

Fuzzy EOQ Model for Time-Deteriorating Items Using Penalty Cost

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Abstract In this paper, a fuzzy inventory model for time-deteriorating items using penalty cost under the conditions of infinite production rate is formulated and solved. Penalty cost is assumed to be linear and exponential. Fuzziness is introduced in the cost component of holding cost and set up cost. Demand rate is also assumed to be fuzzy. In fuzzy environment all related parameters are assumed to be trapezoidal. Representing these three costs by trapezoidal fuzzy numbers, the optimum order quantity is calculated using signed distance method and graded mean integration method for defuzzification. Numerical examples have been given in order to show the applicability of the proposed model. Sensitivity analysis is also carried out to detect the shift in the variables of interest of the system.

Keywords Fuzzy Inventory Model, Trapezoidal Fuzzy Number, Defuzzification, Penalty Cost

1. Introduction

Inventory control is very important for both real world applications and research purpose. In conventional inventory models, the uncertainties are treated as randomness and handled by using probability theory. The most widely used inventory model is the Economic order quantity (EOQ) model. This model was developed by Harris [1], Wilson [2]. Later Hadley [3] analyzed many inventory systems. But uncertainties due to fuzziness primarily introduced by Zadeh [4]. Zadeh et al [5] proposed some strategies for decision making in fuzzy environment. Kacprzyk et al [6] discussed some long-term inventory policy making through fuzzy-decision making models.

Products like fresh vegetables, fruits, bakery items etc. do not deteriorate at the beginning of the period but they continuously deteriorate after some time. As a result, the selling price of such product decreases which can be considered as a penalty cost. Srivastava and Gupta [10] have proposed an EOQ model for time-deteriorating items using penalty cost.

Fujiwara and Perera [8] have proposed an EOQ model for time continuously deteriorating items using linear and exponential penalty cost. Pevekar and Nagare [14] developed an inventory model for timely deteriorating products considering penalty cost and shortage cost. Park [7] and Vujosevic et al [9] developed the inventory model in fuzzy sense whereas ordering cost and holding cost are

represented by fuzzy numbers. Margatham and Lakshmi Devi [13] have proposed a fuzzy inventory model for deteriorating items with price dependent demand.

Previously Park [7] has represented cost as trapezoidal fuzzy numbers, wherein Vujosevic et al [9] represented ordering cost by triangular fuzzy number and holding cost by trapezoidal fuzzy number. Jaggi et al [11] applied the extension principle to obtain the fuzzy total cost and they defuzzified the fuzzy total cost by using graded mean integration method and signed distance method.

In this article, fuzzy EOQ model for time deteriorating items using penalty cost is considered where holding cost, set up cost and demand rate are assumed as trapezoidal fuzzy numbers. For defuzzification of the total cost function, signed distance method and graded mean integration method are used.

2. Definitions and Preliminaries

Definition 2.1. A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented with membership function $\mu_{\tilde{A}}$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, & \text{when } a \leq x \leq b; \\ 1, & \text{when } b \leq x \leq c; \\ R(x) = \frac{d-x}{d-c}, & \text{when } c \leq x \leq d; \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.2. Suppose $\tilde{A} = (a, b, c, d)$ and

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$\tilde{B} = (a, b, c, d)$ are two trapezoidal fuzzy number, then arithmetic operations are defined as

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
3. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\tilde{A} \phi \tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1} \right)$
5. $\alpha a_1 = \begin{cases} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), & \alpha \geq 0 \\ (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), & \alpha < 0 \end{cases}$

Definition 2.3. Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number, then the signed distance method of \tilde{A} is defined as

$$d(\tilde{A}, 0) = \frac{1}{2} \int_0^1 [A_L \alpha + A_R \alpha] d\alpha$$

Where $A_\alpha = [A_L \alpha, A_R \alpha]$

$$= [a + (b - a)\alpha, d - (d - c)\alpha], \alpha \in [0, 1]$$

is a α -cut of fuzzy set \tilde{A} , which is a close interval.

$$d(\tilde{A}, 0) = \frac{a + b + c + d}{4}$$

Definition 2.4. Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number, then the graded mean integration representation of \tilde{A} is defined as

$$P(\tilde{A}) = \frac{\frac{1}{2} \int_0^1 \alpha [A_L \alpha + A_R \alpha] d\alpha}{\int_0^1 \alpha d\alpha} = \frac{a + 2b + 2c + d}{6}$$

3. Assumptions and Notations

The model is developed on the following assumptions and notations.

