

Collage Quality of Two Different Plates by Using Ultrasonic

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Abstract The collage quality of the plates is very important and decisive in the industry and more particularly in that of aeronautics. As much as the safety of the structures with respect to the constraints to which they are subjected is decisive in the industry. The control of the mechanical state of the bonded structures in service or not is essential. For this purpose, the control of the reflection coefficient of an ultrasonic beam on a bonded structure makes it possible to study theoretically and then experimentally the mechanical behavior. The PILARSKI model is applied in this work and will clarify the quality of the collage of two plates of different natures. Indeed, the amplitude variation of the reflection coefficient depends on the quality of the collage.

Keywords Ultrasonic, Non-destructive testing, Reflection coefficient, Collage

1. Introduction

When two solid media are coupled together with a layer of glue, the mechanical behavior of the structure can be studied from the tracking of the reflection coefficient or transmission of an ultrasonic beam. Indeed, the amplitude variations of these coefficients as a function of the frequency depend strongly on the nature of the coupling. The choice of boundary conditions that must describe the effects of coupling is very important in the theoretical and experimental approach of this problem.

Constraints at the collage joint have a direct influence on the behavior of the layers [1] [2]. As a result, an estimate of the allowable stress levels by the in-service structure is necessary to reduce the probability of failure.

The evaluation of residual stresses is the subject of an increasing demand of the metal trades, thus favoring the development of various methods such as X-ray diffraction, incremental drilling and more recently the ultrasonic method and noise Barkhausen [20]. Numerous studies have clearly shown that there is no universal or absolute method that gives complete satisfaction in the field of control of mechanical components in service [18]. Each method has its area of use and validity. The choice of the method used depends on the material, the geometry of the piece, the surface condition, and the nature of the glue as well as the accuracy of the desired result.

In this study, we will focus on the ultrasonic measurement method. The latter, despite its sensitivity to the effects of microstructure and operating conditions, remains potentially one of the most promising [3]. It is non-destructive and allows to control the quality of a collage of a Plexiglas plate and a glass plate. This method is based on the behavior of the resonance frequencies of the structure of the two plates joined by a layer of glue. The cost of its equipment is the main advantage that explains and encourages its development. We will calculate theoretically the frequency variations of the reflection coefficients in the case of a collage of two plates using the conditions of PILARSKI [13]. This author supposed that one could pass continuously a perfect contact (continuity of displacements and normal and tangential stresses) to a sliding contact (continuity of the normal and tangential stress, continuity of the normal displacement and discontinuity of the tangential displacement) by postulating that the tangential stress is proportional to the difference of displacements on both sides of the interface. We will also present the experimental results based on the measurement of the reflection coefficients of the Plexiglas / Glue / Glass structure for different collage qualities.

2. Presentation of the Assembly of the Experimental Technique: Monostatic Method

In this arrangement, measurements are made by an ultrasonic transducer of the 5 MHz center frequency. The transducer used operates alternately as transmitter and

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receiver. The pulse generator electrically excites the transducer. The latter excites, under normal incidence, the Plexiglas / Glue / Glass structure. The backscattered signal is picked up by the same transducer. The signal passes through the same cable as the transmission signal and arrives at the T / R connector from which the transmission signal was sent. The received signal is visualized by connecting the output signal connector to the input of a picoscope (350 MHz) via a coaxial cable. The latter is synchronized by the generator thanks to a coaxial cable connecting between its connector (EXTERNAL TRIGGER) and the entrance (AUX IN Connector) picoscope. The ultrasonic signal is finally visualized on a laptop connected to the picoscope by a USB cable. Figure 1 below shows the diagram of the experimental setup used in this work to measure the reflection coefficient of the Plexiglas / Glue / Glass structure.

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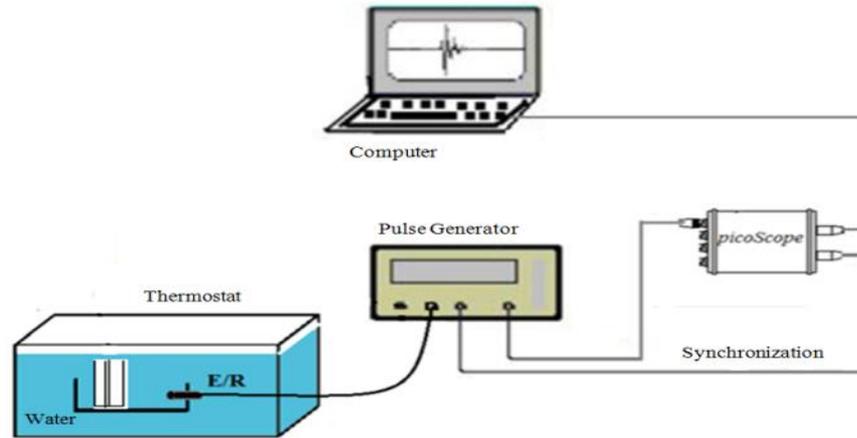


