Structural Identification of Static Systems with Distributed Lags

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Abstract   The approach to structural identification of static systems with the distributed lags is offered. The criterion of an estimation of linearity of system in parametrical space is introduced. The criterion is based on construction of set of secants for system. The special space for an estimation of structural parameters of system is selected. The concept of level of nonlinearity of system is introduced and the method of his estimation is reduced. The analogue of criterion of Darbin-Watson is reduced. Criteria of decision-making in the presence of a lag as in an output variable, and input variables are offered. It is shown that as magnitude of a lag performance on an output variable one can to use an estimation of parameters of coefficient of structural properties on the specified variable. Estimations of parameters of coefficient of structural properties are based on an evaluation of Lyapunov characteristic indicators.

Keywords   Structural identification, Distributed lags, Linearity level, Secant

1. Introduction

With the distributed delays the set of publications is devoted problems of identification of static plants. Such models are widely applied in econometrics and economy[1-4], to the technician[5-7], medicine[8-10]. Delay can have both independent, and dependent variables. The account of the distributed lags leads to autocorrelation between variables[1,2,4]. Autocorrelation hampers identification of parameters of plant. Various models of approximation of parameters are applied to a abatement of influence of autocorrelation on identification process at the distributed lags. It allows to reduce number of estimated parameters of plant. The Koyck scheme[2,12], based on a change of factors of model on a decreasing geometrical progression is most widely applied. The model I. Fisher[1, 11] is based on a change of factors of model on the set decreasing arithmetical progression. Arithmetical and geometrical models are applied in that case when plant parameters decrease from the first members of a progression. Considering it, S. Almon[13] has added more flexibilities of model of I. Fisher, having applied polynomial the law of a modification of factors. In[11] the law of a change of factors is offered at the distributed lags according to logarithmic normal distribution. The same idea based on application the exponential law of distribution, is considered in[14]. In[15] Pascal distribution which leads to the simple form for factors

is applied.

The a priori representation of correlation between the distributed lags in the form of some rational polynomial is considered in[16]. Other approaches to description of factors at the distributed lags are studied in[4].

The considered models of factors minimise number of unknown parameters. To an estimation of parameters apply a method least-squares method or its modifications[1-4]. In these works the model structure is postulated a priori and the problem of parametrical identification is considered. In[17] the interactive algorithm of an estimation of parameters of static plant with the distributed lags is offered. The length of a lag is set and does not become any suppositions about correlation between plant parameters. The case of a piecewise monotonic change of parameters of plant is considered. In a number of works methods of a choice of maximum length of a lag are offered. At the heart of applied approaches the statistics which are based on the analysis of residuals[1,15,18] lie. In[19, 20] various methods of an estimation of parameters of model with the distributed lag in a case of a priori set structure of model are considered. In[21] influence of the a priori information received from the analysis of the empirical data, on a choice of structure of model with the distributed lag on an output variable is analyzed. In work[22] are described algorithm of an estimation of parameters of model with the distributed lag. On the basis of results of modelling the structure of model which explains a mismatch of model applied now in the American interest rate on federal funds is selected. In[23] process of inflation with the help autoregressive models is researched. The choice of length of a lag is carried out on the basis of consecutive magnification of delays and an

estimation of adequacy of the received model. Then the criterion of Akaike and Bayesian Information Criteria are applied and the solution is made on model structure. The case of a priori uncertainty concerning structure and plant parameters was not study.

In the given work the approach to structural identification of static plants in the conditions of uncertainty is offered. It is based on application of the static structures describing properties of plant[24] in special space. The criterion of linearity (autocorrelation) of variables of plant is introduced. Algorithms of decision-making on maximum length of the distributed lag are offered. They do not demand an evaluation of the statistics. The analogue of criterion Darbina-Watson criterion for a considered case is reduced.

