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Sen's Improved Multi Goal Programming Technique- An Extension

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Abstract Multi Goal Programming (MGP) techniques are used to achieve multiple goals simultaneously. The goals may be conflicting and are achievable only at the cost of other goals. Several goal programming techniques have been developed during past three decades. These are the minimization of sum of deviations between goals and the achievements, weighted goal programming and Preemptive goal programming. Observing the problem of multi-dimensional aggregation, an improved MGP technique has been proposed by Sen, which was found efficient in generating a compromising solution for achieving many goals at a time. In this study a modification in the Sen's improved MGP is proposed to solve MGP problems. The modified MGP technique has been tested with the suitable examples.

Keywords Single Goal Programming, Multi Goal Programming, Improved Multi Goal Programming

1. Introduction

Charnes and Cooper [1] proposed the multi goal programming in the year 1961. MGP has been extensively applied [2], [3], [4] for achieving multiple conflicting goals. Lee [5] has made technical improvements in multi goal programming technique. The application of multi goal programming has been emphasized [6] for improving the decision making process. Hokey and James [7] suggested new ideas for improving research in multi goal programming. Schniederjan [8] pointed out the decline in the theoretical development of multi goal programming. Several variants of MGP have also been developed [9], [10], [11] during the recent past. The purpose of all these MGP techniques to achieve the multiple conflicting goals. It may not be possible to achieve all the goals perfectly by any MGP technique. However, a MGP technique is said to be superior whose achievements of goals are closest to their aspiration levels. The existing MGP techniques were unable to neutralize the effect of multidimensional aggregation in formulation multi goal optimization function. The high deviations in the coefficients of decision variables amongst different goals was another problem in obtaining appropriate solutions of MGP problems. Recently, an improved MGP technique has been introduced by Sen [12] and found efficient in generating satisfactory solutions inspire of above mentioned problems. The Sen's improved MGP technique has been modified in this study. The modified improved MGP technique has been tested with three examples and found efficient in solving MGP problems. The results of modified improved MGP technique has been compared with the existing and Sen's improved multi goal programming techniques.

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2. Methodology

2.1. Existing MGP Model

The existing multi goal programming model can be expressed as:

$$Minimize\ Z = \sum_{i=1}^{m} (d_i^+ + d_i^-) \tag{1}$$

Subject to:
Goal Constraints

$$\sum_{j=1}^{n} a_{ij} X_j - d_i^+ + d_i^- = g_i \tag{2}$$

$$for\ i = 1 \ldots .m$$

System constraints

$$\sum_{j=1}^{n} a_{ij} X_j = b_i \tag{3}$$

$$for\ i = m + 1 \ldots \ldots p$$

There are 'm' Goals, 'p' System constraints and 'n' decision variables

Z= Objective function/ Summation of all deviations

a_{ij}= the coefficient associated with j^{th} variable in i^{th} Goal/constraint

X_j= the j^{th} decision variable

g_i = the right hand side value of i[th] goal
b_i= the right hand side value of i[th] constraint
d_i^-= negative deviational variation from i[th] goal (under achievement)
d_i^+= positive deviational variation from i[th] goal (over achievement)

2.2. Improved MGP Technique

The improved technique is formulated as described below:

$$Minimize\ Z = \sum_{i=1}^{m} (d_i^+ + d_i^-)/g_i \qquad (4)$$

Subject to:
Goal Constraints

$$\sum_{j=1}^{n} a_{ij} X_j - d_i^+ + d_i^- = g_i \qquad (5)$$

$$for\ i = 1 \ldots .m$$

System constraints

$$\sum_{j=1}^{n} a_{ij} X_j = b_i \qquad (6)$$

$$for\ i = m+1 \ldots \ldots p$$

There are 'm' Goals, 'p' System constraints and 'n' decision variables
Z= Objective function/ Summation of all deviations
a_{ij}= the coefficient associated with j[th] variable in i[th] Goal/constraint
X_j= the j[th] decision variable
g_i = the right hand side value of i[th] goal
b_i= the right hand side value of i[th] constraint
d_i^-= negative deviational variation from i[th] goal (under achievement)
d_i^+= positive deviational variation from i[th] goal (over achievement)

