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# A Fuzzy Inventory Model for Deteriorating Items Based on Different Defuzzification Techniques

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**Abstract** In traditional inventory models, parameters are considered to be constants. But in reality, the system parameters cannot be considered as constants. In this paper, a production Inventory model with deterioration is developed to determine optimal inventory policy in fuzzy environment. It is assumed that some of the parameters of the model are fuzzy number due to their impreciseness. An effort is given to study the effect of different defuzzification techniques on the optimal value of the variable and associated cost function. An algorithm is proposed to solve the developed model and coded in MATLAB. To validate optimal policy derived, numerical examples along with graphical representations of the results are presented. Finally sensitivity analysis has been carried out.

**Keywords** Fuzzy Production, Fuzzy Demand, Deterioration, Defuzzification, Optimal Inventory Policy

## 1. Introduction

The deterioration plays an important role in the study of inventory control. Several researchers developed inventory models with deterioration in precise and imprecise environment. Some of the researchers considered demand rate, deterioration, production rate etc. as constants and others considered those parameters as fuzzy in nature. Jaggi et al. [1] developed fuzzy inventory model with deterioration where demand was taken as time-varying. Nagar and Surana [5] developed fuzzy inventory model for deteriorating items with fluctuating demand. Nezhad et al. [4] developed periodic and continuous review model by taking fuzzy set up cost, holding cost and shortage cost. Jain et al. [3] gave a fuzzy generic algorithmic approach for inventory model for deteriorating items with back orders under fuzzy inflation and discounting over random planning horizon. Kumar and Rajput [6] developed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging where demand rate, deterioration rate and backlogging rate were considered triangular fuzzy numbers. Roy et al. [8] developed inventory model for seasonal deteriorating item with linearly displayed stock dependent demand in imprecise environment under inflation and time value of money. Saha and Chakra borty [11] developed inventory model with time dependent demand and deterioration with shortages. Recently Jaggi et al. [2] developed inventory model for optimal replenishment policy under inflationary condition. In the field of fuzzy inventory modeling for deteriorating items, the work of Roy [9], Rong et al. [10], Panda et al. [7] are worth mentioning. All the above mentioned work discussed inventory models for deteriorating items in fuzzy environment and defuzzification were done by various methods.

In present paper, a production inventory model for deteriorating item is discussed. The rate of deterioration along with other parameters is taken as triangular fuzzy numbers. For defuzzification, we have used signed distance method, centroid method and graded mean integration method separately. To the best of authors' knowledge, this type of defuzzification approach at a time has not done in literature in order to see the effect on the optimal results.

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## 2. Preliminaries on Fuzzy Numbers and Defuzzification Methods

It is always better to discuss decision making problems in fuzzy sense. The use of fuzzy number is better than the probabilistic approach as mentioned by Zadeh [12, 13].

**Fuzzy Number:** A fuzzy number is a fuzzy set which is both convex and normal.

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 < a_2 < a_3$ is triangular if its membership function is defined as

$$\mu_{\tilde{A}} = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

In the same manner, trapezoidal fuzzy number, parabolic fuzzy number, pentagonal fuzzy number, hexagonal fuzzy numbers etc. can be defined. There are number of methods available for defuzzification of fuzzy numbers. The mostly used methods for defuzzification are graded mean integration method, centroid method and signed distance method. Graded mean integration representation of triangular fuzzy number is defined as $d_F(\tilde{A}) = \frac{a_1+4a_2+a_3}{6}$. The centroid method for triangular fuzzy number is $d_F(\tilde{A}) = \frac{a_1+a_2+a_3}{3}$ and signed distanced method for triangular fuzzy number is defined as $d_F(\tilde{A}) = \frac{a_1+2a_2+a_3}{4}$.

## 3. Assumptions

I. The inventory system involves production of single item.

II. Lead time is zero and shortages are not allowed.

III. The set-up cost, deterioration rate, holding cost are fuzzy.

IV. The production rate, demand rate are fuzzy.

V. Replenishment is instantaneous.

## 4. Notations

$A \to$ Set-up cost per cycle.
$\tilde{A} \to$ Fuzzy set-up cost.
$\theta \to$ Deterioration rate independent of time, $0 < \theta \ll 1$.
$\tilde{\theta} \to$ Fuzzy deterioration.
$T \to$ Cycle length.
$P \to$ Production rate.
$\tilde{P} \to$ Fuzzy production rate.
$h \to$ Holding cost per unit per unit time.
$\tilde{h} \to$ Fuzzy holding cost per unit per unit time.
$d \to$ Deterioration cost per unit per unit time.
$\tilde{d} \to$ Fuzzy deterioration cost per unit per unite time.
$D \to$ Demand rate is constant.
$\tilde{D} \to$ Fuzzy demand rate.
$t_1 \to$ Duration of production.
$I_1(t) \to$ Inventory level at time $t, 0 \le t \le t_1$.
$I_2(t) \to$ Inventory level at time $t,\ t_1 \le t \le T$.
$TC \to$ Total cost per unit time.
$\widetilde{TC} \to$ Fuzzy value of $TC$.
$d_F\widetilde{TC} \to$ Defuzzfied value of $\widetilde{TC}$.