Figure 1. Diagram of the assembly of the monostatic method

Table 1. Elastic Parameters and Thicknesses of Plexiglas, Glass and Glue Layers

Medium	Longitudinal velocity C_L (m/s)	Transverse velocity C_T (m/s)	Density μ (kg/m ³)	Thickness d (mm)	Attenuation α (Np/m)
Plexiglas	2746	1100	1320	3	60
Paste/glue	1298	950	875	2,4	60
Glass	5867	3050	2530	3,6	10

3.2. Theoretical Study

In this work, we will resume the work of BREKHOVSKIKH [7] and other authors [8] [9] on the diffusion of a planar stratified medium composed of n solid layers. There are two computational methods. The first one consists in writing the continuity equations in terms of potentials at each interface of the structure, and then solving linear systems of p equations with unknown p [10]. The number p increases with the number of layers. This method is expensive in terms of calculation (for n layers, $4n+2$ equations are needed). The second method, presented in this paper, is based on recurrence formulas linking the wave

3. Materials and Methodology

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3.1. Sample Preparation

The preparation of the samples of the Plexiglas / Glue / Glass structures to be characterized is very important and this step is decisive. To create the sample well, the same thicknesses of the adhesive must be kept between Plexiglas plates and glass. The thicknesses of the Plexiglas, the adhesive layer and the glass of the Plexiglas / Glue / Glass structure are respectively [3 mm / 2.4 mm / 3.6 mm].

The elastic parameters of Plexiglas, glass and glue are grouped in the table below [6].

amplitudes in two neighboring layers "Fig. 2". It is assumed that a harmonic plane wave arrives in the milieu $n+1$ on the multilayer structure at an angle of incidence θ_{n+1} . The wave vector of the incident wave is assumed to be contained in the xOz plane, which makes the problem two-dimensional. Each layer i of the multilayer medium is defined by its density ρ_i , its LAME coefficients λ_i and μ_i longitudinal velocities c_{Li} and transversal c_{Ti} and its thickness d_i . The layers are assumed to be infinitely extended in the xOy plane. The origin of the direct orthonormal coordinates system (O, x, y, z) is chosen on the interface between the n^{th} layer and the $(n-1)^{\text{th}}$ layer.

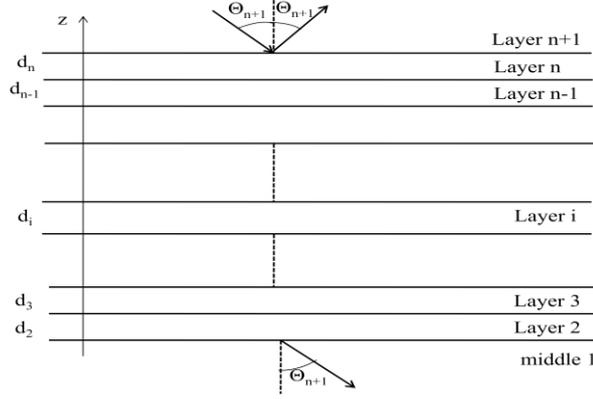


Figure 2. Reflection and transmission through a plane laminated medium composed of $(n - 1)$ solid layers. Geometry of the problem

The conditions of continuity at the interfaces between two neighboring layers imply the continuity of the tangential and normal components of displacements and the stresses. It is conventional to consider the continuity of the components of speed rather than that of displacements. For a harmonic plane wave of pulsation ω whose term of temporal dependence is taken from the form $\exp(-j\omega t)$. Velocity and movement are linked simply by the relation $V = -j\omega u$.

