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Developing Efficient Ratio and Product Type Exponential Estimators of Population Mean under Two Phase Sampling for Stratification

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Abstract This manuscript deals with the estimation of population mean of the main variable Y under study using auxiliary information under two phase sampling for stratification. When the main variable under consideration is homogeneous and the mean of the auxiliary variable is not known, double sampling is used to estimate the population mean of the main variable. Stratified random sampling is used when the main variable under study is not homogeneous. In the present manuscript we have proposed the efficient ratio and product type exponential estimators under two phase (double) sampling scheme for stratification for estimating the population mean. The expressions for the biases and mean square errors of proposed estimators have been obtained up to the first order of approximation. The minimum mean square errors have also been obtained for the proposed estimators. A comparison has been made with the existing estimators of population mean in double sampling for stratification. An empirical study is carried out to meet out the theoretical findings.

Keywords Auxiliary variable, Double sampling, Stratification, Bias, Mean square error

1. Introduction

The proper use of auxiliary information increases the efficiencies of the estimators of the population parameters. This information is supplied by the auxiliary variable (X) and it is completely known to the experimenter. The auxiliary variable is highly (positively or negatively) correlated with the main variable (Y) under study. When X is highly positively correlated with that of Y, ratio type estimators are used to estimate the population parameters while product type estimators are used when X is highly negatively correlated with Y, otherwise regression estimators are used to estimate the population parameters.

Cochran [1] utilized the positively correlated auxiliary information and proposed the traditional ratio estimator of population mean of study variable. Robson [2] proposed the traditional product estimator of population mean using negatively correlated auxiliary information. Later many authors including Upadhyaya and Singh [3], Singh [4], Singh and Tailor [5], Singh *et.al* ([6], [7], [8]), Singh *et al.* [9], Kadilar and Cingi ([10], [11]), Tailor and Sharma [12], Yan and Tian [13], Yadav [14], Pandey *et al.* [15], Subramani and Kumarapandiyan [16], Solanki *et al.* [17], Onyeka [18], Jeelani *et.al* [19], Yadav and Kadilar [20] etc. used auxiliary information for improved estimation of population mean of the study variable.

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2. Methods and Material

Let the finite population consist of N distinct and identifiable heterogeneous units for the characteristic under study. In this situation simple random sampling is not an appropriate technique for estimating population mean as it will have a very large variance. To overcome this problem we divide the whole population into relatively homogeneous groups known as strata. Let the whole population is divided into L strata of size N_h ($h = 1,2,...,L$) with $W_h = \frac{N_h}{N}$ as strata weights. When these strata weights are not known, two phase sampling scheme for stratification is used. Following procedure has been followed for two phase sampling scheme for stratification as given by Tailor et.al [21],

(1) A sample of size n' is drawn at first phase using simple random sampling without replacement technique and the observations are taken on auxiliary variable X only.
(2) This sample of size n' is stratified into L strata based on auxiliary variable. Let n'_h be the number of units in hth stratum ($h = 1,2,...,L$) such that

$\sum_{h=1}^{L} n'_h = n'$.

(3) From these n' units, a sample of size $n_h = \nu_h n'_h$ is drawn, where $0 < \nu_h < 1$ is the predetermined probability of selecting a sample of size n_h from strata of size n'_h and it constitutes a sample of size $\sum_{h=1}^{L} n_h = n$. From this sample observations on both the variables Y and X are taken.

Following notations have been used in this manuscript,

$n = \sum_{h=1}^{L} n_h$ - Size of the sample at second phase

$w_h = \frac{n_h}{n}$ - h^th^ stratum weight for the second phase sample

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$ - h^{th} stratum mean for the study variable Y

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$ - h^{th} stratum mean for the auxiliary variable X

$S_y^2 = \frac{1}{N-1}\sum_{h=1}^{L}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$ - Population mean square deviation for the study variable Y

$S_x^2 = \frac{1}{N-1}\sum_{h=1}^{L}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$ - Population mean square deviation for the auxiliary variable X

$S_{yh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)^2$ - h^{th} stratum mean square deviation for the study variable Y

$S_{xh}^2 = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(x_{hi} - \bar{X}_h)^2$ - h^{th} stratum mean square deviation for the auxiliary variable X

$S_{yxh} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h)$ - Covariance between y and x in h^{th} stratum

$\rho_{yxh} = \frac{S_{yxh}}{S_{yh}S_{xh}}$ - Correlation coefficient between y and x in the h^{th} stratum

