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Optimal Search of Developed Class of Modified Ratio Estimators for Estimation of Population Variance

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Abstract This manuscript deals with the estimation of population variance using auxiliary variable through optimally searching developed class of modified ratio type estimators of population mean of the study variable. We have proposed a generalized class of estimators of population variance of the main variable under study. The proposed estimator is the generalization of the Subramani and Kumarapandiyan (2015) estimator with a characterizing scalar constant α which is to be obtained. The expressions for the bias and mean squared error of the proposed class of estimators have been obtained up to the first order of approximation. The optimum value of the characterizing scalar has been obtained by minimizing the mean squared error of the proposed class of estimators. The minimum value of the mean squared error has also been obtained for this optimum value of the characterizing scalar. At last an empirical study is also carried out through which we infer that the proposed estimator has minimum mean square error among the class of estimators of population variance as an optimal search condition.

Keywords Auxiliary variable, Parameter, Estimator, Bias, Mean Squared Error, Efficiency

1. Introduction

It has been seen in practice that the variance is one of the important measure of dispersion. For example we can see how the variation in production, variation in sales, variation in blood pressure etc affects the human beings. It plays a very important role in the field of Medical sciences, Biological sciences, Agriculture, Industry etc. So it is of great significance in above fields to estimate the variation. To estimate any parameter of the variable under study, the most appropriate estimator for the parameter is the corresponding statistic and in case of variance estimation, the most suitable estimator is the sample variance of the study variable. Being unbiased estimator of the population parameter, the sample variance should be preferred for the estimation but it has a big amount of variation among its values. Therefore we seek for such estimator which may be biased but it should have minimum mean squared error. Auxiliary variable which is generally highly positively or negatively correlated with the main variable under study fulfill the need in searching such estimator and its use lift up the efficiency of the estimator. When the main variable and the auxiliary variable are positively correlated, ratio type estimators are used for the estimation of population parameters under considerations. While the Product type estimators are used when main and auxiliary variables are negatively correlated to each other and the line of regression Y on X passes through origin and in other case regression estimators are used for the estimation of population parameters. In the present manuscript we have considered only positively correlated case using ratio type estimators of population variance.

Let us consider that the finite population under investigation consist of N distinct and identifiable units and let (x_i, y_i), $i = 1, 2, \ldots\ldots\ldots, n$ be a bivariate sample of size n taken from (X, Y) using a simple random sampling without replacement (*SRSWOR*) scheme. Let $\overline{X}$ and $\overline{Y}$ respectively be the population means of the auxiliary and the study variables, and let $\overline{x}$ and $\overline{y}$ be the corresponding sample means. $\overline{x}$ and $\overline{y}$ are the unbiased estimators of the population means $\overline{X}$ and $\overline{Y}$ respectively. In the present manuscript we are searching for such an estimator of population variance which should have minimum mean squared error among the class of such estimators.

The following notations early used by Subramani and Kumarapandiyan (2015) have been used in this manuscript as:

N - Size of the population
n - Size of the sample
Y - Study variable
X - Auxiliary variable

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M_d - Median of the auxiliary variable

ρ - Correlation coefficient between X and Y

$\bar{Y}, \bar{X}$ - Population means

$\bar{y}, \bar{x}$ - Sample means

S_y^2, S_x^2 - Population variances

s_y^2, s_x^2 - Sample variances

C_y, C_x - Coefficient of variations

$\gamma = \frac{(1-f)}{n}$

$f = \frac{n}{N}$

$\lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$

$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{Y})^r (x_i - \bar{X})^s$

$\beta_{1(x)} = \frac{\mu_{03}^2}{\mu_{02}^3}$, Skewness of the auxiliary variable

$\beta_{2(x)} = \frac{\mu_{04}}{\mu_{02}^2}$, Kurtosis of the auxiliary variable

Q_1 - First quartile of the auxiliary variable

Q_3 - Third quartile of the auxiliary variable

Q_r - Inter quartile range of the auxiliary variable

Q_d - Semi quartile range of the auxiliary variable

Q_a - Semi quartile average of the auxiliary variable

D_i - ith Decile of the auxiliary variable

$B(.)$ - Bias of the estimator

$V(.)$ - Variance of the estimator

$MSE(.)$ - Mean squared error of the estimator

$PRE(t_e, t_p) = \frac{MSE(t_e)}{MSE(t_p)} * 100$ - Percentage relative efficiency of the estimator t_p over t_e.

2. Review of Literature of Variance Estimators

The simplest and the most appropriate estimator of population variance is the sample variance given by:

$$\hat{S}_0^2 = s_y^2, \tag{2.1}$$

It is unbiased, and its variance up to the first degree of approximation is:

$$V(\hat{S}_0^2) = \gamma S_y^4(\lambda_{40} - 1) \tag{2.2}$$

Isaki (1983) proposed the following ratio estimator of population variance using auxiliary information as:

$$\hat{S}_R^2 = s_y^2\left(\frac{S_x^2}{s_x^2}\right), \tag{2.3}$$

where

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2,$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N}X_i, \bar{Y} = \frac{1}{N}\sum_{i=1}^{N}Y_i,$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n}x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n}y_i.$$

The *Bias* and Mean Square Error (*MSE*) of the estimator in (2.3), up to the first order of approximation, are respectively given by

$$B(\hat{S}_R^2) = \gamma S_y^2[(\lambda_{04} - 1) - (\lambda_{22} - 1)], \tag{2.4}$$

$$MSE(t_R) = \gamma S_y^4\begin{bmatrix}(\lambda_{40} - 1) + (\lambda_{04} - 1) \\ -2(\lambda_{22} - 1)\end{bmatrix} \tag{2.5}$$

Various authors in the literature have proposed so many estimators of population variance latest being Singh and Solanki (2013), Subramani and Kumarapandiyan (2012a, b, c, 2013), tailor and Shrama (2012), and Yadav and Kadilar (2013a, b), Yadav et al (2015a, b) utilizing different parameters of auxiliary variable. Table-1, early given by Subramani and Kumarpandiyan (2015), represents different estimators of population variance with their *Bias* and *MSE.*