2. Problem Statement

Consider plant

$$y_n = A^T U_n + B^T X_n + \xi_n \qquad (1)$$

where $y_n \in R$ is an exit, $U_n \in R^k$ is the input vector which elements are limited, is limiting nondegenerate functions, $X_n = X(u_{i,n} \in U_n) \in R^m$ is a vector of the distributed lags on $u_{i,n} \in U_n$, $X_n = [u_{i,n-1}, u_{i,n-2}, \ldots, u_{i,n-m}]^T$, $n \in J_N = [0,N]$ is discrete time, $N < \infty$, $A \in R^k, B \in R^m$ are vectors of constant parameters, $\xi_n \in R$ is a perturbation, $|\xi_n| < \infty$ for all $n \in J_N$.

Consider that general case $\xi_n \in R$ and $u_{i,n} \in U_n$ $(i = \overline{1,N})$ are irregular functions of time.

For (1) the set of the measured values is known

$$I_o = \{y_n, U_n \ \forall n \in J_N\} \qquad (2)$$

and map $\Gamma_o : \{U_n\} \times \{y_n\}$ $\forall n \in J_N$ describing an observable informational portrait[7,24].

It is necessary I_o, Γ_o to estimate on the basis of analysis structure of plant (1). It means that it is necessary to estimate degree of linearity and dimension of vector X_n.

3. An Estimation of Level of Linearity of System (1)

Consider a contraction of observable informational portrait $\Gamma_o^{u_i} \subset \Gamma_o |_{u_i \in U}$ $\forall i = \overline{1,k}$ and for everyone $\Gamma_o^{u_i}$ construct a secant

$$\overline{\gamma}(y, u_i) = a_i u_{i,n}$$

where a_i are some real numbers.

Introduce set on (2) set of secants for Γ_o

$$S(U, y) = \{\overline{\gamma}(y, u_i) \ \forall i = \overline{1,k}\}$$

Definition 1[24]. Field of structures S_S of system (1) name set of maps $\overline{\gamma}(y, u_i) \subset \{u_i\} \times \{y\}$ $\forall i = \overline{1,k}$ on Euclidean plane E

$$S_S = S(U, y)$$

Designate

$$\Phi_n = [\overline{\gamma}(y_n, u_{1,n}), \overline{\gamma}(y_n, u_{2,n}), \ldots, \overline{\gamma}(y_n, u_{k,n})]^T \quad \text{and}$$

consider the equation

$$\hat{y}_n = \Psi^T \Phi_n \qquad (3)$$

where vector $\Psi \in R^k$ define by means of a least-squares method. Estimation Ψ exists on the basis of the suppositions made in section 2 concerning input U_n.

Completeness of system (1) in the field of structures S_S estimate on the basis of the following statement[24].

Theorem 1. Consider a vector of informative variables $U_n \in R^k$ and a field of structures $S_S \subset S(U, y)$ for (1). Then the field of structures S_S of system (1) is full, if

$$\sum_{i=1}^{k} \psi_i = 1 \qquad (4)$$

where ψ_i there is i-th element of vector Ψ in (4).

The theorem 1 gives linearity sufficient conditions (nonlinearities, collinearity) systems (1) on the set field of structures S_S. If the condition (4) is fulfilled, that field S_S is full. Hence S_S is a linear span of an exit of system (1). Otherwise make a solution on presence of nonlinearity or collinearity (autocorrelation) in system (1).

Let

$$\sum_{i=1}^{k} \psi_i = \chi$$

Magnitude $\Delta(\Psi) = \chi - 1$ name level of nonlinearity of system (1) in parametrical space $\mathcal{P}_{\mathcal{P}} = (A, B)$. As nonlinearities and lags lead to occurrence multicollinearity in (1) $\Delta(\Psi)$ will accept small values.

