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Partial Backlogging EOQ Model for Queued Customers with Power Demand and Quadratic Deterioration:Computational Approach

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Abstract In this paper, an EOQ model for perishable items for queued customers is developed in which shortages are allowed and partially backlogged. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. Demand follows power pattern on time t. The model is fairly general due to dynamic nature of demand. When fresh and new items arrive in stock they begin to decay after a fixed time interval called the life period of items. The total cost function is constructed and subjected to the optimization which in turn gives us the non linear equation. Further, a computing algorithm is proposed to find the solution of the system by using the N-R method. We compute the optimal inventory period and total optimal average cost as most important performance measures for the model. Finally, numerical examples are provided to illustrate the problem and sensitivity analysis has been carried out.

Keywords Computational Approach, Power Demand, Queued Customer, Partial Backlogging, Deterioration

2000 Mathematical Classification 90Bxx, 90 B05

1. Introduction

Kaleidoscopic pattern of inventory control system with deterioration is much more difficult to be investigated by the researchers engaged in the field. In its investigative design and framework, mathematical ideas have exceedingly shown well for dwelling upon the concern of inventory control system with aforesaid pattern of inventory. Deterioration of inventory encompasses commodities such as such as foods, vegetable, drugs, pharmaceuticals, medicine, gasoline, blood and radioactive substances deterioration takes place during the normal storage period of the units. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored.

In the literature of inventory theory, the deteriorating inventory models have been continually modified so as to accumulate more practical features of the real inventory systems. A number of authors have discussed inventory models for non deteriorating items. However, there are certain substances in which deterioration plays an important role and items cannot be stored for a long time. When the items of the commodity are kept in stock as an inventory for fulfilling the future demand, there may be the deterioration of items in the inventory system. Various types of inventory models for items deteriorating at a constant rate were discussed by[1],[2],[3] and[4] etc.

In practice it can be observed that constant rate of deterioration occurs rarely. Most of the items deteriorate fast as the time passes. Therefore, it is much more realistic to consider the variable deterioration rate. In a realistic product life cycle, demand is increasing with time during the growth phase.[5] investigated an inventory system with power demand pattern for items with variable rate of deterioration. [6] studied the inventory system with two-parameter exponential distributed hazardous items in which production and demand rate were constant.[7] considered an EOQ model in which inventory is depleted not only by demand, but also by deterioration at a Weibull distributed rate, assuming the demand rate with a ramp type function of time.

[8] developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. They had taken a two-parameter Weibull distribution to represent the time to deterioration.[9] considered an inventory model for deteriorating items in which demand increases with respect to time, deterioration rate, inventory holding cost and ordering cost are all continuous functions of time. Shortages are completely backlogged. The planning horizon is finite.

[10] formulated an order-level lot-size inventory model for a time-dependent deterioration and exponentially declining demand.[11] reviewed the recent studies about the deteriorating items inventory management research status.

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They provided a comprehensive introduction, compared with the extant reviews and proposed some key factors which should be considered in the deteriorating inventory studies. This survey provides a clear overview of the deteriorating inventory study field, which can be used as a starting point for further study.

A key assumption of the basic EOQ model is that stock outs are not permitted. Relaxing the basic EOQ that stock outs are not permitted led to the development of EOQ model for the two basic stock out cases: backorders and lost sales. What took longer to develop was a model that recognized that, while some customers are willing to wait for delivery, others are not. Either these customers will cancel their orders or the supplier will have to fill them within the normal delivery time by using more expensive supply methods. While there have been a number of models developed for the EOQ model with partial backordering, most of them incorporate considerably more complicated assumption sets than the classic EOQ model do. Furthermore, when the shortage occurs, some customers are willing to wait for back order and others would turn to buy from other sellers.

[12] developed economic order quantity models that focused on deteriorating items having a deterministic demand pattern with a linear trend and shortages. They assumed that the inventory deteriorates over time at a constant rate. The inventory replenishment policy was considered over a finite time-horizon.[13] considered an inventory model for items with Weibull distributed deterioration. They assumed that the demand rate is a power function of time and allowed for shortages.[14] considered the variable lead-time EPQ model with shortages. He presented a new approach, without reference to the use of derivatives but with algebraic derivation, to solve the deterministic EOQ models with/without shortages.

[15] developed an inventory model with ramp type demand, starting with shortage and three – parameter Weibull distribution deterioration.[16] developed an inventory model with linear demand rate. Shortages in the inventory are allowed and were completely backlogged. They had assumed that the production rate is finite and proportional to the demand rate.[17] developing an inventory model with time dependent Weibull demand rate where shortages are allowed and are completely backlogged. The production rate is assumed to be finite and proportional to the demand rate.

[18] developed an order level inventory system for time dependent linearly deteriorating items with decreasing demand rate. They assumed that the demand rate is time dependent and developed two EOQ models for without shortage case and with shortage case.[19] considered the production inventory problem in which the deterioration is Weibull distribution, production and demand are quadratic function of time. Shortages of cycle are allowed in the inventory system.

Researchers such as[20],[21] and[22] considered the constant partial backlogging rates during the shortage period in their inventory models. In many cases customers are conditioned to a shipping delay and may be willing to wait for a short time in order to get their first choice. In some inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate should be variable and dependent on the length of the waiting time for the next replenishment. When a stock out situation occurs, only a fraction of demand occurring at a given time is backordered. And that fraction is a decreasing function of the waiting time. The approach given is revenue based and does not require specifying the backorder cost or lost sale cost which is very difficult to estimate in reality.

[23] investigated optimal lot sizing for an EOQ model under conditions of perishability allowing shortage and partial backlogging. He modelled the backlogging phenomenon using a new approach in which customers are considered impatient.[24] suggested a continuous review inventory model over a finite-planning horizon with deterministic varying demand and constant deterioration rate. The model allows for shortages, which are partially backlogged at a rate which varies exponentially with time. They established an optimal replenishment policy.[25] developed the deterministic EPQ model with partial backordering when a percentage of stock outs will be backordered.

Many researchers have modified inventory policies for deteriorating items by considering the time proportional partial backlogging rate such as[26],[27],[28],[29],[30],[31],[32] and so on.[33] extended model of[13] by proposing a general class time-proportional backlogging rate to make the theory more complete and provided the necessary condition to find the optimal solution.[34] proposed an EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand. In the model, the shortages are allowed and partially backordered. They show that the minimized objective cost function is jointly convex and derive the optimal solution.

[35] described an EOQ model with time-varying deterioration, partial backlogging which depends on the length of the waiting time for the next replenishment, linearly time-varying demand function over a finite time horizon and variable replenishment cycle.[36] developed a deterministic inventory model for infinite time-horizon incorporating partial backlogging and decrease in demand. Demand at any instant depends linearly on the on-hand inventory level at that instant. Deterioration of items begins after a certain time from the instant of their arrival in stock.[37] studied a deterministic inventory model for deteriorating items under time-dependent partial backlogging and proved that the optimal replenishment solution not only exists but is also unique.

[38] considered an EOQ model for deteriorating items with exponential time varying demand. They assumed that the backlogging rate is dependent on the length of the waiting time for the next replenishment.[39] presented an optimization framework to derive optimal replenishment

policy for perishable items with stock dependent demand rate. The demand rate is assumed to be a function of current level inventory and the inventory deteriorates per unit time with variable deterioration rate. The shortages are allowed and partially backlogged with a variable rate, which depends on the duration of waiting time up to arrival of next lot.[40] developed a single item perishable inventory model assuming that the demand is time dependent accelerated growth-effect of accelerated growth-steady type. The deterioration of inventory starts after a certain time. Shortages are allowed and are partially backlogged.