Where, $R_i = \frac{S_x^2}{S_x^2 + w_i}$, $i = 1, 2, ..., 51$ and $w_1 = C_x$, $w_2 = \beta_{2(x)}$, $w_3 = \beta_{1(x)}$, $w_4 = \rho$, $w_5 = S_x$, $w_6 = M_d$, $w_7 = Q_1$, $w_8 = Q_3$, $w_9 = Q_r$, $w_{10} = Q_d$, $w_{11} = Q_a$, $w_{12} = D_1$, $w_{13} = D_2$, $w_{14} = D_3$, $w_{15} = D_4$, $w_{16} = D_5$, $w_{17} = D_6$, $w_{18} = D_7$, $w_{19} = D_8$, $w_{20} = D_9$, $w_{21} = D_{10}$, $w_{22} = \frac{C_x}{\beta_{2(x)}}$, $w_{23} = \frac{\beta_{2(x)}}{C_x}$, $w_{24} = \frac{C_x}{\beta_{1(x)}}$, $w_{25} = \frac{\beta_{1(x)}}{C_x}$,

$$w_{26}=\frac{C_x}{\rho}\ ,\ w_{27}=\frac{\rho}{C_x}\ ,\ w_{28}=\frac{C_x}{S_x}\ ,\ w_{29}=\frac{S_x}{C_x}\ ,\ w_{39}=\frac{M_d}{\beta_{2(x)}}\ ,\ w_{40}=\frac{\beta_{1(x)}}{\rho}\ ,\ w_{41}=\frac{\rho}{\beta_{1(x)}}\ ,$$

$$w_{30}=\frac{C_x}{M_d}\ ,\ w_{31}=\frac{M_d}{C_x}\ ,\ w_{32}=\frac{\beta_{2(x)}}{\beta_{1(x)}}\ ,\ w_{42}=\frac{\beta_{1(x)}}{S_x}\ ,\ w_{43}=\frac{S_x}{\beta_{1(x)}}\ ,\ w_{44}=\frac{\beta_{1(x)}}{M_d}\ ,$$

$$w_{33}=\frac{\beta_{1(x)}}{\beta_{21(x)}}\ ,\ w_{34}=\frac{\beta_{2(x)}}{\rho}\ ,\ w_{35}=\frac{\rho}{\beta_{2(x)}}\ ,\ w_{45}=\frac{M_d}{\beta_{1(x)}},\ w_{46}=\frac{\rho}{S_x}\ ,\ w_{47}=\frac{S_x}{\rho}\ ,\ w_{48}=\frac{\beta_{1(x)}}{M_d}\ ,$$

$$w_{36}=\frac{\beta_{2(x)}}{S_x}\ ,\ w_{37}=\frac{S_x}{\beta_{2(x)}}\ ,\ w_{38}=\frac{\beta_{2(x)}}{M_d}\ ,\ w_{49}=\frac{M_d}{\beta_{1(x)}},\ w_{50}=\frac{S_x}{M_d},\ w_{51}=\frac{M_d}{S_x}.$$

Table 1. *Bias* and *MSE* of Various Estimators

Estimator	Bias	MSE
$\hat{S}_1^2=s_y^2\left[\frac{S_x^2+C_x}{s_x^2+C_x}\right]$ Kadilar and Cingi (2006b)	$\gamma S_y^2 R_1\left[\begin{array}{r}R_1(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_1^2(\lambda_{04}-1)\\-2R_1(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_2^2=s_y^2\left[\frac{S_x^2+\beta_{2(x)}}{s_x^2+\beta_{2(x)}}\right]$ Upadhyaya and Singh (1999)	$\gamma S_y^2 R_2\left[\begin{array}{r}R_2(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_2^2(\lambda_{04}-1)\\-2R_2(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_3^2=s_y^2\left[\frac{S_x^2+\beta_{1(x)}}{s_x^2+\beta_{1(x)}}\right]$	$\gamma S_y^2 R_3\left[\begin{array}{r}R_3(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_3^2(\lambda_{04}-1)\\-2R_3(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_4^2=s_y^2\left[\frac{S_x^2+\rho}{s_x^2+\rho}\right]$	$\gamma S_y^2 R_4\left[\begin{array}{r}R_4(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_4^2(\lambda_{04}-1)\\-2R_4(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_5^2=s_y^2\left[\frac{S_x^2+S_x}{s_x^2+S_x}\right]$	$\gamma S_y^2 R_5\left[\begin{array}{r}R_5(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_5^2(\lambda_{04}-1)\\-2R_5(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_6^2=s_y^2\left[\frac{S_x^2+M_d}{s_x^2+M_d}\right]$ Subramani and Kumarpandiyan (2012a)	$\gamma S_y^2 R_6\left[\begin{array}{r}R_6(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_6^2(\lambda_{04}-1)\\-2R_6(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_7^2=s_y^2\left[\frac{S_x^2+Q_1}{s_x^2+Q_1}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_7\left[\begin{array}{r}R_7(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_7^2(\lambda_{04}-1)\\-2R_7(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_8^2=s_y^2\left[\frac{S_x^2+Q_3}{s_x^2+Q_3}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_8\left[\begin{array}{r}R_8(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_8^2(\lambda_{04}-1)\\-2R_8(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_9^2=s_y^2\left[\frac{S_x^2+Q_r}{s_x^2+Q_r}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_9\left[\begin{array}{r}R_9(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{r}(\lambda_{40}-1)+R_9^2(\lambda_{04}-1)\\-2R_9(\lambda_{22}-1)\end{array}\right]$

$\hat{S}_{10}^2 = s_y^2\left[\frac{S_x^2+Q_d}{s_x^2+Q_d}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_{10}\begin{bmatrix} R_{10}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{10}^2(\lambda_{04}-1) \\ -2R_{10}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{11}^2 = s_y^2\left[\frac{S_x^2+Q_a}{s_x^2+Q_a}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_{11}\begin{bmatrix} R_{11}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{11}^2(\lambda_{04}-1) \\ -2R_{11}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{12}^2 = s_y^2\left[\frac{S_x^2+D_1}{s_x^2+D_1}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{12}\begin{bmatrix} R_{12}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{12}^2(\lambda_{04}-1) \\ -2R_{12}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{13}^2 = s_y^2\left[\frac{S_x^2+D_2}{s_x^2+D_2}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{13}\begin{bmatrix} R_{13}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{13}^2(\lambda_{04}-1) \\ -2R_{13}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{14}^2 = s_y^2\left[\frac{S_x^2+D_3}{s_x^2+D_3}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{14}\begin{bmatrix} R_{14}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{14}^2(\lambda_{04}-1) \\ -2R_{14}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{15}^2 = s_y^2\left[\frac{S_x^2+D_4}{s_x^2+D_4}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{15}\begin{bmatrix} R_{15}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{15}^2(\lambda_{04}-1) \\ -2R_{15}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{16}^2 = s_y^2\left[\frac{S_x^2+D_5}{s_x^2+D_5}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{16}\begin{bmatrix} R_{16}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{16}^2(\lambda_{04}-1) \\ -2R_{16}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{17}^2 = s_y^2\left[\frac{S_x^2+D_6}{s_x^2+D_6}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{17}\begin{bmatrix} R_{17}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{17}^2(\lambda_{04}-1) \\ -2R_{17}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{18}^2 = s_y^2\left[\frac{S_x^2+D_7}{s_x^2+D_7}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{18}\begin{bmatrix} R_{18}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{18}^2(\lambda_{04}-1) \\ -2R_{18}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{19}^2 = s_y^2\left[\frac{S_x^2+D_8}{s_x^2+D_8}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{19}\begin{bmatrix} R_{19}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{19}^2(\lambda_{04}-1) \\ -2R_{19}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{20}^2 = s_y^2\left[\frac{S_x^2+D_9}{s_x^2+D_9}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{20}\begin{bmatrix} R_{20}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{20}^2(\lambda_{04}-1) \\ -2R_{20}(\lambda_{22}-1)\end{bmatrix}$

