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Simultaneous Effects of Control Measures on the Transmission Dynamics of Chikungunya Disease

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Abstract Chikungunya is a vector borne communicable disease which is transmitted in human population through the bite of an infected *Aedes-Aegeypti* mosquito. In order to study the spread of Chikungunya disease a model has been proposed and analyzed in this paper. In the proposed model the human population and the mosquito population have been divided into three and two classes respectively. For controlling the disease, vector control measures such as, reduction in the breeding of vector population, killing of mosquitoes and isolation of infected humans have been also taken in to account in the model. Linear and non-linear stability analysis of the model has been carried out. From the analysis we have derived a threshold condition involving control reproductive number R_c , and we have found that the disease free equilibrium point is locally asymptotically stable when $R_c < 1$ and unstable when $R_c > 1$ .We have also proved that a unique endemic equilibrium point exists and is locally asymptotically stable when $R_c > 1$. Thus, we have concluded from the analysis of the model that the disease will either die out or will remain endemic depending on the value of control reproductive number. This study will assist the health department in controlling the spread of Chikungunya disease by introducing the control measures such as increasing the awareness in the society, killing of mosquitoes and isolating the infected individuals.

Keywords Chikungunya Disease, Epidemic Model, Control Reproductive Number, Stability, Disease Free Equilibrium Point, Endemic Equilibrium Point

1. Introduction

Chikungunya is a vector borne communicable disease. It is caused by a Buggy creek virus or better known as Chikungunya virus. This virus is an enveloped positive-strand RNA virus capable of replicating in a mosquito species known as a*des-aegeypti*. Aedes-aegeypti mosquitoes usually dwell in human habitats. Chikungunya virus is transmitted in the human individuals through the bite of an Aedes-Aegeypti mosquito. Chikungunya disease has become a global concern due to an escalation in the disease outbreaks, in Africa, India and South East Asian countries. The epidemic is a consequence of heavy rains favoring the active breeding of these mosquitoes in urban habitats that synchronize with humans, who serve as reservoir host for Chikungunya virus.

Aedes-aegeypti is a household container breeder and aggressive anthrophilic day time biter. Apart from other containers like clay jars, drums, cement tanks and coolers, west bottles are also found to be positive for *aedes* breeding. According to a survey report[15] more than 80% breeding has been found in small and midsized containers.

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Since no vaccine or specific antiviral treatment available for Chikungunya fever, vector avoidance and control is at present the only way to limit the disease transmission. As we know that most of the Chikungunya vector population grows in household containers therefore education of society can play an effective role to curb the vector breeding. Simultaneously chemical spray can eliminate the vector presented in the surroundings of human habitat. A new ULV technique consisting of aerosol spray of ultra low volume quantities of insecticide has been found to be effective in killing the mosquito in the air as well as on the water. With the help of ULV treatment the Aedes Research unit Bangkok was able to reduce adult mosquito density by more then 98% for several weeks[12]. Further, spread of the infection can be reduced by isolating infected humans from mosquitoes. This can be done by staying indoors and sleeping under mosquito nets.

Dynamics of various vector borne diseases has been studied by various researchers such as N.T.J.Bailey[3], Aron[2], Anderson and May[1], Esteva and Vardos[9,10], Diekmann and Heesterbeek[7], Chitnis et. al[5,6] and Bacaër[4] using mathematical models.

Ramchurn et. al[13] have specifically studied the transmission dynamics of chikungunya epidemic outbreak with the help of a SI model. In the present work we have developed and extended the model in light of the dynamics of the

chikungunya disease.

Thus in this paper we have studied an epidemic model for chikungunya disease by considering SIR dynamics for human population and SI dynamics for mosquito population. Through this model we have discussed effects of control measures on the transmission dynamics of chikungunya disease.

2. Model Formulation

In the formulation of the proposed model the human population have been classified into three categories, namely susceptible, infected and removed. Similarly mosquito population has been classified into two categories namely susceptible and infected mosquitoes. When an infected mosquito bites a susceptible human then he gets infection and goes to the infected human class. As chikungunya is not a killer disease[14] and after recovery infected human acquires immunity for significant time therefore after recovery infected humans go to removed class. It has been also assumed in the model that the infected human individuals are kept away from the contact of the mosquitoes by different means such as use of mosquito net, staying at isolated place in the house. Similarly when a susceptible mosquito bites an infected human then it gets infected and joins the infected mosquito class. The transfer diagram of this process is shown in following figure 1.

