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Robust Adaptive Fuzzy Control of MIMO Nonlinear Systems with Higher-Order and Unmatched Uncertainties

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Abstract In this paper, a robust adaptive fuzzy controller is designed for the output tracking control problem of multi-input multi-output (MIMO) nonlinear systems with higher-order and unmatched uncertainties. For real mechanical systems, the strength of the unmatched uncertainties is bounded by the pth-order polynomials in states, in contrast to other works assuming that these uncertainties are bounded by the first-order polynomials in states. Based on the combination of the H^{∞} optimal control with fuzzy systems and some feasible adaptation laws, the proposed robust adaptive fuzzy controller can not only guarantee that all the signals in the whole closed-loop systems are bounded, but also obtain that the output tracking performance of MIMO nonlinear uncertain systems is well established. A series of computer simulations are illustrated to demonstrate the validity of the proposed control scheme.

Keywords H^{∞} Optimal Control, Fuzzy Systems, Output Tracking, Unmatched Uncertainties, Adaptation Law

1. Introduction

The adaptive control of nonlinear systems has attracted a lot of attention, and significant progress has been made in recent years and novel techniques facilitated by advances in geometric nonlinear control theory and in particular input-output feedback linearization method[1,2]. The central concept of this approach is to transform the nonlinear system dynamics into an equivalent linear system, so that the conventional linear control techniques can be applied.

The approach requires a perfect model of the plant in order to achieve linearization of the closed-loop system. However, there often exist inevitable uncertainties in the constructed models of many real systems. In addition, some uncertain parameters are not exactly known or difficult to estimate. Generally speaking, uncertainties in the feedback linearization include the matched uncertainties[3,4] and the unmatched (mismatched) uncertainties[5,6]. For many practical systems, unmatched uncertainties are common in control practice. Therefore, the design of a robust adaptive controller that deals with unmatched uncertainties of a nonlinear system is an important subject.

So far, the systematic design of input-output linearizable systems with the unmatched uncertainties has been conducted, and three main extension systems have been proposed: (1) adaptive control[7]; (2) Lyapunov-based control[8]; (3) variable structure control[9,10], to increase the robustness and to improve the performance of the controlled nonlinear system. Recently H^{∞} optimal control theory has been well developed and found extensive application to efficiently treat the robust stabilization and disturbance rejection problems[11-13]. In this paper, we combine H^{∞} control technique with adaptive control to deal with unmatched uncertainties.

In the past few years, there has been rapidly growing interest in fuzzy control of nonlinear systems, and there have been many successful applications. The most important issue for fuzzy control systems is how to get a system design with the guarantee of stability and control performance. Meanwhile, recently there have been significant research efforts on the issue of stability in fuzzy control systems[15-18]. In[19, 20], based on feedback linearization technique, adaptive fuzzy control schemes have been introduced to deal with nonlinear systems. However, their results were valid while the controlled nonlinear systems did not include the unmatched uncertainties.

In this paper, the main objective is to design a robust adaptive fuzzy controller for a class of unknown MIMO nonlinear systems with higher-order and unmatched uncertainties. Also, the strength of the unmatched uncertainties is bounded by the pth-order polynomials in states, unlike other works[14,21,22] based on the assumption that these uncertainties are bounded by the first-order polynomials in states. First, the theory of state feedback input output linearization is applied to the MIMO nonlinear systems with unmatched uncertainties. The resulting normal systems with unmatched uncertainties will be in a minimum phase system. Next, the fuzzy systems and some adaptive laws are applied to approximate the unknown nonlinear functions and estimate the upper bounds of the

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unknown uncertainties, respectively. However, these unmatched uncertainties including modeling errors (fuzzy system approximation errors) and parameter variations, etc., can lead to the instability of the closed-loop system; therefore, a robust compensator is designed by H^{∞} control technique to reject this kind of uncertainties. The proposed control scheme not only guarantees the uniform ultimate boundedness, but also makes the maximum tracking error less than or equal to a desired attenuation level due to the unmatched uncertainties.

This paper is organized as follows. First, Section 2 describes the problem of robust output tracking for MIMO nonlinear system with higher-order and unmatched uncertainties. Then, necessary preliminaries on feedback linearization are presented, and some necessary assumptions are introduced. In Section 3, a brief description of fuzzy systems is made. In Section 4, a robust adaptive fuzzy controller is proposed such that the output of the controlled system with higher-order and unmatched uncertainties exponentially tracks the given desired trajectory. In Section 5, the simulations and discussions are presented to confirm the validity of the proposed control scheme. Finally, a conclusion is given in Section 6.

2. Problem Formulation

Consider a class of MIMO nonlinear systems in the presence of the unmatched uncertainties of the following form:

$$\dot{x} = f(x) + \sum_{j=1}^{m} g_j(x) u_j + \Theta(x) \tag{1}$$
$$y_i = h_i(x) \qquad i = 1, \cdots, m$$

where $x \in R^n$ is the measured state vector, $u \in R^m$, $y \in R^m$ are the system input vector and output vector, respectively. $f(x), g_j(x) \in R^n$, $j = 1, \cdots, m$ are sufficiently smooth vector fields, $h_i(x) \in R,\ i = 1, \cdots, m$ are sufficiently smooth output functions, and $\Theta(x) \in R^n$ represent uncertainties continuously differentiable with respect to x.

First, we pursue the input-output linearization process for a MIMO dynamics. In the following definitions, the notation $L_f h_i(x)$ and $L_{g_j} h_i(x)$ denote, the Lie derivatives of the function $h_i(x)$ with respect to the vector field f and g_j, receptively. Higher-order Lie derivative can be defined recursively such as $L_f^k h_i \equiv L_f\left(L_f^{k-1} h_i\right), \quad k > 1$.

Definition 1[1]**:** A multivariable nonlinear system of the form (1) is said to have a (vector) relative degree $r \equiv (r_1, \cdots, r_m)$ at a point x_0 if

(i) $L_{g_j} L_f^k h_i = 0$ for all $1 \le j \le m$, for all $1 \le i \le m$, for all $k < r_i - 1$, and for all x in a neighborhood of x_0.

(ii) The $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{2}$$

is nonsingular at $x = x_0$

Throughout this paper, we assume that the system (1) possesses a relative degree in its domain of definition. Based on this assumption we have the following proposition.

Proposition 1[1]**:** Suppose a system has a (vector) relative degree $r \equiv (r_1, \cdots, r_m)$ at x_0, then $r_1 + \cdots + r_m \le n$. Let

$$\begin{aligned} z_{i1} &= h_i(x) \\ &\vdots \\ z_{ir_i} &= L_f^{r_i-1} h_i(x) \qquad i = 1, \cdots, m \end{aligned} \tag{3}$$

if $r = r_1 + \cdots + r_m$ is strictly less than n, it is always to find $n - r$ smooth functions $\eta_{r+1}, \cdots, \eta_n$ such that the mapping

$$\overline{z} = [\, z_{11}, \cdots, z_{1r_1}, \cdots, z_{m1}, \cdots, z_{mr_m}, \eta_{r+1}, \cdots, \eta_n \,]^T \tag{4}$$

has a Jacobean matrix that is nonsingular at x_0. The $\eta_{r+1}, \cdots, \eta_n$ are chosen to satisfy

$$L_{g_j} \eta_k = 0 \tag{5}$$

for all $1 \le j \le m,\ r+1 \le k \le n$, and all x around x_0.