## 5. The Proposed Model

The objective of this work is to formulate and solve production inventory model considering the aforesaid assumptions.

At $t = 0$, the inventory level is zero. It increases in the time period $[0, t_1]$ due to production at the constant rate $P$. After that inventory level decreases and reaches to zero at $t = T$. This depletion is due to demand and deterioration of the item. This situation is represented in fig 1.

For $0 \le t \le t_1$

The differential equation governing the situation is

$$\frac{d}{dt} I_1(t) = P - \{D + \theta I_1(t)\}$$
$$\Rightarrow \frac{d}{dt} I_1(t) + \theta I_1(t) = P - D \quad (5.1)$$

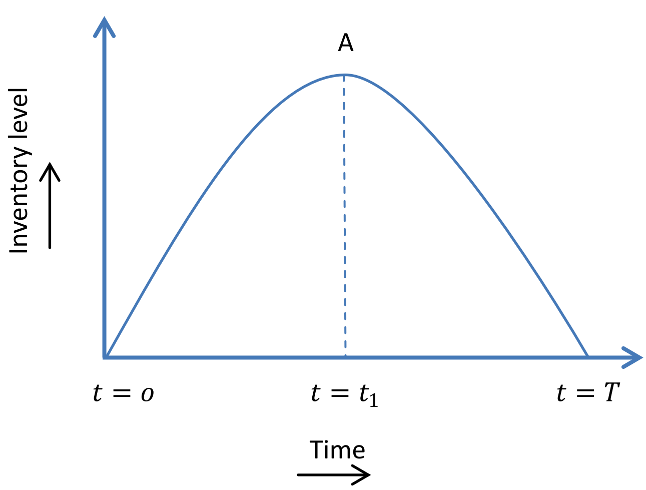


**Figure 1.**

which is linear. Therefore the solution is given by

$$I_1(t)e^{\theta t} = \frac{1}{\theta}(P-D)e^{\theta t} + c_1$$

$$\Rightarrow I_1(t) = \frac{1}{\theta}(P-D) + c_1 e^{-\theta t}$$

Using the condition that $I_1(t) = 0$ at $= 0$, we get

$$c_1 = -\frac{1}{\theta}(P-D)$$

$$\therefore I_1(t) = \frac{1}{\theta}(P-D)(1-e^{-\theta t}) \qquad (5.2)$$

For $t_1 \le t \le T$, the differential equation governing the situation is

$$\frac{d}{dt}I_2(t) = -\{D + \theta I_2(t)\}$$
$$\Rightarrow \frac{d}{dt}I_2(t) + \theta I_2(t) = -D \qquad (5.3)$$

Which is linear and therefore, the solution is given by

$$I_2(t)e^{\theta t} = -\frac{D}{\theta}e^{\theta t} + c_2$$

$$\Rightarrow I_2(t) = -\frac{D}{\theta} + c_2 e^{-\theta t}$$

Using the condition that $I_2(t) = 0$ at $t = T$, we get

$$c_2 = \frac{D}{\theta}e^{\theta T}$$

$$\therefore I_2(t) = \frac{D}{\theta}\{e^{\theta(T-t)} - 1\} \qquad (5.4)$$

Now we find $t_1$ by using $I_2(t_1) = I_1(t_1)$

$$\frac{D}{\theta}\{e^{\theta(T-t_1)} - 1\} = \frac{P-D}{\theta}(1-e^{-\theta t_1})$$

$$\Rightarrow De^{\theta(T-t_1)} - D = P - D - Pe^{-\theta t_1} + De^{-\theta t_1} \qquad (5.5)$$

$$\Rightarrow De^{\theta(T-t_1)} = P - Pe^{-\theta t_1} + De^{-\theta t_1}$$

$$\Rightarrow De^{\theta T} = Pe^{\theta t_1} - (P-D)$$

$$\Rightarrow Pe^{\theta t_1} = P - D + De^{\theta T}$$

$$\Rightarrow Pe^{\theta t_1} = P + D(e^{\theta T} - 1) \qquad (5.6)$$

$$\Rightarrow e^{\theta t_1} = 1 + \frac{D}{P}(e^{\theta T} - 1)$$

$$\Rightarrow t_1 = \frac{1}{\theta}\log\{1 + \frac{D}{P}(e^{\theta T} - 1)\}$$

Now we find different inventory costs namely
1. Holding cost per cycle
2. Deterioration cost per cycle
3. Total cost per unit time