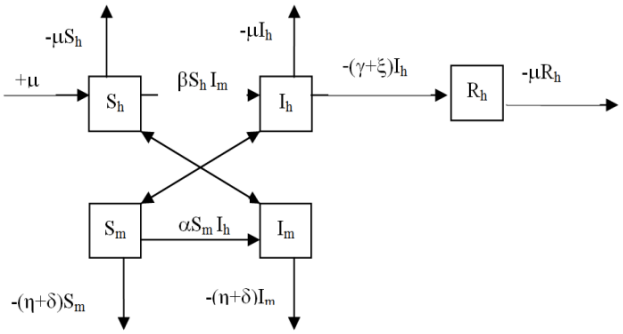

Figure 1. Transfer diagram of Chikungunya

Let S_h, I_h, R_h are the proportions of the susceptible, infectious and removed individuals in the human population and S_m, I_m are the proportions of the susceptible and infected mosquitoes in the mosquito population. The dynamical system representing the epidemic spread in human and vector population are then given by the following system of non linear ordinary differential equations.

$$\frac{dS_h}{dt} = \mu - \mu S_h - \beta S_h I_m \qquad (2.1a)$$

$$\frac{dI_h}{dt} = \beta S_h I_m - (\gamma + \mu + \xi) I_h \qquad (2.1b)$$

$$\frac{dR_h}{dt} = (\gamma + \xi) I_h - \mu R_h \qquad (2.1c)$$

$$\frac{dS_m}{dt} = (1-\varepsilon)\eta - \alpha S_m I_h - (\eta + \delta) S_m \qquad (2.1d)$$

$$\frac{dI_m}{dt} = \alpha S_m I_h - (\eta + \delta) I_m \qquad (2.1e)$$

With initial conditions

$S_h(0)=S_{h0}, I_h(0)=I_{h0}, R_h(0) = R_{h0}, S_m(0)=S_{m0}, I_m(0)=I_{m0}$ (2.2)

The description of parameters is as follows

μ = birth and death rates of humans.

η = birth and death rates of mosquitoes.

β= rate of transmission from infected mosquito to susceptible human.

α= rate of transmission from infected human to susceptible mosquito.

γ = recovery rate of humans.

ξ= rate at which infected humans are isolated from mosquitoes.

ε = Rate of awareness programme for reducing vector breeding. ($0 \le \varepsilon \le 1$).

δ = Killing rate of mosquitoes by spraying chemicals.

Let $N_h = S_h + I_h + R_h$ and $N_m = S_m + I_m$ are total population sizes of human and mosquitoes respectively. Then

$$\frac{dN_h}{dt} = \mu(1 - N_h) \qquad (2.3a)$$

$$\frac{dN_m}{dt} = (1-\varepsilon)\eta - (\eta + \delta) N_m \le \eta\,(1 - N_m) \qquad (2.3b)$$

This implies that $Lim_{t\to\infty} N_h(t) = 1$ and $Lim_{n\to\infty} N_m(t) \le 1$

It can be easily seen that the feasible region $\Omega = \{(S_h, I_h, R_h, S_m, I_m) : S_h, I_h, R_h, S_m, I_m \ge 0; S_h + I_h + R_h = 1 \,\&\, S_m + I_m \le 1$ is positively invariant for the model(2.1).

Thus, we restrict our attention to the dynamics of the model in Ω .Using $R_h = 1 - S_h - I_h$ in Ω equation(2.1c) can be removed from the model. Therefore we have to study only following four equations.

$$\frac{dS_h}{dt} = \mu - \mu S_h - \beta S_h I_m \qquad (2.4a)$$

$$\frac{dI_h}{dt} = \beta S_h I_m - (\gamma + \mu + \xi) I_h \qquad (2.4b)$$

$$\frac{dS_m}{dt} = (1-\varepsilon)\eta - \alpha S_m I_h - (\eta + \delta) S_m \qquad (2.4c)$$

$$\frac{dI_m}{dt} = \alpha S_m I_h - (\eta + \delta) I_m \qquad (2.4d)$$

3. Disease Free Equilibrium Point and Reproductive Number

The model (2.4) has exactly one equilibrium point $E_1(1, 0, \frac{(1-\varepsilon)\eta}{\eta+\delta}, 0)$ in the region Ω , with no disease in the population.

We use the next generation matrix approach as described by Diekmann et. al[8] and Hefferman et. al[11] to define the reproductive number R_c which we call *control reproductive number*, as the number of secondary infections that one infectious individual would create over the duration of the infectious period in the presence of control measures, pro-

vided that everyone else is susceptible. For the model (2.4)

$$R_c = \frac{1}{(\eta+\delta)}\sqrt{\frac{\alpha\beta(1-\varepsilon)\eta}{(\gamma+\mu+\xi)}} \tag{3.1}$$

When there is no control measure applied, then $\varepsilon = \delta = \xi = 0$. In this condition the control reproductive number become *basic reproductive number* R_0 and

$$R_0 = \sqrt{\frac{\alpha\beta}{\eta(\gamma+\mu)}} \tag{3.2}$$

it is clear that $R_c \le R_0$.