Now, we set

$$\begin{aligned} z &= [\, z_{11}, \cdots, z_{1r_1}, \cdots, z_{m1}, \cdots, z_{mr_m} \,]^T \\ &= \left[\, h_1, L_f h_1, \cdots, L_f^{r_1-1} h_1, \cdots, h_m, L_f h_m, \cdots, L_f^{r_m-1} h_m \,\right]^T \end{aligned} \tag{6}$$
$$\eta = [\, \eta_{r+1}, \cdots, \eta_n \,]^T$$

According to Proposition 1, there exists a diffeomorphic coordinate transformation $(z, \eta) = T(x)$ which transforms the system (1) into the following normal form:

$$\dot{z}_{i1} = z_{i2} + \Delta\varphi_{i1}(z, \eta),$$
$$\vdots$$
$$\begin{aligned} \dot{z}_{ir_i} &= b_i(z, \eta) + \sum_{j=1}^{m} a_{ij}(z, \eta) u_j + \Delta\varphi_{ir_i}(z, \eta) \\ &= v_i + \Delta\varphi_{ir_i}(z, \eta) \end{aligned} \tag{7}$$
$$\dot{\eta} = Q(z, \eta) + \Delta\Omega(z, \eta) \tag{8}$$
$$y_i = h_i = z_{i1},\ i = 1, \cdots, m \tag{9}$$
$$u = [u_1, \ldots, u_m]^T = A^{-1}[v - b] \tag{10}$$

where

$$\Delta\varphi_{id}(z, \eta) = L_\Theta L_f^{l-1} h_i \qquad i = 1, \ldots, m, \quad l = 1, \ldots, r_i$$
$$Q(z, \eta) = \left[L_f \eta_{r+1}, \ldots, L_f \eta_n \right]^T \circ T^{-1}(z, \eta)$$
$$\Delta\Omega(z, \eta) = [\, L_\Theta \eta_{r+1}, \cdots, L_\Theta \eta_n \,]^T \circ T^{-1}(z, \eta).$$

$$A(z,\eta)=\begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{bmatrix}=\begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix}\circ T^{-1}(z,\eta)$$

$$v=[v_1,\ldots,v_m]^T$$

$$b(z,\eta)=\left[\ L_f^{r_1}h_1,\cdots,L_f^{r_m}h_m\ \right]^T=[b_1,\cdots,b_m]^T$$

Assumption 1: The zero dynamics (8) is exponentially stable in the domain of definition, and the function $Q(z,\eta)$ is Lipschitz in z and uniformly in η. Moreover, the norm of the uncertainty vector $\Delta\Omega$ satisfies the following condition: $\|\Delta\Omega\|\le L$.

Assumption 2: Let $y_d=[y_{d1},\cdots,y_{dm}]^T$ be the desired output trajectories. The desired trajectories y_{di}, $i=1,\cdots,m,$ and their first r_i derivatives are uniformly bounded, that is $\left\|\left(y_{di},y_{di}^{(1)},\cdots,y_{di}^{(r_i)}\right)\right\|\le B_{di}$ for a bounded positive constant B_{di}.

Define the output tracking errors to be

$$e_{ij}=y_{di}^{(j-1)}-y_i^{(j-1)},\quad i=1,\ldots,m;\ \ j=1,\ldots,r_i \tag{11}$$

and

$$e=[\ e_{11},\cdots,e_{1r_1},\cdots,e_{m1},\cdots,e_{mr_m}\ ]^T \tag{12}$$

Then the output of the system and its derivatives can be expressed as

$$\begin{aligned} y_i&=h_i=z_{i1}\\ y_i^{(1)}&=\dot z_{i1}=z_{i2}+\Delta\varphi_{i1}\\ y_i^{(2)}&=\dot z_{i2}+(\Delta\varphi_{i1})^{(1)}=z_{i3}+\Delta\varphi_{i2}+(\Delta\varphi_{i1})^{(1)}\\ &\vdots\\ y_i^{(r_i)}&=\dot z_{ir_i}+\left(\Delta\varphi_{i(r_i-1)}\right)^{(1)}+\left(\Delta\varphi_{i(r_i-2)}\right)^{(2)}+\cdots+\left(\Delta\varphi_{i2}\right)^{(r_i-2)}\\ &\quad+\left(\Delta\varphi_{i1}\right)^{(r_i-1)}\\ &=b_i+\sum_{j=1}^{m}a_{ij}u_j+\Delta\Phi_i \end{aligned} \tag{13}$$

where

$\Delta\Phi_i=\Delta\phi_{i1}+\Delta\phi_{i2}+\cdots+\Delta\phi_{i(r_i-1)}+\Delta\phi_{ir_i}$ and

$\Delta\phi_{ij}=\Delta\varphi_{ij}^{(r_i-j)}$, for $i=1,\ldots,m$ and $j=1,\ldots,r_i$.

Assumption 3: If the $\Delta\varphi_{ij}$ is the function that has continuous derivatives in the domain of definition. These derivatives are bounded by the polynomial which is combined with both $\|e\|^p$ and $\|\eta\|^k$, $p=0,1,\cdots,N$, $k=1,\cdots,M$. That is,

$$\|\Delta\Phi_i\|\le\sum_{p=0}^{N}\sigma_{ip}\|e\|^p+\sum_{k=1}^{M}\delta_{ik}\|\eta\|^k,\ \text{for}\ \ i=1,\cdots,m \tag{14}$$

where σ_{ip}, δ_{ik} are unknown positive constants.

In accordance with (14), we will choose the simple adaptive laws to estimate the upper bounds of these higher-order uncertainties. The simple adaptive laws can be represented as follows:

$$\dot{\tilde\Psi}_{1ip}(e)=-q_{1ip}\|e\|^p e_i^T P_i B_i;\ \ \dot{\tilde\Psi}_{2ik}(\eta)=-q_{2ik}\|\eta\|^k e_i^T P_i B_i \tag{15}$$

where

$$\tilde\Psi_{1ip}(e)=\bar\Psi_{1ip}(e)-\sigma_{ip},\quad p=0,1,2,\cdots,N,\quad i=1,\cdots,m,$$

$$\tilde\Psi_{2ik}(\eta)=\bar\Psi_{2ik}(\eta)-\delta_{ik},\quad k=1,2,\cdots,M,\quad i=1,\cdots,m,$$

are the parameter adaptation errors and q_{1ip}, q_{2ik} are the adaptation gains with the positive values. Because σ_{ip} and δ_{ik} are unknown positive constants, the adaptive laws can be written as

$$\begin{aligned} \dot{\bar\Psi}_{1ip}(e)&=-q_{1ip}\|e\|^p e_i^T P_i B_i,\ \ p=0,1,2,\cdots,N\ \ i=1,\cdots,m\\ \dot{\bar\Psi}_{2ik}(\eta)&=-q_{2ik}\|\eta\|^k e_i^T P_i B_i,\ \ k=1,2,\ldots,M\ \ i=1,\ldots,m \end{aligned} \tag{16}$$

Control objectives: Determine a robust adaptive fuzzy controller such that the following conditions are satisfied:

(i) The states of the closed-loop system are uniformly ultimately bounded. Furthermore, the output tracking errors asymptotically converge to the bounded region.