[41] considered a deterministic inventory model in which items are subject to constant deterioration and shortages are allowed. The unsatisfied demand is backlogged which is a function of time.[42] considered an order level inventory model for seasonable/fashionable products subject to ramp type demand rate. The unsatisfied demand is partially backlogged with a time dependent backlogging rate. In addition, the product deteriorates with a time dependent deterioration rate.[43] developed a deterministic inventory model for deteriorating items in which shortages are allowed and partially backlogged. Recently, authors[44],[45] and[46] have attempted computational approach to various inventory models and discussed their applications.

[47] developed a partial backlogging inventory model. They proposed the prediction method and algorithms for ordering period as well as for minimum total cost.[48] presented Economic Production Lot Size model with constant deterioration. Shortages are permitted in inventory with partial backlogging.[49] described an EOQ model for a deteriorating item considering general time-dependent demand, time-dependent partial backlogging over a finite time horizon and variable replenishment cycle.[50] developed an inventory model with time dependent two-parameter Weibull demand rate whose deterioration rate increases with time. Each cycle has shortages, which have been partially backlogged.

There are a number of situations in which a customers or vendors of some sort are assumed to receive the demand in bulk of inventory are subject to put in queue at a service facility. The goal of queuing is essentially to trade-off the cost of providing a level of service capacity and the customers waiting for service.

With this motivation, in the present paper an attempt is made to formulate a partial backlogging inventory model by incorporating the deterioration effect and time-dependent power pattern demand rate. Deterioration of items begins after a certain time from the instant of their arrival in stock, we name it as life time of items, and deterioration rate is a quadratic function of time. Unsatisfied demand is partially backlogged with a variable rate. To suit present day competition in the market, the backlogging rate is inversely proportional to the duration of waiting time up to arrival of next lot. The differential equations are derived and the instantaneous state of inventory is obtained analytically. The total cost function, which consists of setup cost, holding cost, backordering cost, lost sale cost, deterioration cost, waiting cost and procurement cost is constructed and subjected to the optimization which in turn gives us the system of non linear equations. Further, a computing algorithm is proposed to find the solution of the system by using the N-R method. We compute the optimal inventory period and total optimal average cost as most important performance measures for the model. Numerical demonstration and sensitivity analysis have been carried out for the model to identify the most sensibilities of various parameters involved in the system leading to interesting observations which seem to be consistent with its economic insights. This model is much useful for analysing the planning of the seasonal and fashionable products with the notion of decay or obsolete.

2. Assumptions and Notations

In this paper we have made the following notations and assumptions in the formulation of proposed mathematical model of the inventory system.

2.1. Notations

$I(t)$: the inventory level at any time t, $t \geq 0$.
S : the initial inventory level.
μ : the life time of items.
λ : the average arrival rate.
δ : the average service rate.
L_S : the number of customers waiting for inventory.
C_O : the set up cost for each replenishment.
C_H : inventory holding cost per unit time.
C_D : deterioration cost per unit.
C_S : shortage cost for backlogged items.
C_L : the unit cost of lost sales.
C_W : waiting cost per customer per unit time.
C_P : procurement cost.
T : the planning horizon.
$TAC(t_1)$: the total average cost.

2.2. Assumptions

I. A single item is considered over the fixed period T units of time.

II. Customers are waiting in line for getting inventory items. They arrive on a precise schedule (deterministic) of evenly spaced intervals and service process is well scheduled.

III. λ and δ are assumed to be constant and are related by $L_S = \dfrac{\lambda}{\delta - \lambda}$.

IV. Deterioration of the items takes place after the life time of items.

V. The variable deterioration rate $\theta(t)$ is time dependent quadratic function such that $\theta(t) = \theta t^2$, $0<\theta<<1$.

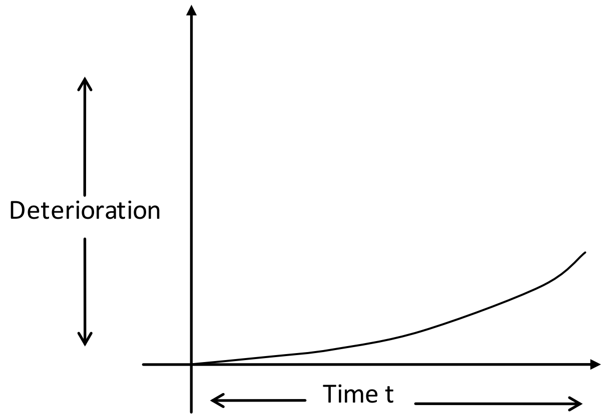

Figure 1. Deterioration-time relationship

VI. There is no replenishment or repair of deteriorated items during a given cycle.

VII. The replenishment occurs instantaneously at an infinite rate.

VIII. Lead time is negligible.

IX. The demand rate is D(t) at any time t such that $D(t) = \dfrac{dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}$, where d is the fixed quantity, n is the parameter of power demand pattern, the value of n may be any positive number.

X. Shortages are allowed and backlogging rate is $\dfrac{dt^{\frac{1-n}{n}}/nT^{\frac{1}{n}}}{1+k(T-t)}$, when inventory is in shortage. The backlogging parameter k is positive constant and 0<k<<1.

XI. All of the cost parameters are positive constants.

3. Mathematical Model

3.1. Model Formulation and Solution

Let us assume that Q be the total amount of inventory produced or purchased at the beginning of each cycle. After fulfilling the backorders let we get an amount S (>0) as initial inventory. During the period (0, μ) the inventory level gradually decreases due to market demand only. After life time deterioration can take place, therefore during the period (μ, t_1) the inventory level gradually abates due to market demand and deterioration of items and falls to zero at time t_1. Shortages take place in the periods (t_1, T) which are partially backlogged. The depletion of inventory level is shown in the following figure.

The differential equations governing the inventory level $I(t)$ at any time t during the cycle (0, T) are given as following,

$$\frac{dI(t)}{dt} = -D(t), \qquad 0 \le t \le \mu \tag{3.1}$$

$$with \qquad I(0) = S$$

$$\frac{dI(t)}{dt} = -D(t) - \theta(t)I(t), \quad \mu \le t \le t_1 \tag{3.2}$$

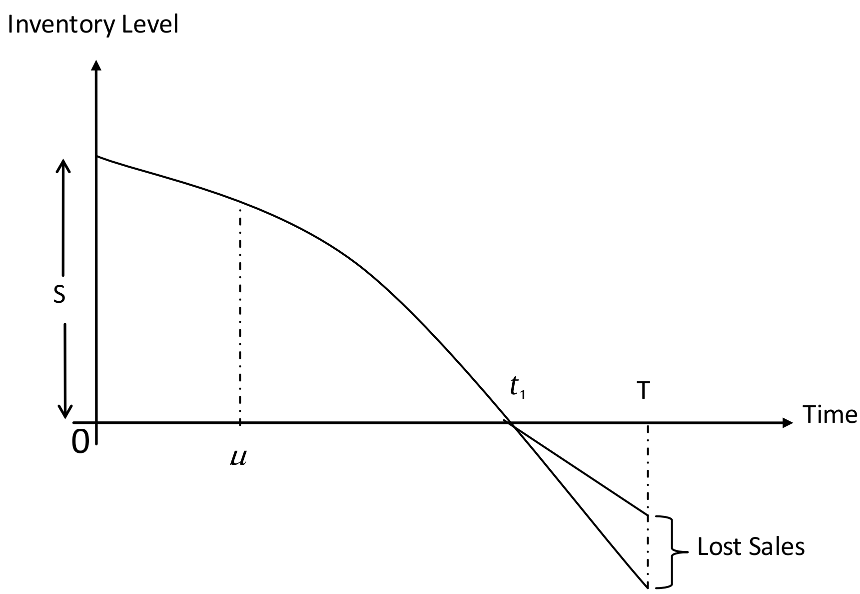

Figure 2. Partial Backlogging Inventory Model

with $I(t_1)=0$ and

$$\frac{dI(t)}{dt}=\frac{-D(t)}{1+k(T-t)},\quad t_1\le t\le T \tag{3.3}$$

The solution of equation (3.1) is

$$I(t)=S-\frac{dt^{\frac{1}{n}}}{T^{\frac{1}{n}}},\qquad 0\le t\le\mu \tag{3.4}$$