2.3. Modified Improved MGP Technique

The improved technique is formulated as described below:

$$Minimize\ Z = \sum_{i=1}^{m} (d_i^+ + d_i^-) \qquad (7)$$

Subject to:
Goal Constraints

$$\sum_{j=1}^{n} a_{ij} X_j / g_i - d_i^+ + d_i^- = 1 \qquad (8)$$

$$for\ i = 1 \ldots .m$$

System constraints

$$\sum_{j=1}^{n} a_{ij} X_j = b_i \qquad (9)$$

$$for\ i = m+1 \ldots \ldots p$$

There are 'm' Goals, 'p' System constraints and 'n' decision variables
Z= Objective function/ Summation of all deviations
a_{ij}= the coefficient associated with j[th] variable in i[th] Goal/constraint
X_j= the j[th] decision variable
g_i = value of i[th] goal
b_i= the right hand side value of i[th] constraint
d_i^-= negative deviational variation from i[th] goal (under achievement)
d_i^+= positive deviational variation from i[th] goal (over achievement)

3. Mathematical Examples

Two examples used in the study by **Sen** [12] and one new example have been solved by the single, existing, improved and modified improved MGP techniques for the comparative analysis.

Example 1

Goal-I: $16500X_1 + 18100X_2 + 15800X_3 + 17400X_4 + 14800X_5 = 73000$
Goal-II: $41X_1 + 35X_2 + 32X_3 + 39X_4 + 31X_5 = 165$
Goal-III: $430X_1 + 470X_2 + 380X_3 + 410X_4 + 440X_5 = 1500$
Goal-IV: $2300X_1 + 2400X_2 + 2100X_3 + 1900X_4 + 1800X_5 = 7000$
Subject to:
$X_1 + X_2 + X_3 + X_4 + X_5 = 4$
$2X_3 \geq 1$
$X_1, X_2, X_3, X_4, X_5 \geq 0$

Example 2

Goal-I: $6X_1 + 5X_2 + 3X_3 + 4X_4 = 55$
Goal-II: $700X_1 + 800X_2 + 900X_3 + 500X_4 = 9000$
Goal-III: $50X_1 + 55X_2 + 40X_3 + 60X_4 = 600$
Subject to:
$X_1 + X_2 + X_3 + X_4 = 11$
$X_1 \geq 1$
$2X_3 \geq 1$
$X_1, X_2, X_3, X_4 \geq 0$

Example 3

Goal-I: $11X_1 + 12X_2 + 13X_3 + 15X_4 + 16X_5 + 14X_6 + 17X_7 + 18X_8 = 50$
Goal-II: $29X_1 + 22X_2 + 24X_3 + 26X_4 + 27X_5 + 23X_6 + 28X_7 + 21X_8 = 80$
Goal-III: $30X_1 + 28X_2 + 31X_3 + 33X_4 + 36X_5 + 34X_6 + 37X_7 + 35X_8 = 100$
Goal-IV: $90X_1 + 91X_2 + 99X_3 + 96X_4 + 97X_5 + 94X_6 + 98X_7 + 95X_8 = 250$
Goal-V: $42X_1 + 41X_2 + 44X_3 + 48X_4 + 45X_5 + 40X_6 + 46X_7 + 43X_8 = 120$
Goal-VI: $69X_1 + 73X_2 + 72X_3 + 74X_4 + 82X_5 + 71X_6 + 75X_7 + 70X_8 = 200$
Goal-VII: $2X_1 + 6X_2 + 5X_3 + 4X_4 + 7X_5 + 9X_6 + 8X_7 + 3X_8 = 25$

Subject to:
$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 = 2.5$
$2X_1 \geq 1$
$X_7 \geq 0.2$
$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8 \geq 0$

4. Solution

All the three examples have been solved using single goal programming, existing, improved and modified improved MGP techniques. The results of example 1 have been presented in table 1. The achievements of the goals in the individual optimizations were all the different. This was due to conflicts amongst the goals. The optimization of first goal achieved its value 71250 which is closer to its goal of 73000. The similar results in the optimization of remaining three goals have been observed. The results of multi goal programming have also been given in the table. The existing MGP has no improvement in the solution. It has reproduced the solution achievement of the second goal. However, the modified improved MGP has generated same results as of improved MGP. The achievement of the first goal was 68800 which is as good as the achievement of the first goal optimization The achievements of remaining three goals were also superior over single MGP solutions.

The solution of example 2 has been arranged in table 2. In all the three single goal programming, only one goal has been achieved fully ignoring remaining two goals. The achievements first. second and third goals were 55, 9000 and 600 respectively with the lesser achievements of remaining goals. The existing MGP has achieved the second goal only and ignored the other two goals. However, all the three goals have been achieved simultaneously by both improved and its modified improved MGP techniques.

