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Robust Asymptotically Stabilization of Special Uncertain Descriptor Fractional-Order Systems with Fractional Feedback Control

Sameer Qasim Hasan*, Ala Muhsien Abd

Department of Mathematics, Almustansiriyah University, Baghdad, Iraq

Abstract In this paper we investigate the asymptotically stabilization of a special type of singular fractional order α belongs to interval (0,1) with uncertain parameter as time-invariant and norm-bounded appearing in the state matrix and suitable feedback fractional control by using Dynamics decomposition form.

Keywords Fractional order, Descriptor system, Fractional, System, Fractional control, Feedback control

1. Introduction

Recently, fractional-order control systems have attracted increasing interest [15, 9, 11]. On the one hand, this is mainly due to the fact that many real-world physical systems are well characterized by fractional-order state equations [15], i.e., equations involving the so-called fractional derivatives and integrals. On the other hand, with the success in the synthesis of real noninteger differentiators and the emergence of a new electrical circuit element called "fractance" [10, 21], fractional-order controllers [16, 18, 12] have been designed and applied to control a variety of dynamical processes, including integer-order and fractional-order systems, so as to enhance the robustness and performance of the control systems. Singular fractional systems (known as generalized, descriptor of Fractional systems) describe a large class of systems, which are not only theoretical interest but also have a great importance in practice.

Stability is fundamental to all control systems, certainly including fractional-order control systems [20, 19]. Recently, stability and stabilization problems of fractional-order linear time-invariant interval systems have been investigated in [1], [2, 4]. For example, for fractional-order linear time-invariant interval systems described in the transfer function form, the stability issue was discussed first in [13] and then further in [14]. In this paper we consider the problem of the robust asymptotical Stabilization for uncertain descriptor fractional-order systems.

* Corresponding author:
dr.sameer_kasim@yahoo.com (Sameer Qasim Hasan)
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The descriptor multi-fractional-order systems by applying a derivative multi-controller and a state feedback.

Controller is given to achieve the robust asymptotical stabilization of the obtained two sub system, first is fractional-order systems and the second is zero state.

We using canonical form for the descriptor fractional-order systems and by applying a derivative controller and a state feedback controller is given to achieve the robust asymptotical stabilization of the fractional-order systems.

In section II, the paper are organized as follow in section II, we introduce the definition of fractional derivative in brief; we present also some mathematical results. In section III, we propose robust linear uncertainty descriptor multi-fractional controller for the stabilization of system.

2. Preliminaries

A. Some definition

Now we review some important and definition:

The Caputo derivative on the other, defined [17],

$$D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_{a}^{t}\frac{\frac{d^{\alpha}f(t)}{dt^{\alpha}}}{(t-\tau)^{\alpha-n+1}}d\tau, (n-1 \le \alpha < n)$$

For $(n-1 \le \alpha < n)$ and $\Gamma(x)$ is the well-known Euler's gamma function.

Definition (2.1), [22]:

Let $A = [a_{ij}B] \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$. Their Kronecker product (i.e., the direct product or tensor product), denoted as $(A \otimes B)$, is defined by

$$(A \otimes B) = [a_{ij}B] = \begin{bmatrix} a_{11}B & \dots & a_{1n}B \\ \dots & \dots & \dots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix} \in \mathbb{R}^{mp\times nq}$$

Now we consider the fractional –order linear system.

$$\begin{cases} D^{\alpha}x(t) = Ax(t), \quad (0 \le \alpha < 1) \\ x(0) = x_0 \end{cases} \tag{1}$$

Where $A \in \mathbb{R}^{n\times n}, x \in \mathbb{R}^n$ is the state vector.

The system (1) is stable if the condition is satisfied $(0 < \alpha \le 1)$ [5], with the condition(2),

$$|\arg(\text{spec}(A))| > \alpha\frac{\pi}{2}, \tag{2}$$

Where spec(A) represents the eigenvalues of matrix A.

B.Some mathematical inequalities.