4. Stability of the Disease Free Equilibrium Point

Applying the transformations,

$S_h = 1 + x_1, I_h = x_2, S_m = \frac{(1-\varepsilon)\eta}{\eta+\delta} + x_3$ & $I_m = x_4$ in (2.4) we have,

$$\frac{dx_1}{dt} = -\mu x_1 - \beta x_4 - \beta x_1 x_4 \tag{4.1a}$$

$$\frac{dx_2}{dt} = -(\gamma+\mu+\xi)x_2 + \beta x_4 + \beta x_1 x_4 \tag{4.1b}$$

$$\frac{dx_3}{dt} = -\frac{\alpha(1-\varepsilon)\eta}{\eta+\delta}x_2 - (\eta+\delta)x_3 - \alpha x_3 x_2 \tag{4.1c}$$

$$\frac{dx_4}{dt} = \frac{\alpha(1-\varepsilon)\eta}{\eta+\delta}x_2 - (\eta+\delta)x_4 + \alpha x_3 x_2 \tag{4.1d}$$

4.1. Local Stability Analysis of Disease Free Equilibrium Point

The linearized system of (2.4) around E_0 is

$$\frac{dx_1}{dt} = -\mu x_1 - \beta x_4 \tag{4.2a}$$

$$\frac{dx_2}{dt} = -(\gamma+\mu+\xi)x_2 + \beta x_4 \tag{4.2b}$$

$$\frac{dx_3}{dt} = -\frac{\alpha(1-\varepsilon)\eta}{\eta+\delta}x_2 - (\eta+\delta)x_3 \tag{4.2c}$$

$$\frac{dx_4}{dt} = \frac{\alpha(1-\varepsilon)\eta}{\eta+\delta}x_2 - (\eta+\delta)x_4 \tag{4.2d}$$

The Jacobian of linearized system around E_0 is

$$J_0 = \begin{bmatrix} -\mu & 0 & 0 & -\beta \\ 0 & -(\gamma+\mu+\xi) & 0 & \beta \\ 0 & -\frac{\alpha(1-\varepsilon)\eta}{\eta+\delta} & -(\eta+\delta) & 0 \\ 0 & \frac{\alpha(1-\varepsilon)\eta}{\eta+\delta} & 0 & -(\eta+\delta) \end{bmatrix}$$

Two characteristic roots of J_0 are $-\mu$ and $-(\eta+\delta)$ which are negative and remaining two characteristic roots are obtained by solving quadratic equation.

$$\lambda^2 + \frac{(\mu+\xi+\gamma+\delta+\eta)}{(\mu+\xi+\gamma)(\eta+\delta)}\lambda + (1-R_c^2) = 0 \tag{4.3}$$

Roots of the equation (4.3) are either negative or have negative real parts only when $R_c < 1$ and has exactly one positive root if $R_c > 1$.

Thus we have established the following result.

Theorem 4.1. The disease free equilibrium point E_0 is locally asymptotically stable if $R_c < 1$ and unstable if $R_c > 1$.

4.2. Non Linear Stability Analysis of Disease Free Equilibrium Point

Consider a positive definite function $V = x_1^2 + x_2^2 + x_3^2 + x_4^2$ (4.4)

then using the non-linear system (4.1) in $\frac{dV}{dt}$, we get

$$\frac{dV}{dt} = -[2\mu x_1^2 + 2\beta(1+x_1)x_1x_4 + 2(\gamma+\mu+\xi)x_2^2 - 2\beta(1+x_1)x_2x_4 + 2(\eta+\delta)x_3^2 + 2\alpha\left\{\frac{(1-\varepsilon)\eta}{\eta+\delta} + x_3\right\}x_2x_3 + 2(\eta+\delta)x_4^2 - 2\alpha\left\{\frac{(1-\varepsilon)\eta}{\eta+\delta} + x_3\right\}x_2x_4]$$

Now using the inequality $\pm 2ab \le (a^2+b^2)$ on the right hand side of $\frac{dV}{dt}$ we find that

$$\begin{aligned}\frac{dV}{dt} \le -[&2\mu x_1^2 - \beta(1+x_1)(x_1^2+x_4^2) + \\ &2(\gamma+\mu+\xi)x_2^2 - \beta(1+x_1)(x_2^2+x_4^2) + \\ &2(\eta+\delta)x_3^2 - \alpha\left\{\frac{(1-\varepsilon)\eta}{\eta+\delta} + x_3\right\}(x_2^2+x_3^2) + \\ &2(\eta+\delta)x_4^2 - \alpha\left\{\frac{(1-\varepsilon)\eta}{\eta+\delta} + x_3\right\}(x_2^2+x_4^2)]\end{aligned} \tag{4.5}$$

Again using the region Ω on the right hand side of the above inequality (4.1) we get

$$\frac{dV}{dt} \le -[(2\mu-\beta)x_1^2 + \{2(\gamma+\mu+\xi) - \beta - 2\alpha\}x_2^2 + \{2(\eta+\delta) - \alpha\}x_3^2 + \{2(\eta+\delta) - \alpha - 2\beta\}x_4^2]$$

Now, it can be easily seen that $\frac{dV}{dt}$ is negative definite under the following conditions.

$$2\mu - \beta > 0 \tag{4.6a}$$

$$2(\gamma+\mu+\xi) - \beta - 2\alpha > 0 \tag{4.6b}$$

$$2(\eta+\delta) - \alpha - 2\beta > 0 \tag{4.6c}$$

Therefore by Lyapunov's second method of stability we state the following result.