$\bar{y}_{ds} = \frac{1}{n}\sum_{h=1}^{L} w_h \bar{y}_h$ - Unbiased estimator of population mean $\bar{Y}$ in second phase sampling

$\bar{x}_{ds} = \frac{1}{n}\sum_{h=1}^{L} w_h \bar{x}_h$ - Unbiased estimator of population mean $\bar{X}$ in second phase sampling

$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} \bar{y}_{hi}$ - h^{th} stratum mean at second phase sampling for the study variable Y

$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} \bar{x}_{hi}$ - h^{th} stratum mean at second phase sampling for the auxiliary variable X

$\bar{x}'_h = \frac{1}{n'_h}\sum_{i=1}^{n'_h} \bar{x}_{hi}$ - h^{th} stratum mean at first phase sampling for the auxiliary variable X

$f = \frac{n'}{N}$ - First phase sampling fraction

Hansen et.al [22] proposed the classical combined ratio estimator for population mean under stratified random sampling as,

$$\hat{\bar{Y}}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{2.1}$$

Ige and Tripathi [23] proposed the double sampling version estimator of (1.1) as,

$$\hat{\bar{Y}}_{CRD} = \bar{y}_{ds}\frac{\bar{x}'}{\bar{x}_{ds}} \tag{2.2}$$

The combined product estimator of population mean in stratified random sampling is defined as,

$$\hat{\bar{Y}}_{PC} = \bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \tag{2.3}$$

The double sampling version estimator of (1.3) is given by,

$$\hat{\bar{Y}}_{CPD} = \bar{y}_{ds}\frac{\bar{z}_{ds}}{\bar{z}'} \tag{2.4}$$

where z is an auxiliary variable which is highly negatively correlated with main variable y under study and $\bar{z}_{ds}$, $\bar{z}'$ have their usual meanings.

The biases and mean square errors of the estimators in (1.2) and (1.4) respectively are,

$$B(\hat{\bar{Y}}_{CRD}) = \frac{1}{\bar{X}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{\nu_h}-1\right)\left(R_x S_{yh}^2 - S_{yxh}\right)\right] \tag{2.5}$$

$$B(\hat{\bar{Y}}_{CPD}) = \frac{1}{\bar{Z}}\left[\sum_{h=1}^{L}\frac{W_h}{n'}\left(\frac{1}{\nu_h}-1\right)S_{yzh}\right] \tag{2.6}$$

$$MSE(\hat{\bar{Y}}_{CRD}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2 + R_x^2 S_{xh}^2 - 2R_x S_{yxh}\right) \quad (2.7)$$

$$MSE(\hat{\bar{Y}}_{CPD}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2 + R_z^2 S_{zh}^2 + 2R_z S_{yzh}\right) \quad (2.8)$$

where $R_x = \frac{\bar{Y}}{\bar{X}}$, $R_z = \frac{\bar{Y}}{\bar{Z}}$ and $W_h = \frac{N_h}{N}$.

Singh et.al [24] suggested following exponential ratio and product type estimators in stratified random sampling based on Bahl and Tuteja [25] estimators of population mean under simple random sampling as,

$$\hat{\bar{Y}}_{\mathrm{Re}}^{st} = \bar{y}_{st}\exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right) = \bar{y}_{st}\exp\left(\frac{\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h + \bar{x}_h)}\right) \quad (2.9)$$

$$\hat{\bar{Y}}_{\mathrm{Pe}}^{st} = \bar{y}_{st}\exp\left(\frac{\bar{z}_{st}-\bar{Z}}{\bar{z}_{st}+\bar{Z}}\right) = \bar{y}_{st}\exp\left(\frac{\sum_{h=1}^{L} W_h(\bar{z}_h - \bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}_h + \bar{Z}_h)}\right) \quad (2.10)$$

Tailor et.al [21] proposed the double sampling estimators of the estimators in (2.9) and (2.10) respectively as,

$$\hat{\bar{Y}}_{\mathrm{Re}d}^{st} = \bar{y}_{ds}\exp\left[\frac{\bar{x}'-\bar{x}_{ds}}{\bar{x}'+\bar{x}_{ds}}\right] \quad (2.11)$$

$$\hat{\bar{Y}}_{Ped}^{st} = \bar{y}_{ds}\exp\left[\frac{\bar{z}_{ds}-\bar{z}'}{\bar{z}_{ds}+\bar{z}'}\right] \quad (2.12)$$