Taking the first two terms of the exponential series and then integrating we get the solution of equation (3.2) as

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-t^{\frac{1}{n}+3}\right)}{3+9n}\right]e^{-\frac{\theta}{3}t^3},$$

$$\mu\le t\le t_1 \tag{3.5}$$

Now taking the first two terms of the exponential series and neglecting the term containing α^2 the equation (3.5) becomes

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}t^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-t^{\frac{1}{n}+3}\right)}{3+9n}\right],\mu\le t\le t_1 \tag{3.6}$$

Similarly the solution of equation (5.3) is

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)(1-kT)+\frac{k\left(t_1^{\frac{1}{n}+1}-t^{\frac{1}{n}+1}\right)}{1+n}\right],t_1\le t\le T \tag{3.7}$$

From equation (3.5) and (3.6), we have

$$I(\mu)=S-\frac{d\mu^{\frac{1}{n}}}{T^{\frac{1}{n}}}=\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-\mu^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}\mu^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-\mu^{\frac{1}{n}+3}\right)}{3+9n}\right]$$

So that, the value of initial inventory level (S) is given by

$$\Rightarrow S=\frac{d\mu^{\frac{1}{n}}}{T^{\frac{1}{n}}}+\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-\mu^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}\mu^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-\mu^{\frac{1}{n}+3}\right)}{3+9n}\right] \tag{3.8}$$

Using equation (3.8) in equation (3.5) we get

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left[\mu^{\frac{1}{n}}-t^{\frac{1}{n}}+\left(t_1^{\frac{1}{n}}-\mu^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}\mu^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-\mu^{\frac{1}{n}+3}\right)}{3+9n}\right],$$

$$0\le t\le\mu$$

$$I(t)=\frac{d}{T^{\frac{1}{n}}}\left[t_1^{\frac{1}{n}}-t^{\frac{1}{n}}-\frac{\theta\mu^3}{3}\left(t_1^{\frac{1}{n}}-\mu^{\frac{1}{n}}\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-\mu^{\frac{1}{n}+3}\right)}{3+9n}\right],0\le t\le\mu \tag{3.9}$$

During period (0, T) total number of units holding I_H is

$$I_H=\int_0^{\mu}I(t)dt+\int_{\mu}^{t_1}I(t)dt$$

Using equation (3.9) and equation (3.6) we get

$$I_H=\int_0^{\mu}\frac{d}{T^{\frac{1}{n}}}\left[\mu^{\frac{1}{n}}-t^{\frac{1}{n}}+\left(t_1^{\frac{1}{n}}-\mu^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}\mu^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-\mu^{\frac{1}{n}+3}\right)}{3+9n}\right]dt$$

$$+\int_{\mu}^{t_1}\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}t^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-t^{\frac{1}{n}+3}\right)}{3+9n}\right]dt$$

Calculating further we get

$$I_H=\frac{d}{T^{\frac{1}{n}}}\left[\frac{(5+9n)\theta\mu^{\frac{1}{n}+4}}{(3+9n)\left(\frac{1}{n}+4\right)}-\frac{\theta t_1^{\frac{1}{n}}\mu^4}{4}+\frac{t_1^{1+\frac{1}{n}}}{1+n}+\frac{\theta t_1^{\frac{1}{n}+4}(3+9n)}{3n}\right] \tag{3.10}$$

Total amount of deteriorated items I_D, during the period (0, T) is

$$I_D=\int_{\mu}^{t_1}\theta(t)I(t)dt$$

$$=\int_{\mu}^{t_1}\theta t^2\frac{d}{T^{\frac{1}{n}}}\left[\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)\left(1-\frac{\theta}{3}t^3\right)+\frac{\theta\left(t_1^{\frac{1}{n}+3}-t^{\frac{1}{n}+3}\right)}{3+9n}\right]dt$$

Integrating this, neglecting the term containing θ^2 or higher degree of it as

$0<\theta<<1$, we get

$$I_D=\frac{d}{T^{\frac{1}{n}}}\left[\frac{\theta t_1^{\frac{1}{n}+3}}{3+9n}+\frac{n\theta\mu^{\frac{1}{n}+3}}{1+3n}-\frac{\theta t_1^{\frac{1}{n}}\mu^3}{3}\right], \tag{3.11}$$

Total amount of shortage units I_S during the period (0,T) is given as

$$I_S=-\int_{t_1}^{T}I(t)dt$$

$$=-\frac{d}{T^{\frac{1}{n}}}\int_{t_1}^{T}\left[\left(t_1^{\frac{1}{n}}-t^{\frac{1}{n}}\right)(1-kT)+\frac{k\left(t_1^{\frac{1}{n}+1}-t^{\frac{1}{n}+1}\right)}{1+n}\right]dt$$

$$= -\frac{d}{T^{\frac{1}{n}}}\left[\begin{array}{c}\left(T-kT^2\right)t_1^{\frac{1}{n}} + t_1^{\frac{1}{n}+1}\left(\dfrac{2kT-1}{1+n}\right) - \dfrac{k\,t_1^{\frac{1}{n}+2}}{1+2n} + \\ \dfrac{2n^2kT^{\frac{1}{n}+2}}{(1+n)(1+2n)} - \dfrac{nT^{\frac{1}{n}+1}}{(1+n)}\end{array}\right] \quad (3.12)$$

Total amount of lost sales I_L during the period $(0,\ T)$ is given by

$$I_L = \int_{t_1}^{T}\left(1-\frac{1}{1+k(T-t)}\right)D(t)\,dt$$

$$= \frac{dk}{T^{\frac{1}{n}}}\left[\frac{nT^{\frac{1}{n}+1}}{(1+n)} + \frac{t_1^{\frac{1}{n}+1}}{1+n} - t_1^{\frac{1}{n}}T\right], \quad (3.13)$$

3.2. Cost Analysis and Optimization

The total waiting cost for the customers in the system $= C_W L_S = C_W\left(\dfrac{\lambda}{\delta-\lambda}\right)$.

Total average cost of the system per unit time is given by

$$TAC(t_1) = \frac{1}{T}\left[C_O + C_H I_H + C_W L_S + C_D I_D + C_P S + C_S I_S + C_L I_L\right]$$

$$TAC(t_1) = \frac{C_O}{T} + \frac{C_H d}{T^{\frac{1}{n}+1}}$$

$$\left[\frac{(5+9n)\theta\mu^{\frac{1}{n}+4}}{(3+9n)\left(\frac{1}{n}+4\right)} - \frac{\theta t_1^{\frac{1}{n}}\mu^4}{4} + \frac{t_1^{1+\frac{1}{n}}}{1+n} + \frac{\theta t_1^{\frac{1}{n}+4}(1+3n)}{n}\right]$$

$$+\frac{C_W}{T}\left(\frac{\lambda}{\delta-\lambda}\right) + \frac{C_D d}{T^{\frac{1}{n}+1}}\left[\frac{\theta t_1^{\frac{1}{n}+3}}{3+9n} + \frac{n\theta\mu^{\frac{1}{n}+3}}{1+3n} - \frac{\theta t_1^{\frac{1}{n}}\mu^3}{3}\right] + \frac{C_P S}{T}$$

$$-\frac{C_S d}{T^{\frac{1}{n}+1}}\left[\begin{array}{c}\left(T-kT^2\right)t_1^{\frac{1}{n}} + t_1^{\frac{1}{n}+1}\left(\dfrac{2kT-1}{1+n}\right) - \\ \dfrac{kt_1^{\frac{1}{n}+2}}{1+2n} + \dfrac{2n^2kT^{\frac{1}{n}+2}}{(1+n)(1+2n)} - \dfrac{nT^{\frac{1}{n}+1}}{(1+n)}\end{array}\right]$$

$$+\frac{C_L dk}{T^{\frac{1}{n}+1}}\left[\frac{nT^{\frac{1}{n}+1}}{(1+n)} + \frac{t_1^{\frac{1}{n}+1}}{1+n} - t_1^{\frac{1}{n}}T\right], \quad (3.14)$$