Theorem 4.2. The disease free equilibrium point E_0 is non-linearly asymptotically stable if the following three conditions are being satisfied.

(i) $2\mu - \beta > 0$

(ii) $2(\gamma+\mu+\xi) - \beta - 2\alpha > 0$

(iii) $2(\eta+\delta) - \alpha - 2\beta > 0$

To predict the behaviour of I_h we have solved the linearized system (4.2) around E_0.

From equations (4.2b) and (4.2d) we get

$$\frac{d^2x_2}{dt^2}+(\gamma+\mu+\xi+\eta+\delta)\frac{dx_2}{dt}+\left\{(\gamma+\mu+\xi)(\eta+\delta)-\beta\alpha\frac{(1-\varepsilon)\eta}{(\eta+\delta)}\right\}x_2=0 \quad (4.7)$$

Above equation is a second order ordinary differential equation with constant coefficient.

The auxiliary equation of (4.7) is

$$m^2+Pm+Q=0 \quad (4.8)$$

Where $P=(\gamma+\mu+\xi+\eta+\delta)\ \&\ Q=(\gamma+\mu+\xi)(\eta+\delta)-\beta\alpha\frac{(1-\varepsilon)\eta}{(\eta+\delta)}$

If two roots of (4.7) are m_1 and m_2 then, $m_1=-P-\sqrt{P^2-4Q}$ and $m_2=-P+\sqrt{P^2-4Q}$

Since $P^2-4Q=\{(\gamma+\mu+\xi)-(\eta+\delta)\}^2+4\alpha\beta\frac{(1-\varepsilon)\eta}{(\eta+\delta)}>0$

Therefore both the roots of (4.8) are real. Root m_1 is always negative and m_2 is negative if $Q>0$ and positive if $Q<0$.

Again $Q>0$

$$\Leftrightarrow\quad (\gamma+\mu+\xi)(\eta+\delta)-\beta\alpha\frac{(1-\varepsilon)\eta}{(\eta+\delta)}>0$$

$$\Leftrightarrow 1-R_c^2>0$$

$$\Leftrightarrow R_c<1$$

Thus m_2 is negative if $R_c<1$ and is positive if $R_c>1$.

Now solution of (4.7) is $x_2=C_1e^{m_1t}+C_2e^{m_2t}$ (4.9)

Where, $C_1\ \&\ C_2$ are constants of integration. Now if m_2 is negative then $x_2\to 0$ as $t\to\infty$ and if m_2 is positive then $x_2\to\infty$ as $t\to\infty$, i.e. as $t\to\infty$ $x_2\to 0$ if $R_c<1$ and $x_2\to\infty$ if $R_c>1$.

Thus we can say that for $R_c<1$ infective population decreases with time and after a sufficient large time disease will die out due to the unavailability of infected human.

5. Endemic Equilibrium Point

Endemic equilibrium point is the steady state solution when the disease persists in the population i.e. $I_h\ \&\ I_m$ are positive. In the model (2.4), a unique endemic equilibrium point $E_1(S_h^*, I_h^*, S_m^*, I_m^*)$, where

$$S_h^*=\frac{(\gamma+\mu+\xi)(\eta+\delta)[\mu(\eta+\delta)R_c^2+\beta(1-\varepsilon)\eta]}{\beta(1-\varepsilon)\eta[(\gamma+\mu+\xi)(\eta+\delta)R_c^2+\mu\alpha]}$$

$$I_h^*=\frac{\mu(\eta+\delta)[R_c^2-1]}{(\eta+\delta)(\gamma+\mu+\xi)R_c^2+\alpha\mu}$$

$$S_m^*=\frac{[\alpha\mu+(\eta+\delta)(\gamma+\mu+\xi)R_c^2](1-\varepsilon)\eta}{\alpha[(1-\varepsilon)\eta\beta+(\eta+\delta)\mu R_c^2]}$$

$$I_m^*=\frac{\mu[R_c^2-1](1-\varepsilon)\eta}{\mu(\eta+\delta)R_c^2+\beta(1-\varepsilon)\eta}$$

exists only when $R_c>1$.