$\hat{S}_{21}^2 = s_y^2\left[\frac{S_x^2 + D_{10}}{s_x^2 + D_{10}}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{21}\begin{bmatrix} R_{21}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{21}^2(\lambda_{04}-1) \\ -2R_{21}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{22}^2 = s_y^2\left[\frac{S_x^2\beta_{2(x)} + C_x}{s_x^2\beta_{2(x)} + C_x}\right]$ Kadilar and Cingi (2006b)	$\gamma S_y^2 R_{22}\begin{bmatrix} R_{22}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{22}^2(\lambda_{04}-1) \\ -2R_{22}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{23}^2 = s_y^2\left[\frac{S_x^2 C_x + \beta_{2(x)}}{s_x^2 C_x + \beta_{2(x)}}\right]$ Kadilar and Cingi (2006b)	$\gamma S_y^2 R_{23}\begin{bmatrix} R_{23}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{23}^2(\lambda_{04}-1) \\ -2R_{23}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{24}^2 = s_y^2\left[\frac{S_x^2\beta_{1(x)} + C_x}{s_x^2\beta_{1(x)} + C_x}\right]$	$\gamma S_y^2 R_{24}\begin{bmatrix} R_{24}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{24}^2(\lambda_{04}-1) \\ -2R_{24}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{25}^2 = s_y^2\left[\frac{S_x^2 C_x + \beta_{1(x)}}{s_x^2 C_x + \beta_{1(x)}}\right]$	$\gamma S_y^2 R_{25}\begin{bmatrix} R_{25}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{25}^2(\lambda_{04}-1) \\ -2R_{25}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{26}^2 = s_y^2\left[\frac{S_x^2\rho + C_x}{s_x^2\rho + C_x}\right]$	$\gamma S_y^2 R_{26}\begin{bmatrix} R_{26}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{26}^2(\lambda_{04}-1) \\ -2R_{26}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{27}^2 = s_y^2\left[\frac{S_x^2 C_x + \rho}{s_x^2 C_x + \rho}\right]$	$\gamma S_y^2 R_{27}\begin{bmatrix} R_{27}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{27}^2(\lambda_{04}-1) \\ -2R_{27}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{28}^2 = s_y^2\left[\frac{S_x^2 S_x + C_x}{s_x^2 S_x + C_x}\right]$	$\gamma S_y^2 R_{28}\begin{bmatrix} R_{28}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{28}^2(\lambda_{04}-1) \\ -2R_{28}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{29}^2 = s_y^2\left[\frac{S_x^2 C_x + S_x}{s_x^2 C_x + S_x}\right]$	$\gamma S_y^2 R_{29}\begin{bmatrix} R_{29}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{29}^2(\lambda_{04}-1) \\ -2R_{29}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{30}^2 = s_y^2\left[\frac{S_x^2 M_d + C_x}{s_x^2 M_d + C_x}\right]$	$\gamma S_y^2 R_{30}\begin{bmatrix} R_{30}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{30}^2(\lambda_{04}-1) \\ -2R_{30}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{31}^2 = s_y^2\left[\frac{S_x^2 C_x + M_d}{s_x^2 C_x + M_d}\right]$ Subramani and Kumarpandiyan (2013)	$\gamma S_y^2 R_{31}\begin{bmatrix} R_{31}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{31}^2(\lambda_{04}-1) \\ -2R_{31}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{32}^2 = s_y^2\left[\frac{S_x^2\beta_{1(x)} + \beta_{2(x)}}{s_x^2\beta_{1(x)} + \beta_{2(x)}}\right]$	$\gamma S_y^2 R_{32}\begin{bmatrix} R_{32}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{32}^2(\lambda_{04}-1) \\ -2R_{32}(\lambda_{22}-1) \end{bmatrix}$
$\hat{S}_{33}^2 = s_y^2\left[\frac{S_x^2\beta_{2(x)} + \beta_{1(x)}}{s_x^2\beta_{2(x)} + \beta_{1(x)}}\right]$	$\gamma S_y^2 R_{33}\begin{bmatrix} R_{33}(\lambda_{04}-1) \\ -(\lambda_{22}-1) \end{bmatrix}$	$\gamma S_y^4\begin{bmatrix} (\lambda_{40}-1)+R_{33}^2(\lambda_{04}-1) \\ -2R_{33}(\lambda_{22}-1) \end{bmatrix}$

$\hat{S}_{34}^2 = s_y^2\left[\dfrac{S_x^2\rho+\beta_{2(x)}}{s_x^2\rho+\beta_{2(x)}}\right]$	$\gamma S_y^2 R_{34}\begin{bmatrix} R_{34}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{34}^2(\lambda_{04}-1) \\ -2R_{34}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{35}^2 = s_y^2\left[\dfrac{S_x^2\beta_{2(x)}+\rho}{s_x^2\beta_{2(x)}+\rho}\right]$	$\gamma S_y^2 R_{35}\begin{bmatrix} R_{35}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{35}^2(\lambda_{04}-1) \\ -2R_{35}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{36}^2 = s_y^2\left[\dfrac{S_x^2 S_x+\beta_{2(x)}}{s_x^2 S_x+\beta_{2(x)}}\right]$	$\gamma S_y^2 R_{36}\begin{bmatrix} R_{36}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{36}^2(\lambda_{04}-1) \\ -2R_{36}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{37}^2 = s_y^2\left[\dfrac{S_x^2\beta_{2(x)}+S_x}{s_x^2\beta_{2(x)}+S_x}\right]$	$\gamma S_y^2 R_{37}\begin{bmatrix} R_{37}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{37}^2(\lambda_{04}-1) \\ -2R_{37}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{38}^2 = s_y^2\left[\dfrac{S_x^2 M_d+\beta_{2(x)}}{s_x^2 M_d+\beta_{2(x)}}\right]$	$\gamma S_y^2 R_{38}\begin{bmatrix} R_{38}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{38}^2(\lambda_{04}-1) \\ -2R_{38}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{39}^2 = s_y^2\left[\dfrac{S_x^2\beta_{2(x)}+M_d}{s_x^2\beta_{2(x)}+M_d}\right]$	$\gamma S_y^2 R_{39}\begin{bmatrix} R_{39}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{39}^2(\lambda_{04}-1) \\ -2R_{39}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{40}^2 = s_y^2\left[\dfrac{S_x^2\rho+\beta_{1(x)}}{s_x^2\rho+\beta_{1(x)}}\right]$	$\gamma S_y^2 R_{40}\begin{bmatrix} R_{40}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{40}^2(\lambda_{04}-1) \\ -2R_{40}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{41}^2 = s_y^2\left[\dfrac{S_x^2\beta_{1(x)}+\rho}{s_x^2\beta_{1(x)}+\rho}\right]$	$\gamma S_y^2 R_{41}\begin{bmatrix} R_{41}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{41}^2(\lambda_{04}-1) \\ -2R_{41}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{42}^2 = s_y^2\left[\dfrac{S_x^2 S_x+\beta_{1(x)}}{s_x^2 S_x+\beta_{1(x)}}\right]$	$\gamma S_y^2 R_{42}\begin{bmatrix} R_{42}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{42}^2(\lambda_{04}-1) \\ -2R_{42}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{43}^2 = s_y^2\left[\dfrac{S_x^2\beta_{1(x)}+S_x}{s_x^2\beta_{1(x)}+S_x}\right]$	$\gamma S_y^2 R_{43}\begin{bmatrix} R_{43}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{43}^2(\lambda_{04}-1) \\ -2R_{43}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{44}^2 = s_y^2\left[\dfrac{S_x^2 M_d+\beta_{1(x)}}{s_x^2 M_d+\beta_{1(x)}}\right]$	$\gamma S_y^2 R_{44}\begin{bmatrix} R_{44}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{44}^2(\lambda_{04}-1) \\ -2R_{44}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{45}^2 = s_y^2\left[\dfrac{S_x^2\beta_{1(x)}+M_d}{s_x^2\beta_{1(x)}+M_d}\right]$	$\gamma S_y^2 R_{45}\begin{bmatrix} R_{45}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{45}^2(\lambda_{04}-1) \\ -2R_{45}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{46}^2 = s_y^2\left[\dfrac{S_x^2 S_x+\rho}{s_x^2 S_x+\rho}\right]$	$\gamma S_y^2 R_{46}\begin{bmatrix} R_{46}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{46}^2(\lambda_{04}-1) \\ -2R_{46}(\lambda_{22}-1)\end{bmatrix}$