To minimize total average cost per unit time $TAC(t_1)$, the optimal value of t_1 can be obtained by solving the following equation

$$\frac{dTAC(t_1)}{dt_1} = 0$$

$$\Rightarrow C_H\left[t_1^{\frac{1}{n}} + \left(\frac{1}{n}+4\right)(3n+1)\theta t_1^{\frac{1}{n}+3} - \frac{\theta t_1^{\frac{1}{n}-1}\mu^4}{4}\right]$$

$$+C_D\frac{\theta}{3}t_1^{\frac{1}{n}-1}\left(t_1^3-\mu^3\right)$$

$$-C_S\left[\left(T-kT^2\right)t_1^{\frac{1}{n}-1} + (2kT-1)t_1^{\frac{1}{n}} - kt_1^{\frac{1}{n}+1}\right]$$

$$+C_L k\left(t_1^{\frac{1}{n}} - Tt_1^{\frac{1}{n}-1}\right) = 0 \quad (3.15)$$

$$\Rightarrow C_H t_1^{\frac{1}{n}} + C_H\left(\frac{1}{n}+4\right)(3n+1)\theta t_1^{\frac{1}{n}+3} - C_H\frac{\theta\mu^4}{4}t_1^{\frac{1}{n}-1}$$

$$+C_D\frac{\theta}{3}t_1^{\frac{1}{n}+2} - C_D\frac{\theta}{3}\mu^3 t_1^{\frac{1}{n}-1} - C_S\left(T-kT^2\right)t_1^{\frac{1}{n}-1}$$

$$-C_S(2kT-1)t_1^{\frac{1}{n}} + C_S kt_1^{\frac{1}{n}+1} + C_L kt_1^{\frac{1}{n}} - C_L kTt_1^{\frac{1}{n}-1} = 0$$

$$\Rightarrow C_H + C_H\left(\frac{1}{n}+4\right)(3n+1)\theta t_1^3 - C_H\frac{\theta\mu^4}{4}t_1^{-1}$$

$$+C_D\frac{\theta}{3}t_1^2 - C_D\frac{\theta}{3}\mu^3 t_1^{-1} - C_S\left(T-kT^2\right)t_1^{-1}$$

$$-C_S(2kT-1) + C_S kt_1 + C_L k - C_L kTt_1^{-1} = 0$$

$$\Rightarrow C_H t_1 + C_H\left(\frac{1}{n}+4\right)(3n+1)\theta t_1^4 - C_H\frac{\theta\mu^4}{4} + C_D\frac{\theta}{3}t_1^3 - C_D\frac{\theta}{3}\mu^3$$

$$-C_S\left(T-kT^2\right) - C_S(2kT-1)t_1 + C_S kt_1^2 + C_L kt_1 - C_L kT = 0$$

$$\Rightarrow\left[C_H\left(\frac{1}{n}+4\right)(3n+1)\theta\right]t_1^4 + \left[C_D\frac{\theta}{3}\right]t_1^3 + \left[C_S k\right]t_1^2$$

$$+\left[C_H + C_L k - C_S(2kT-1)\right]t_1$$

$$-\left[C_H\frac{\theta\mu^4}{4} + C_D\frac{\theta}{3}\mu^3 + T\left(C_S - C_S kT + C_L k\right)\right] = 0$$

$$\Rightarrow K_1t_1^4 + K_2t_1^3 + K_3t_1^2 + K_4t_1 - K_5 = 0 = \text{K(say)} \quad (3.16)$$

Where $K_1 = C_H\left(\dfrac{1}{n}+4\right)(3n+1)\theta,\ K_2 = C_D\dfrac{\theta}{3},$

$K_3 = C_S k,\ K_4 = C_H + C_L k - C_S(2kT-1),$

$K_5 = C_H\dfrac{\theta\mu^4}{4} + C_D\dfrac{\theta}{3}\mu^3 + T\left(C_S - C_S kT + C_L k\right)$

Equation (3.16) is a non-linear equation in t_1 and its value is obtained by using N-R method and following algorithm by using C++ provided total cost is minimum and its second derivative is positive for being t_1 optimal. The following value of second derivative in (3.17) is tested true after required parameters are computed

$$\frac{d^2K}{dt_1^2} > 0 \quad (3.17)$$

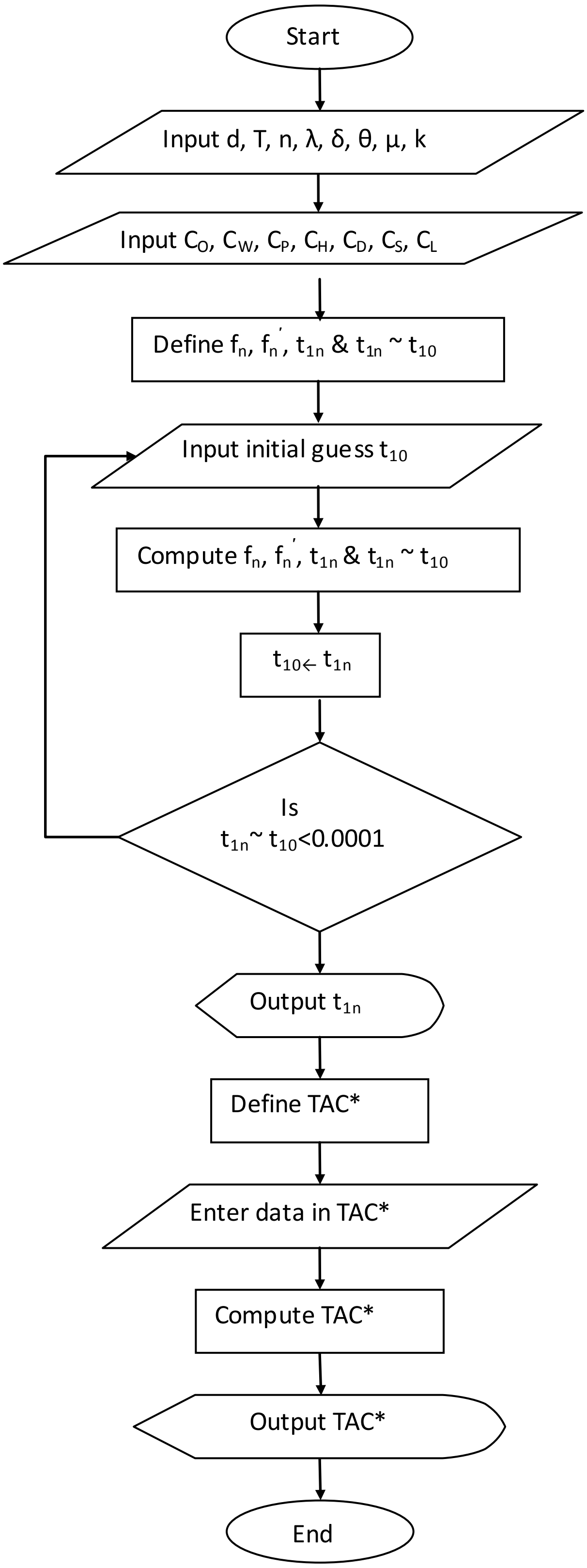

Figure 3. Computing Algorithm

After substituting the optimal value of t_1 in the expressions of initial inventory level (3.8) and minimum total average cost per unit time $TAC(t_1)^*$ are computed finally as optimal.

3.3. Computing Algorithm

Following computing algorithm is developed to find out the optimal inventory period and total optimal average cost of the inventory system.