6. Stability of Endemic Equilibrium Point

Applying the transformations, $S_h=S_h^*+y_1,\quad I_h=I_h^*+y_2,\quad S_m=S_m^*+y_3\ \&\ I_m=I_m^*+y_4$ in (2.4)

We have,

$$\frac{dy_1}{dt}=-(\beta I_m^*+\mu)y_1-\beta S_h^*y_4-\beta y_1y_4 \quad (6.1a)$$

$$\frac{dy_2}{dt}=\beta I_m^*y_1-(\gamma+\mu+\xi)y_2+\beta S_h^*y_4+\beta y_1y_4 \quad (6.1b)$$

$$\frac{dy_3}{dt}=-\alpha S_m^*y_2-(\alpha I_h^*+\eta+\delta)y_3-\alpha y_3y_2 \quad (6.1c)$$

$$\frac{dy_4}{dt}=\alpha S_m^*y_2+\alpha I_h^*y_3-(\eta+\delta)y_4+\alpha y_3y_2 \quad (6.1d)$$

6.1. Local Stability Analysis of Endemic Equilibrium Point

The linearized system of (2.4) around E_1 is

$$\frac{dy_1}{dt}=-(\beta I_m^*+\mu)y_1-\beta S_h^*y_4 \quad (6.2a)$$

$$\frac{dy_2}{dt}=\beta I_m^*y_1-(\gamma+\mu+\xi)y_2+\beta S_h^*y_4 \quad (6.2b)$$

$$\frac{dy_3}{dt}=-\alpha S_m^*y_2-(\alpha I_h^*+\eta+\delta)y_3 \quad (6.2c)$$

$$\frac{dy_4}{dt}=\alpha S_m^*y_2+\alpha I_h^*y_3-(\eta+\delta)y_4 \quad (6.2d)$$

The Jacobian of linearized system around E_1 is

$$J_1=\begin{bmatrix}-(\beta I_m^*+\mu) & 0 & 0 & -\beta S_h^*\\ \beta I_m^* & -(\gamma+\mu+\xi) & 0 & \beta S_h^*\\ 0 & -\alpha S_m^* & -(\alpha I_h^*+\eta+\delta) & 0\\ 0 & \alpha S_m^* & \alpha I_h^* & -(\eta+\delta)\end{bmatrix}$$

One characteristic root of J_1 is $-(\eta+\delta)$ which is negative and remaining three characteristic roots are obtained by solving following cubic equation.

$$\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0 \quad (6.3)$$

Where,

$$a_1=\beta I_m^*+\alpha I_h^*+2\mu+\eta+\delta+\gamma+\xi$$

$$a_2=(\beta I_m^*+\mu)(\alpha I_h^*+\eta+\delta)+(\gamma+\mu+\xi)(\beta I_m^*+\alpha I_h^*+\mu)$$

$$a_3=(\gamma+\mu+\xi)[\alpha\mu I_h^*+(\alpha I_h^*+\eta+\delta)\beta I_m^*] \text{ and}$$

$$a_1a_2-a_3=a_1\{\mu(\eta+\delta)+(\gamma+\mu+\xi)(\beta I_m^*+\alpha I_h^*+\mu)\}$$
$$+(\beta I_m^*+\alpha I_h^*+\mu+\eta+\delta)\{(\alpha I_h^*+\eta+\delta)\beta I_m^*\mu\alpha I_h^*\}$$

Since $I_h^*,\ I_m^*$ exists and are positive only when $R_c>1$, therefore

$a_1>0, a_2>0, a_3>0\ \&\ a_1a_2-a_3>0$ when $R_c>1$.

Thus by Hurwitz criterion all the roots of (6.3) are either negative or have negative real parts if $R_c>1$.

Hence we have established the following result.

Theorem 6.1. The endemic equilibrium point E_1 exists and is locally asymptotically stable if $R_c>1$ and it does not exist if $R_c<1$.

6.2. Non Linear Stability Analysis of Endemic Equilibrium Point

Consider a positive definite function

$$U = y_1^2 + y_2^2 + y_3^2 + y_4^2 \tag{6.4}$$

Then using the non-linear system (6.1) in $\frac{dU}{dt}$, we get

$$\begin{aligned}\frac{dU}{dt} = &-[2(\mu+\beta I_m^*)y_1^2 + 2\beta(S_h^*+y_1)y_1y_4 - \\ &2\beta I_m^* y_1y_2 + 2(\gamma+\mu+\xi)y_2^2 - 2\beta(S_h^*+y_1)y_2y_4 \\ &+2(\alpha I_h^*+\eta+\delta)y_3^2 + 2\alpha(S_m^*+y_3)y_2y_3 \\ &-2\alpha I_h^* y_3y_4 + 2(\eta+\delta)y_4^2 - 2\alpha(S_m^*+y_3)y_2y_4]\end{aligned}$$

