, then the system cannot achieve good tracking performance.

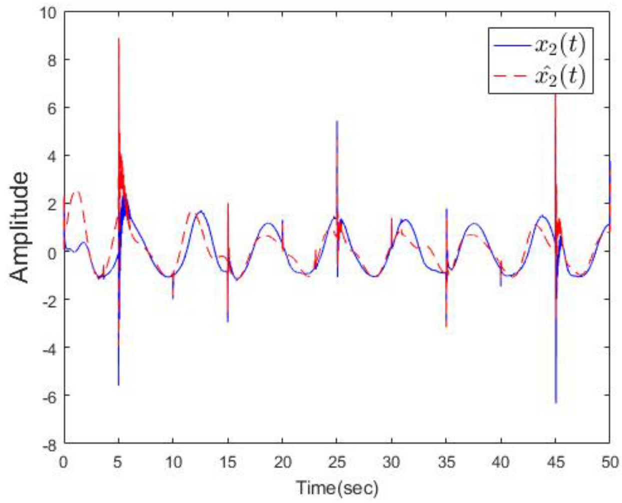

Figure 9. The trajectories of x_2 and $\hat{x}_2$ with dwell time is 5secs

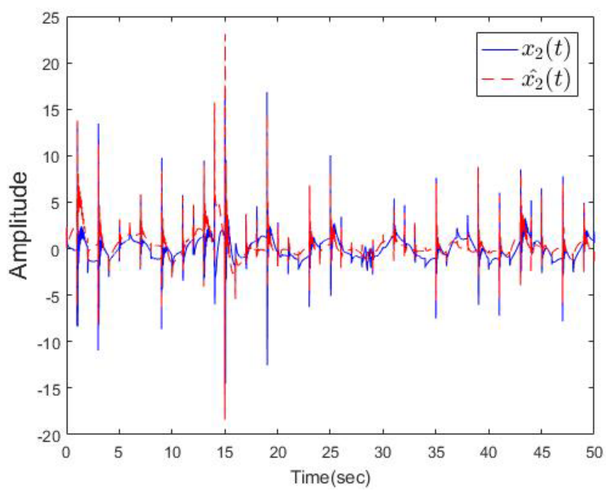

Figure 10. The trajectories of x_2 and $\hat{x}_2$ with dwell time is 1sec

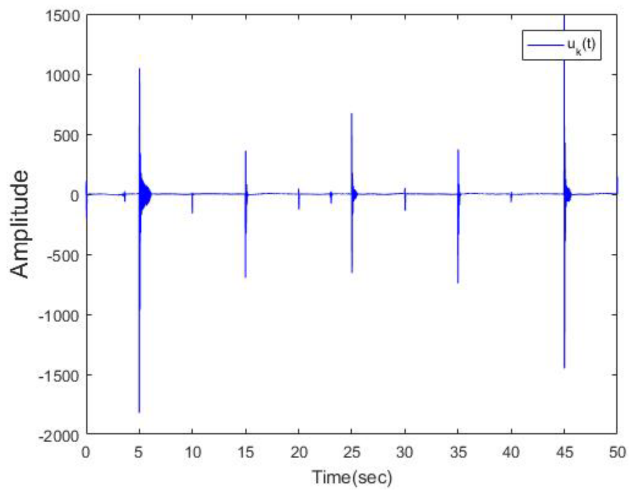

Figure 11. The control signal $u_k(t)$ with dwell time is 5secs

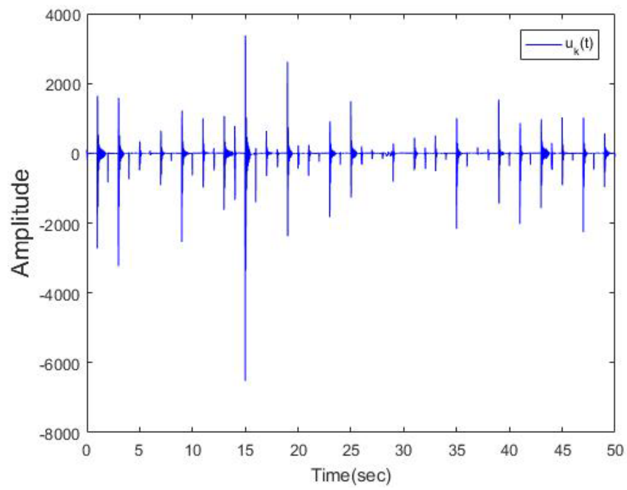

Figure 12. The control signal $u_k(t)$ with dwell time is 1sec

5. Conclusions

An observer-based adaptive fuzzy sliding mode control approach has been proposed for a class of uncertain switched nonlinear systems with dead-zone input in strict-feedback form. A fuzzy state observer has been designed for estimating the unavailable states with the help of FLS approximating the unknown functions. Based on the designed sliding mode controller, the boundedness of the proposed sliding surfaces which realizes the stability of system can be ensured. By choosing an appropriate Lyapunov function, the proposed controller is designed to demonstrate that all the signals of the closed-loop system can not only guarantee uniformly ultimately bounded, but also achieve good tracking performance. Finally, some computer simulation results of a practical example are illustrated to verify the effectiveness of the proposed approach.