4. Sensitivity Analysis

4.1. Numerical Example

We have considered the following parameter values to illustrate the model numerically. d =1000 units, C_O =Rs. 500 per order, C_W =5, C_P =4, C_H =Rs. 35 per unit per year, C_D =100 per unit, C_S =Rs. 80 per unit per year, C_L =Rs. 20 per unit, T =1 year, n =4 units, λ =10, δ =8, θ =0.01 unit, μ =0.3 year, k =0.15 unit. Then to minimize the total average cost, optimal values of the decision variables are obtained as t_1 =0.766271 year, S =1000.108444 units, TAC =Rs. 6933.95 per year.

4.2. Tables, Graphics and Observations

The aim of the sensitivity analysis is to demonstrate the variability of the model based on the simulations or the hypothetical data-input. In this chapter, we prefer the hypothetical data-input to run the search program of the system. It is the process of varying model parameters over a reasonable range and observing the relative changes in the model response. Remarkable are the observed changes in the optimal cycle time and total optimal average cost of the system.

We wrote a program in C++ to apply a one-variable version of N-R method to compute the optimal cycle time and consequently the total optimal average cost of the system is also computed. In sensitivity analysis, variational effect of parameters on the total optimal average cost is presented.

Table 1. Demand Coefficient d Vs. Optimal Total Average Cost (TAC*) $(C_O = 500, C_W = 5, Cp = 4, Ch = 35, C_D = 100, C_S = 80, C_L = 20, T = 1, n = 4, \lambda = 10, \delta = 8, \theta = 0.01, \mu = 0.3, k = 0.15)$

Demand Coefficient (d)	% Change	Inventory Period (t_1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
500	-50	0.766271	500.05	3704.48
700	-30	0.766271	700.08	4996.25
900	-10	0.766271	900.10	6288.06
1000	0	0.766271	1000.11	6933.95
1100	10	0.766271	1100.12	7579.85
1300	30	0.766271	1300.14	8871.64
1500	50	0.766271	1500.16	10163.42

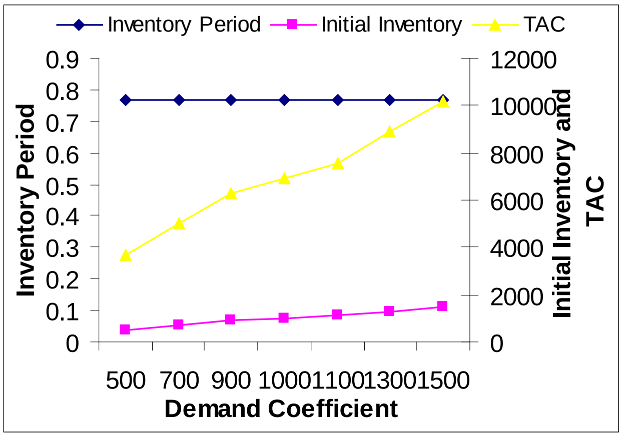

Figure 4. Demand Coefficient d Vs. Optimal Total Average Cost

In Table and Figure 4, we observe that as demand coefficient increases, the optimal inventory period shows a negligible increment, while optimal initial inventory level as well as optimal total average cost shows significant increment. In fact, about 10% increase in demand coefficient amounts to approx. 0.04%, 10% and 9.32% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively. Moreover, demand coefficient shows a positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

Table 2. Setup Cost (C_O) Vs. Optimal Total Average Cost (TAC*)

$(C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,\ C_L = 20,$
$d = 1000, T = 1,\ n = 4,\ \lambda = 10,\ \delta = 8,\ \theta = 0.01, \mu = 0.3,\ k = 0.15)$

Setup Cost (Co)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
200	-60	0.766271	1000.108444	6633.95
300	-40	0.766271	1000.108444	6733.95
400	-20	0.766271	1000.108444	6833.95
500	0	0.766271	1000.108444	6933.95
600	20	0.766271	1000.108444	7033.95
700	40	0.766271	1000.108444	7133.95
800	60	0.766271	1000.108444	7233.95

In Table and Figure 5, we find that as setup cost increases, almost the optimal inventory period and optimal initial inventory level show non-significant increase whereas optimal total average cost increases significantly. Numerically, about 20% increase in setup cost causes about approx. 0.02%, 0.03% and 1.44% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively. Thus, setup cost shows a positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

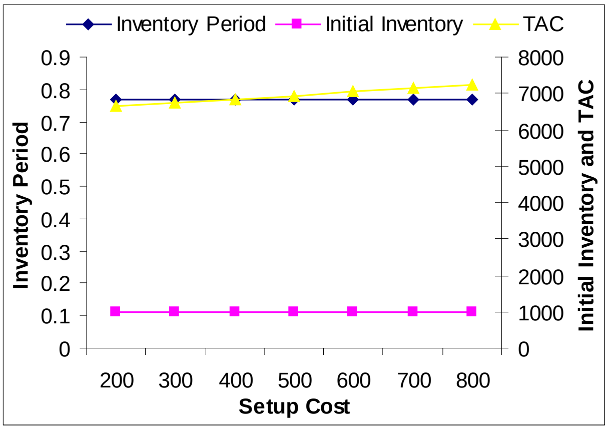

Figure 5. Setup Cost (C_O) Vs. Optimal Total Average Cost (TAC*)

Table 3. Waiting Cost (C_W) Vs. Optimal Total Average Cost (TAC*)

$(C_O = 500,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,\ C_L = 20,$
$d = 1000, T = 1,\ n = 4,\ \lambda = 10,\ \delta = 8,\ \theta = 0.01, \mu = 0.3,\ k = 0.15)$

Waiting Cost (CW)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
2	-60	0.766271	1000.108444	6948.95
3	-40	0.766271	1000.108444	6943.95
4	-20	0.766271	1000.108444	6938.95
5	0	0.766271	1000.108444	6933.95
6	20	0.766271	1000.108444	6928.95
7	40	0.766271	1000.108444	6923.95
8	60	0.766271	1000.108444	6918.95

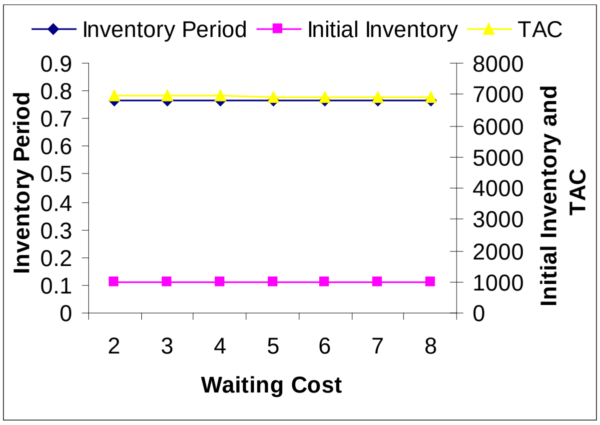

Figure 6. Waiting Cost (C_W) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 6, we find that as waiting cost increases, the optimal total average cost decreases. Numerically about 20% increase in waiting cost creates about % increase in optimal total average cost. Moreover, waiting cost shows a negative correlation with optimal total average cost.

Table 4. Procurement Cost (Cp) Vs. Optimal Total Average Cost (TAC*)
($C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,\ C_L = 20,$
$d = 1000,\ T = 1,\ n = 4,\ \lambda = 10,\ \delta = 8,\ \theta = 0.01, \mu = 0.3,\ k = 0.15$)

Procurement Cost (Cp)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
2	-50	0.766271	1000.108444	4933.73
3	-25	0.766271	1000.108444	5933.84
4	0	0.766271	1000.108444	6933.95
5	25	0.766271	1000.108444	7934.06
6	50	0.766271	1000.108444	8934.17
7	75	0.766271	1000.108444	9934.28
8	100	0.766271	1000.108444	10934.38

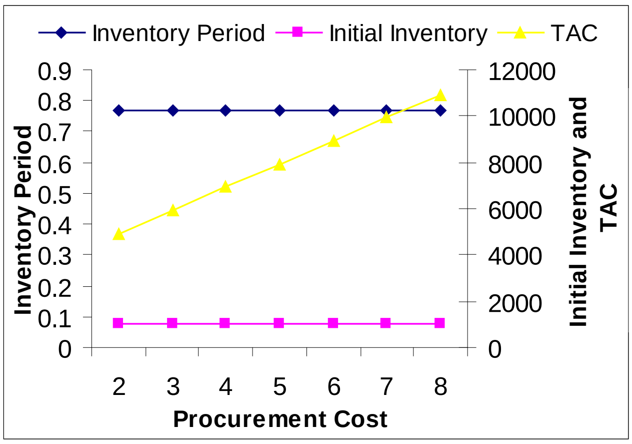

Figure 7. Procurement Cost (Cp) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 7, we find that as the Procurement Cost increases, the optimal total average cost also increases. Further, an increase of 20% in procurement cost creates about % increase in optimal total average cost. Thus there exists a positive correlation between procurement cost and optimal total average cost.