Lemma (2.1), [8]:

Let $A \in \mathbb{R}^{n\times n}$ and $(0 < \alpha < 1)$. The fractional- order system $D^{\alpha}x(t) = Ax(t)$ is asymptotically stable that means

$$|\arg(\text{spec}(A))| > \alpha\frac{\pi}{2}$$

if and only if there exist two real symmetric matrices $P_{k1} \in \mathbb{R}^{n\times n}, k = 1,2$ and two skew-symmetric matrices where $P_{k2} \in \mathbb{R}^{n\times n}, k = 1,2$

such that $\sum_{i=1}^{2}\sum_{j=1}^{2} sym\{\Gamma_{ij} \otimes (AP_{ij})\} < 0$,

$$\begin{bmatrix} P_{11} & P_{12} \\ -P_{12} & P_{11} \end{bmatrix} > 0, \begin{bmatrix} P_{21} & P_{22} \\ -P_{22} & P_{21} \end{bmatrix} > 0$$

where

$$\Gamma_{11} = \begin{bmatrix} \sin(\alpha\frac{\pi}{2}) & -\cos(\alpha\frac{\pi}{2}) \\ \cos(\alpha\frac{\pi}{2}) & \sin(\alpha\frac{\pi}{2}) \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} \cos(\alpha\frac{\pi}{2}) & \sin(\alpha\frac{\pi}{2}) \\ -\sin(\alpha\frac{\pi}{2}) & \cos(\alpha\frac{\pi}{2}) \end{bmatrix},$$

$$\Gamma_{21} = \begin{bmatrix} \sin(\alpha\frac{\pi}{2}) & \cos(\alpha\frac{\pi}{2}) \\ -\cos(\alpha\frac{\pi}{2}) & \sin(\alpha\frac{\pi}{2}) \end{bmatrix}$$

$$\Gamma_{22} = \begin{bmatrix} -\cos(\alpha\frac{\pi}{2}) & \sin(\alpha\frac{\pi}{2}) \\ -\sin(\alpha\frac{\pi}{2}) & -\cos(\alpha\frac{\pi}{2}) \end{bmatrix}.$$

Lemma (2.2), [7]:

For any matrices X and Y with appropriate dimensions, we have

$$X^T Y + Y^T X \le \delta X^T Y + \delta^{-1} Y^T X.$$

For any $\delta > 0$.

Remark (2.1), [5]:

Consider the singular fractional linear system $E\dot{x}(t) = Ax(t)$, if the following conditions hold:

i. the matrix pair (E, A) is regular .
ii. the matrix pair (E, A) is regular an impulse free.

Then there exist there two invertible matrices. $M, N \in \mathbb{R}^{n\times n}$ Satisfying:

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad MAN = \begin{bmatrix} A_0 & 0 \\ 0 & I_{n-r} \end{bmatrix}, \qquad \text{where}$$

$A_0 \in \mathbb{R}^{r\times r}$ is an invertible, I_r, I_{n-r} are the identity matrices of dimensions r, n-r respectively.

Definition (2.2), [5, 6]:

i. A matrix pair (E, A) is called regular if E and A are square and $\det(\lambda E - A) \ne 0$ for some value $\lambda \notin \sigma(E - A)$ it is called singular otherwise. Where $\sigma(E - A)$ is the set of all Finite Spectrum Eigenvalues.
ii. The matrix pair (E, A) is said to be impulse free if $\deg(\det(\lambda E - A)) = rank(E)$.

3. The Main Result

The singular linear fractional order uncertainty control system

$$\begin{cases} E(D^{\alpha} + D^{\beta})x(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \\ x(0) = x_0 \quad ,(0 < \beta < \alpha < 1) \end{cases} \tag{3}$$

where $E \in \mathbb{R}^{n\times n}$ is the singular matrix, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $x \in \mathbb{R}^n$ is the semi-state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $(B + \Delta B)$ is invertible and $\Delta A, \Delta B$ are time-invariant matrix representing norm-bounded parameter uncertainty, with the following conditions:

i. The singular matrix E has the form, where nonsingular matrix

$$E=\begin{bmatrix} E_1 & 0 \\ -[(B_{21}+\Delta B_{21})L_1+(B_{22}+\Delta B_{22})L_3] & 0 \end{bmatrix}$$

$E_1 \in \mathbb{R}^{r\times r}$, $(B_{21}+\Delta B_{21})L_1+(B_{22}+\Delta B_{22})L_3 \in R^{n-r\times r}$, $(B_{11}+\Delta B_{11})L_2+(B_{12}+\Delta B_{12})L_4=0$, $(B_{21}+\Delta B_{21})L_2+(B_{22}+\Delta B_{22})L_4=0$, defined in (iii) and equation (7) later on.