3.1. Assumptions

- (i) A single product is considered over a prescribed period of T unit of time.
- (ii) The replenishment occurs instantaneously at an infinite rate.
- (iii) No back order is permitted.
- (iv) Delivery leads time zero.

3.2. Notations

Q = Number of items received at the beginning of the

period.

D = Demand rate.

H = Inventory holding cost.

A = Set-up cost per cycle.

μ = Time period at which deterioration of product start.

C (T) = Average total variable cost per unit time.

T = Length of replenishment cycle, which will not exceed product lifetime.

T^* = Optimum value of T.

Q^* = Optimum value of Q.

\tilde{H} = Fuzzy inventory holding cost.

\tilde{A} = Fuzzy set up cost per cycle.

\tilde{D} = Fuzzy demand rate \tilde{D}

$\tilde{C}(T)$ = Average total fuzzy variable cost per unit time.

$C_{ds}(T)$ = Defuzzified value of $\tilde{C}(T)$ by applying signed distance method.

$C_{dG}(T)$ = Defuzzified value of $\tilde{C}(T)$ by applying graded mean integration method.

4. Mathematical Model (for Infinite production rate)

4.1. Crisp Model

In this context, we have considered two types of penalty cost function of age

- (i) Linear
- (ii) Exponential penalty cost functions, as a measurement of utility of the product.

A linear penalty cost function

$$P(t) = \pi(t - \mu), \quad t \geq \mu$$

$$= 0; \text{ otherwise}$$

which gives the cost of keeping one unit of product in stock until age t, where μ be the time period at which deterioration of product starts and π is a constant. There will be no penalty cost incurred upon the products up to time period $(0, \mu)$.

An exponential penalty cost function is taken as

$$P(t) = \alpha(e^{\beta(t-\mu)} - 1), \quad t \geq \mu$$

$$= 0, \text{ otherwise}$$

Which also gives the cost of keeping one unit of product in stock until age t, where μ be the time period at which deterioration of product starts and α and β are constants.

The total variable cost per cycle time consists of the inventory holding cost, set up cost and penalty cost. Since the demand rate is D unit per time.

The total demand in one cycle of time-interval is $T = DT$.

The number of items received at the beginning of the period is

$$Q = DT \quad (4.1)$$

Proposed Inventory Model in Crisp Sense

Case I. When linear penalty cost function is used

A linear penalty cost function $P(t) = \pi(t - \mu), t \geq \mu$ which gives the cost of keeping one unit of product in stock until age t , where μ be the time period at which deterioration of product start and π is constant.

The cost due to the deterioration of the product delivered during the period $(t, t+dt)$ is given by $\pi(t - \mu)Ddt$. Thus penalty cost due to the deterioration of the product delivered during the time interval (μ, T) is given by

$$\int_{\mu}^T \pi D(t - \mu)dt = \pi D \left[\frac{T^2}{2} - \mu T + \frac{\mu^2}{2} \right]$$

Now inventory holding cost for the period $(0, T)$ is given by

$$H \cdot \frac{1}{2} QT = \frac{1}{2} HDT^2 \quad (\because Q = DT)$$

Therefore, the average total variable cost per unit time $C(T)$ is given by

$$C(T) = \frac{1}{T} \left[A + \frac{\pi D \mu^2}{2} \right] + \left[\frac{HD}{2} + \frac{\pi D}{2} \right] T - \pi D \mu \quad (4.2)$$

The optimal solution is obtained by differentiating $C(T)$ with respect to T and equating it to zero. Then the optimal cycle time T^* is obtained and expressed as

$$T^* = \sqrt{\frac{2A + \pi D \mu^2}{(\pi + H)D}} \quad (4.3)$$

The optimal economic order quantity Q^* is obtained by putting value of T^* in eq.(4.1),

$$Q^* = \sqrt{\frac{D(2A + \pi D \mu^2)}{(\pi + H)}} \quad (4.4)$$

From the above expressions (4.3) and (4.4), it is clear that if there is no perishability (i.e. $\pi = 0$), then these two expression become same as that of the non-perishable lot size model.