4. Set for an Estimation of Structure of the Distributed Lag of Plant

Following[7,24], generate auxiliary set for an estimation of structure (1). Introduce variable $s_n = I^T U_n$, where $I \in R^k$ is an unit vector, and apply model $\hat{y}_{s,n} = \hat{a}_s s_n$. Define parameter $\hat{a}_s \in R$ from a condition

$$\arg \min_{\hat{a}_s} (\hat{y}_{s,n} - y_n)^2 = \hat{a}_s^* \qquad (5)$$

Let $e_n = \hat{y}_{s,n} - y_n$ is a variable which contains the data about structure of a lag of system (1). As argument $X(u_{i,n})$ use variable $u_{i,n} \in U_n$ which ensures maximum value of coefficient of determination r_{e,u_i}^2 between e and u_i. As shown in[24], set $\{e_n, u_{i,n}\}$ does not allow to solve a problem of structural identification. Therefore introduce coefficient of structural properties (CSP)[7]

$$k_{e,u_i,n} = k_s(e,u_i,n) = \frac{e_n}{u_{i,n}} \qquad (6)$$

and generate set

$$\mathrm{I}_k = \mathrm{I}_k(k,e) = \{e_n, k_n, \ n \in J_N = [0,N]\}, \ k_n \stackrel{\mathrm{df}}{=} k_{e,u_i,n}.$$

Definition 2. Name $\mathscr{P}_s = (k,e)$ structural space of the system (1), allowing to identify structure of vector X_n.

On J_N will order $k_{e,u_i,n}$ on increase. Generate $\{k_q^v\}$, where $k_q^v = k_s(e,u_i,q), \quad q \in J_N^v = [0,N]$. As to everyone k_q^v there corresponds value e_q^v receive $\mathrm{I}_k^v = \{e_q^v, k_q^v, q \in J_N^v\}$. In $\mathscr{P}_s$ define map $\Gamma_{e,k}^v : \{k_q^v\} \times \{e_q^v\}$ and structure $S_{k,e}^v$ corresponding to it. Now the problem is shown to an estimation of structure X_n on the basis of analysis I_k^v and $S_{k,e}^v$. Such approach well works at an estimation of structure of nonlinear static systems[24]. For systems with the distributed lag he demands modification.

5. Decision-Making on Length of a Lag in $X(u_{i,n})$

Set $\mathrm{I}_{e,u} = \{e_n, u_{i,n}\}$ contains uncertainty $\mathscr{N}_L$ and $\mathscr{N}_{Lg}$. $\mathscr{N}_L$ is an incomplete account of linear making system (1). $\mathscr{N}_{Lg}$ is a presence of perturbation from the distributed lag. For elimination $\mathscr{N}_L$ construct a secant for e_n

$$\bar{\gamma}(e,u_{i,n}) = \alpha_0 + \alpha_1 u_{i,n}$$

where α_0, α_1 define as a solution of a problem (5).

Introduce new variable $\varepsilon_n = e_n - \bar{\gamma}(e,u_i)$ which does not contain $\mathscr{N}_L$. For estimation $\mathscr{N}_{Lg}$ analyse set $\{\varepsilon_n, u_{i,n}\}$.

For deriving of a provisional estimate of maximum lag m fulfil following operations. Set admissible level $\rho > 0$ of coefficient of determination r_{e,u_i}^2. Apply the following algorithm.

Algorithm CD_ρ.
1. Suppose $m = 1$.
2. Construct a secant

$$\bar{\gamma}(e,u_{i,n-m}) = \bar{\gamma}_{e,k} = \alpha_0 + \alpha_1 u_{i,n-m}$$

and define $r_{e,u_{i,n-m}}^2$.

3. Verify up condition $r_{e,u_{i,n-m}}^2 \geq \rho$.

4. If the condition is fulfilled, suppose $m = m+1$ and go to a step 2, differently finish work.

In work the multiple-functional approach to structural identification is applied. Therefore known methods of a choice of length of the lag, based on statistical criteria (section 1 see), are inapplicable.

Considering it, to an estimation of independence of elements of vector X_n apply the theorem 1. As the analysis of set $\mathrm{I}_{e,u}$ is in this case ineffective, that use results of section 4 and generate set

$$\mathrm{I}_{\varepsilon,k_x} = \{\varepsilon_n, k_{\varepsilon,x_{i,n}}, i = \overline{1,m} \ \forall n \in J_N\} \qquad (7)$$

where $k_{\varepsilon,x_i,n}$ calculate on the basis of (6), considering $e_n = \varepsilon_n$, $x_{i,n} \in X_n$.