ii. $(A+\Delta A)=\begin{bmatrix} (A_{11}+\Delta A_{11}) & (A_{12}+\Delta A_{12}) \\ (A_{21}+\Delta A_{21}) & (A_{22}+\Delta A_{22}) \end{bmatrix}\in \mathbb{R}^{n\times n}$,
where $(A_{11}+\Delta A_{11})\in \mathbb{R}^{r\times r}$, $(A_{12}+\Delta A_{12})\in \mathbb{R}^{r\times n-r}$, $(A_{21}+\Delta A_{21})\in \mathbb{R}^{n-r\times r}$ $(A_{22}+\Delta A_{22})\in \mathbb{R}^{n-r\times n-r}$, such that $\det(A_{22}+\Delta A_{22})\neq 0$

iii. $(B+\Delta B)=\begin{bmatrix} (B_{11}+\Delta B_{11}) & (B_{12}+\Delta B_{12}) \\ (B_{21}+\Delta B_{21}) & (B_{22}+\Delta B_{22}) \end{bmatrix}\in \mathbb{R}^{n\times m}$,
is invertible, where $(B_{11}+\Delta B_{11})\in \mathbb{R}^{r\times r}$, $(B_{12}+\Delta B_{12})\in \mathbb{R}^{r\times m-r}$ $(B_{21}+\Delta B_{21})\in \mathbb{R}^{n-r\times m}$, $(B_{22}+\Delta B_{22})\in \mathbb{R}^{n-r\times m-r}$

iv. $\Delta A = M_A\bar{\Delta}N_A, \Delta B = M_B\bar{\bar{\Delta}}N_B$
where M_A, N_A, M_B and N_B are known real constant matrices of appropriate dimensions, and the uncertain matrices $\bar{\Delta}$, $\bar{\bar{\Delta}}$ satisfies

$$\bar{\Delta}\bar{\Delta}^T \le I \quad \text{and} \quad \bar{\bar{\Delta}}\bar{\bar{\Delta}}^T \le I \tag{5}$$

v. The pencil matrix $(E, A+\Delta A)$ is regular and impulse free. Consider the feedback control for system (3) in the following form

$$u(t) = -LD^{\alpha}x(t)+(B+\Delta B)^{-1}D^{\beta}x(t)+kx(t) \tag{6}$$

where $L\in\mathbb{R}^{m\times n}$ has the form

$$L=\begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix}, \tag{7}$$

$L_1\in\mathbb{R}^{r\times r}$, $L_2\in\mathbb{R}^{r\times n-r}$, $L_3\in\mathbb{R}^{m-r\times r}$, $L_4\in\mathbb{R}^{m-r\times n-r}$, and $k\in\mathbb{R}^{m\times n}$ are gain matrices such that,

$$K=\begin{bmatrix} K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3 & 0_{r\times(n-r)} \\ 0_{(m-r)\times r} & 0_{(m-r)\times(n-r)} \end{bmatrix} \tag{8}$$

where

$$K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3]\in\mathbb{R}^{r\times r}$$

and $[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3]$ is invertible matrix. We substituting (6) into system (3), we obtain $(ED^{\alpha}+D^{\beta})x(t)x(t)=(A+\Delta A)x(t)$
$+(B+\Delta B)[-LD^{\alpha}x(t)+(B+\Delta B)^{-1}D^{\beta}x(t)+kx(t)]$

We have

$$(ED^{\alpha}+D^{\beta})x(t)x(t)=(A+\Delta A)x(t)-(B+\Delta B)LD^{\alpha}x(t)$$
$$+D^{\beta}x(t)+(B+\Delta B)kx(t)]$$

We gets

$$ED^{\alpha}x(t)+(B+\Delta B)LD^{\alpha}x(t)=(A+\Delta A)x(t)+(B+\Delta B)kx(t)$$