(ii) For the given attenuation level $\rho>0$ such that the following H^∞ tracking performance index is achieved.

$$\begin{aligned} \int_0^T e^T Qe\,dt&\le e^T(0)Pe(0)+\frac{1}{\gamma}\tilde\theta^T(0)\tilde\theta(0)\\ &+\sum_{p=0}^{N}\left(q_{1p}^{-1}\tilde\Psi_{1p}(0)\right)^T\tilde\Psi_{1p}(0)\\ &+\sum_{k=1}^{M}\left(q_{2k}^{-1}\tilde\Psi_{2k}(0)\right)^T\tilde\Psi_{2k}(0)+\rho^2\int_0^T w^T w\,dt \end{aligned} \tag{17}$$

where $T\in[0,\infty)$, $w\in L_2[0,T]$ is the combined fuzzy approximation errors, Q, P are positive matrix of proper dimension, γ is a designed parameter. If the system stars with initial conditions $e(0)=0$, $\tilde\theta(0)=0$, $\tilde\Psi_{1p}(0)=0$, and $\tilde\Psi_{2k}(0)=0$ then performance in (17) can be rewritten as

$$\sup_{w\in L_2[0,T]}\frac{\|e\|_Q}{\|w\|}\le\rho \tag{18}$$

where $\|e\|_Q^2=\int_0^T e^T Qe\,dt$ and $\|w\|^2=\int_0^T w^T w\,dt$, i.e., the L_2 gain from w to the tracking error e must be equal to or less than ρ.

3. Robust Adaptive Controller Design Using Fuzzy Systems

The fuzzy systems are universal approximations from the viewpoint of human experts and can uniformly approximate nonlinear continuous functions to arbitrary accuracy[14]. The basic configuration of the fuzzy system consists of four main components: fuzzy rule base, fuzzy inference engine, fuzzifier and defuzzifier . The fuzzy system performs a mapping from $U \subset R^n$ to $V \subset R$. Let $U = U_1 \times \cdots \times U_n$ where $U_i \subset R$, $i = 1,2,\ldots,n$. Wang[16] presents a detailed description of each of the four blocks in the fuzzy system. The fuzzy rule base consists of a collection of fuzzy IF-THEN rules:

$$R^{(l)}: \text{ IF } x_1 \text{ is } F_1^l \text{ and } \cdots \text{ and } x_n \text{ is } F_n^l,$$
$$\text{THEN } y \text{ is } G^l,\ l = 1,2,\ldots,M$$

where $\mathbf{x} = \left[x_1, x_2, \cdots, x_n\right]^T \in U$ and $y \in V \subset R$ are the input and output of the fuzzy system, F_i^l and G^l are fuzzy sets in U_i and V , respectively. M is the number of rules. The fuzzifier maps a crisp point $\mathbf{x} = \left[x_1, x_2, \cdots, x_n\right]^T$ into a fuzzy set in U . The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V , based upon the fuzzy IF-THEN rules in the fuzzy rule base and the compositional rule of inference. The defuzzifier maps a fuzzy set in V to a crisp point in V . The fuzzy systems with center-average defuzzifier, product inference, and singleton fuzzifier are of the following form:

$$y(\mathbf{x}) = \frac{\sum_{l=1}^{M} \theta^l \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} \tag{19}$$

where θ^l is the point at which fuzzy membership function $\mu_{G^l}(\theta^l)$ of fuzzy sets G^l achieves its maximum value, and it is assumed that $\mu_{G^l}(\theta^l) = 1$. Eq. (19) can be rewritten as

$$y(\mathbf{x}) = \boldsymbol{\theta}^T \boldsymbol{\xi}(\mathbf{x}) \tag{20}$$

where $\boldsymbol{\theta} = \left[\theta^1, \theta^2, \cdots, \theta^M\right]^T$ is a parameter vector, $\boldsymbol{\xi}(\mathbf{x}) = \left[\xi^1(\mathbf{x}), \cdots, \xi^M(\mathbf{x})\right]^T$ is a regressive vector with the regressor $\xi^l(\mathbf{x})$, which is defined as fuzzy basis function

$$\xi^l(\mathbf{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} \tag{21}$$

In order to achieve the proposed control objectives, these unknown nonlinear functions $b_i(z,\eta)$ and $a_{ij}(z,\eta)$ will be approximated by tuning the parameters of the corresponding fuzzy systems. In this situation, $b_i(z,\eta)$ and $a_{ij}(z,\eta)$ in (7) will be approximated by the following fuzzy systems $\hat{b}_i\left(z \mid \theta_{ib}\right)$ and $\hat{a}_{ij}\left(z \mid \theta_{ija}\right)$ respectively.

$$\hat{b}_i\left(z \mid \theta_{ib}\right) = \theta_{ib}^T \xi\left(z\right) = \xi^T\left(z\right) \theta_{ib} \tag{22}$$

$$\hat{a}_{ij}\left(z \mid \theta_{ija}\right) = \theta_{ija}^T \xi\left(z\right) = \xi^T\left(z\right) \theta_{ija} \tag{23}$$

where $\theta_{ib} = \left(\theta_{ib}^1, \ldots, \theta_{ib}^M\right)^T$, $\theta_{ija} = \left(\theta_{ija}^1, \ldots, \theta_{ija}^M\right)^T$, $i = 1,\ldots,m$, $j = 1,\ldots,m$

Due to the existence of fuzzy approximation errors and unmatched uncertainties, the resulting robust adaptive fuzzy controller can be chosen as

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = \begin{bmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1m} \\ \vdots & \ddots & \vdots \\ \hat{a}_{m1} & \cdots & \hat{a}_{mm} \end{bmatrix}^{-1} \left(-\begin{bmatrix} \hat{b}_1\left(z \mid \theta_1\right) \\ \vdots \\ \hat{b}_m\left(z \mid \theta_m\right) \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} - \begin{bmatrix} u_{c1} \\ \vdots \\ u_{cm} \end{bmatrix} \right) \tag{24}$$

where u_{ci} is the robust compensator defined as

$$u_{ci} = -\frac{1}{\lambda_i} B_i^T P_i e_{i1} + \sum_{p=0}^{N} \bar{\Psi}_{1ip} \left\|e\right\|^p + \sum_{k=1}^{M} \bar{\Psi}_{2ik} \left\|\eta\right\|^k \tag{25}$$

and λ_i , P_i are the solutions of the following Riccati equation[19]

$$P_i A_i + A_i^T P_i + Q_i - \left(\frac{2}{\lambda_i} - \frac{1}{\rho^2}\right) P_i B_i B_i^T P_i = 0 \tag{26}$$