On $\mathrm{I}_{\varepsilon,k_X}$ will introduce transformation

$$\Gamma_{\varepsilon,k_{\varepsilon,x_i}} : \{k_{\varepsilon,x_{i,n}}\} \times \{\varepsilon_n\}$$

to which in space $\mathscr{P}_{s,\varepsilon} = (k_{\varepsilon,x_{i,n}}, \varepsilon)$ there corresponds structure $S_{k\varepsilon,x,\varepsilon}$, $k_{\varepsilon,x} = k_{\varepsilon,x_i}$. Construct secants for $S_{k\varepsilon,x,\varepsilon}$

$$\bar{\gamma}(\varepsilon_n, k_{\varepsilon,x_{i,n}}) = \beta_i k_{\varepsilon,x_{i,n}}, \ \forall i = \overline{1,m_*}, \qquad (8)$$

where β_i are the coefficients defined as a result of a solution of a problem (5), m_* is value m received on the basis of application of algorithm CD_ρ.

Generate a vector

$$\Phi_{\varepsilon,n} = [\bar{\gamma}(\varepsilon_n, k_{\varepsilon,x_{1,n}}), \bar{\gamma}(\varepsilon_n, k_{\varepsilon,x_{2,n}}), \ldots, \bar{\gamma}(\varepsilon_n, k_{\varepsilon,x_{m_*,n}})]^T \ (9)$$

and apply model $\hat{\varepsilon}_n = \Psi_\varepsilon^T \Phi_{\varepsilon,n}$ to forecasting of change ε_n, where vector $\Psi_\varepsilon \in R^{m_*}$ is defined on the basis of outcomes of section 3.

The statement following directly from the theorem 1 is fair.

Theorem 2. Let on set $\mathrm{I}_{\varepsilon,k_x}$ the field of secants for ε_n is constructed

$$S_\varepsilon = \left\{\bar{\gamma}(\varepsilon_n, k_{\varepsilon,x_{i,n}}) \ (\forall i = \overline{1,m_*}) \& (n \in J_N)\right\}$$

and model $\hat{\varepsilon}_n = \Psi_\varepsilon^T \Phi_{\varepsilon,n}$ is applied to forecasting of a change of variable ε_n. Then vector $X_n \in R^{m_*}$ is an element of structure of system (1), if

$$\sum_{i=1}^{m_*} \psi_{\varepsilon,i} \neq 1$$

where m_* it is defined by means of algorithm CD_ρ, $\psi_{\varepsilon,i} \in \Psi_\varepsilon$.

Remark. As ε_n contains the information on influence of vector X_n at statistical interpretation U_n, ξ_n use analog of criterion of criterion of Darbin-Watson

$$d = \frac{\sum_{n=1}^{N} \varepsilon_n^2}{\sum_{n=1}^{N} e_n^2}$$

Level of nonlinearity of the system (1), generated by correlation of elements of vector X_n is equal

$$\Delta(\Psi_\varepsilon) = \sum_{i=1}^{m_*} \psi_{\varepsilon,i} - 1$$

Theorem 3. Consider the set of secants S_ε set on I_{ε,k_x} and secant $\bar\gamma(e, k_{e,u_i}) = \alpha_1 k_{e,u_i}$ for $S_{k_{e,u_i},e}$. Let for them coefficients of determination $r^2_{\varepsilon,k_{\varepsilon,xz}}$, $z = \overline{1,m}$ and $r^2_{e,k_{e,ui}}$ are known. Then vector $X(u_{i,n}) \in R^m$ is an element of structure of system (1), if $\forall z = \overline{1,m}$

$$\left| r^2_{\varepsilon,k_{\varepsilon,x_z}} - r^2_{e,k_{e,ui}} \right| \le \delta_x$$

where $\delta_x \ge 0$ is a specified magnitude.

At structure $S_{k,e}^v$ analysis, the distributed lag of system (1) interpret as a nonlinear component (1). Therefore design the corresponding procedure, allowing to make the solution on a class of uncertainty $\mathcal{N}_{Lg}$. Apply the following approach.