We obtain

$$(E+(B+\Delta B)L)D^{\alpha}x(t)=(A+\Delta A)x(t)+(B+\Delta B)kx(t) \tag{9}$$

We have

$$(E+(B+\Delta B)L)=\begin{bmatrix} E_1 & 0 \\ -[(B_{21}+\Delta B_{21})L_1+(B_{22}+\Delta B_{22})L_3] & 0 \end{bmatrix}+\begin{bmatrix} (B_{11}+\Delta B_{11}) & (B_{12}+\Delta B_{12}) \\ (B_{21}+\Delta B_{21}) & (B_{22}+\Delta B_{22}) \end{bmatrix}\begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix}$$

$$=\begin{bmatrix} E_1 & 0 \\ -[(B_{21}+\Delta B_{21})L_1+(B_{22}+\Delta B_{22})L_3] & 0 \end{bmatrix}+\begin{bmatrix} (B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3 & (B_{11}+\Delta B_{11})L_2+(B_{12}+\Delta B_{12})L_4 \\ (B_{21}+\Delta B_{21})L_1+(B_{22}+\Delta B_{22})L_3 & (B_{21}+\Delta B_{21})L_2+(B_{22}+\Delta B_{22})L_4 \end{bmatrix}$$

$$\begin{bmatrix} E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3 & (B_{11}+\Delta B_{11})L_2+(B_{12}+\Delta B_{12})L_4 \\ 0 & (B_{21}+\Delta B_{21})L_2+(B_{22}+\Delta B_{22})L_4 \end{bmatrix}$$

By condition (i), we have

$$(E+(B+\Delta B)L)=\begin{bmatrix} E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3 & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

Such that

$$E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3\in\mathbb{R}^{r\times r}$$
$$(B_{11}+\Delta B_{11})L_2+(B_{12}+\Delta B_{12})L_4=0\in\mathbb{R}^{r\times n-r},\ 0\in\mathbb{R}^{r\times n-r},$$
$$(B_{21}+\Delta B_{21})L_2+(B_{22}+\Delta B_{22})L_4=0\in\mathbb{R}^{n-r\times n-r}$$

The right side of equation (9) has the form

$$((A+\Delta A)+(B+\Delta B)k)=\begin{bmatrix}(A_{11}+\Delta A_{11}) & (A_{12}+\Delta A_{12})\\ (A_{21}+\Delta A_{21}) & (A_{22}+\Delta A_{22})\end{bmatrix}+\begin{bmatrix}(B_{11}+\Delta B_{11}) & (B_{12}+\Delta B_{12})\\ (B_{21}+\Delta B_{21}) & (B_{22}+\Delta B_{22})\end{bmatrix}$$

$$\times\begin{bmatrix}K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] & 0_{r\times(n-r)}\\ 0_{(m-r)\times r} & 0_{(m-r)\times(n-r)}\end{bmatrix}=\begin{bmatrix}(A_{11}+\Delta A_{11}) & (A_{12}+\Delta A_{12})\\ (A_{21}+\Delta A_{21}) & (A_{22}+\Delta A_{22})\end{bmatrix}$$

$$+\begin{bmatrix}(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] & 0_{r\times(n-r)}\\ (B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] & 0_{(m-r)\times(n-r)}\end{bmatrix}$$

$$=\begin{bmatrix}(A_{11}+\Delta A_{11})+(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] & (A_{12}+\Delta A_{12})\\ (A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] & (A_{22}+\Delta A_{22})\end{bmatrix}(A_{12}+\Delta A_{12})\in R^{r\times n-r}, \tag{11}$$

$(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3]\in\mathbb{R}^{n-r\times r}$ $(A_{22}+\Delta A_{22})\in\mathbb{R}^{n-r\times n-r}$. Since

$[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3]\in\mathbb{R}^{n-r\times r}$

is invertible sub-matrix of the system (9) and by using the remark (2.1), then there exist two invertible matrices $M, N\in\mathbb{R}^{n\times n}$ satisfying:

$$M(E+(B+\Delta B)L)N=\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix},\quad M((A+\Delta A)+(B+\Delta B)k)N=\begin{bmatrix}\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1 & 0\\ 0 & I_{n-r\times n-r}\end{bmatrix} \tag{12}$$

where $\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1\in\mathbb{R}^{r\times r}$ is an invertible which defined later on. One can get:

$$\left|\lambda(E+(B+\Delta B)L)-((A+\Delta A)+(B+\Delta B)k)\right|=0$$

Then

$$\left|M^{-1}M(\lambda(E+(B+\Delta B)L)-((A+\Delta A)+(B+\Delta B)k))NN^{-1}\right|$$

$$=\left|M^{-1}(\lambda M(E+(B+\Delta B)L)N-M((A+\Delta A)+(B+\Delta B)k)N)N^{-1}\right|$$

$$=\left|M^{-1}\begin{bmatrix}\lambda I_r-(\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1) & 0\\ 0 & I_{n-r\times n-r}\end{bmatrix}N^{-1}\right|=\left|M^{-1}\right|.\left|\lambda I_r-(\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1)\right|.\left|N^{-1}\right|$$