The biases and mean square errors of above estimators, up to the first order of approximation are respectively,

$$B(\hat{\bar{Y}}_{\mathrm{Re}d}^{st}) = \frac{1}{8\bar{X}}\left[3S_{yx}\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h}-1\right)\left(3S_{xh}^2 - S_{yxh}\right)\right] \quad (2.13)$$

$$B(\hat{\bar{Y}}_{Ped}^{st}) = \frac{1}{8\bar{Z}}\left[\frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h}-1\right)\left(4S_{yzh} - R_z S_{zh}^2\right)\right] \quad (2.14)$$

$$MSE(\hat{\bar{Y}}_{\mathrm{Re}d}^{st}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2 + \frac{R_x^2}{4}S_{xh}^2(1-\frac{\beta_{yxh}}{R_x})\right) \quad (2.15)$$

$$MSE(\hat{\bar{Y}}_{Ped}^{st}) = S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2 + \frac{R_z^2}{4}S_{zh}^2(1+\frac{\beta_{yzh}}{R_z})\right) \quad (2.16)$$

where,

$$S_z^2 = \frac{1}{N-1}\sum_{h=1}^{L}\sum_{i=1}^{N_h}(z_{hi}-\bar{Z}_h)^2 \;\; S_{yzh} = \frac{1}{N_h-1}\sum_{i=1}^{N_h}(y_{hi}-\bar{Y}_h)(z_{hi}-\bar{Z}_h) \;\; \beta_{yxh} = \rho_{yxh}\left(\frac{S_{yh}}{S_{zh}}\right).$$

3. Proposed Estimators

Motivated by Tailor et.al [21] and Prasad [26], we proposed the following ratio and product type estimators as,

$$\tau_{\mathrm{Re}d}^{st} = \kappa_1 \overline{y}_{ds} \exp\left[\frac{\overline{x}' - \overline{x}_{ds}}{\overline{x}' + \overline{x}_{ds}}\right] \tag{3.1}$$

$$\tau_{Ped}^{st} = \kappa_2 \overline{y}_{ds} \exp\left[\frac{\overline{z}_{ds} - \overline{z}'}{\overline{z}_{ds} + \overline{z}'}\right] \tag{3.2}$$

where κ_1 and κ_2 are suitably chosen constant to be determined such that the mean square errors of the estimators (3.1) and (3.2) are minimum respectively.

To study the large sample properties of the proposed estimators, we define

$\overline{y}_{ds} = \overline{Y}(1+e_0)$, $\overline{x}_{ds} = \overline{X}(1+e_1)$ and $\overline{x}' = \overline{X}(1+e_1')$ such that $E(e_0) = E(e_1) = E(e_1') = 0$ and

$$E(e_0^2) = \frac{V(\overline{y}_{ds})}{\overline{Y}^2} = \frac{1}{\overline{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h} - 1\right)S_{yh}^2\right],$$

$$E(e_1^2) = \frac{V(\overline{x}_{ds})}{\overline{X}^2} = \frac{1}{\overline{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h} - 1\right)S_{xh}^2\right],$$

$$E(e_1'^2) = \frac{1}{\overline{X}^2}\left[\left(\frac{1-f}{n'}\right)S_x^2\right],\ E(e_0 e_1) = \frac{Cov(\overline{y}_{ds}, \overline{x}_{ds})}{\overline{Y}\,\overline{X}} = \frac{1}{\overline{Y}\,\overline{X}}\left[S_y^2\left(\frac{1-f}{n'}\right) + \frac{1}{n'}\sum_{h=1}^{L} W_h\left(\frac{1}{\nu_h} - 1\right)S_{yxh}\right],$$

$$E(e_0 e_1') = \frac{1}{\overline{Y}\,\overline{X}}\left[\left(\frac{1-f}{n'}\right)S_{yxh}\right],\ E(e_1 e_1') = \frac{1}{\overline{X}^2}\left[\left(\frac{1-f}{n'}\right)S_x^2\right].$$

Expressing $\tau_{\mathrm{Re}d}^{st}$ in terms of e_i's, we have

$$\tau_{\mathrm{Re}d}^{st} = \kappa_1 \overline{Y} \exp\left[\frac{\overline{X}(1+e_1') - \overline{X}(1+e_1)}{\overline{X}(1+e_1') + \overline{X}(1+e_1)}\right]$$