The velocities and stresses in each layer i can be expressed in terms of the following potentials:

$$\phi_i = [A_i \exp(j\alpha_i z) + B_i \exp(-j\alpha_i z)] \exp j(\sigma x - \omega t) \quad (1)$$

$$\psi_i = [C_i \exp(j\beta_i z) + D_i \exp(-j\beta_i z)] \exp j(\sigma x - \omega t) \quad (2)$$

The potential ϕ_i describes the longitudinal waves in layer i and the potential ψ_i transverse waves.

$$\sigma = k_{Li} \sin \theta_i = k_{Ti} \sin \gamma_i$$

is the tangential component of the wave vectors of the waves propagating in each layer, according to the law of SNELL-DESCARTES.

$$\alpha_i = k_{Li} \cos \theta_i = \sqrt{k_{Li}^2 - \sigma^2}$$

is the normal component of the wave vector of the longitudinal wave in the i^{th} layer.

$$\beta_i = k_{Ti} \cos \gamma_i = \sqrt{k_{Ti}^2 - \sigma^2}$$

is the normal component of the wave vector of the transverse wave in the i^{th} layer.

θ_i , γ_i are respectively the angles that the directions of propagation of the longitudinal and transverse waves make with respect to the normal to the layer i .

The tangential and normal components of velocity and stress in layer i are written as follows [11]:

$$v_x^{(i)} = \frac{\partial \phi_i}{\partial x} - \frac{\partial \psi_i}{\partial z} \quad v_z^{(i)} = \frac{\partial \phi_i}{\partial z} + \frac{\partial \psi_i}{\partial x} \quad (3)$$

$$T_{xz}^{(i)} = j\mu \left(\frac{\partial v_x^{(i)}}{\partial z} + \frac{\partial v_z^{(i)}}{\partial x} \right) / \omega \quad (4)$$

$$T_{zz}^{(i)} = j \left(\lambda \left(\frac{\partial v_x^{(i)}}{\partial x} + \frac{\partial v_z^{(i)}}{\partial z} \right) + 2\mu \frac{\partial v_z^{(i)}}{\partial z} \right) / \omega \quad (5)$$

Its components are represented as a vector:

$$[Y^{(i)}] = [v_x^{(i)}, v_z^{(i)}, T_{zz}^{(i)}, T_{xz}^{(i)}]^t \quad (6)$$

These components will be expressed as a function of the amplitudes of the potentials at the interface. Z_i which represents the upper interface of the i^{th} layer. It is posed like this:

$$[X^{(i)}] = [A_i + B_i, A_i - B_i, C_i - D_i, C_i + D_i]^t \quad (7)$$

This leads to:

$$[Y^{(i)}] = I_i [X^{(i)}] \quad (8)$$

where

$$I_i = \begin{bmatrix} j\sigma \cos P_i & -\sigma \sin P_i & -j\beta_i \cos Q_i & \beta_i \sin Q_i \\ -\alpha_i \sin P_i & j\alpha_i \cos P_i & -\sigma \sin Q_i & j\sigma \cos Q_i \\ -jg_i \cos P_i & e_i \sin P_i & -jg_i \beta_i \cos Q_i & g_i \beta_i \sin Q_i \\ g_i \alpha_i \sin P_i & -jg_i \alpha_i \cos P_i & -e_i \sin Q_i & jg_i \cos Q_i \end{bmatrix} \quad (9)$$

$$P_i = \alpha_i z_i \quad Q_i = \beta_i z_i$$

$$e_i = (\lambda_i k_{Li}^2 + 2\mu_i \alpha_i^2) / \omega = \mu_i (\beta_i^2 - \sigma^2) / \omega$$

$$g_i = 2\mu_i \sigma / \omega$$

Similarly, at the $Z = Z_{(i-1)}$, the components of velocity and stress in the $(i-1)^{\text{th}}$ layer can be written as a function of the amplitudes of the potentials of layer i , due to the conditions of continuity, as follows:

$$[Y^{(i)}]_{z=Z_{i-1}} = [Y^{(i-1)}] = J_i [X^{(i)}] \quad (10)$$

With

$$J_i = \begin{bmatrix} j\sigma \cos P_{i-1} & -\sigma \sin P_{i-1} & -j\beta_i \cos Q_{i-1} & \beta_i \sin Q_{i-1} \\ -\alpha_i \sin P_{i-1} & j\alpha_i \cos P_{i-1} & -\sigma \sin Q_{i-1} & j\sigma \cos Q_{i-1} \\ -jg_i \cos P_{i-1} & e_i \sin P_{i-1} & -jg_i \beta_i \cos Q_{i-1} & g_i \beta_i \sin Q_{i-1} \\ g_i \alpha_i \sin P_{i-1} & -jg_i \alpha_i \cos P_{i-1} & -e_i \sin Q_{i-1} & jg_i \cos Q_{i-1} \end{bmatrix} \quad (11)$$