Since $\left|M^{-1}\right|\neq 0$ and $\left|N^{-1}\right|\neq 0$, hence the finite eigenvalue λ of the matrix pair

$((E+(B+\Delta B)L),((A+\Delta A)+(B+\Delta B)K))$ is also an eigenvalue of matrix

$(\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1)$, with

$\mathrm{Re}(\lambda_i(\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1))<0$

$i=1,\ldots,r$

We assume that M and N has the forms:

$$M=\begin{bmatrix}I_{r\times r} & -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}\\ 0 & (A_{22}+\Delta A_{22})^{-1}\end{bmatrix}\text{ where}$$

$(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}\in\mathbb{R}^{r\times n-r}$, $0\in\mathbb{R}^{n-r\times r}$, $(A_{22}+\Delta A_{22})^{-1}\in\mathbb{R}^{n-r\times n-r}$ and

$$N=\begin{bmatrix} (E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1} & 0 \\ \begin{array}{l} -\{(\mathrm{A}_{22}+\Delta A_{22})^{-1}((\mathrm{A}_{21}+\Delta A_{21}) \\ +(\mathrm{B}_{21}+\Delta B_{21})\mathrm{k}_1(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1 \\ +(\mathrm{B}_{12}+\Delta B_{12})L_3))\times(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1}\} \end{array} & I_{n-r\times n-r} \end{bmatrix} \quad \text{Where } 0\in\mathbb{R}^{r\times n-r},$$

$$(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1}\in\mathbb{R}^{r\times r}$$

$$\{(\mathrm{A}_{22}+\Delta A_{22})^{-1}((\mathrm{A}_{21}+\Delta A_{21})+(\mathrm{B}_{21}+\Delta B_{21})\mathrm{k}_1(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)$$
$$\times(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1}\}\in\mathbb{R}^{n-r\times r}$$

We obtain

$$M(\mathrm{E}+(\mathrm{B}+\Delta B)\mathrm{L})\mathrm{N}\mathrm{D}^{\alpha}\mathrm{x(t)}=\mathrm{M}((\mathrm{A}+\Delta A)+(\mathrm{B}+\Delta B)\mathrm{k})\mathrm{N}\mathrm{x(t)}$$

By using M and N, we have

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \mathrm{D}^{\alpha}\overline{\mathrm{x}}_1(\mathrm{t}) \\ \mathrm{D}^{\alpha}\overline{\mathrm{x}}_2(\mathrm{t}) \end{bmatrix}=\begin{bmatrix} I_{r\times r} & -(\mathrm{A}_{12}+\Delta A_{12})(\mathrm{A}_{22}+\Delta A_{22})^{-1} \\ 0 & (\mathrm{A}_{22}+\Delta A_{22})^{-1} \end{bmatrix}$$
$$\times\begin{bmatrix} (\mathrm{A}_{11}+\Delta A_{11})+(\mathrm{B}_{11}+\Delta B_{11})K_1[E_1+((\mathrm{B}_{11}+\Delta \mathrm{B}_{11})\mathrm{L}_1+(\mathrm{B}_{12}+\Delta \mathrm{B}_{12})\mathrm{L}_3] & (\mathrm{A}_{12}+\Delta A_{12}) \\ (\mathrm{A}_{21}+\Delta A_{21})+(\mathrm{B}_{21}+\Delta B_{21})K_1[E_1+((\mathrm{B}_{11}+\Delta \mathrm{B}_{11})\mathrm{L}_1+(\mathrm{B}_{12}+\Delta \mathrm{B}_{12})\mathrm{L}_3] & (\mathrm{A}_{22}+\Delta A_{22}) \end{bmatrix}$$
$$\times\begin{bmatrix} (E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1} & 0 \\ \begin{array}{l} -\{(\mathrm{A}_{22}+\Delta A_{22})^{-1}((\mathrm{A}_{21}+\Delta A_{21})+(\mathrm{B}_{21}+\Delta B_{21})\mathrm{k}_1(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3) \\ \times(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1} \end{array} & I_{n-r\times n-r} \end{bmatrix}\times\begin{bmatrix} \overline{\mathrm{x}}_1(\mathrm{t}) \\ \overline{\mathrm{x}}_2(\mathrm{t}) \end{bmatrix} \quad (13)$$

