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Preliminary Test Single Stage Shrinkage Estimator for the Scale Parameter of Gamma Distribution

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Abstract In this paper, preliminary test single stage shrinkage (PTSSS) techniques was used for estimation the scale parameter θ of Gamma distribution when the shape parameter α was known as well as a prior knowledge about θ was available in the form of initial estimate θ_0 of θ. It is proposed to estimate θ by a testimator $\tilde{\theta}$ that is based upon the result of a test of the hypothesis H_0: $\theta = \theta_0$ against the hypothesis H_A: $\theta \neq \theta_0$ with level of significance Δ. If H_0 is accepted we used $\tilde{\theta} = \psi_1(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0$, where the weighting factor $\psi_1(\hat{\theta})$ is a function of the test statistic for testing H_0 or may be constant such that $0 \le \psi_1(\hat{\theta}) \le 1$. However if H_0 is rejected we used $\tilde{\theta} = \psi_2(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0$, where $0 \le \psi_2(\hat{\theta}) \le 1$ and $\hat{\theta}$ is the classical estimator of θ (MLE or MVUE). Choosing the weighting factor $\psi_i(\hat{\theta})$, (i =1,2) appropriately, an expression for the Mean Squared Error (MSE) and Bias Ratio [B(.)] of $\tilde{\theta}$ were derived and comparisons were made with classical estimator ($\hat{\theta}$) in the sense of efficiency and with some related earlier studies.

Keywords Gamma Distribution, Maximum Likelihood Estimator, Preliminary Test Single Stage Shrinkage Estimator, Bias Ratio, Mean Squared Error and Relative Efficiency

1. Introduction

The two-parameter gamma distribution has been used quite extensively in reliability and survival analysis, particularly when the data are not censored. The two-parameter Gamma distribution has one shape and one scale parameter. The random variable X follows a Gamma distribution with the shape and scale parameters as $\alpha > 0$ and $\theta > 0$, respectively, if it has the following probability density function (PDF):

$$f(x;\alpha,\theta) = \begin{cases} \dfrac{x^{\alpha-1} e^{\frac{-x}{\theta}}}{\Gamma(\alpha)\theta^{\alpha}} & \text{for } x > 0; \alpha > 0, \theta > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

and it will be denoted by Gamma (α,θ). Here Γ(α) is the Gamma function and it is expressed as

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx \tag{1.2}$$

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It is well known that the (PDF) of Gamma (α,θ) can take different shapes but it is always unimodal.

The Hazard function of Gamma (α,θ) can be increasing, decreasing or constant depending on $\alpha > 1$, $\alpha < 1$ or $\alpha = 1$, respectively. The moments of X can be obtained in explicit form; for example

$$E(X) = \alpha\theta \text{ and } Var(X) = \alpha\theta^2 \tag{1.3}$$

The Gamma, or Pearson [16] Type III, distribution has been used to model a wide range of data types in many disciplines, especially in the context of reliability modeling, life testing and fatigue testing. For example, Birnbaum and Saunders [7] introduced the gamma distribution for modeling the life-length of certain materials, and the use of this distribution for various reliability problems is noted by both Herd [11] and Drenick [9]. Gupta and Groll [10] discuss acceptance sampling based on this distribution, and they derive the operating characteristic function, producer's risk, failure rates and minimum sample sizes for this problem.

Empirical applications of the Gamma distribution arise in a diverse range of fields. For example, Wein and Baveja [20] applied this distribution in analyses of human fingerprint data. Segal et al. [17] used it for matching scores in the context of DNA fingerprint genotyping of tuberculosis, and Keaton [13] adopted it for an inventory control problem.

The Gamma distribution has also been applied in a number of studies in the fields of signal processing (e.g., Martin [15], Jensen et al. [12], and Kim and Stern [14]), hydrology (e.g., Askoy [6], and Bhunya et al. [8]) and meteorology (e.g., Simpson [18]).

The aim of this paper is to estimate the scale parameter of Gamma distribution when the shape parameter is known using preliminary test single stage shrinkage estimation (PTSSS) techniques via study the performance of Bias, Mean Squared Error and Relative Efficiency expressions of the proposed estimator when we set up a selection of shrinkage weight factor $\varphi(\hat{\theta})$ and suitable region R and create comparisons of the numerical results with $\hat{\theta}$ and with some existing studies.

A numerical study is carried out to appraise these effects of proposed estimators.