$$P_{i-1} = \alpha_i z_{(i-1)} \quad Q_{i-1} = \beta_i z_{(i-1)}$$

So the velocities and stresses in layers i and $i - 1$ are linked by the matrix relation:

$$[Y^{(i)}] = I_i J_i^{-1} [Y^{(i-1)}] = a^{(i)} [Y^{(i-1)}] \quad (12)$$

The elements of the matrix $a^{(i)}$ are expressed as a function of the acoustic impedances of the longitudinal and transverse waves. Z_{Li} and Z_{Ti} in the i -layer and angles θ_i and γ_i giving the direction of propagation of these waves with respect to the normal to the i^{th} layer.

Finally, the speeds and constraints of the n^{th} layers are expressed as a function of those in medium 1 as follows:

$$\begin{bmatrix} v_x^{(n)} \\ v_z^{(n)} \\ T_{zz}^{(n)} \\ T_{xz}^{(n)} \end{bmatrix} = a^{(n)} \cdot a^{(n-1)} \dots a^{(2)} \begin{bmatrix} v_x^{(1)} \\ v_z^{(1)} \\ T_{zz}^{(1)} \\ T_{xz}^{(1)} \end{bmatrix} = A \begin{bmatrix} v_x^{(1)} \\ v_z^{(1)} \\ T_{zz}^{(1)} \\ T_{xz}^{(1)} \end{bmatrix} \quad (13)$$

In experimental applications, the media (n + 1) and 1 are water; thus the tangential components of stresses in media n and 1 are null:

$$T_{xz}^{(n)} = T_{xz}^{(1)} = 0 \quad (14)$$

We deduce from the relation

$$T_{xz}^{(n)} = A_{41}v_x^{(1)} + A_{42}v_z^{(1)} + A_{43}T_{zz}^{(1)} + A_{44}T_{xz}^{(1)} \quad (15)$$

that

$$A_{41}v_x^{(1)} + A_{42}v_z^{(1)} + A_{43}T_{zz}^{(1)} = 0 \quad (16)$$

The tangential component of velocity in medium 1 is then expressed simply as a function of the normal components of velocity and stress of this medium. We can therefore write the following relationships linking the normal components of velocity and stress in media *n* and *1*:

$$\begin{cases} v_z^{(n)} = M_{22}v_z^{(1)} + M_{23}T_{zz}^{(1)} \\ T_{zz}^{(n)} = M_{32}v_z^{(1)} + M_{33}T_{zz}^{(1)} \end{cases} \quad (17)$$

where

$$M_{ij} = A_{ij} - \frac{A_{4j}A_{4i}}{A_{41}}, \quad i, j = 2, 3 \quad (18)$$

Waves propagating in media (*n+1*) and *1* are described by the following potentials [5] [11]:

$$\varphi_{n+1}(z) = [A_{n+1} \exp j\alpha_{n+1}(z - d_n) + B_{n+1} \exp -j\alpha_{n+1}(z - d_n)] \exp j(\sigma x - \omega t)$$

$$\phi_1(z) = B_1 \exp -j\alpha_1(z + h) \exp j(\sigma x - \omega t)$$

where $h = \sum_i d_i$

The reflection coefficients of the solid multilayer are such that:

$$R = \frac{A_{n+1}}{B_{n+1}} \quad (19)$$

The normal components of velocity and stress are written as a function of the amplitudes of the potentials in the media (*n+1*) and *1*:

at the interface $Z=d_n$ between the media (n + 1) and n:

$$v_z^{(n)} = j\alpha_{n+1}(A_{n+1} - B_{n+1}) \quad (20)$$

$$T_{zz}^{(n)} = -j\rho_{n+1}\omega(A_{n+1} + B_{n+1}) \quad (21)$$

At the interface $Z = -h$ between the median 2 and 1

$$v_z^{(1)} = -j\alpha_1 B_1 \quad (22)$$

$$T_{zz}^{(1)} = -j\rho_1\omega B_1 \quad (23)$$

The substitution of the expressions (20), (21) and (22), (23) in relation (16) leads to:

$$R = \frac{(M_{32} + Z_1 M_{33}) - Z_{n+1}(M_{22} + Z_1 M_{23})}{(M_{32} + Z_1 M_{33}) + Z_{n+1}(M_{22} + Z_1 M_{23})} \quad (24)$$

$$\text{With } Z_{n+1} = Z_1 = \frac{\rho_1 c_1}{\cos \theta_1} = \frac{\rho_1 \omega}{\alpha_1}$$

3.3. Experimental Study

In this work, the Plexiglas/Glue/Glass structure, consisting of a Plexiglas plate, a glass plate and a layer of glue is emerged in water and insulated by an ultrasonic pulse.