We have

$$\begin{bmatrix} \mathrm{D}^{\alpha}\overline{\mathrm{x}}_1(\mathrm{t}) \\ 0 \end{bmatrix}=\begin{bmatrix} \begin{array}{l} (\mathrm{A}_{11}+\Delta A_{11})+(\mathrm{B}_{11}+\Delta B_{11})K_1[E_1+((\mathrm{B}_{11}+\Delta \mathrm{B}_{11})\mathrm{L}_1+(\mathrm{B}_{12}+\Delta \mathrm{B}_{12})\mathrm{L}_3] \\ -(\mathrm{A}_{12}+\Delta A_{12})(\mathrm{A}_{22}+\Delta A_{22})^{-1} \\ \times(\mathrm{A}_{21}+\Delta A_{21})+(\mathrm{B}_{21}+\Delta B_{21})K_1[E_1+((\mathrm{B}_{11}+\Delta \mathrm{B}_{11})\mathrm{L}_1+(\mathrm{B}_{12}+\Delta \mathrm{B}_{12})\mathrm{L}_3] \end{array} & \begin{array}{l} (\mathrm{A}_{12}+\Delta A_{12}) \\ -(\mathrm{A}_{12}+\Delta A_{12})(\mathrm{A}_{22}+\Delta A_{22})^{-1}(\mathrm{A}_{22}+\Delta A_{22}) \end{array} \\ (\mathrm{A}_{22}+\Delta A_{22})^{-1}(\mathrm{A}_{21}+\Delta A_{21})+(\mathrm{B}_{21}+\Delta B_{21})K_1[E_1+((\mathrm{B}_{11}+\Delta \mathrm{B}_{11})\mathrm{L}_1+(\mathrm{B}_{12}+\Delta \mathrm{B}_{12})\mathrm{L}_3] & I_{n-r\times n-r} \end{bmatrix}$$
$$\times\begin{bmatrix} (E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1} & 0 \\ \begin{array}{l} -\{(\mathrm{A}_{22}+\Delta A_{22})^{-1}((\mathrm{A}_{21}+\Delta A_{21})+(\mathrm{B}_{21}+\Delta B_{21})\mathrm{k}_1(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3) \\ \times(E_1+(\mathrm{B}_{11}+\Delta B_{11})L_1+(\mathrm{B}_{12}+\Delta B_{12})L_3)^{-1} \end{array} & I_{n-r\times n-r} \end{bmatrix}\times\begin{bmatrix} \overline{\mathrm{x}}_1(\mathrm{t}) \\ \overline{\mathrm{x}}_2(\mathrm{t}) \end{bmatrix}$$

We have

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{l} (A_{11}+\Delta A_{11})+(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1} \\ \times(A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \end{array} & 0 \\ \begin{array}{l} (A_{22}+\Delta A_{22})^{-1}(A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ +\Delta A_{22})^{-1}((A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})k_1(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3) \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \end{array} & I_{n-r\times n-r} \end{bmatrix} \times \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix} \quad (14)$$

0ne can get

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \{(A_{11}+\Delta A_{11})+(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}\} \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ \times(A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \end{array} & 0 \\ \begin{array}{l} \{(A_{22}+\Delta A_{22})^{-1}(A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3]\} \\ (E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \end{array} & I_{n-r\times n-r} \end{bmatrix} \times \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix}$$

Then we have

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \{(A_{11}+\Delta A_{11})+(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}\} \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ \times(A_{21}+\Delta A_{21})+(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \end{array} & 0 \\ 0 & I_{n-r\times n-r} \end{bmatrix} \times \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix}$$

We obtain

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \{(A_{11}+\Delta A_{11})(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ +(B_{11}+\Delta B_{11})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(A_{12}+\Delta A_{12}) \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1} \\ \times(B_{21}+\Delta B_{21})K_1[E_1+((B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3] \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \end{array} & 0 \\ 0 & I_{n-r\times n-r} \end{bmatrix} \times \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix},$$