Holding cost (per cycle) =H.C.=$h$ (area of the region OABO) $= h\,[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt]$

$$= h\left\{\left[\frac{P-D}{\theta}(t + \frac{e^{-\theta t}}{\theta})\right]_0^{t_1} + \left[\frac{D}{\theta}\left\{\frac{e^{\theta(T-t)}}{-\theta} - t\right\}\right]_{t_1}^{T}\right\}$$

$$= h\left[\frac{P-D}{\theta}\left(t_1 + \frac{e^{-\theta t_1}}{\theta}\right) - \frac{P-D}{\theta}\frac{1}{\theta} + \frac{D}{\theta}\left(-\frac{1}{\theta} - T\right) - \frac{D}{\theta}\left\{\frac{e^{\theta(T-t_1)}}{-\theta} - t_1\right\}\right]$$

$$= h\left[\frac{P-D}{\theta^2}\left(\theta t_1 + e^{-\theta t_1} - 1\right) - \frac{D}{\theta^2}\left(1 + \theta T\right) + \frac{D}{\theta^2}\left\{e^{\theta(T-t_1)} + \theta t_1\right\}\right]$$

$$= h\left[\frac{P}{\theta^2}\left(\theta t_1 + e^{-\theta t_1} - 1\right) - \frac{D}{\theta^2}\left(\theta t_1 + e^{-\theta t_1} - 1\right) + \frac{D}{\theta^2}\left\{e^{\theta(T-t_1)} + \theta t_1 - 1 - \theta T\right\}\right]$$

$$= \frac{h}{\theta^2}\left[P(\theta t_1 + e^{-\theta t_1} - 1) + (De^{\theta(T-t_1)} - De^{-\theta t_1}) - D\theta T\right]$$

$$= \frac{h}{\theta^2}\left[P(\theta t_1 + e^{-\theta t_1} - 1) + (P - Pe^{-\theta t_1}) - D\theta T\right\} \text{ (using 5.5)}$$

$$= \frac{h}{\theta^2}\left[P\theta t_1 - D\theta T\right] = \frac{h}{\theta}\left[Pt_1 - DT\right] \qquad (5.7)$$

The detorioration cost (per cycle) =D.C.= $d\left[\int_0^{t_1}\theta I_1(t)dt+\int_{t_1}^{T}\theta I_2(t)dt\right]=d\theta\left(\frac{Pt_1-DT}{\theta}\right)=d(Pt_1-DT)$ using(5.7)

Total cost per unit time $=TC=\frac{1}{T}\left[A+H.C.+D.C.\right]$ (5.8)

$$=\frac{A}{T}+\frac{1}{T}\frac{h}{\theta}(Pt_1-DT)+\frac{d}{\theta}(Pt_1-DT)$$

$$=\frac{A}{T}+\frac{(Pt_1-DT)}{\theta T}(h+d\theta)$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[\frac{P}{\theta}\log\left\{1+\frac{D}{P}(e^{\theta T}-1)\right\}-DT\right] \text{using (5.6)}$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[\frac{P}{\theta}\log\left\{1+\frac{D}{P}(\theta T+\frac{\theta^2T^2}{2})\right\}-DT\right], \text{neglecting higher powers of } \theta$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[\frac{P}{\theta}\left\{\frac{D}{P}\left(\theta T+\frac{\theta^2T^2}{2}\right)-\frac{D^2}{2P^2}\left(\theta T+\frac{\theta^2T^2}{2}\right)^2\right\}-DT\right], \text{neglecting higher powers of } \theta$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[\frac{P}{\theta}\left\{\frac{D}{P}\left(\theta T+\frac{\theta^2T^2}{2}\right)-\frac{D^2}{2P^2}\theta^2T^2\right\}-DT\right], \text{neglecting higher powers of } \theta \quad (5.9)$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[\frac{P}{\theta}\left\{\frac{D\theta T}{P}+\frac{D\theta^2T^2}{2P}-\frac{D^2\theta^2T^2}{2P^2}\right\}-DT\right]$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[DT+\frac{D\theta T^2}{2}-\frac{D^2\theta T^2}{2P}-DT\right]$$

$$=\frac{A}{T}+\frac{(h+d\theta)}{\theta T}\left[\frac{D\theta T^2}{2}-\frac{D^2\theta T^2}{2P}\right]$$

$$=\frac{A}{T}+DT\left(\frac{h+d\theta}{2}\right)\left(1-\frac{D}{P}\right)$$

$$=\frac{1}{T}\left[A+\frac{1}{2}(h+d\theta)DT^2-\frac{1}{2}(h+d\theta)\frac{D^2T^2}{P}\right]$$

## 6. Fuzzy Model

Due to uncertainty in the environment, it is not always possible to define certain parameters with certainty for which we fuzzify some parameters. Here we fuzzify the parameters $A, h, d, \theta, P, D$.