On simplification, we have

$$\tau_{\mathrm{Re}d}^{st} = \kappa_1 \overline{Y} \exp\left[\frac{(e_1' - e_1)}{2}\left\{1 + \frac{(e_1' + e_1)}{2}\right\}^{-1}\right]$$

After expansion and simplification, we get

$$\tau_{\mathrm{Re}d}^{st} = \kappa_1 \overline{Y}\left[1 + e_0 - \frac{e_1}{2} - \frac{e_1'}{2} - \frac{e_0 e_1}{2} + \frac{e_0 e_1'}{2} - \frac{e_1 e_1'}{4} + \frac{3}{8}e_1^2 - \frac{1}{8}e_1'^2\right] \tag{3.3}$$

Subtracting $\overline{Y}$ from both the sides and taking expectation both sides, we get bias of $\tau_{\mathrm{Re}d}^{st}$ as,

$$B(\tau_{\mathrm{Re}d}^{st}) = \overline{Y}\left[(\kappa_1 - 1) + \frac{3}{8}E(e_1^2) - \frac{1}{8}E(e_1'^2) - \frac{1}{2}E(e_0 e_1) + \frac{1}{2}E(e_0 e_1') - \frac{1}{4}E(e_1 e_1')\right] \tag{3.4}$$

Subtracting $\overline{Y}$ from both the of (3.3), squaring and simplifying up to the first order of approximation we have,

$$(\tau_{\mathrm{Re}d}^{st} - \overline{Y})^2 = \left[\kappa_1 \overline{Y}(1 + e_0 - \frac{e_1}{2} - \frac{e_1'}{2} - \frac{e_0 e_1}{2} + \frac{e_0 e_1'}{2} - \frac{e_1 e_1'}{4} + \frac{3}{8}e_1^2 - \frac{1}{8}e_1'^2) - \overline{Y}\right]^2$$

Expanding, simplifying and taking expectations on both the sides, we get the mean square error of $\tau_{\mathrm{Re}d}^{st}$, up to the first order of approximation as,

$$MSE(\tau^{st}_{\mathrm{Re}d}) = \bar{Y}^2\Big[(\kappa_1-1)^2 + \kappa_1^2\left\{E(e_0^2)+E(e_1^2)-2E(e_0e_1)\right\}$$
$$-\kappa_1\left\{\frac{3}{4}E(e_1^2)-\frac{1}{4}E(e_1'^2)-E(e_0e_1)\ +E(e_0e_1')-\frac{E(e_1e_1')}{2}\right\}\Big] \tag{3.5}$$

which is minimum for,

$$\kappa_1 = \frac{1+\frac{3}{8}E(e_1^2)-\frac{1}{8}E(e_1'^2)-\frac{E(e_0e_1)}{2}+\frac{E(e_0e_1')}{2}-\frac{E(e_1e_1')}{4}}{1+E(e_0^2)+E(e_1^2)-2E(e_0e_1)} = \frac{A_1}{B_1} \tag{3.6}$$

where,

$$A_1 = 1+\frac{3}{8}E(e_1^2)-\frac{1}{8}E(e_1'^2)-\frac{E(e_0e_1)}{2}+\frac{E(e_0e_1')}{2}-\frac{E(e_1e_1')}{4}$$

$$= \left[\begin{array}{l} 1+\frac{3}{8}\frac{1}{\bar{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{xh}^2\right]-\frac{1}{8}\frac{1}{\bar{X}^2}\left[\left(\frac{1-f}{n'}\right)S_x^2\right] \\ -\frac{1}{2\bar{Y}\,\bar{X}}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yxh}\right]+\frac{1}{2\bar{Y}\,\bar{X}}\left[\left(\frac{1-f}{n'}\right)S_{yxh}\right]-\frac{1}{4\bar{X}^2}\left[\left(\frac{1-f}{n'}\right)S_x^2\right] \end{array}\right]$$

$$B_1 = 1+E(e_0^2)+E(e_1^2)-2E(e_0e_1)$$

$$= \left[\begin{array}{l} 1+\frac{1}{\bar{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yh}^2\right]+\frac{1}{\bar{X}^2}\left[S_x^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{xh}^2\right] \\ -2\frac{1}{\bar{Y}\,\bar{X}}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yxh}\right] \end{array}\right]$$

And the minimum mean square error is,

$$MSE_{\min}(\tau^{st}_{\mathrm{Re}d}) = \bar{Y}^2\left[1-\frac{A_1^2}{B_1}\right] \tag{3.7}$$