It is noticed that Riccati equation (26) has a solution $P_i = P_i^T \geq 0$ if and only if $2\rho^2 > \lambda_i$. Moreover, the auxiliary control input can be chosen as

$$\begin{aligned} v_1 &= y_{d1}^{(r_1)} + k_{1r_1} e_{11}^{(r_1 - 1)} + \cdots + k_{11} e_{11} \\ &\vdots \\ v_m &= y_{dm}^{(r_m)} + k_{mr_m} e_{m1}^{(r_m - 1)} + \cdots + k_{m1} e_{m1} \end{aligned} \tag{27}$$

It is obvious from (24), (27) and (13) that we can obtain

$$\begin{aligned} &\begin{bmatrix} e_{11}^{(r_1)} + k_{1r_1} e_{11}^{(r_1-1)} + \cdots + k_{11} e_{11} \\ \vdots \\ e_{m1}^{(r_m)} + k_{mr_m} e_{m1}^{(r_m-1)} + \cdots + k_{m1} e_{m1} \end{bmatrix} \\ &= \begin{bmatrix} \hat{b}_1 - b_1 \\ \vdots \\ \hat{b}_m - b_m \end{bmatrix} + \left(\hat{A} - A\right) \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} + \begin{bmatrix} u_{c1} \\ \vdots \\ u_{cm} \end{bmatrix} - \begin{bmatrix} \Delta\Phi_1 \\ \vdots \\ \Delta\Phi_m \end{bmatrix} \end{aligned} \tag{28}$$

Then the state space representation of (28) can be defined as

$$\begin{aligned} \dot{e}_i &= A_i e_i \\ &+ B_i \left[\hat{b}_i\left(z \mid \theta_{ib}\right) - b_i\left(z,\eta\right) + \sum_{j=1}^{m} \left(\hat{a}_{ij}\left(z \mid \theta_{ija}\right) - a_{ij}\left(z,\eta\right) \right) u_j \right] \\ &+ B_i u_{ci} - B_i \Delta\Phi_i \end{aligned} \tag{29}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{i1} & -k_{i2} & -k_{i3} & \cdots & -k_{ir_i} \end{bmatrix}; \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix};$$

$$e_i = \begin{bmatrix} e_{i1} \\ e_{i1}^{(1)} \\ \vdots \\ e_{i1}^{(r_i - 1)} \end{bmatrix} = \begin{bmatrix} e_{i1} \\ e_{i2} \\ \vdots \\ e_{ir_i} \end{bmatrix}$$

Moreover, the coefficients k_{ij} are chosen such that the matrices A_i are Hurwitz for $i=1,2,\cdots,m$ and $j=1,2,\cdots,m.$

Now let the optimal parameter estimates θ_{ib}^* and θ_{ija}^* be defined as follows[16]:

$$\theta_{ib}^* = \arg\min_{\theta_{ib}\in\Omega_{ib}} \left\{ \sup_{z\in\Omega_x} \left| \hat{b}_i\left(z \mid \theta_{ib}\right) - b_i\left(z,\eta\right) \right| \right\},$$

$$\theta_{ija}^* = \arg\min_{\theta_{ija}\in\Omega_{ija}} \left\{ \sup_{z\in\Omega_x} \left| \hat{a}_{ij}\left(z \mid \theta_{ija}\right) - a_{ij}\left(z,\eta\right) \right| \right\} \quad (30)$$

where $\Omega_{ib}, \Omega_{ija}$ and Ω_x denote the sets of suitable bounds on θ_{ib}, θ_{ija}, and z, respectively. The minimum approximation errors[15,16] is defined as

$$w_i = \left[\hat{b}_i\left(z \middle| \theta_{ib}^*\right) - b_i\left(z,\eta\right) \right] + \sum_{j=1}^{m} \left[\hat{a}_{ij}\left(z \middle| \theta_{ija}^*\right) - a_{ij}\left(z,\eta\right) \right] u_j \quad (31)$$

Then (29) can be rewritten as

$$\begin{aligned}
\dot{e}_i &= A_i e_i + B_i \Big\{ \hat{b}_i\left(z \mid \theta_{ib}\right) - b_i\left(z,\eta\right) + \hat{b}_i\left(z \mid \theta_{ib}^*\right) - \hat{b}_i\left(z \mid \theta_{ib}^*\right) \\
&\quad + \sum_{j=1}^{m} \left[\hat{a}_{ij}\left(z \mid \theta_{ija}\right) - a_{ij}\left(z,\eta\right) + \hat{a}_{ij}\left(z \mid \theta_{ija}^*\right) - \hat{a}_{ij}\left(z \mid \theta_{ija}^*\right) \right] u_j \Big\} \\
&\quad + B_i u_{ci} - B_i \Delta\Phi_i \\
&= A_i e_i + B_i \tilde{\theta}_{ib}^T \xi\left(z\right) + B_i \sum_{j=1}^{m} \tilde{\theta}_{ija}^T \xi\left(z\right) u_j \\
&\quad + B_i w_i + B_i u_{ci} - B_i \Delta\Phi_i
\end{aligned} \quad (32)$$

where $\tilde{\theta}_{ib} = \theta_{ib} - \theta_{ib}^*$, $\tilde{\theta}_{ija} = \theta_{ija} - \theta_{ija}^*$

The parameter adaptive laws are chosen as

$$\dot{\theta}_{ib} = -\gamma_i \xi(z) B_i^T P_i e_i, \quad (33)$$

$$\dot{\theta}_{ija} = -\gamma_{ij} \xi(z) B_i^T P_i e_i u_j, \quad (34)$$

where γ_i and γ_{ij} are positive constants.

Theorem 1: Consider the MIMO nonlinear uncertain system (1) and Assumptions 1-3 are satisfied. If the robust adaptive fuzzy control scheme in (24)-(27) with the learning adaptive laws in (16), (33), and (34) are adopted, the following properties are guaranteed:

(i) The states of the closed-loop system are uniformly ultimately bounded. Furthermore, the output tracking errors asymptotically converge to the boundary set.

(ii) For the given attenuation level ρ, the tracking performance index (17) is achieved.