We consider triangular fuzzy numbers

$$\tilde{A}=(A_1,A_2,A_3),\quad \tilde{h}=(h_1,h_2,h_3),\quad \tilde{d}=(d_1,d_2,d_3)$$

$$\tilde{\theta}=(\theta_1,\theta_2,\theta_3),\quad \tilde{P}=(P_1,P_2,P_3),\quad \tilde{D}=(D_1,D_2,D_3)$$

$$\therefore \widetilde{TC}=\frac{1}{T}\left[\tilde{A}+\frac{1}{2}(\tilde{h}+\tilde{d}\tilde{\theta})\tilde{D}T^2-\frac{1}{2}(\tilde{h}+\tilde{d}\tilde{\theta})\frac{\tilde{D}^2T^2}{\tilde{P}}\right]$$

$$=(TC1,TC2,TC3) \text{ say}$$

where,

$$TCi=\frac{1}{T}\left[A_i+\frac{1}{2}(h_i+d_i\theta_i)D_iT^2-\frac{1}{2}(h_{4-i}+d_{4-i}\theta_{4-i})\frac{D_{4-i}^2T^2}{P_i}\right],i=1,2,3$$

Now we find the derivatives

$$\frac{\mathrm{d}}{\mathrm{dT}}TCi=-\frac{1}{T^2}\left[A_i+\frac{1}{2}(h_i+d_i\theta_i)D_iT^2-\frac{1}{2}(h_{4-i}+d_{4-i}\theta_{4-i})\frac{D_{4-i}^2T^2}{P_i}\right]+\left[(h_i+d_i\theta_i)D_i-(h_{4-i}+d_{4-i}\theta_{4-i})\frac{D_{4-i}^2}{P_i}\right]$$

$$=-\frac{A_i}{T^2}+\left[-\frac{1}{2}(h_i+d_i\theta_i)D_i+\frac{1}{2}(h_{4-i}+d_{4-i}\theta_{4-i})\frac{D_{4-i}^2}{P_i}\right]+\left[(h_i+d_i\theta_i)D_i-(h_{4-i}+d_{4-i}\theta_{4-i})\frac{D_{4-i}^2}{P_i}\right]$$

$$=-\frac{A_i}{T^2}+\left[\frac{1}{2}(h_i+d_i\theta_i)D_i-\frac{1}{2}(h_{4-i}+d_{4-i}\theta_{4-i})\frac{D_{4-i}^2}{P_i}\right],i=1,2,3$$

And

$$\frac{\mathrm{d}^2}{\mathrm{dT}^2}TCi=\frac{2Ai}{T^3},i=1,2,3$$

Now we find the defuzzified value of $\widetilde{TC}$ by

1. Graded mean integration method
2. Signed distance method
3. Centroid method

**By Graded Mean Integration Method**

The defuzzified value is:

$$d_F\widetilde{TC}=\frac{1}{6}\left(TC1+4TC2+TC3\right)$$

$$\therefore\frac{d}{dT}d_F\widetilde{TC}=\frac{1}{6}\left(\frac{d}{dT}TC1+4\frac{d}{dT}TC2+\frac{d}{dT}TC3\right)$$

$$=\frac{1}{6}\left[\begin{array}{l}-\dfrac{A_1+4A_2+A_3}{T^2}+\left\{\dfrac{1}{2}\left(h_1+d_1\theta_1\right)D_1+4\times\dfrac{1}{2}\left(h_2+d_2\theta_2\right)D_2+\dfrac{1}{2}\left(h_3+d_3\theta_3\right)D_3\right\}\\-\left\{\dfrac{1}{2}\left(h_1+d_1\theta_1\right)\dfrac{D_1^2}{P_3}+4\times\dfrac{1}{2}\left(h_2+d_2\theta_2\right)\dfrac{D_2^2}{P_2}+\dfrac{1}{2}\left(h_3+d_3\theta_3\right)\dfrac{D_3^2}{P_1}\right\}\end{array}\right]$$

And

$$\frac{d^2}{dT^2}d_F\widetilde{TC}=\frac{1}{6}\left(\frac{d^2}{dT^2}TC1+4\frac{d^2}{dT^2}TC2+\frac{d^2}{dT^2}TC3\right)$$

$$=\frac{1}{6}\left(\frac{2A_1}{T^3}+4\times\frac{2A_2}{T^3}+\frac{2A_3}{T^3}\right)=\frac{1}{3T^3}\left(A_1+4A_2+A_3\right)$$