Consider set I_k^v and static structure $S_{k,e}^v$ corresponding to it. Construct the secants $S_{k,e}^v$

$$\bar\gamma_1(e_q^v, k_q^v) = a_1^v k_q^v$$

$$\bar\gamma_2(e_q^v, k_q^v) = a_{2,0}^v k_q^v + a_{2,1}^v \left(k_q^v\right)^2$$

and define for them coefficients of determination r_1^2, r_2^2.

Consider CSP

$$k_{e,u_i,n,\pi} = k_{s,\pi} = \frac{e_n}{u_{i,n}^\pi}, \quad \pi < \infty \qquad (10)$$

and map $\Gamma_{e,k,\pi}^v : \{k_{s,\pi,q}^v\} \times \{e_q^v\}$, $q \in J_N^v$ to which there

corresponds structure $S_{k_{s,\pi},e}^v$. Construct for $S_{k_{s,\pi},e}^v$ a secant

$$\bar\gamma(e_q^v, k_{s,\pi,q}^v) = \bar\gamma_\pi = a_\pi^v k_{s,\pi,q}^v \qquad (11)$$

and parameter π select so that the factor of determination $r_{\bar\gamma}^2$ for (11) belonged to interval $[r_1^2, r_2^2]$.

Further consider CSP

$$k_{e,x,n} = k_{s,x} = \frac{e_n}{I^T X_n}$$

where $I \in R^{m_*}$ is an unit vector. Construct a secant for $S_{k_{s,x},e}^v$

$$\bar\gamma(e_q^v, k_{s,x,q}^v) = \bar\gamma_x = a_\pi^v k_{s,x,q}^v \qquad (12)$$

Designate coefficient of determination for $\bar\gamma_x$ through $r_{\bar\gamma_x}^2$.

Theorem 4. Consider structures $S_{k_{s,\pi},e}^v$, $S_{k_{s,x},e}^v$ and secants (11), (12) corresponding to them. If for structure $S_{k_{s,\pi},e}^v$, $S_{k_{s,x},e}^v$ secants the condition is satisfied

$$r_{e,k_x}^2 \ge r_{e,k_{s,\pi}}^2$$

then X_n is an element of structure of system (1).

The proof of the theorem 4 is obvious. It is based that structures $S_{k_{s,\pi},e}^v$, $S_{k_{s,x},e}^v$ describe a change of same variable e_q^v. Therefore a solution about inclusion in structure of system (1) accept concerning that variable X_n, $u_{i,n}^\pi$ which ensures greater coefficient of determination.

Statement. If coefficient of determination $r_{\varepsilon,u_i}^2 \approx 0$, then $r_{\varepsilon,ks_{u_i}}^2 \approx 1$.

Proof. From $r_{\varepsilon,u_i}^2 \approx 0$ follows that parameter $\hat a$ of model $\hat\varepsilon_n = \hat a u_{i,n}$ predicting a change of variable ε_n, is equal to zero. Then for $r_{\varepsilon,ks_{u_i}}^2$ receive

$$r_{\varepsilon,ks_{u_i}}^2 = 1 - \frac{\sum_{n=1}^{N} \left(\hat\alpha k_{s,u_{i,n}} - \varepsilon_n\right)^2}{\sum_{n=1}^{N} \varepsilon_n^2} \qquad (13)$$

where $k_{s,u_{i,n}} = \varepsilon_n / u_{i,n}$. Substituting $k_{s,u_{i,n}}$ in (13), for fraction numerator receive

$$\sum_{n=1}^{N} \left(\hat\alpha k_{s,u_{i,n}} - \varepsilon_n\right)^2 = \sum_{n=1}^{N} \frac{\varepsilon_n^2}{u_{i,n}^2} \left(\hat\alpha - u_{i,n}\right)^2 \qquad (14)$$

If $\hat{\alpha} = \bar{u}_{i,n}$ where $\bar{u}_{i,n}$ there is average value $u_{i,n}$, then $r^2_{\varepsilon,ks,u_i} \approx 1$.

Results of modelling confirm the made statement.

Consider a case when vector X_n in (1) contains the distributed lags on y_n, that is $X_n = X(y_n)$.