Similarly we can get the bias and mean square error of the estimator in (3.2) up to the first order of approximation as,

$$B(\tau^{st}_{Ped}) = \bar{Y}\left[(\kappa_2-1)+\frac{3}{8}E(e_2'^2)-\frac{1}{8}E(e_2^2)+\frac{1}{2}E(e_0e_2)-\frac{1}{2}E(e_0e_2')-\frac{1}{4}E(e_2e_2')\right] \tag{3.8}$$

where,

$\bar{z}_{ds} = \bar{Z}(1+e_2)$ and $\bar{z}' = \bar{Z}(1+e_2')$ such that $E(e_2)=E(e_2')=0$ and

$$E(e_2^2) = \frac{V(\bar{z}_{ds})}{\bar{Z}^2} = \frac{1}{\bar{Z}^2}\left[S_z^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{zh}^2\right],$$

$$E(e_2'^2) = \frac{1}{\bar{Z}^2}\left[\left(\frac{1-f}{n'}\right)S_z^2\right],\ E(e_0e_2) = \frac{Cov(\bar{y}_{ds},\bar{z}_{ds})}{\bar{Y}\,\bar{Z}} = \frac{1}{\bar{Y}\,\bar{Z}}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yzh}\right],$$

$$E(e_0e_2') = \frac{1}{\bar{Y}\,\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{yzh}\right],\ E(e_2e_2') = \frac{1}{\bar{Z}^2}\left[\left(\frac{1-f}{n'}\right)S_z^2\right].$$

$$MSE(\tau_{Ped}^{st}) = \bar{Y}^2\Big[(\kappa_2-1)^2+\kappa_2^2\left\{E(e_0^2)+E(e_2'^2)+2E(e_0e_2)-2E(e_0e_2')-E(e_2e_2')\right\}$$
$$-2\kappa_2\left\{\frac{3}{8}E(e_2'^2)-\frac{1}{8}E(e_2^2)+\frac{1}{2}E(e_0e_2)-\frac{1}{2}E(e_0e_2')-\frac{1}{4}E(e_2e_2')\right\}\Big] \tag{3.9}$$

which is minimum for,

$$\kappa_2 = \frac{1-\frac{1}{8}E(e_2^2)+\frac{3}{8}E(e_2'^2)+\frac{E(e_0e_2)}{2}-\frac{E(e_0e_2')}{2}-\frac{E(e_2e_2')}{4}}{1+E(e_0^2)+E(e_2'^2)+2E(e_0e_2)-2E(e_0e_2')-E(e_2e_2')} = \frac{A_2}{B_2} \tag{3.10}$$

where,

$$A_2 = 1-\frac{1}{8}E(e_2^2)+\frac{3}{8}E(e_2'^2)+\frac{E(e_0e_2)}{2}-\frac{E(e_0e_2')}{2}-\frac{E(e_2e_2')}{4}$$
$$=\left[\begin{array}{l} 1-\frac{1}{8}\frac{1}{\bar{Z}^2}\left[S_z^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{zh}^2\right]+\frac{3}{8}\frac{1}{\bar{Z}^2}\left[\left(\frac{1-f}{n'}\right)S_z^2\right] \\ +\frac{1}{2\bar{Y}\bar{Z}}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yzh}\right]-\frac{1}{2\bar{Y}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{yzh}\right]-\frac{1}{4\bar{Z}^2}\left[\left(\frac{1-f}{n'}\right)S_z^2\right]\end{array}\right]$$

$$B_2 = 1+E(e_0^2)+E(e_2'^2)+2E(e_0e_2)-2E(e_0e_2')-E(e_2e_2')$$
$$=\left[\begin{array}{l} 1+\frac{1}{\bar{Y}^2}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yh}^2\right]-2\frac{1}{\bar{Y}\bar{Z}}\left[\left(\frac{1-f}{n'}\right)S_{yzh}\right] \\ +2\frac{1}{\bar{Y}\bar{Z}}\left[S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)S_{yzh}\right]\end{array}\right]$$

And the minimum mean square error is,

$$MSE_{\min}(\tau_{Ped}^{st}) = \bar{Y}^2\left[1-\frac{A_2^2}{B_2}\right] \tag{3.11}$$

4. Efficiency Conditions

From (2.7) and (3.7) we have

$MSE(\hat{\bar{Y}}_{CRD})-MSE_{\min}(\tau_{\mathrm{Re}d}^{st})>0$ If,

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2+R_x^2S_{xh}^2-2R_xS_{yxh}\right)-\bar{Y}^2\left[1-\frac{A_1^2}{B_1}\right]>0 \tag{4.1}$$