Now,

$$\frac{d}{dT}d_F\widetilde{TC}=0$$

$$\Rightarrow T^2 = \frac{A_1 + 4A_2 + A_3}{\left\{\frac{1}{2}(h_1 + d_1\theta_1)D_1 + 4\times\frac{1}{2}(h_2 + d_2\theta_2)D_2 + \frac{1}{2}(h_3 + d_3\theta_3)D_3\right\} - \left\{\frac{1}{2}(h_1 + d_1\theta_1)\frac{D_1^2}{P_3} + 4\times\frac{1}{2}(h_2 + d_2\theta_2)\frac{D_2^2}{P_2} + \frac{1}{2}(h_3 + d_3\theta_3)\frac{D_3^2}{P_1}\right\}}$$

$$\Rightarrow T^2 = \frac{2(A_1 + 4A_2 + A_3)}{\left\{(h_1 + d_1\theta_1)D_1 + 4(h_2 + d_2\theta_2)D_2 + (h_3 + d_3\theta_3)D_3\right\} - \left\{(h_1 + d_1\theta_1)\frac{D_1^2}{P_3} + 4(h_2 + d_2\theta_2)\frac{D_2^2}{P_2} + (h_3 + d_3\theta_3)\frac{D_3^2}{P_1}\right\}}$$

$$\Rightarrow T^2 = \frac{2(A_1 + 4A_2 + A_3)}{(h_1 + d_1\theta_1)D_1\left(1 - \frac{D_1}{P_3}\right) + 4(h_2 + d_2\theta_2)D_2\left(1 - \frac{D_2}{P_2}\right) + (h_3 + d_3\theta_3)D_3\left(1 - \frac{D_3}{P_1}\right)}$$

$$\Rightarrow T = \sqrt{\frac{2(A_1 + 4A_2 + A_3)}{(h_1 + d_1\theta_1)D_1\left(1 - \frac{D_1}{P_3}\right) + 4(h_2 + d_2\theta_2)D_2\left(1 - \frac{D_2}{P_2}\right) + (h_3 + d_3\theta_3)D_3\left(1 - \frac{D_3}{P_1}\right)}}$$

**By Signed Distance Method**

The defuzzified value is:

$$d_F\widetilde{TC} = \frac{1}{4}(TC1 + 2TC2 + TC3)$$

$$\therefore \frac{d}{dT}d_F\widetilde{TC} = \frac{1}{4}\left(\frac{d}{dT}TC1 + 2\frac{d}{dT}TC2 + \frac{d}{dT}TC3\right)$$

$$= \frac{1}{4}\left[\begin{array}{l} -\frac{A_1 + 2A_2 + A_3}{T^2} + \left\{\frac{1}{2}(h_1 + d_1\theta_1)D_1 + 2\times\frac{1}{2}(h_2 + d_2\theta_2)D_2 + \frac{1}{2}(h_3 + d_3\theta_3)D_3\right\} \\ -\left\{\frac{1}{2}(h_1 + d_1\theta_1)\frac{D_1^2}{P_3} + 2\times\frac{1}{2}(h_2 + d_2\theta_2)\frac{D_2^2}{P_2} + \frac{1}{2}(h_3 + d_3\theta_3)\frac{D_3^2}{P_1}\right\} \end{array}\right]$$

And

$$\frac{d^2}{dT^2}d_F\widetilde{TC} = \frac{1}{4}\left(\frac{d^2}{dT^2}TC1 + 2\frac{d^2}{dT^2}TC2 + \frac{d^2}{dT^2}TC3\right)$$

$$= \frac{1}{4}\left(\frac{2A_1}{T^3} + 2\times\frac{2A_2}{T^3} + \frac{2A_3}{T^3}\right) = \frac{1}{2T^3}(A_1 + 2A_2 + A_3)$$

Now,

$$\frac{d}{dT}d_F\widetilde{TC} = 0$$

$$\Rightarrow T = \sqrt{\frac{2(A_1 + 2A_2 + A_3)}{(h_1 + d_1\theta_1)D_1\left(1 - \frac{D_1}{P_3}\right) + 2(h_2 + d_2\theta_2)D_2\left(1 - \frac{D_2}{P_2}\right) + (h_3 + d_3\theta_3)D_3\left(1 - \frac{D_3}{P_1}\right)}}$$