Shrinkage technique was introduced by Thompson [19] as follows

$$\tilde{\theta} = \varphi(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0 \tag{1.4}$$

where θ_0 is a prior estimate(initial value) about (θ) from the past experiences and $0 \le \varphi(\hat{\theta}) \le 1$, represents a shrinkage weight factor specifying the degree of belief in $\hat{\theta}$ and 1-$\varphi(\hat{\theta})$ specifying the degree of belief in θ_0. We used the form (1.4) above to estimate the scale parameter θ of Gamma distribution in case $\varphi(\hat{\theta})$ is chosen as follows:-

$$\varphi(\hat{\theta}) = \begin{cases} \psi_1(\hat{\theta}), & \text{if } \hat{\theta} \in R \\ \psi_2(\hat{\theta}), & \text{if } \hat{\theta} \notin R \end{cases} \tag{1.5}$$

where R is the preliminary test region for acceptance of size Δ for testing the hypothesis H_0: $\theta = \theta_0$ against the hypothesis H_A: $\theta \neq \theta_0$ using the test statistic $T(\hat{\theta}/\theta) = \frac{2n\alpha_0\hat{\theta}}{\theta_0}$ and $\hat{\theta}$ is the classical estimator of θ (MLE or MVUE), then the estimator which is defined in (1.4) will be written as below

$$\tilde{\theta} = \begin{cases} \psi_1(\hat{\theta} - \theta_0) + \theta_0, & \text{if } \hat{\theta} \in R \\ \psi_2(\hat{\theta} - \theta_0) + \theta_0, & \text{if } \hat{\theta} \notin R \end{cases} \tag{1.6}$$

where $\psi_i(\hat{\theta})$, i = 1,2; $0 \le \psi_i(\hat{\theta}) \le 1$ represents as shrinkage weight factors which may be a functions of $\hat{\theta}$ or may be constants. The resulting estimator (1.6) is known as preliminary test single stage shrinkage estimator (PTSSSE).

Several authors had studied the estimator defined in (1.6) for special distribution for different parameters and suitable regions (R) as well as for estimate the parameters of linear regression model. For example see [1], [2], [3], [4], [5] and [19].

2. Maximum Likelihood Estimator $\hat{\theta}$

Let $t_1, t_2, \ldots, t_n$ be a random sample of size n from the two parameter Gamma distribution with scale parameter θ and shape parameter α.

i.e. ; $t_i \sim G(\alpha,\theta)$ for i=1,2,3,...n.

And assume that the shape parameters α is known (say $\alpha = \alpha_0$)

The maximum likelihood function L (t_i; α_0 , θ) is defined as below : -

$$L(t_i, \alpha_0, \theta) = \prod_{i=1}^{n} f(t_i, \alpha_0, \theta) \quad , i = 1, 2, \ldots, n \,. \tag{1.7a}$$

Where, f(.) is defined in equation(1.1).

Therefore, The maximum likelihood function will be

$$L(t_i, \alpha_0, \theta) = \frac{1}{[\Gamma(n)]^n \theta^{n\alpha_0}} \left(\prod_{i=1}^{n} t_i^{\alpha_0 - 1}\right) EXP\left(-\sum_{i=1}^{n} \frac{t_i}{\theta}\right) \tag{1.7b}$$

And the logarithm of L (t_i, α_0, θ) is:-

$$\operatorname{Log} L(t_i, \alpha_0, \theta)$$

$$= -n \log\Gamma(n) - n\alpha_0 \log\theta + (\alpha_0 - 1)\sum_{i=1}^{n} t_i - \frac{\sum_{i=1}^{n} t_i}{\theta} \tag{1.8}$$

The partial derivative of $\operatorname{Log} L(t_i, \alpha_0, \theta)$ with respect to (w.r.t) θ is as below:-

$$\frac{\partial \operatorname{Log} L(t_i, \alpha_0, \theta)}{\partial\theta} = -\frac{n\alpha_0}{\theta} + \frac{\sum_{i=1}^{n} t_i}{\theta^2} \tag{1.9}$$

And equating the equation (1.9) to zero, we obtain the maximum likelihood estimator for θ as below

$$\hat{\theta} = \frac{\sum_{i=1}^{n} t_i}{n\alpha_0} \text{ (and sometimes symbolized by } \hat{\theta}_{mle}) \tag{1.10}$$

Noted that $\hat{\theta} \sim$ Gamma ($n\alpha_0, \theta/n\alpha_0$) .

It is easy of note that $\hat{\theta}$ is unbiased estimator.

i.e.; E ($\hat{\theta}$) = θ and MSE ($\hat{\theta}$) = Var ($\hat{\theta}$) = $\theta^2/n\alpha_0$.

3. Preliminary Test Single Stage Shrinkage Estimator (PTSSSE)

Using the form (1.6), we proposed the preliminary test single stage shrinkage estimators $\tilde{\theta}$ for estimate the scale parameter θ of Gamma distribution when a prior information θ_0 available about θ with known shape parameter α_0 as below:-

$$\tilde{\theta}=\begin{cases}\theta_0\,, & \text{if } \hat{\theta}\in R\\ k(\hat{\theta}-\theta_0)+\theta_0\,, & \text{if } \theta\notin R,\end{cases} \tag{1.11}$$

i.e. $\psi_1(\hat{\theta})=0$ and $\psi_2(\hat{\theta})=k$ in equations (1.5).

