

Improved Scalarizing Techniques for Solving Multi-objective Optimization Problems

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Abstract Several scalarizing techniques are used for solving multi-objective optimization (MOO) problems. Most of these scalarizing techniques were found inefficient in obtaining an appropriate solution of MOO problems. The study proposed improved scalarizing techniques for solving multi-objective optimization (MOO) problems. The improved scalarizing techniques using mean, harmonic mean and geometric mean have been applied for solving two MOO problems and generated satisfactory solutions.

Keywords Improved Scalarizing Technique, Multi-Objective Optimization (MOO), Mean, Harmonic Mean, Geometric Mean

1. Introduction

The scalarizing technique was first introduced in year 1983 [1] for solving MOO problems. The technique has been successfully used for resource use planning in agriculture for increasing farm income, increasing farm employment with lesser use of fertilizer, irrigation etc. [2] [3]..[10]. Several new averaging techniques [11] [12]... [27] using mean, harmonic mean and geometric mean have been proposed during last three decades. Most of these scalarizing techniques have been found inefficient in generating the acceptable MOO solutions [28] [29]. The improved scalarizing techniques are proposed for generating compromising and acceptable solutions of MOO problems.

2. Multi-Objective Optimization Problem

For optimizing 'n' objective functions with maximization of 'r' objective functions and minimization of remaining 'n-r' objective functions. The MOO problem is formulated as described below:

$$\text{Optimize } Z = [\text{Max. } Z_1, Z_2, \dots, Z_r, \text{ Min. } Z_{r+1}, \dots, Z_n]$$

Where,

$$Z_1 = \sum_j^n C_j X_j$$

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$$Z_n = \sum_j^n C_j X_j$$

Subject To:

$$AX = b \text{ and } X \geq 0$$

2.1. Existing Scalarizing Techniques

There are many scalarizing techniques for solving MOO problems. The scalarizing techniques using mean, geometric mean and harmonic mean have been explained with an example. The multi-objective function is constructed by scalarizing each objective function by the mean, geometric and harmonic mean of optimal values of objective functions as described below:

$$\text{Maximize } Z = \frac{\sum_{j=1}^r Z_j}{|\theta_1|} - \frac{\sum_{j=r+1}^s Z_j}{|\theta_2|}$$

Subject to:

$$AX = b \text{ and } X \geq 0$$

$$\theta_j \neq 0 \quad \text{for } j=1, 2, \dots, s.$$

Where,

$|\theta_1|$ = Mean, Geometric Mean and Harmonic Mean of optimal values of maximization objective functions.

$|\theta_2|$ = Mean, Geometric Mean and Harmonic Mean of optimal values of Minimization Objective functions.

Note: The estimation of θ_1 and θ_2 is not logical when objective functions are of different dimensions.

2.2. Improved Scalarizing Techniques

The multi-objective function is constructed as explained below:

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$$\text{Max. } Z = \sum_{j=1}^r \frac{Z_j}{|C_1|_{\text{Mean/Hm/Gm}}} - \sum_{r=1}^n \frac{Z_j}{|C_2|_{\text{Mean/Hm/Gm}}}$$

Where,

$|C_1|$ = Mean, Harmonic mean (Hm) or Geometric mean (Gm) of C_j values of Maximization objective functions

$|C_2|$ = Mean, Harmonic mean (Hm) or Geometric mean (Gm) of C_j values of Minimization objective functions

Note: The multi-objective function is free from the problem of multi dimensions aggregation.

3. Solving MOO Problem

Example

The following example for achieving four objective functions has been solved using both existing and improved scalarizing techniques.

$$\text{Max. } Z_1 = 12500X_1 + 25100X_2 + 16700X_3 + 23300X_4 + 20200X_5$$

$$\text{Max. } Z_2 = 21X_1 + 15X_2 + 13X_3 + 17X_4 + 11X_5$$

$$\text{Min. } Z_3 = 370X_1 + 280X_2 + 350X_3 + 270X_4 + 240X_5$$

$$\text{Min. } Z_4 = 1930X_1 + 1790X_2 + 1520X_3 + 1690X_4 + 1720X_5$$

Subject to:

$$X_1 + X_2 + X_3 + X_4 + X_5 = 4.5$$

$$2X_1 \geq 1.0$$

$$3X_4 \geq 1.5$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

All the objective functions have been optimized individually and the solution is presented in Table 1.