Table 5. Holding Cost (C_H) Vs. Optimal Total Average Cost (TAC*)

($C_O = 500,\ C_W = 5,\ Cp = 4,\ C_D = 100,\ C_S = 80,\ C_L = 20,$
$d = 1000,\ T = 1,\ n = 4,\ \lambda = 10,\ \delta = 8,\ \theta = 0.01, \mu = 0.3,\ k = 0.15$)

Holding Cost (C_H)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
25	-50	0.852457	1000.151914	7076.74
30	-40	0.806937	1000.127804	7000.06
40	-20	0.729722	1000.092711	6878.45
50	0	0.666704	1000.069063	6796.76
60	20	0.614296	1000.052516	6748.27
70	40	0.570028	1000.040569	6726.13
80	60	0.532140	1000.031717	6724.59

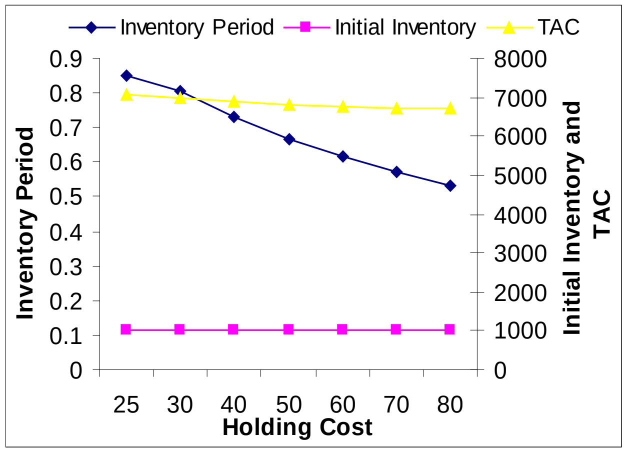

Figure 8. Holding Cost (C_H) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 8, we find that as holding cost increases, the optimal inventory period, optimal initial inventory level as well as optimal total average cost decreases. Moreover, about 20% increase in holding cost causes about 7.86%, 0.002% and 0.71% decrease in optimal inventory period, optimal initial inventory level and optimal total average cost respectively, which indicates that holding cost has negative correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

Table 6. Deterioration Cost (C_D) Vs. Optimal Total Average Cost (TAC*)
($C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_S = 80,\ C_L = 20,$
$d = 1000, T = 1,\ n = 4,\ \lambda = 10,\ \delta = 8,\ \theta = 0.01, \mu = 0.3,\ k = 0.15$)

Deterioration Cost (C_D)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
40	-60	0.763809	1000.107336	6871.06
60	-40	0.764636	1000.107707	6892.15
80	-20	0.765457	1000.108077	6913.11
100	0	0.766271	1000.108444	6933.95
120	20	0.767079	1000.108810	6954.66
140	40	0.767881	1000.109173	6975.25
160	60	0.768676	1000.109534	6995.73

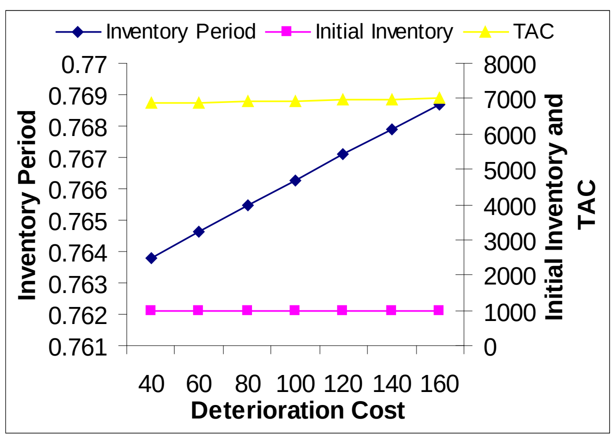

Figure 9. Deterioration Cost (C_D) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 9, we observe that as the deterioration cost increases, the optimal inventory period, optimal initial inventory level and optimal total average cost also increase. Numerically, an increase of 20% in deterioration cost amounts to about 0.11%, 0.0001% and 0.30% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively. Here deterioration cost shows a positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

Table 7. Shortage Cost (Cs) Vs. Optimal Total Average Cost (TAC*) ($C_O = 500$, $C_W = 5$, $Cp = 4$, $Ch = 35$, $C_D = 100$, $C_L = 20$, $d = 1000, T = 1$, $n = 4$, $\lambda = 10$, $\delta = 8$, $\theta = 0.01$, $\mu = 0.3$, $k = 0.15$)

Shortage Cost (Cs)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
56	-30	0.672614	1000.071102	6149.32
64	-20	0.707900	1000.084037	6400.12
72	-10	0.738871	1000.096506	6662.44
80	0	0.766271	1000.108444	6933.95
88	10	0.790685	1000.119826	7212.81
96	20	0.812576	1000.130648	7497.59
104	30	0.832315	1000.140919	7787.12

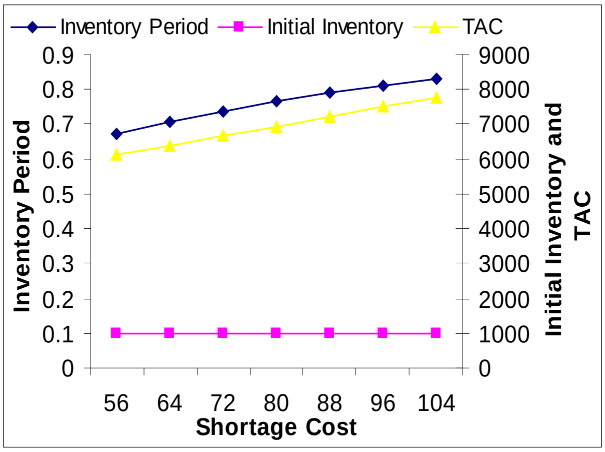

Figure 10. Shortage Cost (Cs) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 10, we find that as the shortage cost increases, the optimal inventory period, optimal initial inventory level and optimal total average cost also increase. More exactly, an increase of 30% in shortage cost amounts to about 8.62%, 0.003% and 12.30% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively. Here shortage cost shows a positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

Table 8. Lost Sales Cost (C_L) Vs. Optimal Total Average Cost (TAC*) ($C_O = 500$, $C_W = 5$, $Cp = 4$, $Ch = 35$, $C_D = 100$, $C_S = 80$, $d = 1000, T = 1$, $n = 4$, $\lambda = 10$, $\delta = 8$, $\theta = 0.01$, $\mu = 0.3$, $k = 0.15$)

Lost Sales Cost (C_L)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
10	-50	0.762606	1000.106797	6921.18
12	-40	0.763348	1000.107129	6923.75
16	-20	0.764818	1000.107790	6928.87
20	0	0.766271	1000.108444	6933.95
24	20	0.767706	1000.109093	6938.99
28	40	0.769123	1000.109737	6943.99
30	50	0.769825	1000.110057	6946.47