Proof. Choose Lyapunov function as

$$V = V_1 + \cdots + V_m$$

$$\begin{aligned}
V_i &= \frac{1}{2} e_i^T P_i e_i + \frac{1}{2\gamma_i} \tilde{\theta}_{ib}^T \tilde{\theta}_{ib} + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}} \tilde{\theta}_{ija}^T \tilde{\theta}_{ija} \\
&\quad + \frac{1}{2} \sum_{p=0}^{N} q_{1ip}^{-1} \tilde{\Psi}_{1ip}^2 + \frac{1}{2} \sum_{k=1}^{M} q_{2ik}^{-1} \tilde{\Psi}_{2ik}^2
\end{aligned} \quad (35)$$

Differentiating V and V_i along the solution of (32), we obtain

$$\dot{V} = \dot{V}_1 + \cdots + \dot{V}_m$$

$$\begin{aligned}
\dot{V}_i &= \frac{1}{2} \dot{e}_i^T P_i e_i + \frac{1}{2} e_i^T P_i \dot{e}_i + \frac{1}{\gamma_i} \dot{\tilde{\theta}}_{ib}^T \tilde{\theta}_{ib} + \sum_{j=1}^{m} \frac{1}{\gamma_{ij}} \dot{\tilde{\theta}}_{ija}^T \tilde{\theta}_{ija} \\
&\quad + \sum_{p=0}^{N} q_{1ip}^{-1} \tilde{\Psi}_{1ip} \dot{\tilde{\Psi}}_{1ip} + \sum_{k=1}^{M} q_{2ik}^{-1} \tilde{\Psi}_{2ik} \dot{\tilde{\Psi}}_{2ik}
\end{aligned}$$

By the fact $\dot{\tilde{\theta}}_{ib} = \dot{\theta}_{ib}$, $\dot{\tilde{\theta}}_{ija} = \dot{\theta}_{ija}$, $\dot{\tilde{\Psi}}_{1ip} = \dot{\bar{\Psi}}_{1ip}$, $\dot{\tilde{\Psi}}_{2ik} = \dot{\bar{\Psi}}_{2ik}$ and (14), (25) and (32), the above equation becomes

$$\begin{aligned}
\dot{V}_i &\le \frac{1}{2} e_i^T \left[A_i^T P_i + P_i A_i - \frac{2}{\lambda_i} P_i B_i B_i^T P_i \right] e_i + \frac{1}{2} \left(w_i^T B_i^T P_i e_i + e_i^T P_i B_i w_i \right) \\
&\quad + \left[e_i^T P_i B_i \xi^T + \frac{1}{\gamma_i} \dot{\tilde{\theta}}_{ib}^T \right] \tilde{\theta}_{ib} + \sum_{j=1}^{m} \left[e_i^T P_i B_i \xi^T u_j + \frac{1}{\gamma_{ij}} \dot{\tilde{\theta}}_{ija}^T \right] \tilde{\theta}_{ija} \\
&\quad + \sum_{p=0}^{N} \tilde{\Psi}_{1ip} \left(\left\| e \right\|^p e_i^T P_i B_i + q_{1ip}^{-1} \dot{\tilde{\Psi}}_{1ip} \right) \\
&\quad + \sum_{k=1}^{M} \tilde{\Psi}_{2ik} \left(\left\| \eta \right\|^k e_i^T P_i B_i + q_{2ik}^{-1} \dot{\tilde{\Psi}}_{2ik} \right)
\end{aligned} \quad (36)$$

From the adaptive laws, (15) and (33) and (34), and the Riccati equation (26), we get

$$\begin{aligned}
\dot{V}_i &\le -\frac{1}{2} e_i^T Q_i e_i - \frac{1}{2} \left(\frac{1}{\rho} B_i^T P_i e_i - \rho w_i \right)^T \left(\frac{1}{\rho} B_i^T P_i e_i - \rho w_i \right) \\
&\quad + \frac{1}{2} \rho^2 w_i^T w_i
\end{aligned}$$

Since $\frac{1}{2} \left(\frac{1}{\rho} B_i^T P_i e_i - \rho w_i \right)^T \left(\frac{1}{\rho} B_i^T P_i e_i - \rho w_i \right) \ge 0$

$$\begin{aligned}
\dot{V}_i &\le -\frac{1}{2} e_i^T Q_i e_i + \frac{1}{2} \rho^2 w_i^T w_i \\
&= -\frac{1}{2} e_i^T Q_i e_i - \frac{1}{2\gamma_i} \tilde{\theta}_{ib}^T \tilde{\theta}_{ib} - \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}} \tilde{\theta}_{ija}^T \tilde{\theta}_{ija} - \frac{1}{2} \sum_{p=0}^{N} q_{1ip}^{-1} \tilde{\Psi}_{1ip}^2 \\
&\quad - \frac{1}{2} \sum_{k=1}^{M} q_{2ik}^{-1} \tilde{\Psi}_{2ik}^2 + \frac{1}{2\gamma_i} \left\| \tilde{\theta}_{ib} \right\|^2 + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}} \left\| \tilde{\theta}_{ija} \right\|^2 \\
&\quad + \frac{1}{2} \sum_{p=0}^{N} q_{1ip}^{-1} \tilde{\Psi}_{1ip}^2 + \frac{1}{2} \sum_{k=1}^{M} q_{2ik}^{-1} \tilde{\Psi}_{2ik}^2 + \frac{1}{2} \rho^2 \left\| w_i \right\|^2
\end{aligned} \quad (37)$$

By the fact $\left\| \theta_{ib} \right\| \le M_{ib}$, $\left\| \theta_{ija} \right\| \le M_{ija}$, $\bar{w}_i = \sup \left\| w_i \right\|$, the above equation becomes

$$\begin{aligned}
\dot{V}_i &\le -\frac{1}{2} e_i^T Q_i e_i - \frac{1}{2\gamma_i} \tilde{\theta}_{ib}^T \tilde{\theta}_{ib} - \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}} \tilde{\theta}_{ija}^T \tilde{\theta}_{ija} \\
&\quad - \frac{1}{2} \sum_{p=0}^{N} q_{1ip}^{-1} \tilde{\Psi}_{1ip}^2 - \frac{1}{2} \sum_{k=1}^{M} q_{2ik}^{-1} \tilde{\Psi}_{2ik}^2 + \frac{1}{2\gamma_i} M_{ib}^2 \\
&\quad + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}} M_{ija}^2 + \frac{1}{2} \sum_{p=0}^{N} q_{1ip}^{-1} \tilde{\Psi}_{1ip}^2 \\
&\quad + \frac{1}{2} \sum_{k=1}^{M} q_{2ik}^{-1} \tilde{\Psi}_{2ik}^2 + \frac{1}{2} \rho^2 \bar{w}_i^2
\end{aligned}$$

Let

$$\mu_i = \frac{1}{2\gamma_i} M_{ib}^2 + \sum_{j=1}^{m} \frac{1}{2\gamma_{ij}} M_{ija}^2 + \frac{1}{2}\sum_{p=0}^{N} q_{1ip}^{-1}\tilde{\Psi}_{1ip}^2 + \frac{1}{2}\sum_{k=1}^{M} q_{2ik}^{-1}\tilde{\Psi}_{2ik}^2 + \frac{1}{2}\rho^2 \overline{w}_i^2$$

$$\dot{V}_i \le -\frac{1}{2}\frac{\inf \lambda_{\min}(Q_i)}{\sup \lambda_{\max}(P_i)} e_i^T P_i e_i - \frac{1}{2\gamma_i}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib} - \sum_{j=1}^{m}\frac{1}{2\gamma_{ij}}\tilde{\theta}_{ija}^T\tilde{\theta}_{ija} - \frac{1}{2}\sum_{p=0}^{N} q_{1ip}^{-1}\tilde{\Psi}_{1ip}^2 - \frac{1}{2}\sum_{k=1}^{M} q_{2ik}^{-1}\tilde{\Psi}_{2ik}^2 + \mu_i$$