$\hat{S}_{47}^2 = s_y^2\left[\dfrac{S_x^2\rho + S_x}{s_x^2\rho + S_x}\right]$	$\gamma S_y^2 R_{47}\begin{bmatrix} R_{47}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{47}^2(\lambda_{04}-1) \\ -2R_{47}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{48}^2 = s_y^2\left[\dfrac{S_x^2 M_d + \rho}{s_x^2 M_d + \rho}\right]$	$\gamma S_y^2 R_{48}\begin{bmatrix} R_{48}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{48}^2(\lambda_{04}-1) \\ -2R_{48}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{49}^2 = s_y^2\left[\dfrac{S_x^2\rho + M_d}{s_x^2\rho + M_d}\right]$	$\gamma S_y^2 R_{49}\begin{bmatrix} R_{49}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{49}^2(\lambda_{04}-1) \\ -2R_{49}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{50}^2 = s_y^2\left[\dfrac{S_x^2 M_d + S_x}{s_x^2 M_d + S_x}\right]$	$\gamma S_y^2 R_{50}\begin{bmatrix} R_{50}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{50}^2(\lambda_{04}-1) \\ -2R_{50}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}_{51}^2 = s_y^2\left[\dfrac{S_x^2 S_x + M_d}{s_x^2 S_x + M_d}\right]$	$\gamma S_y^2 R_{51}\begin{bmatrix} R_{51}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_{51}^2(\lambda_{04}-1) \\ -2R_{51}(\lambda_{22}-1)\end{bmatrix}$

Thus the bias and MSE of 51 estimators mentioned above in the table may be written as,

$$B(\hat{S}_i^2) = \gamma S_y^2 R_i\left[R_i(\lambda_{04}-1) \quad -(\lambda_{22}-1)\right],$$
$$i = 1,2,...,51$$

$$MSE(\hat{S}_i^2) = \gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+R_i^2(\lambda_{04}-1) \\ -2R_i(\lambda_{22}-1)\end{bmatrix}$$
$$i = 1,2,...,51 \tag{2.6}$$

Upadhyaya and Singh (2001) proposed the following estimator of population variance utilizing the population mean of the auxiliary variable as,

$$\hat{S}_{52}^2 = s_y^2\left[\frac{\bar{X}}{\bar{x}}\right] \tag{2.7}$$

The bias and the mean squared error of estimator up to the first order of approximation respectively are,

$$B(\hat{S}_{52}^2) = \gamma S_y^2\left[C_x^2 - \lambda_{21}C_x\right] \tag{2.8}$$

$$MSE(\hat{S}_{52}^2) = \gamma S_y^4\left[(\lambda_{40}-1) + C_x^2 - 2\lambda_{21}C_x\right]$$

$$\text{where } \lambda_{21} = \frac{\mu_{21}}{\mu_{20}\sqrt{\mu_{02}}} \tag{2.9}$$

Subramani and Kumarpandiyan (2015), using population mean of the auxiliary variable proposed the following modified estimators for population variance as,

$$\hat{S}_{Pi}^2 = s_y^2\left[\frac{\bar{X}+w_i}{\bar{x}+w_i}\right],\ i = 1,2,...,51 \tag{2.10}$$

The bias and the mean squared error of the estimator in (2.10), up to the first order of approximations respectively are,

$$B(\hat{S}_{Pi}^2) = \gamma S_y^2\left[\theta_{Pi}^2 C_x^2 - \theta_{Pi}\lambda_{21}C_x\right] \tag{2.11}$$

$$MSE(\hat{S}_{Pi}^2) = \gamma S_y^4\begin{bmatrix}(\lambda_{40}-1)+\theta_{Pi}^2 C_x^2 \\ -2\theta_{Pi}\lambda_{21}C_x\end{bmatrix},$$
$$i = 1,2,...,51 \tag{2.12}$$

Where $\theta_{Pi} = \dfrac{\bar{X}}{\bar{X}+w_i}$, $i = 1,2,...,51$.

3. Proposed Class of Estimators

Motivated by Yadav et al (2015b), we proposed the following class of estimators using a characterizing scalar α as,

$$\hat{S}_{\alpha i}^2 = s_y^2\left[\alpha + (1-\alpha)\left\{\frac{\bar{X}+w_i}{\bar{x}+w_i}\right\}\right], \tag{3.1}$$
$$i = 1,2,...,51$$

Where, α is a characterizing scalar obtained by minimizing the mean squared error of the proposed estimator $\hat{S}_{\alpha i}^2$.

To study the large sample properties of the proposed class of estimators, we define $s_y^2 = S_y^2(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ such that $E(e_0) = E(e_1) = 0$ and $E(e_0^2) = \gamma(\lambda_{40}-1)$,

$E(e_1^2) = \gamma C_x^2$, $E(e_0 e_1) = \gamma \lambda_{21} C_x$.

