Applied Mathematics 2021, 11(2): 21-22
DOI: 10.5923/j.am.20211102.02

A P-Series Formula

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Abstract Let $t, n, p \in N$ such that $m_n, h_n \in R$ then $\sum_{i=1}^{n}\left(m_i{}^p + h_i{}^p\right) = \frac{1}{2^p}\left[2\sum_{a=1}^{t}\sum_{i=1}^{n}\binom{p}{2a}(m_i - h_i)^{2a}(m_i + h_i)^{p-2a} + 2\sum_{i=1}^{n}(m_i + h_i)^p + \sum_{i=1}^{n}(m_i + h_i)^p\left(1+(-1)^p\right)\right]$ Where p=2t+1 for all p odd Natural numbers(N), p=2t+2 for all p even natural numbers(N).

Keywords Power series, Formula

1. Introduction

In the early time fermat, a French number theorist, had stated that the equation $x^n + y^n = z^n$ has no solution in integers, if n>2 [1]. The problem to find the exact value of z is enhanced as the number of terms increases rather than power two. Let us have n terms each of which has a common power p in the series then the series is called a p-series which is applied by the following theorem for p>2 which is also an answer for the claim of fermat.

Theorem 1. Let $t, n, p \in N$ such that $m_n, h_n \in R$ then

$$\sum_{i=1}^{n}\left(m_i{}^p + h_i{}^p\right) = \frac{1}{2^p}\left[2\sum_{a=1}^{t}\sum_{i=1}^{n}\binom{p}{2a}(m_i - h_i)^{2a}(m_i + h_i)^{p-2a} + 2\sum_{i=1}^{n}(m_i + h_i)^p + \sum_{i=1}^{n}(m_i + h_i)^p\left(1+(-1)^p\right)\right]$$

Where p=2t+1 for all p odd Natural numbers(N), p=2t+2 for all p even natural numbers.

Proof. Let $t, n, p \in N$ such that $r_1, r_2, x_n, A_n \in R.$ if $A_n x_n = x_{n+1} - x_n$ where $x_1 = 1$ then multiplying $\left(\frac{x_{n+1}}{x_n}\right)$ up to n times we get $x_{n+1} = \prod_{i=1}^{n}(A_i + 1)$, therefor we can write this expression to p power as $A_n{}^p x_n{}^p = (x_{n+1} - x_n)^p$, $x_{n+1}{}^p = \prod_{i=1}^{n}(A_i + 1)^p$ If the binomial theorem for expansion $(x_{n+1} + x_n)^p$.

Can be written as

$$(x_{n+1} - x_n)^p = \prod_{i=1}^{n}(A_i + 1)^p (r_1) + (-1)^p \prod_{i=1}^{n-1}(A_i + 1)^p$$

Where

$$r_1 = -\binom{p}{1}\left(\frac{1}{(A_n + 1)}\right) + \binom{p}{2}\left(\frac{1}{(A_n + 1)^2}\right) - \binom{p}{3}\left(\frac{1}{(A_n + 1)^3}\right),$$
$$+ \ldots\ldots\ldots(-1)^{p-1}\binom{p}{p-1}\left(\frac{1}{(A_n + 1)^{p-1}}\right)$$

$n \geq 2$

Putting $x_{n+1} = \prod_{i=1}^{n}(A_i + 1)$, $x_n = \prod_{i=1}^{n-1}(A_i + 1)$ and dividing both sides by $\prod_{i=1}^{n-1}(A_i + 1)^p$ we get

$$A_n{}^p - (-1)^p = (A_n + 1)^p (r_1 + 1) \qquad (1)$$

The other way of writing $A_n x_n = x_{n+1} - x_n$ is $(A_n + 2)x_n = x_{n+1} + x_n$ which can be written to p power as $(A_n + 2)^p x_n{}^p = (x_{n+1} + x_n)^p$

If binomial theorem for expansion $(x_{n+1} + x_n)^p$ can be written as

$$(x_{n+1} + x_n)^p = \prod_{i=1}^{n}(A_i + 1)^p + \prod_{i=1}^{n}(A_i + 1)^p (r_2) + \prod_{i=1}^{n-1}(A_i + 1)^p$$

Where

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Received: Jun. 14, 2021; Accepted: Jul. 11, 2021; Published: Jul. 15, 2021
Published online at http://journal.sapub.org/am

$$r_2 = \binom{p}{1}\left(\frac{1}{(A_n+1)}\right) + \binom{p}{2}\left(\frac{1}{(A_n+1)^2}\right) + \binom{p}{3}\left(\frac{1}{(A_n+1)^3}\right),$$
$$+ \ldots\ldots\ldots + \binom{p}{p-1}\left(\frac{1}{(A_n+1)^{p-1}}\right)$$

$n \geq 2$

Putting $x_n = \prod_{i=1}^{n-1}(A_i+1)$ in $(A_n+1)^p\left(x_n{}^p\right)$ and dividing both sides by $\prod_{i=1}^{n-1}(A_i+1)^p$ we get

$$(A_n+2)^p - 1 = (A_n+1)^p\,(r_2+1) \tag{2}$$

Therefor, the summation of EQ(1) and EQ(2) can be put as follow

$$\sum_{i=1}^{n}\left(A_i{}^p + (A_i+2)^p - \left(1+(-1)^p\right)\right)$$
$$= 2\left[\sum_{a=1}^{t}\sum_{i=1}^{n}\binom{p}{2a}(A_i+1)^{p-2a} + \sum_{i=1}^{n}(A_i+1)^p\right]$$

Where p=2t+1 for all p odd natural numbers, whereas p=2t+2 for all p even natural numbers

Let put $A_n = h_n$ and if h_n be divided by $\frac{m_n - h_n}{2}$ such that m_n, $h_n \in R$ (Real numbers) then we get the following result

$$\sum_{i=1}^{n}\left(m_i{}^p + h_i{}^p\right) = \frac{1}{2^p}\left[2\sum_{a=1}^{t}\sum_{i=1}^{n}\binom{p}{2a}(m_i-h_i)^{2a}(m_i+h_i)^{p-2a}\right.$$
$$\left. + 2\sum_{i=1}^{n}(m_i+h_i)^p + \sum_{i=1}^{n}(m_i+h_i)^p\left(1+(-1)^p\right)\right]$$

This result led us to a conclusion that every couple terms which have common power can undergo this formula.

REFERENCES

[1] Gerhard Frey, The way to the proof of Fermat's Last Theorem, Annales de la faculte des sciences de Toulouse, Vol. XVIII, 2009, pp.5-23.