6. Decision-Making on Length of a Lag for $X(y_n)$

Let $X_n = [x_{1,n}, x_{1,n}, \ldots, x_{l,n}]^T$, where $x_{i,n} = y_{n-i}$, $i = \overline{1,l}$. To tentative estimation l apply algorithm CD_ρ. Decision-making in space $\mathcal{P}_s = (k_{\varepsilon,x_{i,n}}, e_n)$, can appear ineffective because of connections between $x_{i,n}$. Therefore consider space $\mathcal{P}_{s,X} = (X, e)$ set on set $\mathbb{I}_{e,X} = \{X_n, e_n \; \forall n \in J_N\}$. Consider structure $S_{X,e}$ and its projections $S_{x_i,e}$ to plane (x_i, e). For everyone $S_{x_i,e}$ define secants $\bar{\gamma}(e, x_i) = \alpha_{0,i} + \alpha_{1,i} x_{i,n}$, $i = \overline{1,l}$ Also generate a vector

$$\Phi_{e,n} = \left[\bar{\gamma}(e_n, x_{1,n}), \bar{\gamma}(e_n, x_{2,n}), \ldots, \bar{\gamma}(e_n, x_{l,n}) \right]^T \quad (15)$$

Theorem 5. Let on set $\mathbb{I}_{e,X}$ the field of secants for e_n is constructed

$$S_e = \left\{ \bar{\gamma}(e_n, x_{i,n}) \; (\forall i = \overline{1,l}) \& (n \in J_N) \right\}$$

and model $\hat{e}_n = \Psi_e^T \Phi_{e,n}$ is applied to forecasting of variable e_n. Then vector $X_n \in R^l$ is an element of structure of system (1), if

$$\sum_{i=1}^{l} \psi_{e,i} \neq 1 \quad (16)$$

where l define by means of algorithm CD_ρ, $\psi_{e,i} \in \Psi_e$.

The theorem 5 is analogue of the theorem 2. Performance (16) specifies on presence of dependence between $x_{i,n}$. At statistical interpretation of a problem (16) speaks about autocorrelation of residuals[1]. Level of nonlinearity of system (1) estimate by means of $\Delta(\Psi_e)$.

For an estimation of existence of a lag on y_n analyse a change CSP

$$k_s(e, x_i, n) = k_{s,x_i,e,n} = \frac{e_n}{x_{i,n}}$$

As the log on y_n is considered, then

$$k_{s,x_i,e,n} = e^{\lambda n\tau} \quad (17)$$

where λ is an eigenvalue of system (1) with $X_n = X(y_n)$, τ is an interval of measurement of the data.

The problem consists in estimation $k_s(e, x_i, n)$ on the basis of analysis $\mathbb{I}_{e,x_i}$ and identification of parameter λ on set $\{k_s(e, x_i, n), n\tau\}$. To estimation λ apply Lyapunov characteristic indicators.

Apply model $\hat{e}_n = \beta x_{i,n}$ to forecasting of change e_n and define parameter β by means of a least-squares method. To description e_n also apply model $\hat{e}_n = \hat{\alpha} k_{s,x_i,e}$, where $\hat{\alpha} = \bar{x}_{i,n}$. From comparison of these two models receive

$$k_{s,x_i,e,n} = \beta \frac{x_{i,n}}{\bar{x}_{i,n}}, \quad \forall n \in J_N \quad (18)$$

Arrange values $k_{s,x_i,e,n}$ in ascending order and receive set $\mathbb{I}_{k_{x,e}}^v = \{k_{s,x_i,e,q}^v, q \in J_N\}$. Define for $k_{s,x_i,e,q}^v$ of Lyapunov characteristic indicator[25]

$$\mu[k_{s,x_i,e,q}^v] = \varlimsup_{t_q \to \infty} \frac{1}{t_q} \ln \left| \frac{k_{s,x_i,e,t_q}^v}{\max_{t_q} k_{s,x_i,e,t_q}^v} \right| \quad (19)$$

where $\varlimsup_{t_q \to \infty}$ is an limit superior, $t_q = q\tau$.