The method implemented consists in measuring the reflex coefficient of the Plexiglas/Glue/Glass structure. The geometry of the problem is schematized in figure 3.

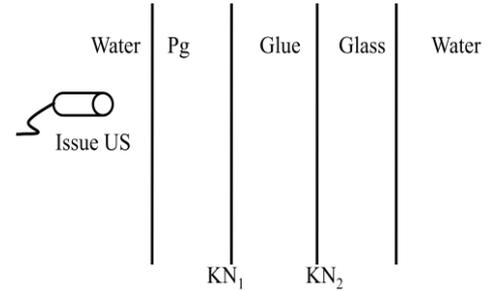


Figure 3. Geometry of the problem

The experiments are carried out by the monostatic technique (case of normal incidence). The Pg/Glass structure is inserted normally to its plane by a transducer with a centre frequency of 5 MHz. The signal backscattered by the Pg/Glue/Glass structure shown in the diagram in Fig. 4 corresponds to the case of good collage.

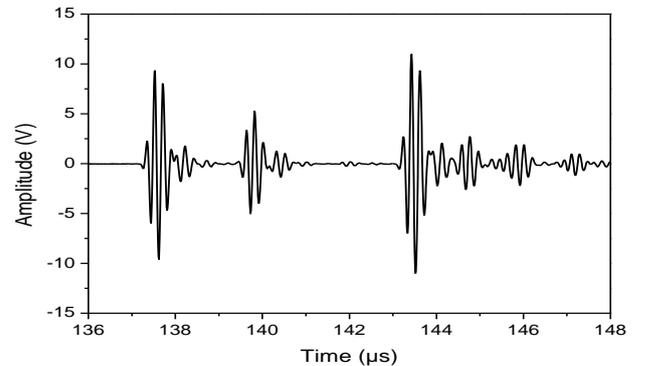


Figure 4. Signal backscattered by the Plexiglas/Glue/Glass structure Emission on the Plexiglas side (Good collage)

In figure 4 we note in particular echo A_1 which corresponds to the reflection at the water/plexiglas interface, this echo is called the specular echo. This echo is followed just afterwards by the A_2 and A_3 echoes which correspond to the round trips in the Plexiglas. The most interesting echo in glue quality control is the A_4 echo which corresponds to the

reflection at the glue/glass interface. This echo is followed by round-trips in the glass.

The experimental reflection coefficient R is calculated using the following formula [19]:

$$R = \frac{A}{A'} R'$$

where A and A' are the spectral amplitudes of the signals reflected from the Pg/Glass structure and the specular respectively. R' corresponds to the reflection coefficient of the water/plexiglas interface [11] [19]:

$$R' = \frac{Z_{Pg} - Z_{Wat}}{Z_{Pg} + Z_{Wat}}$$

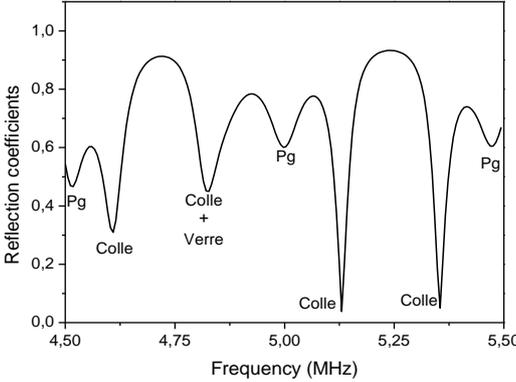
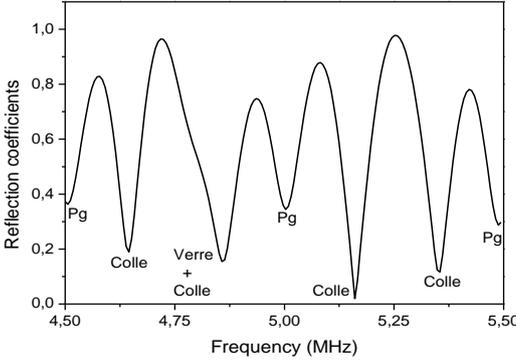
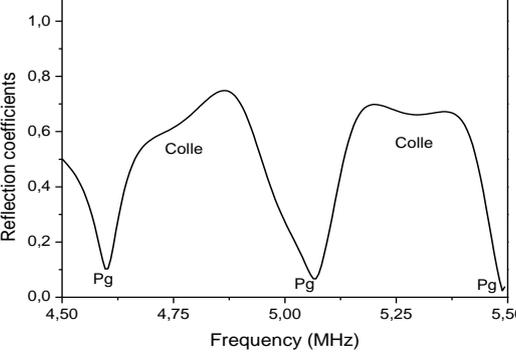
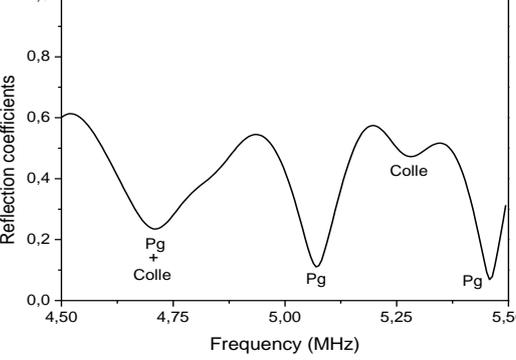
with $Z_{Pg} = \rho_{Pg} C_{LPg}$: Acoustic impedance of Plexiglas.

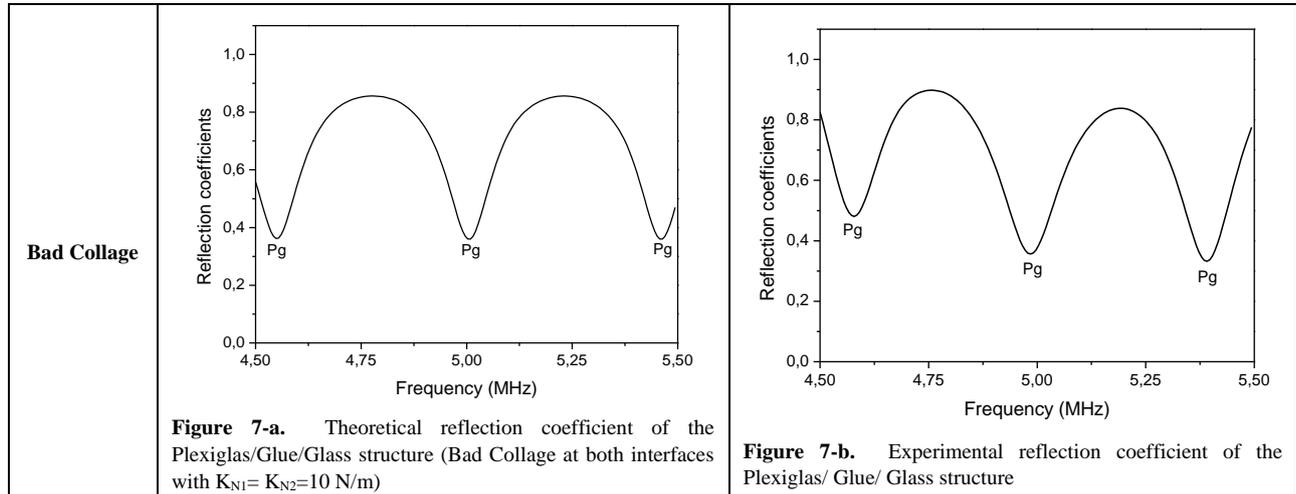
$Z_{Wat} = \rho_{Wat} C_{Wat}$: Acoustic impedance of water.

4. Experimental Results and Discussions

The objective is to compare theoretical results with experimental results. The theoretical and experimental results of reflection coefficients are presented in the 4.5 MHz and 5.5 MHz frequency domain. For the experimental part, a wide band transducer with a center frequency of 5MHz was used. The incident ultrasonic beam is sent from the Plexiglas side and normal to its surface. The transverse velocity has no influence. The quality of the bond is checked by measuring the reflection coefficient.