Let $\lambda_v = \dfrac{\inf \lambda_{\min}(Q_i)}{\sup \lambda_{\max}(P_i)}$

$$\dot{V}_i \le -\frac{1}{2}\lambda_v e_i^T P_i e_i - \frac{1}{2\gamma_i^2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib} - \sum_{j=1}^{m}\frac{1}{2\gamma_{ij}^2}\tilde{\theta}_{ija}^T\tilde{\theta}_{ija} + \frac{1-\gamma_i}{2\gamma_i^2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib} + \frac{1-\gamma_{ij}}{2\gamma_{ij}^2}\tilde{\theta}_{ija}^T\tilde{\theta}_{ija} - \frac{1}{2}\sum_{p=0}^{N} q_{1ip}^{-1}\tilde{\Psi}_{1ip}^2 - \frac{1}{2}\sum_{k=1}^{M} q_{2ik}^{-1}\tilde{\Psi}_{2ik}^2 + \mu_i$$

Let $L_i = \dfrac{1-\gamma_i}{2\gamma_i^2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib} + \dfrac{1-\gamma_{ij}}{2\gamma_{ij}^2}\tilde{\theta}_{ija}^T\tilde{\theta}_{ija} + \mu_i$

$$\dot{V}_i \le -\frac{1}{2}\lambda_v e_i^T P_i e_i - \frac{1}{2\gamma_i^2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib} - \sum_{j=1}^{m}\frac{1}{2\gamma_{ij}^2}\tilde{\theta}_{ija}^T\tilde{\theta}_{ija} - \frac{1}{2}\sum_{p=0}^{N} q_{1ip}^{-1}\tilde{\Psi}_{1ip}^2 - \frac{1}{2}\sum_{k=1}^{M} q_{2ik}^{-1}\tilde{\Psi}_{2ik}^2 + L_i$$

Let $c_i = \min\left\{\lambda_v, \dfrac{1}{\gamma_i}, \dfrac{1}{\gamma_{ij}}, 1\right\}$

$$\dot{V}_i = -c_i V_i + L_i \tag{38}$$

Let $c = \min\{c_i\}$, $L = p.\max\{L_i\}$ from (35) and (38) we have

$$\dot{V} \le -cV + L \tag{39}$$

However, the proposed controller will not only guarantee the asymptotic output tracking but also the uniform ultimate boundary. Thus, the control objective (i) is realized.

In order to achieve the control objective (ii), we integrate (37) from $t = 0$ to $t = T$, we have

$$\frac{1}{2}\int_0^T e_i^T Q_i e_i dt \le V_i(0) - V_i(T) + \frac{1}{2}\rho^2 \int_0^T w_i^T w_i dt \tag{40}$$

Since $V_i(T) \ge 0$, we can write (40) as follows:

$$\frac{1}{2} e_i^T Q_i e_i dt \le V_i(0) + \frac{1}{2}\rho^2 \int_0^T w_i^T w_i dt$$

$$= \frac{1}{2} e_i^T(0) P_i e_i(0) + \frac{1}{2\gamma_i}\tilde{\theta}_{ib}^T(0)\tilde{\theta}_{ib}(0) + \sum_{j=1}^{m}\frac{1}{2\gamma_{ij}}\tilde{\theta}_{ija}^T(0)\tilde{\theta}_{ija}(0) + \frac{1}{2}\sum_{p=0}^{N} q_{1ip}^{-1}\tilde{\Psi}_{1ip}^2(0) + \frac{1}{2}\sum_{k=1}^{M} q_{2ik}^{-1}\tilde{\Psi}_{2ik}^2(0) + \frac{1}{2}\rho^2\int_0^T w_i^T w_i dt \tag{41}$$

Let $Q = diag[Q_1,\ldots,Q_m]$, $P = diag[P_1,\ldots,P_m]$,

$$e = \left[e_1^T,\ldots,e_m^T\right]^T$$

$$q_{1p}^{-1} = diag\left[\ (q_{11p})^{-1},\cdots,(q_{1mp})^{-1}\ \right],$$

$$q_{2k}^{-1} = diag\left[\ (q_{21k})^{-1},\cdots,(q_{2mk})^{-1}\right]$$

$$\tilde{\Psi}_{1p}(0) = \left[\ \tilde{\Psi}_{11p}(0),\cdots,\tilde{\Psi}_{1mp}(0)\right]^T,$$

$$\tilde{\Psi}_{2k}(0) = \left[\ \tilde{\Psi}_{21k}(0),\cdots,\tilde{\Psi}_{2mk}(0)\ \right]^T$$

$$\tilde{\theta}_b(0) = \left[\frac{1}{\sqrt{\gamma_1}}\tilde{\theta}_{1b}^T(0),\ldots,\frac{1}{\sqrt{\gamma_m}}\tilde{\theta}_{mb}^T(0)\right]^T,$$

$$\tilde{\theta}_a(0) = \left[\frac{1}{\sqrt{\gamma_{11}}}\tilde{\theta}_{11a}^T(0),\ldots,\frac{1}{\sqrt{\gamma_{mm}}}\tilde{\theta}_{mma}^T(0)\right]^T$$

$$\tilde{\theta}(0) = \left[\tilde{\theta}_a^T(0),\tilde{\theta}_b^T(0)\right]^T \quad , \quad w = \left[w_1,\ldots,w_m\right]^T$$

then from (41), we obtain

$$\int_0^T e^T Q e\, dt \le e^T(0) P e(0) + \tilde{\theta}^T(0)\tilde{\theta}(0) + \sum_{p=0}^{N}\left(q_{1p}^{-1}\tilde{\Psi}_{1p}(0)\right)^T\tilde{\Psi}_{1p}(0) + \sum_{k=1}^{M}\left(q_{2k}^{-1}\tilde{\Psi}_{2k}(0)\right)^T\tilde{\Psi}_{2k}(0) + \rho^2\int_0^T w^T w\, dt \tag{42}$$

i.e., the control objective (ii) is achieved.

4. An Example and Simulation Results

In this section we provide an example to demonstrate the performance of the proposed robust adaptive fuzzy controller. The two-degrees-of freedom manipulator[23] shown in Fig.1 is illustrated under the assumption of lumped equivalent masses and mass less links, and its dynamics are represented as follows:

$$\underbrace{\begin{bmatrix} \alpha_{11}(\phi) & \alpha_{12}(\phi) \\ \alpha_{12}(\phi) & \alpha_{22}(\phi) \end{bmatrix}}_{D(\phi)}\begin{bmatrix}\ddot{\theta} \\ \ddot{\phi}\end{bmatrix}$$

$$= \begin{bmatrix} \beta_{12}(\phi)\dot{\theta}^2 + 2\beta_{12}(\phi)\dot{\theta}\dot{\phi} \\ -\beta_{12}(\phi)\dot{\phi}^2 \end{bmatrix} + \begin{bmatrix} \gamma_1(\theta,\phi) g \\ \gamma_2(\theta,\phi) g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{43}$$

where θ and ϕ are the first joint angle and the second joint angle, respectively. g is the gravitational constant and u_i , $i = 1,2$ are the applied torques. Suppose that $D^{-1}(\phi)$ exists and let the desired positions be θ_d and ϕ_d . Let the state vector be defined as

$$e_{ij} = y_{di}^{(j-1)} - y_i^{(j-1)}, \quad i = 1,2, \quad j = 1,2$$

and

$(y_1, y_{1d}, y_2, y_{2d}) \equiv (\theta, \theta_d, \phi, \phi_d)$.