Using above approximations, the proposed estimator may be written as,

$$\hat{S}_{\alpha i}^2 = S_y^2(1+e_0)\left[\alpha + (1-\alpha)\left\{\frac{\bar{X}+w_i}{\bar{X}\left(1+e_1\right)+w_i}\right\}\right]$$

$$= S_y^2(1+e_0)\left[\alpha + (1-\alpha)\left\{\frac{\bar{X}+w_i}{\bar{X}+\bar{X}e_1+w_i}\right\}\right]$$

$$= S_y^2(1+e_0)\left[\alpha + (1-\alpha)\left\{\frac{1}{1+\frac{\bar{X}}{\bar{X}+w_i}e_1}\right\}\right]$$

$$= S_y^2(1+e_0)\left[\alpha + (1-\alpha)\left\{\frac{1}{1+\theta_{pi}e_1}\right\}\right]$$

$$= S_y^2(1+e_0)[\alpha + (1-\alpha)(1+\theta_{pi}e_1)^{-1}]$$

$$= S_y^2(1+e_0)\begin{bmatrix}\alpha + (1-\alpha)(1-\theta_{Pi}e_1 \\ +\theta_{Pi}^2 e_1^2 - ...)\end{bmatrix}$$

$$= S_y^2\begin{bmatrix}1+e_0-\theta_{pi}e_1+\theta_{pi}^2e_1^2-\theta_{pi}e_0e_1 \\ +\alpha\theta_{pi}e_1-\alpha\theta_{pi}^2e_1^2+\alpha\theta_{pi}e_0e_1\end{bmatrix}$$

Through the order of approximations, retaining the terms up to the first order, we have

$$\hat{S}_{\alpha i}^2 = S_y^2\begin{bmatrix}1+e_0-\theta_{pi}e_1+\theta_{pi}^2e_1^2-\theta_{pi}e_0e_1 \\ +\alpha\theta_{pi}e_1-\alpha\theta_{pi}^2e_1^2+\alpha\theta_{pi}e_0e_1\end{bmatrix} \quad (3.2)$$

Subtracting S_y^2 on both sides of (3.2), we have

$$\hat{S}_{\alpha i}^2 - S_y^2 = S_y^2\begin{bmatrix}e_0-\theta_{pi}e_1+\theta_{pi}^2e_1^2-\theta_{pi}e_0e_1 \\ +\alpha\theta_{pi}e_1-\alpha\theta_{pi}^2e_1^2+\alpha\theta_{pi}e_0e_1\end{bmatrix} \quad (3.3)$$

Taking expectation on both the sides of equation (3.3), we get

$$E(\hat{S}_{\alpha i}^2 - S_y^2) = S_y^2\begin{bmatrix}E(e_0)-\theta_{pi}E(e_1)+\theta_{pi}^2(e_1^2) \\ -\theta_{pi}(e_0e_1)+\alpha\theta_{pi}(e_1) \\ -\alpha\theta_{pi}^2(e_1^2)+\alpha\theta_{pi}(e_0e_1)\end{bmatrix}$$

Putting the values of different expectations in above equation, we get the bias of the proposed class of estimators as,

$$B(\hat{S}_{\alpha i}^2) = \gamma S_y^2\begin{bmatrix}\theta_{pi}^2C_x^2-\theta_{pi}\lambda_{21}C_x \\ -\alpha\theta_{pi}^2C_x^2+\alpha\theta_{pi}\lambda_{21}C_x\end{bmatrix} \quad (3.4)$$

Squaring on both sides of (3.3), simplifying and retaining the terms up to the first order of approximation, we have,

$$[\hat{S}_{\alpha i}^2 - S_y^2]^2 = S_y^4\begin{bmatrix}e_0^2+\theta_{pi}^2e_1^2+\alpha^2\theta_{pi}^2e_1^2 \\ -2\theta_{pi}e_0e_1+2\alpha\theta_{pi}e_0e_1 \\ -2\alpha\theta_{pi}^2e_1^2\end{bmatrix} \quad (3.5)$$

Taking expectations on both the sides of equation (3.5), we get the mean squared error of the proposed class of estimators up to the first order of approximation as,

$$MSE(\hat{S}_{\alpha i}^2) = S_y^4\begin{bmatrix}E(e_0^2)+\theta_{pi}^2E(e_1^2) \\ +\alpha^2\theta_{pi}^2E(e_1^2)-2\theta_{pi}E(e_0e_1) \\ +2\alpha\theta_{pi}E(e_0e_1)-2\alpha\theta_{pi}^2E(e_1^2)]\end{bmatrix} \quad (3.6)$$

Putting the values of different expectations in above equations and simplifying, we have,

$$MSE(\hat{S}_{\alpha i}^2) = \gamma S_y^4\begin{bmatrix}\left(\lambda_{40}-1\right)+\theta_{pi}^2C_x^2 \\ +\alpha^2\theta_{pi}^2C_x^2-2\theta_{pi}\lambda_{21}C_x \\ +2\alpha\theta_{pi}\lambda_{21}C_x-2\alpha\theta_{pi}^2C_x^2\end{bmatrix} \quad (3.7)$$

The optimum value of the characterizing scalar α is obtained by minimizing MSE in (3.7) using the method of maxima-minima as,

$$2\alpha\theta_{pi}^2C_x^2+2\theta_{pi}\lambda_{21}C_x-2\theta_{pi}^2C_x^2=0$$

$$\alpha = \frac{\theta_{pi}^2C_x^2-\theta_{pi}\lambda_{21}C_x}{\theta_{pi}^2C_x^2} = \frac{\mu}{\beta} \quad (3.8)$$

Where $\mu = \theta_{pi}^2C_x^2-\theta_{pi}\lambda_{21}C_x$ and $\beta = \theta_{pi}^2C_x^2$

The minimum value of bias of the proposed class of estimators is obtained by putting optimum value of α in (3.4) as,

$$B(\hat{S}_{\alpha i}^2) = S_y^2\left[\gamma\,(\theta_{pi}^2C_x^2-\theta_{pi}\lambda_{21}C_x)-\frac{\mu^2}{\beta}\right] \quad (3.9)$$

Minimum value of the MSE of the proposed class of estimators is obtained by putting the optimum value of α in (3.7) and thus the minimum MSE is given as,

$$MSE(\hat{S}_{\alpha i}^2) = S_y^4\begin{bmatrix}\gamma\{(\lambda_{40}-1)+\theta_{pi}^2C_x^2 \\ -2\theta_{pi}\lambda_{21}C_x\}-\frac{\mu^2}{\beta}\end{bmatrix} \quad (3.10)$$

4. Efficiency of the Proposed Class of Estimators

From (2.2) and (3.10), we have:

$$V(t_0) - MSE_{\min}(\hat{S}^2_{\alpha i})$$
$$= S_y^4 \left[\frac{\mu^2}{\beta} - \gamma\{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} \right] > 0$$

if $\gamma\{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} < \frac{\mu^2}{\beta}$ (4.1)