The reflection coefficient is represented as a function of frequency at normal incidence for the Pg/Glue/Glass structure and the only parameter that changes is the quality of the glue. The minima in the reflection coefficients correspond to the resonance frequencies of the structure. These resonance frequencies are relative to the three materials: Plexiglas, glue and glass. The modules of the theoretical and experimental reflection coefficient by the structure are shown in the following figures. The study makes it possible to isolate and identify the resonance frequencies of the structure.

Type of collage	Theoretical study	Experimental study
<p>Good collage</p>	 <p>Figure 5-a. Theoretical reflection coefficient of the Plexiglas /Glue/Glass structure (Good collage at both interfaces with $K_{N1}= K_{N2}=10^8$ N/m)</p>	 <p>Figure 5-b. Experimental reflection coefficient of the Plexiglas/ Glue/ Glass structure(Good collage)</p>
<p>Intermediate Collage</p>	 <p>Figure 6-a. Theoretical reflection coefficient of the Plexiglas/Glue/Glass structure (intermediate Collage with $K_{N1}= K_{N2}=2.5.10^6$ N/m)</p>	 <p>Figure 6-b. Experimental reflection coefficient of the Plexiglas/ Glue/ Glass structure (intermediate Collage)</p>



Figures 5-a and 5-b represent the reflection coefficient (theoretical and experimental study) as a function of frequency for good collage at the two interfaces (Plexiglas/Glue and Glue/Glass). Each interface is characterized by the PILARSKI coefficient [13] [14] K_{N1} and K_{N2} which are, in the case of good collage, very large. In this frequency domain there are four longitudinal modes relative to the adhesive layer, three longitudinal modes relative to the Plexiglas plate and only one longitudinal mode relative to the glass plate.

The representations of the theoretical (Figure 5-a) and experimental (Figure 5-b) reflection coefficients in the case of the Plexiglas/Glue/Glass structure show that the modes relative to the glue layer and the glass plate are relatively very deep compared to the other qualities of the glue as will be seen for the other qualities of the glue.

In the case of an intermediate collage the theoretical and experimental reflection coefficients are shown in Figure 6-a and Figure 6-b. Each interface is characterized by the PILARSKI coefficients [13] [14] K_{N1} and K_{N2} equal to $2.5 \cdot 10^6$ N/m. The disappearance of the mode relative to the glass plate is noted. However, the existence of the modes relative to the Plexiglas plate and to the adhesive layer is still noticeable. However, the depth of the modes relative to the glue layer for this case is not great compared to that of good gluing [17]. Due to the low ultrasonic amplitude at the Plexiglas/glue interface on the one hand and the decrease of the ultrasonic amplitude at the glue/glass interface on the other hand. The low ultrasonic transmission at both interfaces causes the glass mode to disappear.

In the case of poor collage the theoretical and experimental reflection coefficients are shown in Figure 7-a and Figure 7-b. For poor collage, each interface is characterized by the PILARSKI coefficients [13] [14] K_{N1} and K_{N2} equal to 10 N/m. For this glueing quality, we notice in figures 7-a and 7-b a disappearance of the modes relative to the glue and the glass. This disappearance of the glue and glass modes is due to poor collage at the two interfaces

(Plexiglas/Glue and Glue/Glass) [15]. Poor collage decreases the amplitude of the ultrasonic wave on the other side of the interface. The low amplitudes do not allow the ultrasound to go back and forth either in the adhesive layer or in the glass layer.

Experimentally, the study of the reflection of an incident wave on a Plexiglas/Glued/Glass structure highlights the resonant character of the structure. The results obtained theoretically are relatively in agreement with the experimental results. The differences between the theoretical and experimental curves are due, on the one hand, to the mechanical adjustments of the experimental set-up and, on the other hand, to the PILARSKI theoretical model which assumes that the transfer at the interfaces is carried out without energy loss [16].

5. Conclusions

For quality control of the collage of the structure (Plexiglas/glue/glass), a non-destructive ultrasonic pulse-echo technique has been proposed. This technique is based on the control of the ultrasonic reflection coefficient. The amplitude of the reflection coefficient of the structure immersed in water varies as a function of the frequency and depends on the quality of the collage. The experimental and theoretical results of the amplitude of the reflection coefficient are concordant for the three types of collage. The quality of the collage can therefore be determined easily and quickly by checking the reflection coefficient. The study is based on the control of resonance modes and their amplitudes. The analysis of the experimental and theoretical results shows that it is possible to control the collage quality of certain structures during the industrial production process or after their use. The use of ultrasonic techniques offers a reliable, non-destructive and instantaneous means of control.

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