**By Centroid Method**

The defuzzified value is:

$$d_F\widetilde{TC}=\frac{1}{3}\left(TC1+TC2+TC3\right)$$

$$\therefore\frac{d}{dT}d_F\widetilde{TC}=\frac{1}{3}\left(\frac{d}{dT}TC1+\frac{d}{dT}TC2+\frac{d}{dT}TC3\right)$$

$$=\frac{1}{3}\left[\begin{array}{l}-\dfrac{A_1+A_2+A_3}{T^2}+\left\{\dfrac{1}{2}\left(h_1+d_1\theta_1\right)D_1+\dfrac{1}{2}\left(h_2+d_2\theta_2\right)D_2+\dfrac{1}{2}\left(h_3+d_3\theta_3\right)D_3\right\}\\-\left\{\dfrac{1}{2}\left(h_1+d_1\theta_1\right)\dfrac{D_1^2}{P_3}+\dfrac{1}{2}\left(h_2+d_2\theta_2\right)\dfrac{D_2^2}{P_2}+\dfrac{1}{2}\left(h_3+d_3\theta_3\right)\dfrac{D_3^2}{P_1}\right\}\end{array}\right]$$

and

$$\frac{d^2}{dT^2}d_F\widetilde{TC}=\frac{1}{3}\left(\frac{d^2}{dT^2}TC1+\frac{d^2}{dT^2}TC2+\frac{d^2}{dT^2}TC3\right)$$

$$=\frac{1}{3}\left(\frac{2A_1}{T^3}+\frac{2A_2}{T^3}+\frac{2A_3}{T^3}\right)=\frac{2}{3T^3}\left(A_1+A_2+A_3\right)$$

Now,

$$\frac{d}{dT}d_F\widetilde{TC}=0$$

$$\Rightarrow T=\sqrt{\frac{2\left(A_1+A_2+A_3\right)}{\left(h_1+d_1\theta_1\right)D_1\left(1-\dfrac{D_1}{P_3}\right)+\left(h_2+d_2\theta_2\right)D_2\left(1-\dfrac{D_2}{P_2}\right)+\left(h_3+d_3\theta_3\right)D_3\left(1-\dfrac{D_3}{P_1}\right)}}$$

*Let* $\frac{\widetilde{D}}{\widetilde{P}}<1$ i.e. $\frac{D_1}{P_3}<1$, $\frac{D_2}{P_2}<1$, $\frac{D_3}{P_1}<1$, then in each case, $T$ exists and clearly for this value of $T$, we see that $\frac{d^2}{dT^2}d_F\widetilde{TC}>0$ so that defuzzified value of total cost per unit time i.e. $d_F\widetilde{TC}$ is minimum. And the minimum value is obtained by putting the value $T$ in $d_F\widetilde{TC}$ for the respective cases.

## 7. Solution Procedure

To solve the proposed model, an algorithm is developed and coded in MATLAB. The proposed algorithm is as below.

**Algorithm:**

Set fuzzy variables: $\tilde{A},\tilde{h},\tilde{D},\tilde{\theta},\tilde{P}$

Repeat:

for $T\in[0,1]$ (Cycle length)

Evaluate Average Total Cost: TC

set: TC1, TC2,TC3

Evaluate: $TC_{gm},TC_{sd},TC_{cen}$

Find the optimal cost: $\min\{TC_{gm}\},\min\{TC_{sd}\},\min\{TC_{cen}\}$

End loop

## 8. Results and Discussion

The following numerical values of the parameters in appropriate units are considered to analyze the model numerically and graphically,

$$\tilde{A} = (50,54,58)\ \ \tilde{h} = (6,8,10)\ \ \tilde{\theta} = (0.006,0.010,0.014)\ \ \tilde{P} = (500,550,600)\ \ \tilde{D} = (450,500,550)$$

$$\tilde{d} = (1.2,1.5,1.8)$$

The output of the model using MATLAB software under different defuzzification methods are given below $TC_{gm}$=168.7322, T=0.6400; $TC_{sd}$=151.7654, T=0.7100 and $TC_{cen}$=132.6469, T=0.8100.

From the above results indicate that total cost is minimum with corresponding value of T when centroid method of defuzzification is used. On the other hand T is minimum with corresponding total cost when graded mean integration value is used. The cost functions are plotted against T in the following figure.