Case II. When exponential penalty cost function is used

An exponential penalty cost function $P(t) = \alpha(e^{\beta(t-\mu)} - 1), t \geq \mu$ which gives the cost of keeping one unit of product in stock until age t , where μ be the time period at which deterioration of product starts and α and β are constants.

The cost due to the deterioration of the product delivered during the period $(t, t + dt)$ is given by $\alpha(e^{\beta(t-\mu)} - 1)Ddt$.

The penalty cost due to the deterioration of the product

delivered during the time interval (μ, T) is given by

$$\int_{\mu}^T D \alpha (e^{\beta(t-\mu)} - 1) dt = \frac{\alpha D}{\beta} [(e^{\beta(T-\mu)} - 1) - \beta(T - \mu)]$$

Therefore, total variable cost per unit time is given by

$$C(T) = \frac{A}{T} + \frac{1}{2} HDT + \frac{\alpha D}{\beta T} [(e^{\beta(T-\mu)} - 1) - \beta(T - \mu)]$$

By using second order approximation of the exponential term $e^{\beta(T-\mu)}$ in $C(T)$

We get,

$$C(T) = \frac{A}{T} + \frac{1}{2} HDT + \frac{1}{2} \alpha D \beta T + \frac{1}{2T} \alpha D \mu^2 \beta - \alpha D \mu \beta$$

The optimal solution is obtained by differentiating $C(T)$ with respect to T and equating it to zero.

Then, the optimal cycle time T^* is obtained and expressed as

$$T^* = \sqrt{\frac{2A + \alpha D \beta \mu^2}{D(H + \alpha \beta)}} \quad (4.5)$$

The optimal economic order quantity Q^* is obtained by putting value of T^* in eq.(4.1),

$$Q^* = \sqrt{\frac{D(2A + \alpha D \beta \mu^2)}{(H + \alpha \beta)}} \quad (4.6)$$

From the expression (4.5) and (4.6), it is clear that if $\mu = 0$ and $\alpha \beta = \pi$, then these two expressions are same as (4.3) and (4.4).

4.2. Fuzzy Model

Case I. When linear penalty cost function is used

Due to uncertainty in the environment, it is not easy to define all the parameters precisely. Accordingly we assume some of these parameters \tilde{H} , \tilde{A} and \tilde{D} may change with some limit.

Let $\tilde{H} = (H_1, H_2, H_3, H_4)$, $\tilde{A} = (A_1, A_2, A_3, A_4)$ and $\tilde{D} = (D_1, D_2, D_3, D_4)$ are trapezoidal fuzzy numbers.

The total variable cost per unit time in fuzzy sense is given by

$$\tilde{C}(T) = \frac{1}{T} \left[\tilde{A} + \frac{\pi \tilde{D} \mu^2}{2} \right] + \left[\frac{\tilde{H} \tilde{D}}{2} + \frac{\pi \tilde{D}}{2} \right] T - \pi \tilde{D} \mu$$

We defuzzify the fuzzy total cost $\tilde{C}(T)$ by using signed distance method and graded mean integration method.

(i) By Signed distance method, total cost is given by

$$C_{ds}(T) = \frac{1}{4} [C_{ds_1}(T), C_{ds_2}(T), C_{ds_3}(T), C_{ds_4}(T)]$$

Where,

$$C_{ds_1}(T) = \frac{1}{T} \left[A_1 + \frac{\pi D_1 \mu^2}{2} \right] + \left[\frac{H_1 D_1}{2} + \frac{\pi D_1}{2} \right] T - \pi D_1 \mu$$

$$C_{ds_2}(T) = \frac{1}{T} \left[A_2 + \frac{\pi D_2 \mu^2}{2} \right] + \left[\frac{H_2 D_2}{2} + \frac{\pi D_2}{2} \right] T - \pi D_2 \mu$$

$$C_{ds_3}(T) = \frac{1}{T} \left[A_3 + \frac{\pi D_3 \mu^2}{2} \right] + \left[\frac{H_3 D_3}{2} + \frac{\pi D_3}{2} \right] T - \pi D_3 \mu$$

$$C_{ds_4}(T) = \frac{1}{T} \left[A_4 + \frac{\pi D_4 \mu^2}{2} \right] + \left[\frac{H_4 D_4}{2} + \frac{\pi D_4}{2} \right] T - \pi D_4 \mu$$

$$C_{ds}(T) = \frac{1}{4} [C_{ds_1}(T) + C_{ds_2}(T) + C_{ds_3}(T) + C_{ds_4}(T)]$$