Now using the inequality, $\pm 2ab \le (a^2+b^2)$ on the right hand side of $\frac{dU}{dt}$, we find that

$$\begin{aligned}\frac{dU}{dt} \le &-[2(\mu+\beta I_m^*)y_1^2 - \beta S_h(y_1^2+y_4^2) - \\ &\beta I_m^*(y_1^2+y_2^2) + 2(\gamma+\mu+\xi)y_2^2 - \beta S_h(y_2^2+y_4^2) \\ &+2(\alpha I_h^*+\eta+\delta)y_3^2 - \alpha S_m(y_2^2+y_3^2) - \\ &\alpha I_h^*(y_3^2+y_4^2) + 2(\eta+\delta)y_4^2 - \alpha S_m(y_2^2+y_4^2)]\end{aligned} \tag{6.5}$$

Again using the region Ω on the right hand side of the above inequality (6.5), we get

$$\begin{aligned}\frac{dU}{dt} \le &-[(\beta I_m^* + 2\mu - \beta)y_1^2 + \{2(\gamma+\mu+\xi) - \beta I_m^* - \beta - 2\alpha\}y_2^2 \\ &+\{2(\eta+\delta) + \alpha I_h^* - \alpha\}y_3^2 + \{2(\eta+\delta) - \alpha I_h^* - \alpha - 2\beta\}y_4^2]\end{aligned}$$

Thus,

$\frac{dU}{dt}$ is negative definite if the following conditions hold good.

$$\beta\frac{\mu[R_c^2-1](1-\varepsilon)\eta}{\mu(\eta+\delta)R_c^2 + \beta(1-\varepsilon)\eta} + 2\mu - \beta > 0$$

$$2(\gamma+\mu+\xi) - \beta\frac{\mu[R_c^2-1](1-\varepsilon)\eta}{\mu(\eta+\delta)R_c^2 + \beta(1-\varepsilon)\eta} - \beta - 2\alpha > 0$$

$$2(\eta+\delta) - \alpha\frac{\mu(\eta+\delta)[R_c^2-1]}{(\eta+\delta)(\gamma+\mu+\xi)R_c^2 + \alpha\mu} - \alpha - 2\beta > 0$$

Hence, we have reached the following result.

Theorem 6.2 The endemic equilibrium point E_1 is non-linearly asymptotically stable if following conditions are being satisfied.

(i) $\beta\frac{\mu[R_c^2-1](1-\varepsilon)\eta}{\mu(\eta+\delta)R_c^2 + \beta(1-\varepsilon)\eta} + 2\mu - \beta > 0$

(ii) $2(\gamma+\mu+\xi) - \beta\frac{\mu[R_c^2-1](1-\varepsilon)\eta}{\mu(\eta+\delta)R_c^2 + \beta(1-\varepsilon)\eta} - \beta - 2\alpha > 0$

(iii) $2(\eta+\delta) - \alpha\frac{\mu(\eta+\delta)[R_c^2-1]}{(\eta+\delta)(\gamma+\mu+\xi)R_c^2 + \alpha\mu} - \alpha - 2\beta > 0$

7. Discussion

In this paper, we have analyzed a model to study the impact of various vector control measures on the transmission dynamics of chikungunya disease. We have shown that there exists a feasible region where the model is well posed and for which a unique disease free equilibrium point is obtained. We defined a control reproductive number R_c, and it has been concluded that DFE is linearly asymptotically stable if $R_c < 1$ and unstable if $R_c > 1$ (Theorems 4.1&6.1). We have also shown that an endemic equilibrium point exists which is linearly asymptotically stable if $R_c > 1$ .When no vector control measure is applied then the control reproductive number R_c becomes the basic reproductive number R_0 and it has been shown that $R_c < R_0$ .Thus we have verified that the intervention policy by means of vector control, decreases the reproductive number and level of disease.

Also the conditions for non linear stability of both the equilibrium points have been derived (Theorems 4.2 &6.2).The solution of linearized system (4.2) shows that the disease will die out after some time if $R_c < 1$ and it will remain in the population if $R_c > 1$ .Finally, with the help of simulations the graphs between state variables and time have been plotted (fig.2,3,4&5), and phase plane plots between I_h and I_m have been also shown(fig.6,7,8 & 9) for different values of R_c . It has been shown from the figures that if we are able to reduce the control reproductive number to less than 1 using various control measures independently or simultaneously then disease likely to vanish.