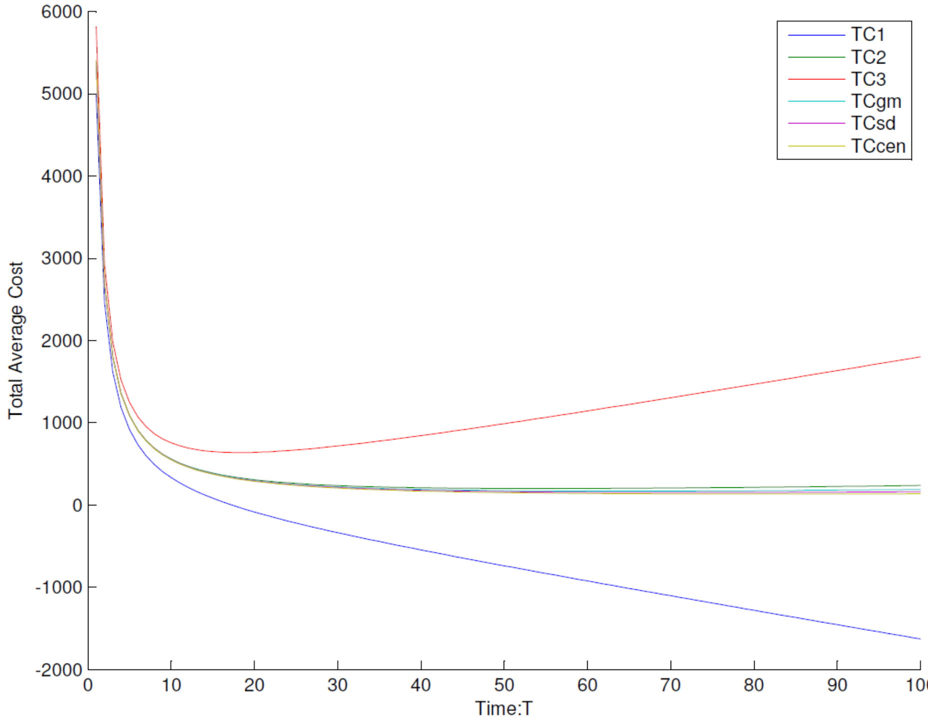


**Figure 2.** Representation of Cost against Time under Different Defuzzification Methods

**Table 1.** Sensitivity on $\tilde{A}$

| $\tilde{A}$ | Graded Mean Method | | Signed Distance Method | | Centroid method | |
|---|---|---|---|---|---|---|
| | Total Cost ($TC_{gm}$) | Time(Yrs) | Total Cost ($TC_{sd}$) | Time(Yrs) | Total Cost ($TC_{cen}$) | Time(Yrs) |
| (60,64.8,68.4) | 184.5514 | 0.7000 | 165.8657 | 0.7800 | 144.8563 | 0.8900 |
| (55,49.4,63.6) | 166.6899 | 0.6300 | 152.2584 | 0.7100 | 135.0792 | 0.8300 |
| (50,54,58) | 168.7322 | 0.6400 | 151.7654 | 0.7100 | 132.6469 | 0.8100 |
| (45,48.6,52.2) | 160.0750 | 0.6100 | 143.9807 | 0.6800 | 125.8388 | 0.7700 |
| (40,43.2,46.4) | 150.9201 | 0.5700 | 135.7448 | 0.6400 | 118.6417 | 0.7300 |

**Table 2.** Sensitivity on $\tilde{h}$

| $\tilde{h}$ | Graded Mean Method | | Signed Distance Method | | Centroid method | |
|---|---|---|---|---|---|---|
| | Total Cost ($TC_{gm}$) | Time(Yrs) | Total Cost ($TC_{sd}$) | Time(Yrs) | Total Cost ($TC_{cen}$) | Time(Yrs) |
| (7.2,9.6,12) | 184.8211 | 0.5800 | 166.2375 | 0.6500 | 145.3042 | 0.7400 |
| (6.6,8.8,11) | 176.9569 | 0.6100 | 159.1662 | 0.6800 | 139.1195 | 0.7800 |
| (6,8,10) | 168.7322 | 0.6400 | 151.7654 | 0.7100 | 132.6469 | 0.8100 |
| (5.4,7.2,9) | 160.0893 | 0.6700 | 143.9844 | 0.7500 | 125.8400 | 0.8600 |
| (4.8,6.4,8) | 150.9473 | 0.7200 | 135.7606 | 0.8000 | 118.6446 | 0.9100 |

**Table 3.** Sensitivity on $\tilde{\theta}$

| $\tilde{\theta}$ | Graded Mean Method | | Signed Distance Method | | Centroid method | |
|---|---|---|---|---|---|---|
| | Total cost ($TC_{gm}$) | Time(Yrs) | Total Cost ($TC_{sd}$) | Time(Yrs) | Total Cost ($TC_{cen}$) | Time(Yrs) |
| (0.0072,0.0120,0.0168) | 168.7551 | 0.6400 | 151.7794 | 0.7100 | 132.6497 | 0.8100 |
| (0.0066,0.0110,0.0154) | 168.7436 | 0.6400 | 151.7724 | 0.7100 | 132.6483 | 0.8100 |
| (0.006,0.010,0.014) | 168.7322 | 0.6400 | 151.7654 | 0.7100 | 132.6469 | 0.8100 |
| (0.0054,0.0090,0.0126) | 168.7207 | 0.6400 | 151.7585 | 0.7100 | 132.6454 | 0.8100 |
| (0.0048,0.0080,0.0112) | 168.7092 | 0.6400 | 151.7515 | 0.7100 | 132.6440 | 0.8100 |