To minimize total cost function per unit time $C_{ds}(T)$, the optimal value of T can be obtained by solving the following equation:

$$\frac{dC_{ds}(T)}{dT} = 0 \quad (4.7)$$

Provided

$$\frac{d^2 C_{ds}(T)}{d^2(T)} > 0 \quad (4.8)$$

Equation (4.7) is equivalent to

$$\Rightarrow \frac{1}{4} \left[\begin{aligned} & \left(-\frac{1}{T^2} \left(A_1 + \frac{\pi D_1 \mu^2}{2} \right) + \frac{H_1 D_1}{2} + \frac{\pi D_1}{2} \right) \\ & + \left(-\frac{1}{T^2} \left(A_2 + \frac{\pi D_2 \mu^2}{2} \right) + \frac{H_2 D_2}{2} + \frac{\pi D_2}{2} \right) \\ & + \left(-\frac{1}{T^2} \left(A_3 + \frac{\pi D_3 \mu^2}{2} \right) + \frac{H_3 D_3}{2} + \frac{\pi D_3}{2} \right) \\ & + \left(-\frac{1}{T^2} \left(A_4 + \frac{\pi D_4 \mu^2}{2} \right) + \frac{H_4 D_4}{2} + \frac{\pi D_4}{2} \right) \end{aligned} \right] = 0$$

After simplification we get the optimal cycle time T^* is obtained and expressed as

$$T^* = \sqrt{\frac{2(A_1 + A_2 + A_3 + A_4) + \pi \mu^2 (D_1 + D_2 + D_3 + D_4)}{(H_1 D_1 + H_2 D_2 + H_3 D_3 + H_4 D_4) + \pi (D_1 + D_2 + D_3 + D_4)}}$$

The optimal economic order quantity Q^* is

$$Q^* = \tilde{D}^* T^* = (D_1 T^*, D_2 T^*, D_3 T^*, D_4 T^*)$$

(ii) By Graded mean integration method, total cost is given by

$$C_{dG}(T) = \frac{1}{6} [C_{dG_1}(T), C_{dG_2}(T), C_{dG_3}(T), C_{dG_4}(T)]$$

Where,

$$C_{dG_1}(T) = \frac{1}{T} \left[A_1 + \frac{\pi D_1 \mu^2}{2} \right] + \left[\frac{H_1 D_1}{2} + \frac{\pi D_1}{2} \right] T - \pi D_1 \mu$$

$$C_{dG_2}(T) = \frac{1}{T} \left[A_2 + \frac{\pi D_2 \mu^2}{2} \right] + \left[\frac{H_2 D_2}{2} + \frac{\pi D_2}{2} \right] T - \pi D_2 \mu$$

$$C_{dG_3}(T) = \frac{1}{T} \left[A_3 + \frac{\pi D_3 \mu^2}{2} \right] + \left[\frac{H_3 D_3}{2} + \frac{\pi D_3}{2} \right] T - \pi D_3 \mu$$

$$C_{dG_4}(T) = \frac{1}{T} \left[A_4 + \frac{\pi D_4 \mu^2}{2} \right] + \left[\frac{H_4 D_4}{2} + \frac{\pi D_4}{2} \right] T - \pi D_4 \mu$$

$$C_{dG}(T) = \frac{1}{6} [C_{dG_1}(T) + 2C_{dG_2}(T) + 2C_{dG_3}(T) + C_{dG_4}(T)]$$

To minimize total cost function per unit time $C_{dG}(T)$ the optimal value of T can be obtained by solving the following equation.