Table 1. Goal Achievements in Single and Multi-Goal Programming

Goals		Single Goal Programming				Multi-Goal Programming		
		I	II	III	IV	Existing MGP	Improved MGP	Modified MGP
X_i		0, 3.5, 0.5, 0, 0	3.5, 0, 0.5, 0, 0	0, 0, 4, 0, 0	0, 0, 0.5, 0, 3.5	0, 3.5, 0.5, 0,0	0, 0, 0.5, 3.5, 0	0, 0, 0.5, 3.5, 0
I	73000	71250	65650	63200	59700	71250	68800	68800
II	165	138.5	159.5	128	124.5	138.5	152.5	152.5
III	1500	1835	1695	1520	1730	1835	1625	1625
IV	7000	9450	9100	8400	7350	9450	7700	7700

Table 2. Goal Achievements in Single and Multi-Goal Programming

Goals		Single Goal Programming			Multi-Goal Programming		
		I	II	III	Existing MGP	Improved MGP	Modified MGP
X_i		5.75, 0, 0.5, 4.75	1, 7, 3, 0	5, 0, 0.5, 5.5	1, 7, 3, 0	1, 9.5, 0.5, 0	1, 9.5, 0.5, 0
I	55	55	50	53.5	50	55	55
II	9000	6850	9000	6700	9000	8750	8750
III	600	592.50	555	600	555	592.50	592.50

Table 3. Goal Achievements in Single Goal Programming

Goals		**Single Goal Programming**						
		$\mathbf{Z_1}$	$\mathbf{Z_2}$	$\mathbf{Z_3}$	$\mathbf{Z_4}$	$\mathbf{Z_5}$	$\mathbf{Z_6}$	$\mathbf{Z_7}$
$\mathbf{X_i}$		0.5, 0, 0 0, 0, 0, 0.2, 1.8	2.3, 0, 0, 0, 0, 0, 0.2, 0	0.5, 1.8, 0, 0, 0, 0, 0.2, 0	0.5, 0, 1.8, 0, 0, 0, 0.2, 0	0.5, 0, 0, 1.8, 0, 0, 0.2, 0	0.5, 0, 0, 0, 1.8, 0, 0.2, 0	0.5, 0, 0, 0, 0, 1.8, 0.2, 0
I	50	41.3	28.7	30.5	32.3	35.9	37.7	34.1
II	80	57.9	72.3	59.7	63.3	66.9	68.7	61.5
III	100	85.4	76.4	90.8	78.2	81.8	87.2	83.6
IV	250	235.6	226.6	228.4	242.8	237.4	239.2	233.8
V	120	107.6	105.8	104	109.4	116.6	111.2	102.2
VI	200	175.5	173.7	180.9	179.1	182.7	197.1	177.3
VII	25	8	6.2	13.4	11.6	9.8	15.2	18.8

Tables 3 and 4 presents the results of the third example. The single goal programming has achieved the only one goal ignoring the remaining six goals as mentioned in table 3. The solution of multi goal programming using existing. improved and modified improved techniques are given in table 4. The existing MGP has made no improvement in achieving all the goals. It has reproduce the solution of single goal achievement of sixth goal only. However, the improved and the modified improved MGP have achieved all the seven goals simultaneously.

Table 4. Goal Achievements in Multi-Goal Programming

Goals		**Multi Goal Programming**		
		Existing MGP	**Improved MGP**	**Modified MGP**
X_i		0.5, 0, 0, 0, 1.8, 0, 0.2, 0	0.5, 0, 0, 0, 0, 0, 2, 0	0.5, 0, 0, 0, 0, 0, 2, 0
I	50	37.7	39.5	39.5
II	80	68.7	70.5	70.5
III	100	87.2	89	89
IV	250	239.2	241	241
V	120	111.2	113	113
VI	200	197.1	184.5	184.5
VII	25	15.2	17	17

5. Conclusions

The improved MGP technique has been modified and applied to solve three examples of multiple conflicting goals. The examples have also been solved with single, existing, and improved MGP techniques also for the comparative analysis. The modified improved MGP technique has achieved all the goals simultaneously in all the three examples. The solutions of modified improved MGP technique were same as of improved MGP technique. It can be concluded that the modified improved technique is also efficient in solving the MGP problems.

Compliance with Ethical Standards

Conflict of interest: The author declares that there is no conflict of interest.

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