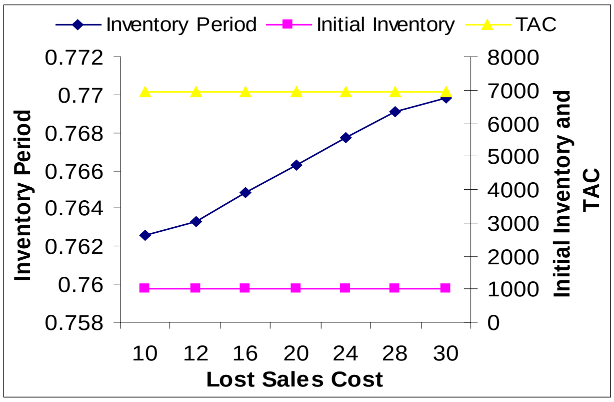

Figure 11. Lost Sales Cost (C_L) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 11, we find that as lost sales cost increases, the optimal inventory period, optimal initial inventory level as well as optimal total average cost also increases. Moreover, about 40% increase in lost sales cost causes about 0.37%, 0.0001% and 0.15% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively, which indicates that lost sales cost has a positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

Table 9. Cycle Time (T) Vs. Optimal Total Average Cost (TAC*) ($C_O = 500$, $C_W = 5$, $Cp = 4$, $Ch = 35$, $C_D = 100$, $C_S = 80$, $C_L = 20, d = 1000, n = 4, \lambda = 10, \delta = 8$, $\theta = 0.01, \mu = 0.3, k = 0.15$)

Cycle Time (T)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
0.5	-50	0.384319	1000.007632	10823.61
0.6	-40	0.458336	1000.017765	9319.84
0.7	-30	0.533458	1000.032003	8322.55
0.8	-20	0.609763	1000.051209	7651.19
0.9	-10	0.687336	1000.076338	7207.34
1.0	0	0.766271	1000.108444	6933.95
1.1	10	0.846672	1000.148702	6796.85

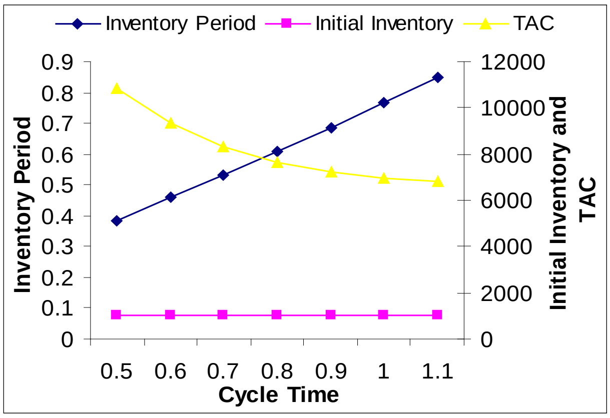

Figure 12. Cycle Time (T) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 12, we see that as cycle time increases, the optimal inventory period and optimal initial inventory level show an increasing trend whereas optimal total average cost shows a diminishing trend. In fact, about 10% increase in cycle time amounts to approx. 10.49% and 0.004% increase in optimal inventory period and optimal initial inventory level respectively but 1.98% decrease in optimal total average cost. Thus, cycle time is positively correlated with optimal inventory period and optimal initial inventory level whereas it is negatively correlated with optimal total average cost.

Table 10. Demand Parameter (n) Vs. Optimal Total Average Cost (TAC*) $(C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,$ $C_L = 20, d = 1000, T = 1, \lambda = 10, \delta = 8, \theta = 0.01, \mu = 0.3, k = 0.15)$

Demand Parameter (n)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
2	50	0.766271	1000.201396	8270.73
3	-25	0.766271	1000.140977	7441.61
4	0	0.766271	1000.108444	6933.95
5	25	0.766271	1000.088111	6591.56
6	50	0.766271	1000.074199	6345.14
7	75	0.766271	1000.064081	6159.36
8	100	0.766271	1000.056391	6014.29

In Table and Figure 13, we see that as demand parameter increases, the optimal inventory period remains unaffected whereas optimal initial inventory level as well as optimal total average cost show diminishing trend. Further, an increase of 50% in demand parameter creates about 0.003% and 8.49% decrease in optimal initial inventory level and optimal total average cost respectively, which indicates that demand parameter is negatively correlated with optimal initial inventory level and optimal total average cost.

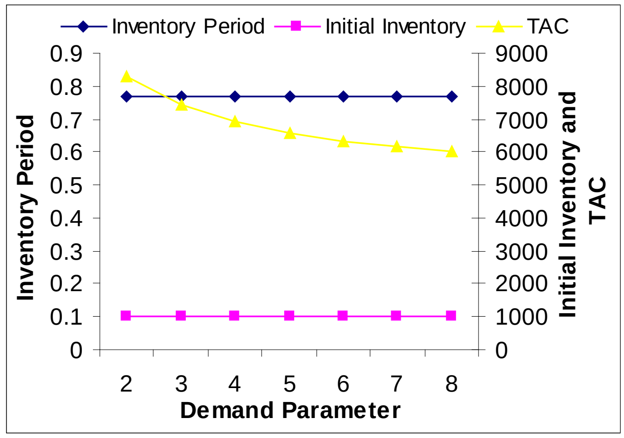

Figure 13. Demand Parameter (n) Vs. Optimal Total Average Cost (TAC*)

Table 11. Average Arrival Rate(λ) Vs. Optimal Total Average Cost (TAC*) $(C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,$ $C_L = 20, d = 1000,\ T = 1,\ n = 4,\ \delta = 18,\ \theta = 0.01, \mu = 0.3,\ k = 0.15)$

Average Arrival Rate(λ)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
19	-13.62	0.766271	1000.108444	6863.95
20	-9.08	0.766271	1000.108444	6908.95
21	-4.54	0.766271	1000.108444	6923.95
22	0.00	0.766271	1000.108444	6933.95
23	4.54	0.766271	1000.108444	6938.95
25	13.62	0.766271	1000.108444	6943.95
28	27.24	0.766271	1000.108444	6948.95

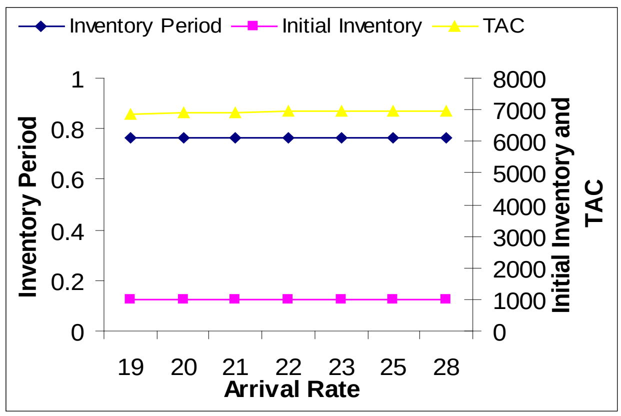

Figure 14. Average Arrival Rate(λ) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 14, we observe that as average arrival rate increases, the optimal total average cost also increases. In fact, about 27.24% increase in average arrival rate amounts to approx. 0.22% increase in optimal total average cost. Moreover, average arrival rate shows a positive correlation with optimal total average cost.