Equation (43) in state space representation becomes

$$\dot{y}_1 = \dot{z}_{11} = z_{12} + \mu_{11}\theta\phi \equiv z_{12} + \Delta\phi_{11} \tag{44}$$

$$\ddot{y}_1 = \dot{z}_{12} + \Delta\dot{\phi}_{11} = \frac{\alpha_{22}}{d}(\beta_{12}\dot{\theta}^2 + 2\beta_{12}\dot{\theta}\dot{\phi} + \gamma_1 g + u_1)$$
$$-\frac{\alpha_{12}}{d}(-\beta_{12}\dot{\phi}^2 + \gamma_2 g + u_2) + \mu_{12}\theta\dot{\theta}\phi + \mu_{11}(\dot{\theta}\phi + \theta\dot{\phi})$$
$$\equiv b_1 + a_{11}u_1 + a_{12}u_2 + \Delta\phi_{12} + \left(\Delta\dot{\phi}_{11}\right) \tag{45}$$

$$\dot{y}_2 = \dot{z}_{21} = z_{22} + \mu_{11}\phi^2 \equiv z_{22} + \Delta\phi_{21} \tag{46}$$

$$\ddot{y}_2 = \dot{z}_{22} + \Delta\dot{\phi}_{21} = -\frac{\alpha_{12}}{d}(\beta_{12}\dot{\theta}^2 + 2\beta_{12}\dot{\theta}\dot{\phi} + \gamma_1 g + u_1)$$
$$+\frac{\alpha_{11}}{d}(-\beta_{12}\dot{\phi}^2 + \gamma_2 g + u_2) + \mu_{12}\dot{\theta}\dot{\phi}^2 + \mu_{21}(2\phi\dot{\phi})$$
$$\equiv b_2 + a_{21}u_1 + a_{22}u_2 + \Delta\phi_{22} + \left(\Delta\dot{\phi}_{21}\right) \tag{47}$$

with

$$d = \alpha_{11}\alpha_{22} - \alpha_{12}^2$$
$$\alpha_{11} = (m_1 + m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos\phi + J_1$$
$$\alpha_{12} = m_2 r_2^2 + m_2 r_1 r_2 \cos\phi$$
$$\alpha_{22} = m_2 r_2^2 + J_2$$
$$\gamma_1 = -[(m_1 + m_2)r_1 \cos\phi + m_2 r_2 \cos(\theta + \phi)]$$
$$\gamma_2 = -m_2 r_2 \cos(\theta + \phi)$$
$$\beta_{12} = m_2 r_1 r_2 \sin\phi$$

where $m_i, r_i, J_i, i = 1,2$ denote the point mass, length of link, and additional constant inertia with respect to axis of rotation. For better comparison to the simulation results of earlier parameter values used are the same as those of[23]: m_1=0.5 kg, m_2=6.35 kg, r_1=1 m, r_2=0.8 m, J_1=5 kg-m, J_2=5 kg-m, $\theta_d = \sin(t)$ rad, $\phi_d = \sin(t)$ rad, $\theta(0) = 0.5$ rad, and $\phi(0) = 0.5$ rad. By tracking a sine function, the unmatched uncertainties will be violent change with time varying and raised. This situation could make us more difficult to track the desired output, but the trajectory still can be arrived by the proposed controller in this paper.

Choose fuzzy membership functions as follows:

$$\mu_{F_i^1}(x_i) = \frac{1}{1 + \exp(5(x_i + 0.6))},$$
$$\mu_{F_i^2}(x_i) = \exp\left(-(x_i + 0.4)^2\right),$$
$$\mu_{F_i^3}(x_i) = \exp\left(-(x_i + 0.2)^2\right),$$
$$\mu_{F_i^4}(x_i) = \exp\left(-x_i^2\right),\ \mu_{F_i^5}(x_i) = \exp\left(-(x_i - 0.2)^2\right),$$
$$\mu_{F_i^6}(x_i) = \exp\left(-(x_i - 0.4)^2\right),$$
$$\mu_{F_i^7}(x_i) = \frac{1}{1 + \exp(-5(x_i - 0.6))},\quad \text{for } i = 1,2,3,4$$

Let

$$\xi^l(x) = \frac{\mu_{F_1^l}(x_1)\mu_{F_2^l}(x_2)\mu_{F_3^l}(x_3)\mu_{F_4^l}(x_4)}{\sum_{l=1}^{7}\mu_{F_1^l}(x_1)\mu_{F_2^l}(x_2)\mu_{F_3^l}(x_3)\mu_{F_4^l}(x_4)}$$

and

$$\xi(x) = \left(\xi^1(x), \xi^2(x), \xi^3(x), \xi^4(x), \xi^5(x), \xi^6(x), \xi^7(x)\right)^T$$

Using (22) and (23) to approximate the unknown b_1, b_2, a_{11} and a_{22} (here we assume a_{12}, a_{21} are known a priori). For the $Q_1 = Q_2 = diag[10,10]$, $\rho_1 = \rho_2 = 0.5$, $\lambda_1 = \lambda_2 = 0.05$, solving Riccati equation (26), we get

$$P_1 = P_2 = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

Let $k_{11} = k_{12} = 1$, $k_{21} = k_{22} = 2$, $\gamma_1 = \gamma_2 = 0.1$, $\gamma_{11} = \gamma_{22} = 0.01$, $q_{110} = 3$, $q_{111} = 3$, $q_{120} = 1$, $q_{121} = 1$, $\bar{\Psi}_{110}(0) = 2$, $\bar{\Psi}_{111}(0) = 2$, $\bar{\Psi}_{120}(0) = 2$, $\bar{\Psi}_{121}(0) = 2$. We set the uncertain coefficients $\mu_{11} = -0.1$, $\mu_{12} = -0.1$, $\mu_{21} = 0.1$, and $\mu_{22} = 0.1$. Results of this simulation are given in Figs 2-6. Figs. 2-3 show the tracking trajectories of joint 1 and joint 2, respectively. Figs. 4-5 show the torques of joint 1 and joint 2, respectively. Fig. 6 shows the curve of the H^∞ tracking performance index in (17). It is easily shown from these simulation results that the proposed robust adaptive fuzzy control algorithm (24) can achieve the excellent output tracking performances of the nonlinear system with higher-order and unmatched uncertainties.