Suppose $\hat{\lambda} = \mu$ where $\hat{\lambda}$ there is estimation λ. To an improving of received estimation λ apply the approach offered in[24].

Remark. Estimation λ serves as the indicator of presence at system (1) distributed lags on y_n. Exact estimation λ define only at a step of parametrical identification.

Consider the more general case when vector X_n in (1) is equal $X(y_n, u_{i,n}) = [X_{y,n}^T, X_{u,n}^T]^T$, where $X_{y,n} = X(y_n)$, $X_{u,n} = X(u_{i,n})$.

7. Decision-Making on Length of a Lag for $X(y_n, u_{i,n})$

Spread the approach stated above to system (1) with $X_n = X(y_n, u_{i,n})$. The provisional estimate of level of linearity of system (1) receive by means of the theorem 1. Introduce variable $e_n \in R$ which depends on uncertainty $\mathcal{N}_L$. Apply algorithm CD_ρ to definition of dimension of vectors $X_{y,n}$, $X_{u,n}$. For elimination of influence $\mathcal{N}_L$

construct a secant for e_n

$$\bar{\gamma}(e, u_i, y) = \alpha_0 + \alpha_1 u_{i,n} + \alpha_2 y_n$$

where $\alpha_0, \alpha_1. \alpha_2$ define as a solution of a problem (5).

Introduce new variable $\varepsilon_n = e_n - \bar{\gamma}(e, u_i, y)$ which does not contain $\mathcal{N}_L$. To decision-making on lag presence apply results of sections 5, 6.
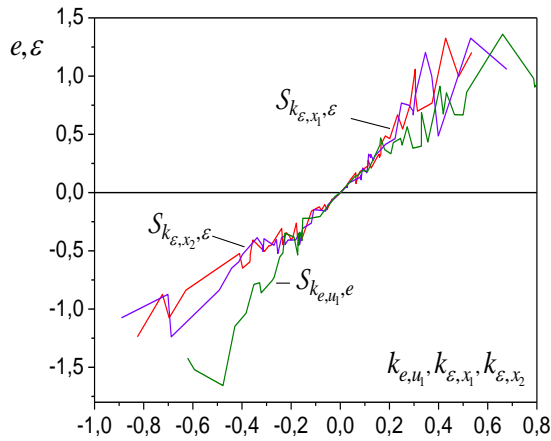
8. Examples

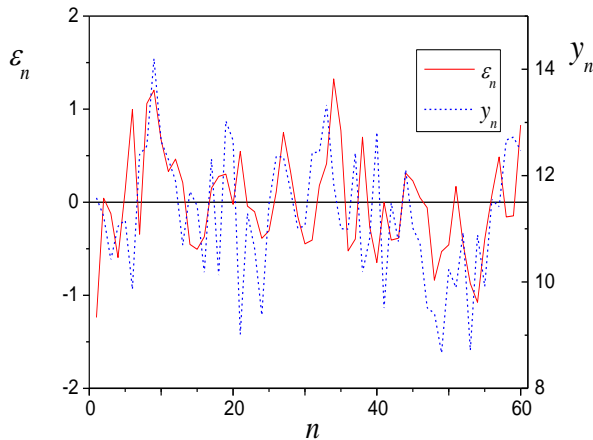**Figure 1.** Structures for decision-making on length of lag $X(u_{1,n})$

Figure 2. Variables y_n, ε_n

Consider system (1) with $A = [1.5; 2; 2.8]^T$, $B = [0.7; 0.6]^T$, $X_n \in R^2$, $X_n = X(u_{1,n})$. $u_{i,n} \in U_n$ are limited random functions, ξ_n is an stochastic variable with zero expectation and a final variance, $|\xi_n| \le 0.3$. Application of the theorem 1 has shown that the system is not linear, $\Delta(\Psi) = 0.03$. Receive set

$$\mathbf{I}_{e,u} = \{e_n, u_{i,n} \ (\forall i = \overline{1,3}) \& (\forall n \in J_N)\}$$