Table 1. Parameter values and initial conditions used in the simulation

$\mu = 1.5X10^{-3}$ / day
$\eta = 5.0X10^{-2}$ / day
$\gamma = 5.0X10^{-4}$ / day
$\alpha = 3.5X10^{-2}$ / day
$\beta = 2.5X10^{-2}$ / day
$S_h(0) = 0.9, I_h(0) = 0.1$
$S_m(0) = 0.9, I_m(0) = 0.1$

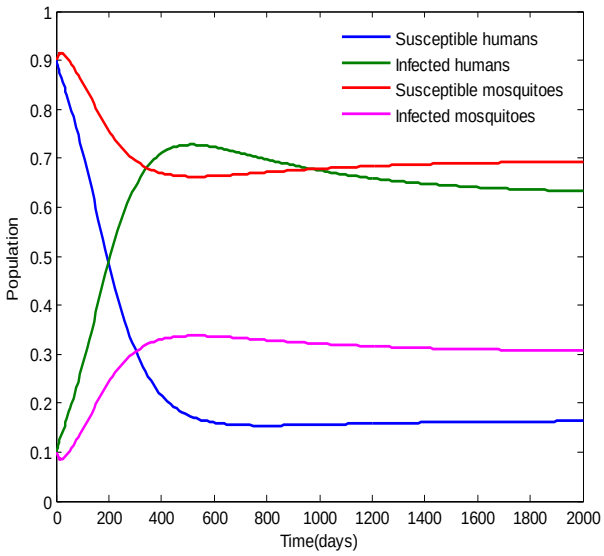

Figure 2. Solution of chikungunya model without control measures and with parameter values given in table 1 which corresponds to $R_c = 2.9580$

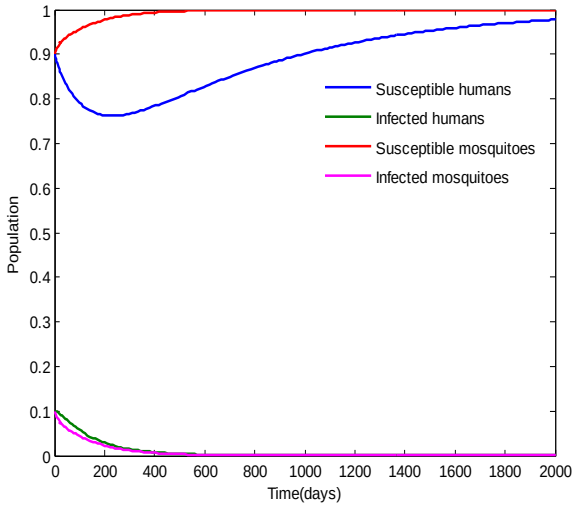

Figure 3. Solution of chikungunya model with parameter values given in table 1 and with $\xi = 0.02, \varepsilon = 0, \delta = 0$ which corresponds to $R_c = 0.8919$

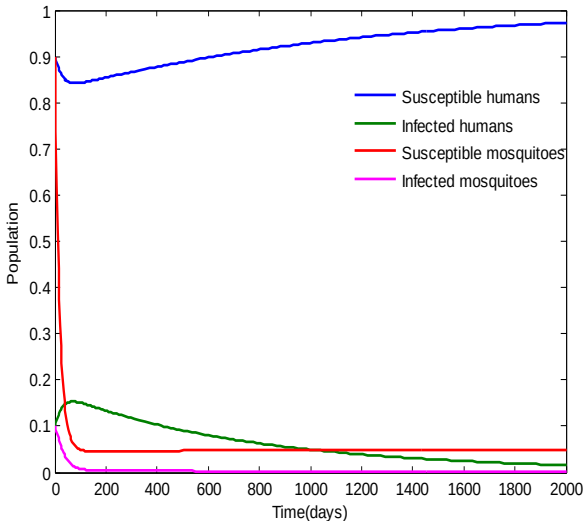

Figure 4. Solution of chikungunya model with parameter values given in table 1 and with $\xi = 0., \varepsilon = 0.95, \delta = 0$ which corresponds to $R_c = 0.6614$

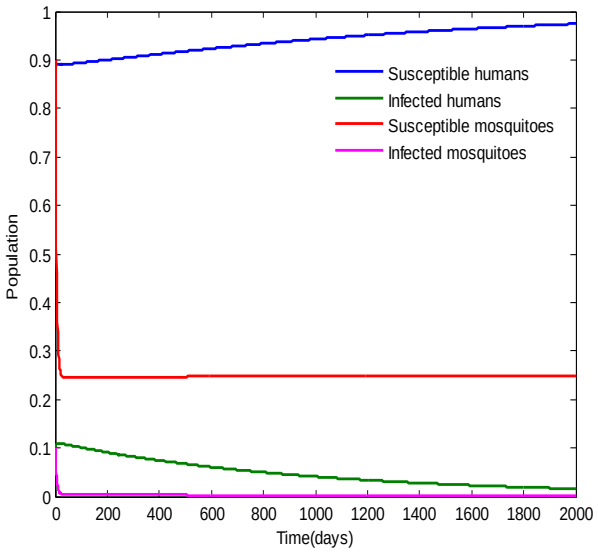

Figure 5. Solution of chikungunya model with parameter values given in table 1 and with $\xi = 0., \varepsilon = 0, \delta = 0.15$ which corresponds to $R_c = 0.7395$