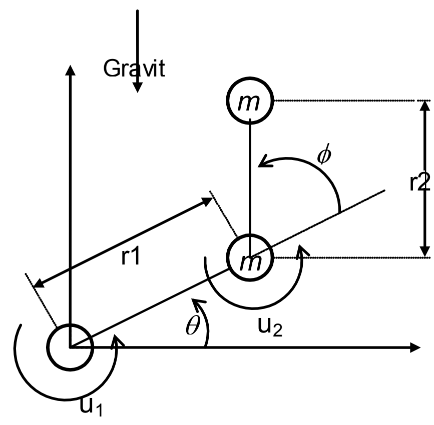

Figure 1. A two-degree of freedom manipulator

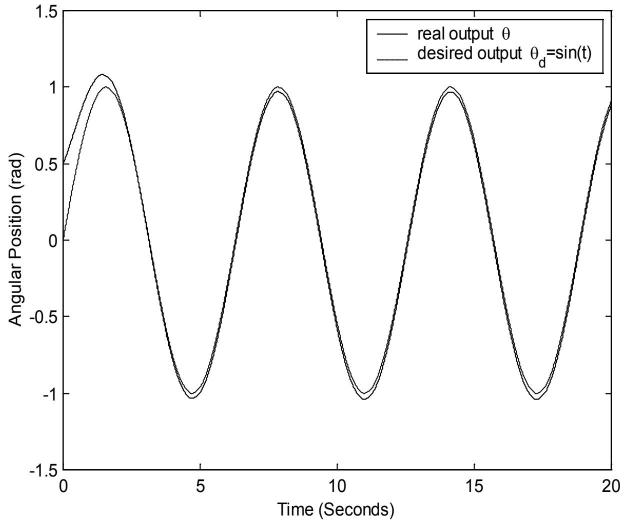

Figure 2. The tracking trajectory of joint 1

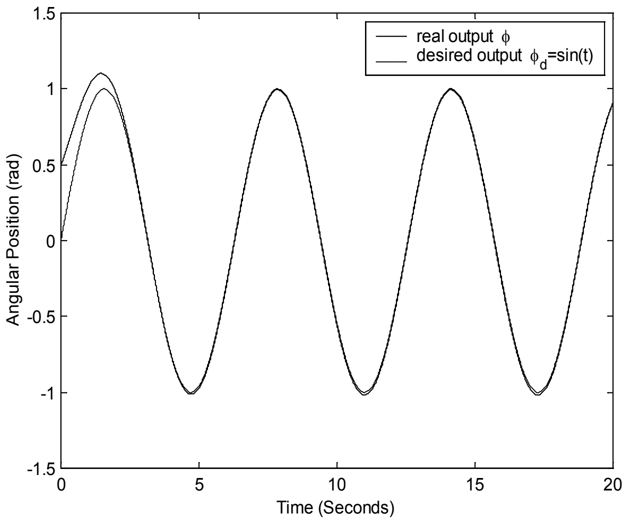

Figure 3. The tracking trajectory of joint 2

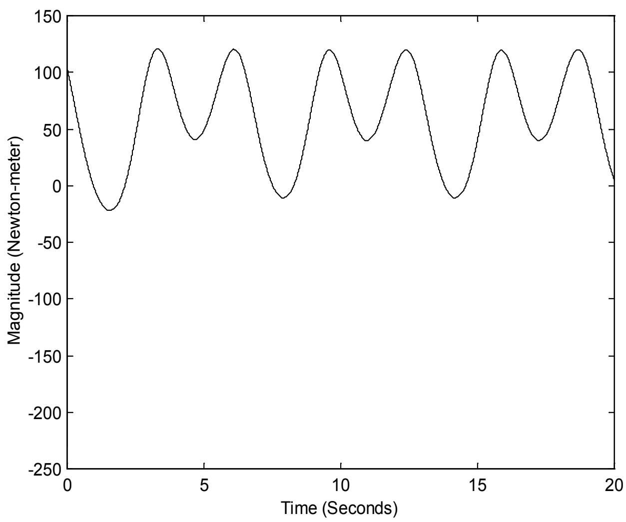

Figure 4. The torque of joint 1

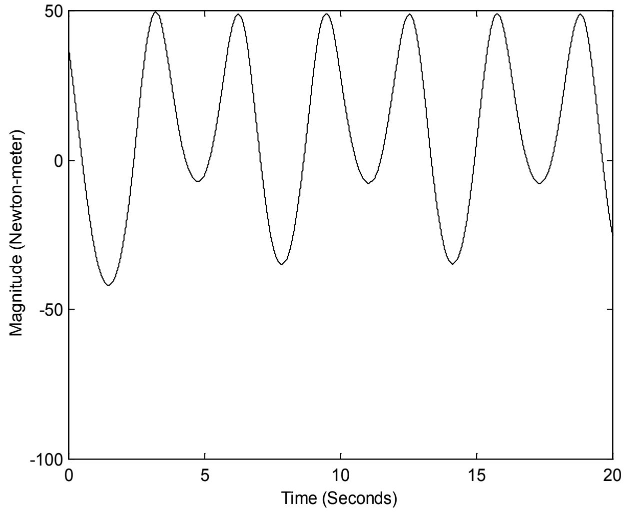

Figure 5. The torque of joint 2

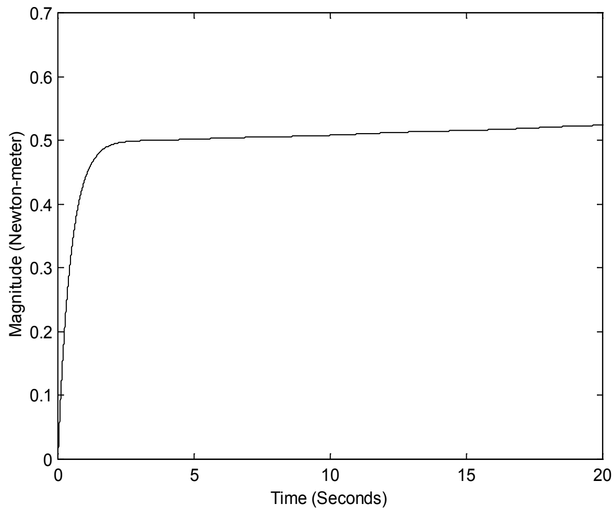

Figure 6. Curves of $\int_0^T e^2(t)dt$

5. Conclusions

The output tracking control problem of MIMO nonlinear systems with higher-order and unmatched uncertainties has been studied in this paper. In the proposed design method, fuzzy logic systems are used to estimate the part of unknown nonlinear functions, and the robust controller that combines the H^∞ optimal control with adaptive laws can deal with unmatched uncertainties and fuzzy approximation errors. Because of the complexity of the structure of the uncertainties, the upper bounds on the norm of the uncertainties can be estimated by the proposed adaptation laws. Simulation results demonstrate that the overall control system guarantees that all signals involved are uniformly ultimate bounded, and that the tracking performance index can be achieved.

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