From (2.5) and (3.10), we have:

$$MSE(t_R) - MSE_{\min}(\hat{S}^2_{\alpha i})$$
$$= S_y^4 \begin{bmatrix} \gamma\{(\lambda_{04}-1) - 2(\lambda_{22}-1)\} \\ -\gamma\{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} + \frac{\mu^2}{\beta} \end{bmatrix} > 0$$

If $\gamma \begin{bmatrix} \{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} \\ -\{(\lambda_{04}-1) - 2(\lambda_{22}-1)\} \end{bmatrix} < \frac{\mu^2}{\beta}$ (4.2)

From (2.6) and (3.10), we have:

$$MSE(\hat{S}_i^2) - MSE_{\min}(\hat{S}^2_{\alpha i})$$
$$= S_y^4 \begin{bmatrix} \gamma\{R_i^2(\lambda_{04}-1) - 2R_i(\lambda_{22}-1)\} \\ -\gamma\{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} + \frac{\mu^2}{\beta} \end{bmatrix} > 0,$$

If $\gamma \begin{bmatrix} \{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} \\ -\{R_i^2(\lambda_{04}-1) - 2R_i(\lambda_{22}-1)\} \end{bmatrix} < \frac{\mu^2}{\beta},$

$(i = 1, 2, ..., 51)$ (4.3)

From (2.9) and (3.10), we have:

$$MSE(\hat{S}^2_{52}) - MSE_{\min}(\hat{S}^2_{\alpha i})$$
$$= S_y^4 \begin{bmatrix} \gamma\{C_x^2 - 2\lambda_{21}C_x\} \\ -\gamma\{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} + \frac{\mu^2}{\beta} \end{bmatrix} > 0,$$

If $\gamma \begin{bmatrix} \{\theta_{pi}^2 C_x^2 - 2\theta_{pi}\lambda_{21}C_x\} \\ -\{C_x^2 - 2\lambda_{21}C_x\} \end{bmatrix} < \frac{\mu^2}{\beta}$ (4.4)

From (2.12) and (3.10), we have:

$$MSE(\hat{S}^2_{Pi}) - MSE_{\min}(\hat{S}^2_{\alpha i}) = S_y^4 \left[\frac{\mu^2}{\beta} \right] > 0,$$

if $\mu^2 > \beta,$ (4.5)

5. Numerical Study

To judge the performances of the proposed class of estimators along with the existing modified ratio type estimators of population variance given in this manuscript, we have considered two natural populations, the population-1 and Population-2. Both the populations have been taken from the book by Singh and Chaudhary (1986) at page 177. The population parameters of above populations are given below as:

Population-1: Singh and Chaudhary (1986)

$N = 34$, $n = 20$, $\bar{Y} = 85.6412$, $\bar{X} = 20.8882$, $S_y = 73.3141$, $\lambda_{22} = 1.1525$, $S_x = 15.0506$, $C_x = 0.7205$, $\lambda_{40} = 13.3666$, $\lambda_{04} = 2.9123$, $\beta_1(x) = 0.8732$, $\lambda_{21} = -0.3104$, $M_d = 15$, $Q_1 = 9.4250$, $Q_3 = 25.4750$, $Q_r = 16.05$, $Q_d = 16.05$, $Q_a = 17.45$, $D_1 = 7.03$, $D_2 = 7.68$, $D_3 = 10.82$, $D_4 = 12.94$, $D_5 = 15$, $D_6 = 22.72$, $D_7 = 25.04$, $D_8 = 33.56$, $D_9 = 43.61$, $D_{10} = 56.40$.

Population-2: Singh and Chaudhary (1986)

$N = 34$, $n = 20$, $\bar{Y} = 85.6412$, $\bar{X} = 19.9441$, $S_y = 73.3141$, $\lambda_{22} = 1.2244$, $S_x = 15.0215$, $C_x = 0.7532$, $\lambda_{40} = 13.3666$, $\lambda_{04} = 3.7257$, $\beta_1(x) = 1.2758$, $\lambda_{21} = -0.2946$, $M_d = 14.25$, $Q_1 = 9.925$, $Q_3 = 27.80$, $Q_r = 17.875$, $Q_d = 8.9375$, $Q_a = 18.8625$, $D_1 = 6.06$, $D_2 = 8.30$, $D_3 = 10.27$, $D_4 = 1.12$, $D_5 = 14.25$, $D_6 = 21.02$, $D_7 = 26.45$, $D_8 = 30.44$, $D_9 = 37.32$, $D_{10} = 63.40$.

The following table represents the Bias and the Mean Squared Error (MSE) of proposed class of estimators with the existing modified ratio type estimators of population variance mentioned in the manuscript as,

Table 2. *Bias* and *MSE* of Various Estimators for Population-1

Estimator	Bias (.)	MSE (.)	Estimator	Bias (.)	MSE (.)
$\hat{S}_1^2$	274.8748	8305040.9247	$\hat{S}_{P1}^2$	77.6142	7901409.5289
$\hat{S}_2^2$	267.4800	8285277.5233	$\hat{S}_{P2}^2$	65.9785	7827023.2643
$\hat{S}_3^2$	273.5533	8303644.9272	$\hat{S}_{P30}^2$	76.6955	7895569.2207
$\hat{S}_4^2$	275.6578	8307529.5562	$\hat{S}_{P4}^2$	79.2931	7912069.8376
$\hat{S}_5^2$	241.8679	8185653.1155	$\hat{S}_{P5}^2$	33.7963	7614602.8707
$\hat{S}_6^2$	243.5012	8186036.5850	$\hat{S}_{P6}^2$	33.8714	7615115.3167
$\hat{S}_7^2$	276.2783	8309384.7128	$\hat{S}_{P7}^2$	80.5802	7920230.8059
$\hat{S}_8^2$	264.5267	8275317.6357	$\hat{S}_{P8}^2$	61.0748	7795375.0862
$\hat{S}_9^2$	275.2885	8304084.3017	$\hat{S}_{P9}^2$	76.9829	7897396.7792
$\hat{S}_{10}^2$	272.5028	8300558.7975	$\hat{S}_{P10}^2$	74.7213	7883000.6155
$\hat{S}_{11}^2$	272.5089	8297000.4445	$\hat{S}_{P11}^2$	72.5377	7869069.1599
$\hat{S}_{12}^2$	275.2863	8305931.4127	$\hat{S}_{P39}^2$	78.2088	7905186.9248
$\hat{S}_{13}^2$	276.6674	8311225.1070	$\hat{S}_{P13}^2$	81.8881	7928514.0560
$\hat{S}_{14}^2$	231.8128	8143027.5287	$\hat{S}_{P42}^2$	26.7408	7565872.9971
$\hat{S}_{15}^2$	276.6604	8311223.6155	$\hat{S}_{P15}^2$	81.8870	7928507.2658
$\hat{S}_{16}^2$	233.8546	8143521.8877	$\hat{S}_{P16}^2$	26.8101	7566357.2506
$\hat{S}_{17}^2$	269.4550	8281531.8967	$\hat{S}_{P17}^2$	64.0680	7814716.2766
$\hat{S}_{18}^2$	275.9200	8308901.9409	$\hat{S}_{P18}^2$	80.2422	7918089.0587
$\hat{S}_{19}^2$	256.5008	8254285.7975	$\hat{S}_{P47}^2$	52.3462	7738509.3354
$\hat{S}_{20}^2$	276.4897	8310243.7205	$\hat{S}_{P20}^2$	81.1867	7924073.4637
$\hat{S}_{21}^2$	276.1611	8309881.4741	$\hat{S}_{P21}^2$	80.9301	7922448.0103
$\hat{S}_{22}^2$	266.7349	8265536.8722	$\hat{S}_{P22}^2$	56.7674	7767404.1126
$\hat{S}_{23}^2$	276.1268	8309875.4575	$\hat{S}_{P23}^2$	80.9259	7922421.0747
$\hat{S}_{24}^2$	267.2385	8265686.6605	$\hat{S}_{P24}^2$	56.8299	7767811.3938
$\hat{S}_{25}^2$	269.5908	8293932.9140	$\hat{S}_{P25}^2$	70.7307	7857516.3340
$\hat{S}_{26}^2$	275.9032	8306930.9379	$\hat{S}_{P26}^2$	78.8843	7909475.9332