We have

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \{(A_{11}+\Delta A_{11})(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ -(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(A_{12}+\Delta A_{12}) \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ -[(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(B_{21}+\Delta B_{21})K_1 \\ +(B_{11}+\Delta B_{11})K_1] \end{array} & 0 \\ 0 & I_{n-r\times n-r} \end{bmatrix} \times \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix}$$

We have

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \begin{array}{l} \{(A_{11}+\Delta A_{11})-(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(A_{12}+\Delta A_{12})\} \\ \times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1} \\ -[(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(B_{21}+\Delta B_{21})K_1 \\ +(B_{11}+\Delta B_{11})K_1] \end{array} & 0 \\ 0 & I_{n-r\times n-r} \end{bmatrix} \times \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix} \tag{15}$$

We get

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} (\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix} \tag{16}$$

Where

$$\tilde{c} = (A_{11}+\Delta A_{11})-(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(A_{12}+\Delta A_{12})\}\times(E_1+(B_{11}+\Delta B_{11})L_1+(B_{12}+\Delta B_{12})L_3)^{-1}$$

$$(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1) = -[(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(B_{21}+\Delta B_{21})K_1+(B_{11}+\Delta B_{11})K_1]$$

also

$$\tilde{B}_1 = -[(A_{12}+\Delta A_{12})(A_{22}+\Delta A_{22})^{-1}(B_{21}+\Delta B_{21})K_1]$$

Then, we have

$$\begin{bmatrix} D^{\alpha}\overline{x}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} (\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \overline{x}_1(t) \\ \overline{x}_2(t) \end{bmatrix}$$

One can get

$$\begin{cases} D^{\alpha}\overline{x}_1(t) = (\tilde{c}+(\tilde{B}_1+(B_{11}+\Delta B_{11})K_1)\overline{x}_1(t) \\ 0 = I_{n-r}\overline{x}_2(t) \end{cases}$$

We have

$$\begin{cases} D^{\alpha}\overline{x}_1(t) = (\tilde{c}+(\tilde{B}_1+B_{11})K_1)+\Delta B_{11})K_1)\overline{x}_1(t) & (17.a) \\ 0 = I_{n-r}\overline{x}_2(t) & (17.b) \end{cases}$$

We obtain the formula as follows:

$$\begin{cases} D^{\alpha}\overline{x}_1(t) = (\tilde{c} + (\tilde{B}_1 + B_{11})K_1) + \widehat{\Delta B}_{11})\overline{x}_1(t) & (18a) \\ 0 = I_{n-r}\,\overline{x}_2(t) & (18b) \end{cases}$$

where $\widehat{\Delta}B_{11} = \Delta B_{11}K_1$, The design of the gain matrix K which robustly stabilization the descriptor fractional-order system (3) for the fractional order α belonging to (18.a), $0 < \alpha < 1$ are derived.

Theorem (3.1)

Assume that (3) is regular and impulse free, then there exists again matrix K_1 such that descriptor fractional order (3) with fractional-order $0 < \alpha < 1$ controlled by the control (6) is asymptotically stable, if there exist matrices $X \in \mathbb{R}^{m\times n}$, $P_0 = P_0^T > 0 \in \mathbb{R}^{n\times n}$, and two real scalars $\delta_i > 0$, $(i = 1,2)$, such that

$$\begin{bmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{21} & \varpi_{22} \end{bmatrix} < 0 \qquad (19)$$

Where

$$\varpi_{11} = \sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (\tilde{c} + \tilde{B}_1 + B_{11})X\}$$

$$+\sum_{i=1}^{2} \delta_i\{\{I_2 \otimes (IM_{B_{11}})(I_2 \otimes (IM_{B_{11}})^T\}\}$$

$$\varpi_{12} = \left[I_2 \otimes (N_{B11}P_0)^T \quad I_2 \otimes (N_{B11}P_0)^T \right]$$

$$\varpi_{22} = -diag(\delta_i, \delta_i) \otimes I_2$$

$\Gamma_{i1}(i = 1,2)$, Satisfy Lemma (2.1).