$$\frac{dC_{dG}(T)}{dT} = 0 \quad (4.9)$$

Provided

$$\frac{d^2 C_{dG}(T)}{d^2(T)} > 0 \quad (4.10)$$

Equation (4.9) is equivalent to

$$\Rightarrow \frac{1}{6} \left[\begin{aligned} & \left(-\frac{1}{T^2} \left(A_1 + \frac{\pi D_1 \mu^2}{2} \right) + \frac{H_1 D_1}{2} + \frac{\pi D_1}{2} \right) \\ & + 2 \left(-\frac{1}{T^2} \left(A_2 + \frac{\pi D_2 \mu^2}{2} \right) + \frac{H_2 D_2}{2} + \frac{\pi D_2}{2} \right) \\ & + 2 \left(-\frac{1}{T^2} \left(A_3 + \frac{\pi D_3 \mu^2}{2} \right) + \frac{H_3 D_3}{2} + \frac{\pi D_3}{2} \right) \\ & + \left(-\frac{1}{T^2} \left(A_4 + \frac{\pi D_4 \mu^2}{2} \right) + \frac{H_4 D_4}{2} + \frac{\pi D_4}{2} \right) \end{aligned} \right] = 0$$

After simplification we get the optimal cycle time T^* is obtained and expressed as

$$T^* = \sqrt{\frac{2(A_1 + 2A_2 + 2A_3 + A_4) + \pi \mu^2 (D_1 + 2D_2 + 2D_3 + D_4)}{(H_1 D_1 + 2H_2 D_2 + 2H_3 D_3 + H_4 D_4) + \pi (D_1 + 2D_2 + 2D_3 + D_4)}}$$

The optimal economic order quantity Q^* is

$$Q^* = \tilde{D}^* T^* = (D_1 T^*, D_2 T^*, D_3 T^*, D_4 T^*)$$

Case II. when exponential penalty cost function is used,

Due to uncertainty in the environment, it is not easy to define all the parameters precisely, Accordingly we assume some of these parameters \tilde{H} , \tilde{A} and \tilde{D} may change

with some limit.

Let $\tilde{H} = (H_1, H_2, H_3, H_4)$, $\tilde{A} = (A_1, A_2, A_3, A_4)$ and $\tilde{D} = (D_1, D_2, D_3, D_4)$ are as trapezoidal fuzzy numbers.

The total variable cost per unit time in fuzzy sense is given by

$$\tilde{C}(T) = \frac{\tilde{A}}{T} + \frac{1}{2}\tilde{H}\tilde{D}T + \frac{1}{2}\alpha\tilde{D}\beta T + \frac{1}{2T} + \alpha\tilde{D}\mu^2\beta - \alpha\tilde{D}\mu\beta$$

We defuzzify the fuzzy total cost $\tilde{C}(T)$ by signed distance and graded mean representation methods.

(i) By Signed distance method, total cost is given by

$$C_{ds}(T) = \frac{1}{4}[C_{ds_1}(T), C_{ds_2}(T), C_{ds_3}(T), C_{ds_4}(T)]$$

Where,

$$C_{ds_1}(T) = \frac{A_1}{T} + \frac{1}{2}H_1D_1T + \frac{1}{2}\alpha D_1\beta T + \frac{1}{2T}\alpha D_1\mu^2\beta - \alpha D_1\mu\beta$$

$$C_{ds_2}(T) = \frac{A_2}{T} + \frac{1}{2}H_2D_2T + \frac{1}{2}\alpha D_2\beta T + \frac{1}{2T}\alpha D_2\mu^2\beta - \alpha D_2\mu\beta$$

$$C_{ds_3}(T) = \frac{A_3}{T} + \frac{1}{2}H_3D_3T + \frac{1}{2}\alpha D_3\beta T + \frac{1}{2T}\alpha D_3\mu^2\beta - \alpha D_3\mu\beta$$

$$C_{ds_4}(T) = \frac{A_4}{T} + \frac{1}{2}H_4D_4T + \frac{1}{2}\alpha D_4\beta T + \frac{1}{2T}\alpha D_4\mu^2\beta - \alpha D_4\mu\beta$$

$$C_{ds}(T) = \frac{1}{4}(C_{ds_1}(T) + C_{ds_2}(T) + C_{ds_3}(T) + C_{ds_4}(T))$$