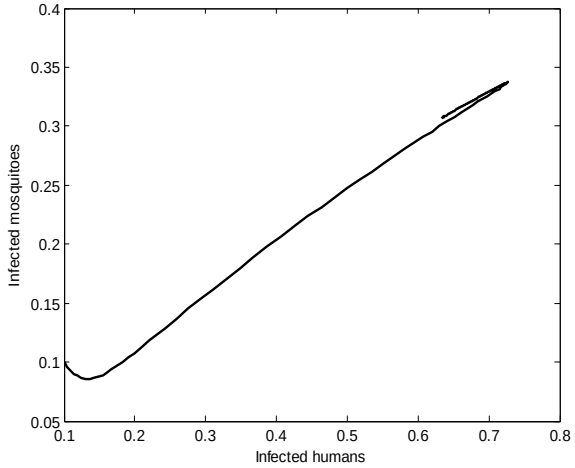

Figure 6. Phase plane plot between Ih and Im of chikungunya model without control measures and with parameter values given in table 1 which corresponds to $R_c = 2.9580$

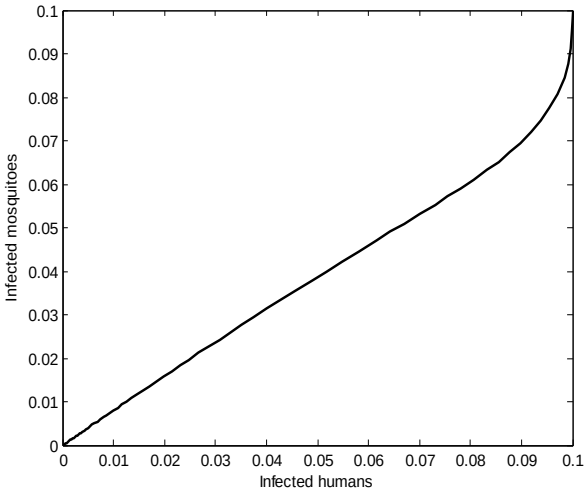

Figure 7. Phase plane plot between Ih and Im of chikungunya model with parameter values given in table 1 and with $\xi = 0.02, \varepsilon = 0, \delta = 0$ which corresponds to $R_c = 0.8919$

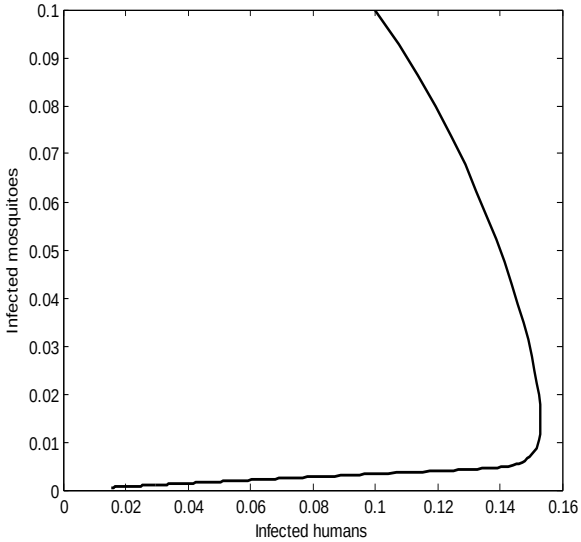

Figure 8. Phase plane plot between I_h and I_m of chikungunya model with parameter values given in table 1 and with $\xi = 0., \varepsilon = 0.95, \delta = 0$ which corresponds to $R_c = 0.6614$

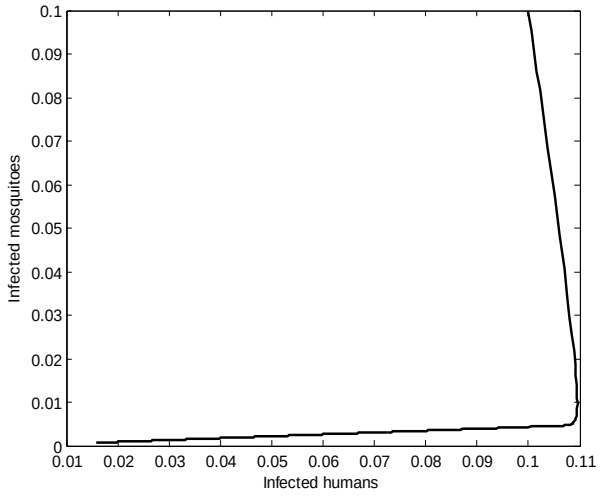

Figure 9. Phase plane plot between Ih and Im of chikungunya model with parameter values given in table 1 and with $\xi = 0., \varepsilon = 0, \delta = 0.15$ which corresponds to Rc = 0.7395

Table 2. Endemic values of state variables for parameter values given in table 1, where $\xi = \varepsilon = \delta = 0$

$S_h^* = 0.16442$
$I_h^* = 0.62668$
$S_m^* = 0.69507$
$I_m^* = 0.30491$

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