Proof:-

Under the assumption regular and impulse free that system(3), then there exists a gain matrix L such that system (3)can be written in the form (18), in this case the matrix K can be determined from the stability of system (18).It follows from Lemma (2.1) that $\left|\arg(\mathrm{spec}(A))\right| > \alpha\dfrac{\pi}{2}$ is equivalent to

$$\sum_{i=1}^{2}\sum_{j=1}^{2} sym\{\Gamma_{ij} \otimes (\widehat{A}P_{ij})\} < 0 \qquad (20)$$

Where $\widehat{A} = (\tilde{c} + (\tilde{B}_1 + B_{11})K_1) + \widehat{\Delta}B_{11})$ and $\Gamma_{i1}(i = 1,2)$, Satisfy Lemma (2.1). By assume $P_{11} = P_{21} = P_0, P_{12} = P_{22} = 0$ in (20) one can conclude that

$$sym\{\Gamma_{I1} \otimes (\widehat{A}P_0)\} + sym\{\Gamma_{21} \otimes (\widehat{A}P_0)\} < 0 \qquad (21)$$

Suppose that there exists matrices $X \in \mathbb{R}^{m\times n}$ and $P_0 = P_0^T > 0 \in \mathbb{R}^{n\times n}$, Such that

$$\sum_{i=1}^{2} sym\{\Gamma_{I1} \otimes (\widehat{A}P_0)\} < 0 \qquad (22)$$

Substituting $\widehat{A} = (\tilde{c} + (\tilde{B}_1 + B_{11})K_1) + \widehat{\Delta}B_{11})$ in (22) with $K = XP_0^{-1}$ we obtain

$$\widehat{A} = (\tilde{c} + (\tilde{B}_1 + B_{11})K_1) + \widehat{\Delta}B_{11})$$

$$\sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (\tilde{c}P_0 + (\tilde{B}_1 + B_{11})X)\}$$
$$+\sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (I\widehat{\Delta}B_{11})P_0\} < 0 \qquad (23)$$

By equation (5) $\overline{\overline{\Delta}}\,\overline{\overline{\Delta}}^T \le I$, then we obtain

$$(I_2 \otimes \overline{\overline{\Delta}})(I_2 \otimes \overline{\overline{\Delta}})^T = (I_2 \otimes \overline{\overline{\Delta}})(I_2 \otimes \overline{\overline{\Delta}}^T) = (I_2 \otimes \overline{\overline{\Delta}}\,\overline{\overline{\Delta}}^T) < I \qquad (24)$$

Also

$$\Gamma_{ij}\Gamma_{ij}^T\,(i = 1,2) = I_2$$

Then by (24) and Lemma (2.2) that for any real scalar $\delta > 0$

$$\sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (I\widehat{\Delta}B_{11})P_0)\}$$

$$= \sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (IM_{B_{11}}))(I_2 \otimes \overline{\overline{\Delta}})(I_2 \otimes N_{B_{11}}P_0)\}$$

$$\le \sum_{i=1}^{2} \delta_i\{\{\Gamma_{i1} \otimes (IM_{B_{11}})(I_2 \otimes \overline{\overline{\Delta}})(I_2 \otimes \overline{\overline{\Delta}})^T(\Gamma_{i1} \otimes (IM_{B_{11}})^T\}\}$$

$$+\sum_{i=1}^{2} \delta_i^{-1}(I_2 \otimes (N_{B_{11}}P_0)^T(I_2 \otimes (N_{B_{11}}P_0) \qquad (25)$$

By using equation (24), we obtain

$$\sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (I\widehat{\Delta}B_{11})P_0)\}$$

$$\le \sum_{i=1}^{2} \delta_i\{I_2 \otimes (IM_{B_{11}}))I_2 \otimes (IM_{B_{11}})^T\}\}$$

$$+\sum_{i=1}^{2} \delta_i^{-1}(I_2 \otimes (N_{B_{11}}P_0)^T(I_2 \otimes (N_{B_{11}}P_0) \qquad (26)$$

By substituting (26) into (23), we have

$$\sum_{i=1}^{2} sym\{\Gamma_{i1} \otimes (\widehat{A}P_0\} \le \sum_{i=1}^{2} sym\{\Gamma_{I1} \otimes (\tilde{c}P_0 + (\tilde{B}_1 + B_{11})X)\} \qquad (27)$$

Inequality (27) is equivalent to (18) by the well-known Schur Complement by [3].

4. Conclusions

The necessary conditions of robust asymptotically stabilization for special Uncertain singuler fractional-order systems with feedback fractional control for the fractional order α belonging to $0 < \alpha < 1$ with parameter uncertainties in the state matrix have been given in details . The problem of canonical of descriptor fractional-order systems by derivative fractional controller has been proposed with implosive free condition.

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