Table 12. Average Service Rate (δ) Vs. Optimal Total Average Cost (TAC*)
$(C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,$
$C_L = 20, d = 1000, T = 1,\ n = 4, \lambda = 25, \theta = 0.01, \mu = 0.3, k = 0.15)$

Average Service Rate (δ)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
18	-14.29	0.766271	1000.108444	6943.95
19	-9.52	0.766271	1000.108444	6938.95
20	-4.76	0.766271	1000.108444	6933.95
21	0.00	0.766271	1000.108444	6928.95
22	4.76	0.766271	1000.108444	6918.95
23	9.52	0.766271	1000.108444	6898.95
24	14.29	0.766271	1000.108444	6833.95

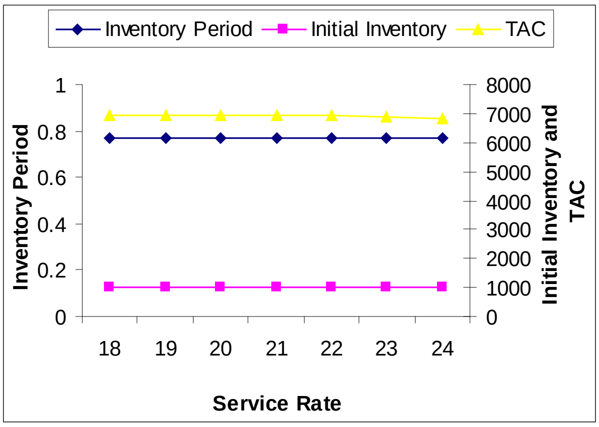

Figure 15. Average Service Rate (δ) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 15, we find that as average service rate increases, the optimal total average cost decreases. Numerically, about 14.29% increase in average service rate amounts to approx. 1.37% decrease in optimal total average cost. Moreover, average service rate shows a negative correlation with optimal total average cost.

Table 13. Deterioration Coefficient (θ)Vs. Optimal Total Average Cost (TAC*) $(C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,$
$C_L = 20, d = 1000, T = 1, n = 4, \lambda = 10, \delta = 8, \mu = 0.3, k = 0.15)$

Deterioration Coefficient (θ)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
0.005	-50	0.760886	1000.053014	6614.41
0.006	-40	0.761977	1000.063909	6677.72
0.008	-20	0.764138	1000.085987	6805.24
0.010	0	0.766271	1000.108444	6933.95
0.012	20	0.768376	1000.131277	7063.85
0.014	40	0.770455	1000.154482	7194.94
0.015	50	0.771484	1000.166222	7260.93

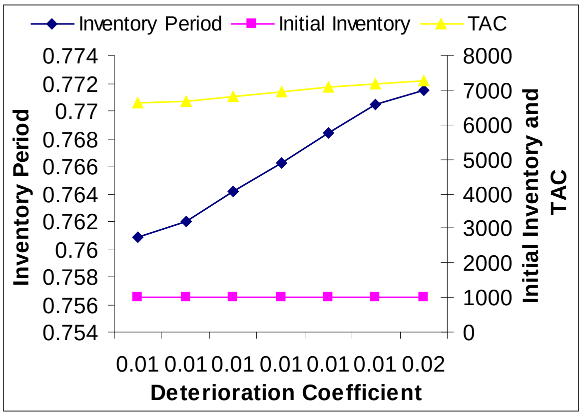

Figure 16. Deterioration Coefficient (θ)Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 16, we observe that as deterioration coefficient increases, the optimal inventory period, optimal initial inventory level and optimal total average cost also increases. Actually, about 40% increase in deterioration coefficient causes about 0.55%, 0.005% and 3.76% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively, which indicates that deterioration coefficient has positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

Table 14. Life Time (μ) Vs. Optimal Total Average Cost (TAC*)
$(C_O = 500,\ C_W = 5,\ Cp = 4,\ Ch = 35,\ C_D = 100,\ C_S = 80,$
$C_L = 20, d = 1000,\ T = 1,\ n = 4,\ \lambda = 10,\ \delta = 8,\ \theta = 0.01,\ k = 0.15)$

Life Time (μ)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
0.1	-67	0.765828	1000.114911	6924.45
0.2	-33	0.765991	1000.113190	6928.00
0.3	0	0.766271	1000.108444	6933.95
0.4	33	0.766676	1000.099140	6942.32
0.5	67	0.767212	1000.083742	6953.17
0.6	100	0.767887	1000.060714	6966.54
0.7	133	0.768709	1000.028523	6982.51

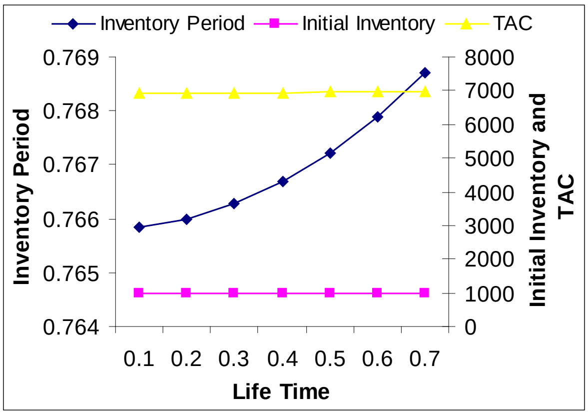

Figure 17. Life Time (μ) Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 17, we observe that as life time of deteriorating items increases, the optimal inventory period and optimal total average cost increases whereas optimal initial inventory level diminishes. Numerically, about 67% increase in life time causes about 0.12% and 0.28% increase in optimal inventory period and optimal total average cost respectively but 0.003% decrease in optimal initial inventory level. Thus life time is positively correlated with optimal inventory period and optimal total average cost but it is negatively correlated with optimal initial inventory level.

Table 15. Backlogging Coefficient (k)Vs. Optimal Total Average Cost (TAC*) $(C_O = 500, C_W = 5, Cp = 4, Ch = 35, C_D = 100, C_S = 80, C_L = 20, d = 1000, T = 1, n = 4, \lambda = 10, \delta = 8, \theta = 0.01, \mu = 0.3)$

Backlogging Coefficient (k)	% Change	Inventory Period (t1*)	Initial Inventory Level (S)	Optimal Total Average Cost (TAC*)
0.06	-60	0.727506	1000.091806	5742.59
0.09	-40	0.739399	1000.096728	6125.36
0.12	-20	0.752279	1000.102239	6521.71
0.15	0	0.766271	1000.108444	6933.95
0.18	20	0.781528	1000.115473	7364.90
0.21	40	0.798229	1000.123489	7818.08
0.24	60	0.816589	1000.132696	8297.89

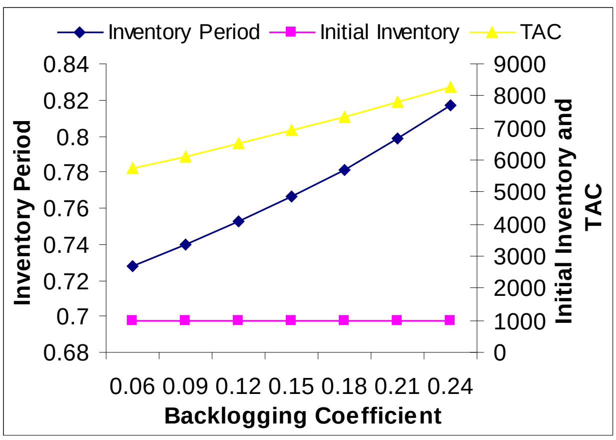

Figure 18. Backlogging Coefficient (k)Vs. Optimal Total Average Cost (TAC*)

In Table and Figure 18, we find that as the backlogging coefficient increases, the optimal inventory period, optimal initial inventory level and optimal total average cost also increase. More exactly, an increase of 40% in backlogging coefficient amounts to about 4.17%, 0.002% and 12.75% increase in optimal inventory period, optimal initial inventory level and optimal total average cost respectively. Here backlogging coefficient shows a positive correlation with optimal inventory period, optimal initial inventory level and optimal total average cost.

5. Conclusions

In the present paper, we have developed an inventory model for perishable items with power form time dependent demand and quadratic deterioration rate. This type of demand if occurs, then managers develop a different policy other than the conventional policy based on general ramp pattern. In cases where large portion of demand occurs at the beginning of the period we use $n > 1$ and if it occurs at the end of the period, we use $0 < n < 1$. Constant demand rate corresponds to $n = 1$ and $n = \infty$ corresponds to instantaneous demand. Shortages are allowed and the backlogging rate is dependent on the duration of waiting time for the next replenishment and varies inversely. Shortages are partially backlogged in this model. Behaviours of different parameters have been discussed through the numerical example and sensitivity analysis. A future study will incorporate more realistic assumptions in the proposed model such as stochastic nature of demand and deterioration.

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