REFERENCES

[1] R. J. Downey, T. H. Cheng, M. J. Bellman and W. E. Dixon, "Switched Tracking Control of the Lower Limb During Asynchronous Neuromuscular Electrical Stimulation: Theory and Experiments," IEEE Transactions on Cybernetics, vol. 47, no. 5, pp. 1251-1262, May 2017.

[2] J. W. Gao, Y. D. Ye, T. Y. Shi, Q. H. Jiang and L. M. Jia, "Intelligent switching system methodology and its application in complex process control system," Proceedings 2002 IEEE International Conference on Artificial Intelligence Systems (ICAIS 2002), pp. 12-17, 2002.

[3] M. P. Aghababa, A. R. Haghighi and M. Roohi, "Stabilisation of unknown fractional-order chaotic systems: an adaptive switching control strategy with application to power systems," IET Generation, Transmission & Distribution, vol. 9, no. 14, pp. 1883-1893, 2015.

[4] W. L. Jin and H. Jin, "Analysis and design of a variable speed limit control system at a freeway lane-drop bottleneck: A switched systems approach," 53rd IEEE Conference on Decision and Control, Los Angeles, CA, pp. 1753-1758, 2014.

[5] C. J. B. Macnab, "Stable neural control of a flexible-joint manipulator subjected to sinusoidal disturbance," 2009 4th International Conference on Autonomous Robots and Agents, Wellington, pp. 698-703, 2009.

[6] J. H. Park and G. T. Park, "Adaptive fuzzy observer with minimal dynamic order for uncertain nonlinear systems," IEE Proceedings - Control Theory and Applications, vol. 150, no. 2, pp. 189-197, March 2003.

[7] W. Liu, J. Lu, Z. Zhang, and S. Xu, "Observer-Based Neural Control for MIMO Pure-Feedback Non-linear Systems with Input Saturation and Disturbances," *IET Control Theory & Applications*, vol. 10, no. 17, pp. 2314-2324, Nov. 2016.

[8] J. Yu, P. Shi, W. Dong and H. Yu, "Observer and -Filter-Based Adaptive Fuzzy Output Feedback Control of Uncertain Nonlinear Systems," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 9, pp. 5962-5970, Sept. 2015.

[9] B. Chen, C. Lin, X. Liu and K. Liu, "Observer-Based Adaptive Fuzzy Control for a Class of Nonlinear Delayed Systems," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 46, no. 1, pp. 27-36, Jan. 2016.

[10] H. Zargarzadeh, T. Dierks, and S. Jagannathan, "Optimal Control of Nonlinear Continuous-Time Systems in Strict-Feedback Form," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, no. 10, pp. 2535-2549, Oct. 2015.

[11] Y. J. Liu, J. Li, S. Tong and C. L. P. Chen, "Neural Network Control-Based Adaptive Learning Design for Nonlinear Systems With Full-State Constraints," IEEE Transactions on Neural Networks and Learning Systems, vol. 27, no. 7, pp. 1562-1571, July 2016.

[12] L. Long and J. Zhao, "Adaptive fuzzy output-feedback control for switched uncertain non-linear systems," IET Control Theory & Applications, vol. 10, no. 7, pp. 752-761, 2016.

[13] Y. Li and S. Tong, "Adaptive fuzzy output-feedback stabilization control for a class of switched nonstrict-feedback nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 4, pp. 1007-1016, Apr 2017.

[14] Z. Zhu, Y. Xia, and M. Fu, "Adaptive Sliding Mode Control for Attitude Stabilization With Actuator Saturation," IEEE Transactions on Industrial Electronics, vol. 58, no. 10, pp. 4898-4907, Oct. 2011.

[15] C. C. Kung and T. H. Chen, "Observer-based indirect adaptive fuzzy integral sliding mode control with state variable filters," Fuzzy Sets and Syst., vol. 155, no. 2, pp. 292–308, 2005.

[16] L. X. Wang, *A course in fuzzy systems and control,* Englewood Cliffs, NJ: Prentice-Hall, 1997.

[17] S. Tong, S. Sui, and Y. Li, "Observed-based adaptive fuzzy tracking control for switched nonlinear systems with dead-zone," IEEE Transactions on Cybernetics, vol. 45, no. 12, pp. 2816-2826, December 2015.

[18] Chiang-Cheng Chiang, "Adaptive Fuzzy Tracking Control for Uncertain Nonlinear Time-Delay Systems with Unknown Dead-Zone Input," Mathematical Problems in Engineering, vol. 2013, Article ID 363748, 13 pages, 2013. doi:10.1155/2013/363748.

[19] S. Yin, H. Yu, R. Shahnazi and A. Haghani, "Fuzzy Adaptive Tracking Control of Constrained Nonlinear Switched Stochastic Pure-Feedback Systems," IEEE Transactions on Cybernetics, vol. 47, no. 3, pp. 579-588, March 2017.

[20] Chih-Lyang Hwang, Chiang-Cheng Chiang, and Yao-Wei Yeh, "Adaptive Fuzzy Hierarchical Sliding-Mode Control for the Trajectory Tracking of Uncertain Under-Actuated Nonlinear Dynamic Systems," IEEE Trans. on Fuzzy Systems, vol. 22, no. 2, pp. 286-299, April 2014.