

# Cancellation Series

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**Abstract** Mathematical series play an important role in various aspects of our life and help to predict, evaluate and monitor the outcome of a situation in order to pass decision. The purpose of this paper is to develop a new kind of series and the sum or deduction of two terms of which fractionally cancel in order to get the cancellation result  $r$ . let for all  $m, d, n \in N$  and let  $m_n, A_n, x_n, x_{m+nd}, r \in R$  such that  $A_n x_n = x_{m+nd} \pm x_{m+(n-1)d}$  and  $x_{m+nd}(m_n \pm m_{n+1}) = r$  then

$$\sum_{i=1}^n m_i(A_i x_i) = m_n x_{m+nd} \pm m_1 x_m + r(n-1).$$

**Keywords** Series, Cancellation series

## 1. Introduction

Telescoping series is a very important series for nowadays technology achievement. It is very simple to use because of its alternative, positive and negative, sign for the same term in order to get cancellation zero [7]. In telescoping, the sum of series ( $s_n$ ) must be represented only by  $a_n - a_{n+1}$  after each term cancelling with part of the next term is known as the method of differences [6] [2]. A telescoping product is a finite product (or the partial product of an infinite product) that can be cancelled by method of quotients to be eventually only a finite number of factors [5] [1]. Unless each term of a sequence related with one to another through succession of the same pattern, there is no formula if the sequence terms are succession of random number [3]. The idea of telescoping a series is widely known, but is not widely trusted. It is often treated as a formalism with no meaning, unless convergence is already established. even for divergent series, the results of telescoping are self-consistent, and consistent with other well-behaved summation operations. Moreover, the summation operations obtained by telescoping are the strongest possible operations with these properties [4]. However, on this paper I want to show that how a new series can be formed rather than telescoping series. The purpose of the method is to get the cancellation  $r$  and to form a new kind of series. The process of cancellation of telescoping series is straight, but the process of cancellation of this paper is fractional. Therefore, this paper does have an effort in solving different mathematical problems.

**Theorem 1:** let for all  $m, d, n \in N$  (Natural numbers) and let  $m_n, A_n, x_n, x_{m+nd}, r \in R$  (Real Numbers) such that  $A_n x_n = x_{m+nd} \pm x_{m+(n-1)d}$  and  $x_{m+nd}(m_n \pm m_{n+1}) = r$  then

$$\sum_{i=1}^n m_i(A_i x_i) = m_n x_{m+nd} \pm m_1 x_m + r(n-1)$$

**Proof.** let for all  $m, d, n \in N$  and let  $m_n, A_n, x_n, x_{m+nd}, r \in R$  such that  $A_n x_n = x_{m+nd} \pm x_{m+(n-1)d}$  and  $x_{m+nd}(m_n \pm m_{n+1}) = r$  then

$$\begin{aligned} \sum_{i=1}^n m_i(A_i x_i) &= m_1(A_1 x_1) + m_2(A_2 x_2) + m_3(A_3 x_3) + \dots + \\ & \quad m_n(A_n x_n) \\ &= m_1(x_{m+d} \pm x_m) + m_2(x_{m+2d} \pm x_{m+d}) + \\ & \quad m_3(x_{m+3d} \pm x_{m+2d}) + \dots + m_n(x_{m+nd} \pm x_{m+(n-1)d}) \end{aligned}$$

Gathering the same terms, we have the following

$$= \pm m_1 x_m + x_{m+d}(m_1 \pm m_2) + x_{m+2d}(m_2 \pm m_3) + \dots + x_{m+(n-1)d}(m_{n-1} \pm m_n) + m_n x_{m+nd}$$

Therefore, we have

$$\sum_{i=1}^n m_i(A_i x_i) = m_n x_{m+nd} \pm m_1 x_m + r(n-1)$$

## 2. Conclusions

The cancellation series can generate different kinds of unknown but viable series, which can be a base for number theory specifically and for mathematics in general. The

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benefit of this series enables us to add up sequences of couple terms having similar pattern. Here the end result of cancellation series is the difference or addition between the first and last terms and the number of cancelled results even though the result of telescoping series leaves the difference between first and last term. It also utilizes the technique of Partial Fractions which is a way for us to express a rational function (algebraic fraction) as a sum of simpler fractions. The method does have a great addition to mathematics by simplifying tedious series into three terms.

**Illustration:**

Let the series be  $3+3+5/2+3+27/8$ , this can be simplified as follows

$$3+3+5/2+3+27/8 = \frac{1}{2}(2+4) + \frac{1}{4}(4+8) + \frac{1}{8}(8+12) +$$

$$\frac{1}{8}(12+12) + \frac{1}{8}(12+15)$$

$$3+3+5/2+3+27/8 = \frac{1}{2}(2) + 4(\frac{1}{2} + \frac{1}{4}) + 8(\frac{1}{4} + \frac{1}{8}) +$$

$$12(\frac{1}{8} + \frac{1}{8}) + 12(\frac{1}{8} + \frac{1}{8}) + \frac{1}{8}(15)$$

$$3+3+5/2+3+27/8 = 1+3+3+3+15/8$$

$$3+3+5/2+3+27/8 = 1+4 \times 3 + 15/8$$

$$\underline{14.875} = \underline{14.875}$$

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## REFERENCES

- [1] Bogomolny, Alexander. Telescoping Sums, Series and Products, Cut the Knot. Retrieved 9 February 2020.
- [2] Brian S. Thomson and Andrew M. Bruckner, 2008, Elementary Real Analysis, Second Edition, CreateSpace, p. 85.
- [3] G. Baley Price, 1973, Telescoping sums and The summation of sequences, The Two-year college Mathematics Journal, Vol. 4, No. 2, PP. 16-29. <https://doi.org/10.2307/3026439>.
- [4] Robert J. Mac G. Dawson, 1998, Formal Summation of Divergent Series, Journal of Mathematical analysis and Applications, Vol. 225, PP. 532-541.
- [5] Telescoping Series – Product, Brilliant Math & Science Wiki. Brilliant.org. Retrieved 9 February 2020.
- [6] Tom M. Apostol, 1962, Calculus, Vol. 1, Blaisdell Publishing Company, pp. 422–423.
- [7] Weisstein, Eric W., Telescoping Sum, MathWorld. Wolfram.