It is very clear that all the four individual optimizations are all different and conflicting to each other. The value of first objective function is 1, 05,750 which is highest among other values of same objective function. Similarly the value of second objective function is also highest and values of third

and fourth objective functions are lowest as desired. This conflicting scenario necessitates the need for multi-objective optimization.

The mean, geometric mean and harmonic mean have been used for scalarizing the multi-objective function. The mean, geometric mean and harmonic mean have been estimated for both existing and improved techniques of scalarizing as given in Table 2.

The mean, geometric mean and harmonic mean of maximum values of objective functions Z_1 and Z_2 are 52921.25, 3127.59 and 184.83 respectively as given in Table 2. Similarly the mean, geometric mean and harmonic mean of minimum values of remaining objective functions 3 and 4 are 4145, 2875.89 and 1995.3 respectively for existing scalarizing techniques. On the other hand, for improved scalarizing techniques, the mean, geometric mean and harmonic mean have been estimated considering all the four (one optimal and three sub optimal) values of each objective function as mentioned in Table 2. The multi-objective function has been formulated by scalarizing the objective functions using mean, geometric mean and harmonic mean as explained in equations 2.1 and 2.2. The existing and improved scalarized multi-objective functions have been optimized and the results are presented in Table 3.

The Table 3 indicates that all the three scalarized multi-objective functions generated unique solution. These solutions are same as individual optimum of objective function Z_1 . The existing scalarizing techniques are unable to generate the compromising solution. However, the improved scalarizing techniques have optimized all the objective functions simultaneously. The value of each objective function is closer to its individual optimum as given in Table 1. The optimal solutions generated by improved scalarizing techniques are compromising and acceptable.

Table 1. Individual Optimization

Objective function	Individual Optimization			
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4
X_i	0.5, 3.5, 0, 0.5, 0	4, 0, 0, 0.5, 0	0.5, 0, 0, 0.5, 3.5	0.5, 0, 3.5, 0.5, 0
Z_1	105750	61650	88600	76350
Z_2	71.5	92.5	57.5	64.5
Z_3	1300	1615	1160	1545
Z_4	8075	8565	7830	7130

Table 2. Mean, Geometric Mean and Harmonic Mean

Objective function	Existing Scalarizing Techniques			Improved Scalarizing Techniques		
	Mean	G. Mean	H. Mean	Mean	G. Mean	H. Mean
Max. Z_1	52921.25	3127.59	184.83	19560	18984.60	18372.61
Max. Z_2				15.4	15.03	14.67
Min. Z_3	4145	2875.89	1995.36	302	297.99	294.09
Min. Z_4				1730	1724.78	1719.51

Table 3. Multi-Objective Optimization

Item	Existing Average Techniques			Improved Average Techniques		
	Mean	Geometric Mean	Harmonic Mean	Mean	Geometric Mean	Harmonic Mean
Xi	0.5, 3.5, 0, 0.5, 0	0.5, 3.5, 0, 0.5, 0	0.5, 3.5, 0, 0.5, 0	0.5, 0, 0, 4, 0	0.5, 0, 0, 4, 0	0.5, 0, 0, 4, 0
Z ₁	105750	105750	105750	99450	99450	99450
Z ₂	71.5	71.5	71.5	78.5	78.5	78.5
Z ₃	1300	1300	1300	1265	1265	1265
Z ₄	8075	8075	8075	7725	7725	7725

4. Conclusions

There are several scalarizing techniques used for solving MOO problems. The mean, geometric mean and harmonic mean have been frequently used for scalarizing the objective functions. The solutions of MOO problems generated using these techniques were not superior. However the improved scalarizing techniques using mean, harmonic mean and geometric mean are found superior and efficient in solving MOO problems for obtaining compromising solutions.

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