To minimize total cost function per unit time $C_{ds}(T)$, the optimal value of T can be obtained by solving the following equation:

$$\frac{dC_{ds}(T)}{dT} = 0 \quad (4.11)$$

Provided

$$\frac{d^2C_{ds}(T)}{d^2(T)} > 0 \quad (4.12)$$

Equation (4.11) is equivalent to

$$\Rightarrow \frac{1}{4} \left[\begin{aligned} &\left(-\frac{A_1}{T^2} + \frac{1}{2}H_1D_1 + \frac{1}{2}\alpha D_1\beta - \frac{1}{2T^2}\alpha D_1\mu^2\beta\right) \\ &+ \left(-\frac{A_2}{T^2} + \frac{1}{2}H_2D_2 + \frac{1}{2}\alpha D_2\beta - \frac{1}{2T^2}\alpha D_2\mu^2\beta\right) \\ &+ \left(-\frac{A_3}{T^2} + \frac{1}{2}H_3D_3 + \frac{1}{2}\alpha D_3\beta - \frac{1}{2T^2}\alpha D_3\mu^2\beta\right) \\ &+ \left(-\frac{A_4}{T^2} + \frac{1}{2}H_4D_4 + \frac{1}{2}\alpha D_4\beta - \frac{1}{2T^2}\alpha D_4\mu^2\beta\right) \end{aligned} \right] = 0$$

After simplification we get the optimal cycle time T^* is obtained and expressed as

$$T^* = \sqrt{\frac{2(A_1 + A_2 + A_3 + A_4) + \alpha\mu^2\beta(D_1 + D_2 + D_3 + D_4)}{(H_1D_1 + H_2D_2 + H_3D_3 + H_4D_4) + \alpha\beta(D_1 + D_2 + D_3 + D_4)}}$$

The optimal economic order quantity Q^* is

$$Q^* = \tilde{D} * T^* = (D_1T^*, D_2T^*, D_3T^*, D_4T^*)$$

(ii) By Graded mean integration method, total cost is given by

$$C_{dG}(T) = \frac{1}{6}[C_{dG_1}(T), C_{dG_2}(T), C_{dG_3}(T), C_{dG_4}(T)]$$

Where,

$$C_{dG_1}(T) = \frac{A_1}{T} + \frac{1}{2}H_1D_1T + \frac{1}{2}\alpha D_1\beta T + \frac{1}{2T}\alpha D_1\mu^2\beta - \alpha D_1\mu\beta$$

$$C_{dG_2}(T) = \frac{A_2}{T} + \frac{1}{2}H_2D_2T + \frac{1}{2}\alpha D_2\beta T + \frac{1}{2T}\alpha D_2\mu^2\beta - \alpha D_2\mu\beta$$

$$C_{dG_3}(T) = \frac{A_3}{T} + \frac{1}{2}H_3D_3T + \frac{1}{2}\alpha D_3\beta T + \frac{1}{2T}\alpha D_3\mu^2\beta - \alpha D_3\mu\beta$$

$$C_{dG_4}(T) = \frac{A_4}{T} + \frac{1}{2}H_4D_4T + \frac{1}{2}\alpha D_4\beta T + \frac{1}{2T}\alpha D_4\mu^2\beta - \alpha D_4\mu\beta$$

$$C_{dG}(T) = \frac{1}{6}[C_{dG_1}(T) + 2C_{dG_2}(T) + 2C_{dG_3}(T) + C_{dG_4}(T)]$$

To minimize total cost function per unit time $C_{dG}(T)$, the optimal value of T can be obtained by solving the following equation:

$$\frac{dC_{dG}(T)}{dT} = 0 \quad (4.13)$$

Provided

$$\frac{d^2C_{dG}(T)}{d^2(T)} > 0 \quad (4.14)$$

Equation (4.13) is equivalent to

$$\Rightarrow \frac{1}{6} \left[\begin{aligned} &\left(-\frac{A_1}{T^2} + \frac{1}{2}H_1D_1 + \frac{1}{2}\alpha D_1\beta - \frac{1}{2T^2}\alpha D_1\mu^2\beta\right) \\ &+ 2\left(-\frac{A_2}{T^2} + \frac{1}{2}H_2D_2 + \frac{1}{2}\alpha D_2\beta - \frac{1}{2T^2}\alpha D_2\mu^2\beta\right) \\ &+ 2\left(-\frac{A_3}{T^2} + \frac{1}{2}H_3D_3 + \frac{1}{2}\alpha D_3\beta - \frac{1}{2T^2}\alpha D_3\mu^2\beta\right) \\ &+ \left(-\frac{A_4}{T^2} + \frac{1}{2}H_4D_4 + \frac{1}{2}\alpha D_4\beta - \frac{1}{2T^2}\alpha D_4\mu^2\beta\right) \end{aligned} \right] = 0$$