$\hat{S}^2_{27}$	276.5782	8311131.4101	$\hat{S}^2_{51}$	81.8207	7928087.7466
$\hat{S}^2_{28}$	248.8531	8169328.3846	$\hat{S}^2_{P28}$	30.8108	7594123.3077
$\hat{S}^2_{29}$	276.5665	8311129.6027	$\hat{S}^2_{P29}$	81.8194	7928079.5282
$\hat{S}^2_{30}$	250.1860	8169755.4807	$\hat{S}^2_{P30}$	30.8840	7594627.7126
$\hat{S}^2_{31}$	276.7199	8311391.6909	$\hat{S}^2_{P31}$	82.0080	7929273.2247
$\hat{S}^2_{32}$	206.6709	8060755.6421	$\hat{S}^2_{P32}$	17.9756	7503320.8713
$\hat{S}^2_{33}$	276.7158	8311390.7610	$\hat{S}^2_{P33}$	82.0073	7929268.9825
$\hat{S}^2_{34}$	209.6060	8061435.6153	$\hat{S}^2_{P34}$	18.0305	7503722.1084
$\hat{S}^2_{35}$	274.1127	8696146.6470	$\hat{S}^2_{P35}$	75.9268	7890677.8152
$\hat{S}^2_{36}$	274.3795	8697518.9442	$\hat{S}^2_{P36}$	75.9662	7890929.0386
$\hat{S}^2_{37}$	252.9612	8587461.1638	$\hat{S}^2_{P37}$	44.3384	7685618.9421
$\hat{S}^2_{38}$	216.9786	8403072.8621	$\hat{S}^2_{P38}$	22.8150	7538189.7352
$\hat{S}^2_{39}$	235.9633	8500271.5781	$\hat{S}^2_{P39}$	32.3710	7604849.6035
$\hat{S}^2_{40}$	255.1960	8598934.7234	$\hat{S}^2_{P40}$	47.8702	7709041.6847
$\hat{S}^2_{41}$	233.9686	8490049.7272	$\hat{S}^2_{P41}$	30.5426	7592273.8712
$\hat{S}^2_{42}$	261.8741	8633234.9291	$\hat{S}^2_{P42}$	50.6830	7727586.6369
$\hat{S}^2_{43}$	256.6540	8606421.4282	$\hat{S}^2_{P43}$	48.8148	7715279.6791
$\hat{S}^2_{44}$	252.1872	8583487.6588	$\hat{S}^2_{P44}$	41.2404	7664933.2877
$\hat{S}^2_{45}$	250.2947	8573774.1884	$\hat{S}^2_{P45}$	37.1913	7637676.2776
$\hat{S}^2_{46}$	243.5012	8538919.0760	$\hat{S}^2_{P46}$	33.8714	7615115.3167
$\hat{S}^2_{47}$	229.6949	8468156.2743	$\hat{S}^2_{P47}$	25.0396	7553934.3985
$\hat{S}^2_{48}$	219.4277	8415600.5132	$\hat{S}^2_{P48}$	23.1425	7540518.2755
$\hat{S}^2_{49}$	212.3024	8379163.5680	$\hat{S}^2_{P49}$	17.9527	7503153.4053
$\hat{S}^2_{50}$	200.7812	8320314.8836	$\hat{S}^2_{P50}$	14.0431	7474185.5247
$\hat{S}^2_{51}$	164.4345	8135286.8030	$\hat{S}^2_{P51}$	10.8871	7450090.5703
$\hat{S}^2_{52}$	87.3340	7957053.4638	$\hat{S}^2_{\alpha 51}$	**22.6402**	**7163549.0143**
$\hat{S}^2_{R}$	210.6359	8709947.6355			