From (2.8) and (3.7) we have that the proposed estimator $\tau_{\mathrm{Re}d}^{st}$ is better than the estimator $\hat{\bar{Y}}_{CPD}$ if

$MSE(\hat{\bar{Y}}_{CPD})-MSE_{\min}(\tau_{\mathrm{Re}d}^{st})>0$ i.e,

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2+R_x^2S_{xh}^2+2R_xS_{yxh}\right)-\bar{Y}^2\left[1-\frac{A_1^2}{B_1}\right]>0 \tag{4.2}$$

From (2.15) and (3.7) we have

$MSE(\hat{\bar{Y}}^{st}_{\mathrm{Re}d}) - MSE_{\min}(\tau^{st}_{\mathrm{Re}d}) > 0$ If,

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2+\frac{R_x^2}{4}S_{xh}^2(1-\frac{\beta_{yxh}}{R_x})\right)-\bar{Y}^2\left[1-\frac{A_1^2}{B_1}\right]>0 \tag{4.3}$$

The estimator $\tau^{st}_{\mathrm{Re}d}$ is better than the estimator $\hat{\bar{Y}}^{st}_{Ped}$ if

$MSE(\hat{\bar{Y}}^{st}_{Ped}) - MSE_{\min}(\tau^{st}_{\mathrm{Re}d}) > 0$ i.e,

$$S_y^2\left(\frac{1-f}{n'}\right)+\frac{1}{n'}\sum_{h=1}^{L}W_h\left(\frac{1}{\nu_h}-1\right)\left(S_{yh}^2+\frac{R_x^2}{4}S_{xh}^2(1+\frac{\beta_{yxh}}{R_x})\right)-\bar{Y}^2\left[1-\frac{A_1^2}{B_1}\right]>0 \tag{4.4}$$

Note: Similar conditions are for the estimator τ^{st}_{Ped}.

5. Empirical Study

We examine the efficiency of the proposed estimator over other estimators. We consider published data sets whose statistics are given in Table1. The MSE and percent relative efficiency (PRE) values are given in Table 2. It is obviously seen that the proposed estimator are quite efficient than the other estimators for the given Population.

Table 1. Population (Source: Gujarati [27]) Y_i (Thousands of wildcats), X_i (per barrel price), Z_i (time)

	Stratum I	Stratum II		Stratum I	Stratum II
N_h	14	14	S_{xh}^2	1.25878	0.25808
n'_h	10	10	S_{zh}^2	81.2088	55.3242
n_h	5	5	S_{yxh}	0.511	0.07806
$\bar{Y}_h$	8.62429	11.6636	S_{yzh}	0.33857	-3.94599
$\bar{X}_h$	4.47	4.40214	S_y^2	3.46138	
$\bar{Z}_h$	21.1429	12.3571	S_x^2	0.731536	
S_{yh}^2	0.86519	1.34992	S_z^2	85.75	

Table 2. MSE and PRE Values of Estimators

Estimators	MSE	PRE	Estimators	MSE	PRE
$\hat{\bar{Y}}_{CRD}$	0.198676	100	$\hat{\bar{Y}}_{CPD}$	1.21037	100
$\hat{\bar{Y}}^{st}_{\mathrm{Re}d}$	0.108891	182.453	$\hat{\bar{Y}}^{st}_{Ped}$	0.36705	329.756
$\tau^{st}_{\mathrm{Re}d}$	**0.043024**	**461.775**	τ^{st}_{Ped}	**0.32361**	**373.940**

6. Results and Conclusions

In this manuscript we proposed the improved ratio and product type exponential estimators of population mean under two phase sampling using the powerful kappa technique. As we observe from the table-2 that the proposed estimators $\tau_{\mathrm{Re}d}^{st}$ and τ_{Ped}^{st} have lesser mean square errors than the other mentioned estimators of population mean under two phase sampling for stratification. It means the estimated values of the parameter (population mean) by these estimators are very close to the actual value of the parameter. The main aim of the study is to find the estimators which predict the values very close to the actual value of the parameter with minimum mean squared error. Here in the present study the major goal is obtained as the proposed estimators have the minimum mean square errors. Thus we infer that the proposed estimators should be preferred for the estimation of population mean in two phase sampling for stratification.

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