After simplification we get the optimal cycle time T^* is obtained and expressed as

$$T^* = \sqrt{\frac{2(A_1 + 2A_2 + 2A_3 + A_4) + \alpha\mu^2\beta(D_1 + 2D_2 + 2D_3 + D_4)}{(H_1D_1 + 2H_2D_2 + 2H_3D_3 + H_4D_4) + \alpha\beta(D_1 + 2D_2 + 2D_3 + D_4)}}$$

The optimal economic order quantity Q^* is

$$Q^* = \tilde{D} * T^* = (D_1T^*, D_2T^*, D_3T^*, D_4T^*)$$

5. Numerical Illustration

The data for the study have been solicited after an

intensive search of the following research papers.

Tripathy, P.K. and Pradhan, S. [12]

Srivastava, M. and Gupta, R. [10]

Jaggi, C. K., Pareek, S., Sharma, A. and Nidhi. [11]

5.1. Crisp Model

Let $P = 45$ units per day, $D = 30$ units per day, $H = \text{Rs. } 0.02$ per day, $\mu = 5$ days, $\alpha = 10$, $\beta = 0.99$, $A = 100$.

Case-1. when linear penalty cost function is used then,

Optimum cycle time $T^* = 5.19$ days

Optimum order quantity $Q^* = 155.7$ units

Case-2. when exponential penalty cost function is used then,

Optimum cycle time $T^* = 4.41$

Optimum order quantity $Q^* = 132.3$

Sensitivity Analysis of Crisp Model

The sensitivity analysis is performed for checking the effectiveness of the EOQ model for infinite production rate with respect to the parameter μ on optimum cycle time T^* and optimum order quantity Q^* . Table 1 and Table 2 depict the values of the optimum policies for different values of the parameter μ in case of linear and exponential penalty cost. Percentage changes of these values are shown with respect to the parameter $\mu = 5$ in the data set are taken.

The Table 1 shows the effect of parameter μ on optimum policies in case of linear penalty cost. If the value of the parameter μ is increased by 60%, the value of optimum cycle time and optimum order quantity are also increased by 56%. Further, if the parameter μ is decreased by 20%, the optimum cycle time and optimum order quantity are decreased by 17%.

Table 1. Effect of parameter μ on optimal policies in case of linear penalty cost

μ	Change (%) in μ	T^*	Q^*
2	-60%	2.46(-52%)	73.8(-52%)
4	-20%	4.24(-17%)	127.2(-17%)
5	0	5.19(0%)	155.7(0%)
6	20%	6.15(18%)	184.5(18%)
8	60%	8.10(56%)	243.0(56%)

The Table 2 shows the effect of parameter μ on optimum policies in case of exponential penalty cost. If the value of the parameter μ is increased by 60%, the value of optimum cycle time and optimum order quantity are also increased by 58%. Further, if the parameter μ is decreased by 20%, the optimum cycle time and optimum order quantity are

decreased by 19%.

Table 2. Effect of parameter μ on optimal policies in case of exponential penalty cost

μ	Change (%) in μ	T^*	Q^*
2	-60%	1.88(-57%)	56.4(-57%)
4	-20%	3.55(-19%)	106.5(-19%)
5	0	4.41(0%)	132.3(0%)
6	20%	5.27(19%)	158.1(19%)
8	60%	7.00(58%)	210.0(58%)

5.2. Fuzzy Model

Let $P = 45$ units per day, $\mu = 5$ days, $\alpha = 10$, $\beta = 0.99$,

$\tilde{H} = (0.01, 0.03, 0.05, 0.08)$, $\tilde{A} = (90, 95, 100, 105)$,

$\tilde{D} = (20, 25, 30, 35)$

Case-1. When linear penalty cost function is used then, (By using signed distance method)