Table 3. *Bias* and *MSE* of Various Estimators for Population-2

Estimator	Bias (.)	MSE (.)	Estimator	Bias (.)	MSE (.)
$\hat{S}_1^2$	274.8748	8700066.3664	$\hat{S}_{P1}^2$	81.9543	7923335.0831
$\hat{S}_2^2$	267.4800	8662043.5448	$\hat{S}_{P2}^2$	65.2611	7817638.5301
$\hat{S}_3^2$	273.5533	8693269.2999	$\hat{S}_{P30}^2$	78.5355	7901826.3797
$\hat{S}_4^2$	275.6578	8704093.4323	$\hat{S}_{P4}^2$	84.0855	7936710.5220
$\hat{S}_5^2$	241.8679	8530542.5925	$\hat{S}_{P5}^2$	34.4309	7616001.5114
$\hat{S}_6^2$	243.5012	8538919.0760	$\hat{S}_{P6}^2$	35.6790	7624408.3942
$\hat{S}_7^2$	276.2783	8707285.4442	$\hat{S}_{P7}^2$	85.8340	7947666.4656
$\hat{S}_8^2$	264.5267	8646864.7198	$\hat{S}_{P8}^2$	59.9815	7783806.2406
$\hat{S}_9^2$	275.2885	8702194.1132	$\hat{S}_{P9}^2$	83.0702	7930341.3699
$\hat{S}_{10}^2$	272.5028	8687867.1404	$\hat{S}_{P10}^2$	75.9674	7885626.4956
$\hat{S}_{11}^2$	272.5089	8687898.6943	$\hat{S}_{P11}^2$	75.9820	7885718.9285
$\hat{S}_{12}^2$	275.2863	8702182.9335	$\hat{S}_{P39}^2$	83.0643	7930304.2040
$\hat{S}_{13}^2$	276.6674	8709286.6597	$\hat{S}_{P13}^2$	86.9579	7954701.1868
$\hat{S}_{14}^2$	231.8128	8479004.9265	$\hat{S}_{P42}^2$	27.9728	7571999.8359
$\hat{S}_{15}^2$	276.6604	8709250.8872	$\hat{S}_{P15}^2$	86.9377	7954574.2911
$\hat{S}_{16}^2$	233.8546	8489465.8908	$\hat{S}_{P16}^2$	29.1347	7579983.9267
$\hat{S}_{17}^2$	269.4550	8672196.4026	$\hat{S}_{P17}^2$	69.1858	7842651.4287
$\hat{S}_{18}^2$	275.9200	8705442.4919	$\hat{S}_{P18}^2$	84.8179	7941301.8389
$\hat{S}_{19}^2$	256.5008	8605634.9323	$\hat{S}_{P47}^2$	48.4540	7709070.5888
$\hat{S}_{20}^2$	276.4897	8708372.8292	$\hat{S}_{P20}^2$	86.4420	7951472.7129
$\hat{S}_{21}^2$	276.1611	8706682.5452	$\hat{S}_{P21}^2$	85.4996	7945572.4829
$\hat{S}_{22}^2$	266.7349	8658213.4456	$\hat{S}_{P22}^2$	63.8659	7808719.1725
$\hat{S}_{23}^2$	276.1268	8706506.0880	$\hat{S}_{P23}^2$	85.4021	7944961.8077
$\hat{S}_{24}^2$	267.2385	8660802.2542	$\hat{S}_{P24}^2$	64.8041	7814718.1590
$\hat{S}_{25}^2$	269.5908	8672894.5544	$\hat{S}_{P25}^2$	69.4685	7844448.5663
$\hat{S}_{26}^2$	275.9032	8705356.1073	$\hat{S}_{P26}^2$	84.7708	7941006.1426

$\hat{S}_{27}^2$	276.5782	8708828.2800	$\hat{S}_{51}^2$	86.6986	7953078.3599
$\hat{S}_{28}^2$	248.8531	8566375.7830	$\hat{S}_{P28}^2$	40.2619	7655051.1228
$\hat{S}_{29}^2$	276.5665	8708767.7148	$\hat{S}_{P29}^2$	86.6644	7952864.4529
$\hat{S}_{30}^2$	250.1860	8573216.2487	$\hat{S}_{P30}^2$	41.5362	7663513.4206
$\hat{S}_{31}^2$	276.7199	8709556.8046	$\hat{S}_{P31}^2$	87.1113	7955660.8324
$\hat{S}_{32}^2$	206.6709	8350388.1091	$\hat{S}_{P32}^2$	17.7903	7500315.7683
$\hat{S}_{33}^2$	276.7158	8709535.6494	$\hat{S}_{P33}^2$	87.0993	7955585.5952
$\hat{S}_{34}^2$	209.6060	8365383.3370	$\hat{S}_{P34}^2$	18.6824	7506746.2177
$\hat{S}_{35}^2$	274.1127	8696146.6470	$\hat{S}_{P35}^2$	79.9563	7910773.1951
$\hat{S}_{36}^2$	274.3795	8697518.9442	$\hat{S}_{P36}^2$	80.6475	7915121.2084
$\hat{S}_{37}^2$	252.9612	8587461.1638	$\hat{S}_{P37}^2$	44.3855	7682352.0994
$\hat{S}_{38}^2$	216.9786	8403072.8621	$\hat{S}_{P38}^2$	21.2121	7524802.2329
$\hat{S}_{39}^2$	235.9633	8500271.5781	$\hat{S}_{P39}^2$	30.4083	7588699.7565
$\hat{S}_{40}^2$	255.1960	8598934.7234	$\hat{S}_{P40}^2$	46.8929	7698842.0318
$\hat{S}_{41}^2$	233.9686	8490049.7272	$\hat{S}_{P41}^2$	29.2015	7580442.3507
$\hat{S}_{42}^2$	261.8741	8633234.9291	$\hat{S}_{P42}^2$	55.7610	7756593.1089
$\hat{S}_{43}^2$	256.6540	8606421.4282	$\hat{S}_{P43}^2$	48.6422	7710301.9522
$\hat{S}_{44}^2$	252.1872	8583487.6588	$\hat{S}_{P44}^2$	43.5628	7676923.4831
$\hat{S}_{45}^2$	250.2947	8573774.1884	$\hat{S}_{P45}^2$	41.6428	7664219.6974
$\hat{S}_{46}^2$	243.5012	8538919.0760	$\hat{S}_{P46}^2$	35.6790	7624408.3942
$\hat{S}_{47}^2$	229.6949	8468156.2743	$\hat{S}_{P47}^2$	26.8362	7564156.3652
$\hat{S}_{48}^2$	219.4277	8415600.5132	$\hat{S}_{P48}^2$	22.1574	7531487.3776
$\hat{S}_{49}^2$	212.3024	8379163.5680	$\hat{S}_{P49}^2$	19.5567	7513015.7817
$\hat{S}_{50}^2$	200.7812	8320314.8836	$\hat{S}_{P50}^2$	16.1672	7488520.5228
$\hat{S}_{51}^2$	164.4345	8135286.8030	$\hat{S}_{P51}^2$	9.4709	7438144.1980
$\hat{S}_{52}^2$	87.3340	7957053.4638	$\hat{S}_{\alpha 51}^2$	22.4704	7125447.0436
$\hat{S}_{R}^2$	210.6359	8709947.6355			

Note: Since the estimator $\hat{S}^2_{P51}$ of the class of estimators proposed by Subramani and Kumarapandiyan (2015) has minimum MSE among its class and the rest estimators mentioned in above table except the proposed estimator. Therefore we have computed only $\hat{S}^2_{\alpha 51}$ estimator of proposed class of estimators for both the populations corresponding to $\hat{S}^2_{P51}$ estimator.

6. Conclusions

In the present paper we develop a generalized improved class of estimators for population variance using population mean of the auxiliary variable by applying the scalar constant to Subramani and Kumarapandiyan (2015) estimator of population variance. The bias and the mean squared error of the proposed general class of estimators have been obtained up to the first order of approximation. The optimum value of the characterizing scalar has been obtained up to the first order of approximation and for this optimum value of the scalar, the minimum mean squared error of the proposed class of estimators have been obtained. From table-2 and table-3, we observe that the estimator $\hat{S}^2_{\alpha 51}$ of the proposed class of estimators has the minimum mean squared error as compared to other mentioned existing estimators of population variance in this manuscript. Therefore it is strongly recommended that the proposed generalized class of estimators should be preferably used for the estimation of population variance using auxiliary variable under simple random sampling scheme.

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