**Table 4.** Sensitivity on $\tilde{d}$

| $\tilde{d}$ | Graded Mean Method | | Signed Distance Method | | Centroid method | |
|---|---|---|---|---|---|---|
| | Total Cost ($TC_{gm}$) | Time(Yrs) | Total Cost ($TC_{sd}$) | Time(Yrs) | Total Cost ($TC_{cen}$) | Time(Yrs) |
| (1.4400,1.8000,2.1600) | 168.7551 | 0.6400 | 151.7794 | 0.7100 | 132.6497 | 0.8100 |
| (1.3200,1.6500,1.9800) | 168.7436 | 0.6400 | 151.7724 | 0.7100 | 132.6483 | 0.8100 |
| (1.2,1.5,1.8) | 168.7322 | 0.6400 | 151.7654 | 0.7100 | 132.6469 | 0.8100 |
| (1.0800,1.3500,1.6200) | 168.7207 | 0.6400 | 151.7585 | 0.7100 | 132.6454 | 0.8100 |
| (0.9600,1.2000,1.4400) | 168.7092 | 0.6400 | 151.7515 | 0.7100 | 132.6440 | 0.8100 |

**Table 5.** Sensitivity on $\tilde{P}$

| $\tilde{P}$ | Graded Mean Method | | Signed Distance Method | | Centroid method | |
|---|---|---|---|---|---|---|
| | Total Cost ($TC_{gm}$) | Time(Yrs) | Total Cost ($TC_{sd}$) | Time(Yrs) | Total Cost ($TC_{cen}$) | Time(Yrs) |
| (600,660,720) | 310.5976 | 0.3500 | 303.7305 | 0.3600 | 296.6831 | 0.3600 |
| (550,505,660) | 135.4223 | 0.8000 | 159.2674 | 0.6800 | 179.9800 | 0.6000 |
| (500,550,600) | 168.7322 | 0.6400 | 151.7654 | 0.7100 | 132.6469 | 0.8100 |
| (450,495,540) | -24.0568 | 1.0000 | -52.9652 | 1.0000 | -81.8737 | 1.0000 |
| (400,440,480) | -286.3878 | 1.0000 | -319.9624 | 1.0000 | -353.5370 | 1.0000 |

**Table 6.** Sensitivity on $\tilde{D}$

| $\tilde{D}$ | Graded Mean Method | | Signed Distance Method | | Centroid method | |
|---|---|---|---|---|---|---|
| | Total Cost ($TC_{gm}$) | Time(Yrs) | Total Cost ($TC_{sd}$) | Time(Yrs) | Total Cost ($TC_{cen}$) | Time(Yrs) |
| (540,600,660) | -241.1384 | 1.0000 | -279.4121 | 1.0000 | -317.6859 | 1.0000 |
| (495,550,605) | -8.7773 | 1.0000 | -40.1660 | 1.0000 | -71.5547 | 1.0000 |
| (450,500,550) | 168.7322 | 0.6400 | 151.7654 | 0.7100 | 132.6469 | 0.8100 |
| (405,450,495) | 249.6870 | 0.4300 | 241.0426 | 0.4500 | 232.0877 | 0.4700 |
| (360,400,440) | 296.7547 | 0.3600 | 291.3141 | 0.3700 | 285.7916 | 0.3800 |

## 9. Sensitivity Analysis

Sensitivity analysis is performed to study the effect on optimal values due to changes in different inventory parameters. The sensitivity analysis of the parameters has been discussed in table (1-6).

The optimal values change significantly with change (+10%,-10%) in $\tilde{A}$, $\tilde{h}$, $\tilde{\theta}$, $\tilde{d}$ in table (1-4). Table (5&6) show some absurd values. On the basis of the sensitivity analysis, the following are noted

1. When fuzzy set up cost is decreased, $TC_{gm}$ and T are decreased.
2. First increased value of fuzzy set up cost show the best optimal value in $TC_{gm}$ where as $TC_{sd}$ and $TC_{cen}$ are increased.
3. When fuzzy deterioration is increased, Cycle length remains same. Similar scenario appears when it is decreased.
4. When fuzzy production rate is decreased, cycle length increase and total cost moves towards negative.
5. When fuzzy demand rate is increased, total cost decreases.

## 10. Conclusions

In real life situation, nothing can be predicted with certainty in production system. This type of uncertainty cannot be handled with probabilistic approach alone. The use of fuzzy set theory is of great importance to handle uncertainty in industrial problems. Thus, the study of production inventory model in fuzzy environment helps the manufacturer to run the production process without any interruption. The proposed fuzzy inventory model extends the model of literature with key parameters as fuzzy numbers. In this model, the production of deteriorating items has thrown light to understand the uncertainty clearly and then to make appropriate decision for optimality. The graphical representations also allow us to understand the scenario clearly. The developed model may be further extended by taking shortage, different types of demand, perishable items.

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