Optimum cycle time $T^* = 5.18$ days

Optimum order quantity $Q^* = (103.6, 129.5, 155.4, 181.3)$ units

(By using graded mean integration method)

Optimum cycle time $T^* = 5.18$ days

Optimum order quantity $Q^* = (103.6, 129.5, 155.4, 181.3)$

Case-2. When exponential penalty cost function is used then, (By using signed distance method)

Optimum cycle time $T^* = 5.05$ days

Optimum order quantity $Q^* = (101.00, 126.25, 151.5, 176.25)$ units

(By using graded mean integration method)

Optimum cycle time $T^* = 5.05$ days

Optimum order quantity $Q^* = (101.00, 126.25, 151.5, 176.25)$ units

Sensitivity Analysis of Fuzzy Model

The sensitivity analysis is performed for checking the effectiveness of the EOQ model for infinite production rate with respect to the parameter μ on optimum cycle time T^* and optimum order quantity Q^* in fuzzy sense by using signed distance method and graded mean integration method. Table-3 and Table-4 depict the values of the optimum policies for different values of the parameter μ in case of linear and exponential penalty cost. Percentage changes of these values are shown with respect to the parameter $\mu = 5$ in the data set are taken.

Table 3. Effect of parameter μ on optimal policies in case of exponential penalty cost

μ	Change (%) in μ	By using Signed distance method		By using Graded mean integration method	
		T^*	Q^*	T^*	Q^*
2	-60%	2.50 (-51%)	2.50 (-51%)	2.50 (-51%)	50.0, 62.5, 75.0, 87.5 (-51%)
4	-20%	4.25 (-17%)	4.25 (-17%)	4.25 (-17%)	85.0, 106.25, 127.5, 148.75 (-17%)
5	0	5.18 (0%)	5.18 (0%)	5.18 (0%)	103.6, 129.5, 155.4, 181.2 (0%)
6	20%	6.04 (18%)	6.04 (18%)	6.04 (18%)	122.8, 153.5, 184.2, 214.2 (18%)
8	60%	8.02 (56%)	8.02 (56%)	8.02 (56%)	161.6, 202.2, 242.4, 282.8 (56%)

Table 4. Effect of parameter μ on optimal policies in case of exponential penalty cost

μ	Change (%) in μ	By using Signed distance method		By using Graded mean integration method	
		T^*	Q^*	T^*	Q^*
2	-60%	2.16 (-57%)	43.2, 54.00, 64.8, 75.6 (-57%)	2.19 (-57%)	43.32, 54.01, 64.8, 75.75 (-57%)
4	-20%	4.07 (-19%)	81.4, 101.75, 122.1, 142.45 (-19%)	4.17 (-19%)	81.40, 101.85, 122.11, 142.55 (-19%)
5	0	5.05 (0%)	101.0, 126.2, 151.5, 176.7 (0%)	5.18 (0%)	101.0, 126.3, 151.6, 176.8 (0%)
6	20%	6.04 (19%)	120.8, 151.0, 181.2, 211.4 (19%)	6.04 (19%)	120.8, 151.01, 181.3, 211.6 (19%)
8	60%	8.02 (57%)	160.4, 200.5, 240.6, 280.7 (57%)	8.02 (59%)	160.3, 200.7, 240.8, 280.9 (59%)

6. Conclusions

In this paper, a fuzzy EOQ model for time-deteriorating items using penalty cost is studied. The demand rate, holding cost and set up cost are represented by trapezoidal fuzzy numbers. It has been also fuzzified by using signed distance method and graded mean integration method.

From the table-1 and table-2, it shows that if it increased the parameter μ in the crisp model then the optimal cycle time and optimal order quantity will also simultaneously increase. Findings from table-3 and table-4 shows that the change in parameter μ will result in the change in T^* and Q^* . With the increased value of parameter μ will also result in increase of optimal cycle time and optimal order quantity. Similarly, with the decreased value of parameter μ will also result in decrease of optimal cycle time and optimal order quantity. It is concluded that the value of the optimal cycle time and optimal order quantity are not much sensitive to change in the value of the parameter μ implying that fuzzy model permits flexibility in the system inputs.

The outcome of this research can be extended in future to the case of discount models.

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