Analysis $\mathbf{I}_{e,u}$ has shown that variable $u_{1,n}$ has a lag. Apply algorithm CD_ρ and the theorem 2. The length of a

lag on $u_{1,n}$ is equal 2. Indicator $\Delta(\Psi_\varepsilon)$ practically coincides with $\Delta(\Psi)$. To an correction of the received estimation of length of a lag apply the theorem 3 with $\delta_x = 0.01$. $r^2_{e,ku_1} = 0.906$, $r^2_{\varepsilon, kx_2} = 0.907$, $r^2_{\varepsilon, kx_1} = 0.914$, $r^2_{\varepsilon, kx_3} = 0.934$. Corresponding structures on which the theorem 3 is based, are shown on fig. 1. Corresponding structures on which the theorem 3 is based, are shown on fig. 1.

On fig. 2 show variables y_n, ε_n. They confirm presence at system of the distributed lag on $u_{1,n}$. As inputs are random calculate criterion d. Receive $d = 0.54$ that following[2], confirms result about presence a lag.

Consider system (1) with $A = [0.5; 1.4; 1.7]^T$, $B = 0.6$, $X_n = x_n = y_{n-1}$. Receive indicator $\Delta(\Psi) = 0.03$. Make a solution on presence multicollinearity (nonlinearity) in systems. Apply algorithm CD_ρ. Receive $x_n = y_{n-1}$. The length of a lag is equal 1. Generate variable ε_n on the basis of a method described in section 5. On fig. 3 show variables y_n, ε_n to estimate delay presence on y_{n-1}.
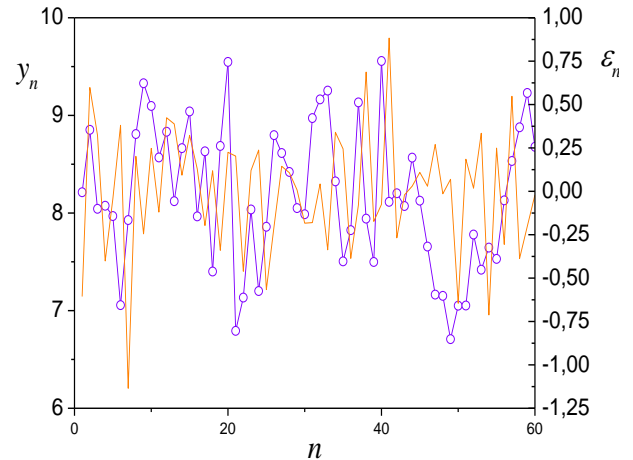

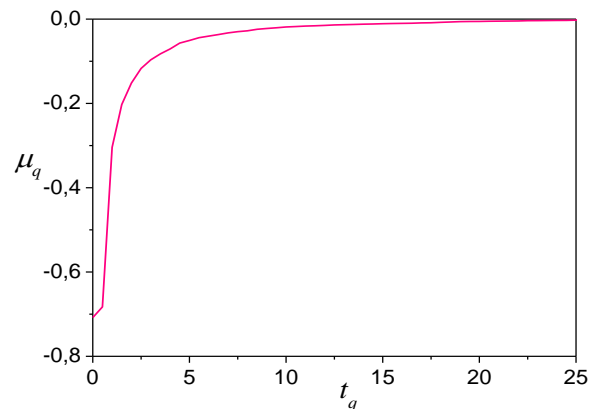**Figure 3.** Delay in system (1) with $X_n = y_{n-1}$

Figure 4. Lyapunov characteristic index μ

For validation of the received conclusion estimate CSP $k_{s,x_i,e,n}$ at $X_n = y_{n-1}$, using dependences (17), (18) and (19). Problem reduce to estimation λ in (17). On fig. 4 show change μ_n on the basis of (19). Suppose $\hat{\lambda} = -0.008$. More exact estimation λ in (17) define, using a method offered in[24]. Receive $\hat{\lambda} = -0.0102$.

9. Conclusions

The is functional-multiple approach to structural identification of static systems with the distributed lag is offered. Solution about structure of the distributed lag of system receive on the basis of the analysis of special static structures. The criterion of an estimation of degree of linearity of system in parametrical space is introduced. It is considered variants of identification of system in the availability of a lag on dependent and independent variables. The approach to an

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