And the region R which is defined in (1.5) will be as below:

$$R=[\frac{\theta_0}{2n\alpha_0}X^2_{1-\Delta/2,2n\alpha_o},\frac{\theta_0}{2n\alpha_0}X^2_{\Delta/2,2n\alpha_o}] \tag{1.12}$$

For simplicity, assume that R = [*a*,*b*], *a* < *b*
i.e.;

$$a=\frac{\theta_0}{2n\alpha_0}X^2_{1-\Delta/2,2n\alpha_o}\,,\ b=\frac{\theta_0}{2n\alpha_0}X^2_{\Delta/2,2n\alpha_o} \tag{1.13}$$

where $X^2_{1-\Delta/2,2n\alpha_o}$ and $X^2_{\Delta/2,2n\alpha_o}$ are respectively the lower and upper 100(Δ/2) percentile point of Chi-square distribution with ($2n\alpha_o$) degree of freedom.

The expressions for Bias of the estimator $\tilde{\theta}$ is as follows:-

$$\begin{aligned}\text{Bias}(\tilde{\theta}|\theta,R)&=E(\tilde{\theta})-\theta\\&=\int_R[\theta_0-\theta]f(\hat{\theta})d\hat{\theta}+\int_{\bar{R}}[k(\theta-\theta_0)+\theta_0]f(\theta)d\theta\end{aligned}$$

where $\bar{R}$ is the complement region of R in real space and $f(\hat{\theta})$ is a (PDF) of $\hat{\theta}$ which has the following forms

$$f(\hat{\theta})=\frac{[\hat{\theta}]^{n\alpha_o-1}\exp[-n\alpha_o\theta/\theta]}{\Gamma(n\alpha_o)(\theta/n\alpha_o)^{n\alpha_o}},\ \ \alpha_o>o \ \text{ for } 0<\hat{\theta}<\infty \tag{1.14}$$

we conclude,

$$\text{Bias}(\tilde{\theta}|\theta,R)=\theta\{(\lambda-1)(1-k)-k[j_1(a^*,b^*)-\lambda j_0(a^*,b^*)]\} \tag{1.15}$$

$$\text{where, } j_\ell(a^*,b^*)=\frac{\theta^\ell}{(n\alpha_o)^\ell\Gamma(n\alpha_o)}\int_{a^*}^{b^*}y^{n\alpha_o+\ell-1}e^{-y}dy\ ,\ \text{for } \ell=0,1,2 \tag{1.16}$$

$$\lambda=\theta_0/\theta\,,\ a^*=\lambda\ X^2_{1-\alpha/2,2n\alpha_o}\ ,b^*=\lambda\ X^2_{\alpha/2,2n\alpha_0}\ \text{ and } y=-n\alpha_o\hat{\theta}/\theta \tag{1.17}$$

The bias ratio B ($\tilde{\theta}$) of the estimator $\tilde{\theta}$ is defined below

$$B(\tilde{\theta})=\text{Bias}(\tilde{\theta}|\theta,R)\ /\theta \tag{1.18}$$

And, the expression for mean square error (MSE) of $\tilde{\theta}$ is given as below:

$$\begin{aligned}\text{MSE}(\tilde{\theta}|\theta,R)=\theta^2\{&k^2[\frac{1}{n\alpha_o}+(\lambda-1)^2]+(\lambda-1)^2-2k(\lambda-1)^2-k^2[J_2(a^*,b^*)\\&-2\lambda J_1(a^*,b^*)+\lambda^2J_0(a^*,b^*)]-2K(\lambda-1)[J_1(a^*,b^*)-\lambda J_0(a^*,b^*)]\}\end{aligned} \tag{1.19}$$

As for the value of k is found by minimizing the mean squared error of $\tilde{\theta}$ as follow

$$\begin{aligned}&\frac{\partial MSE(\tilde{\theta},\theta,R)}{\partial k}=0\\&k^*=\frac{(\lambda-1)^2+(\lambda-1)[J_1(a^*,b^*)-\lambda J_0(a^*,b^*)]}{(n\alpha_0)^{-1}+(\lambda-1)^2-[J_2(a^*,b^*)-2\lambda J_1(a^*,b^*)+\lambda^2J_0(a^*,b^*)]}\end{aligned} \tag{1.20}$$

To be sure that the value of $k^* \in [0, 1]$ as a shrinkage weight factor we put k as follows:-

$$k = \begin{cases} 0 & \text{if } k^* \le 0 \\ k^* & \text{if } 0 < k^* < 1 \\ 1 & \text{if } k^* \ge 1 \end{cases} \tag{1.21}$$

The Relative Efficiency of estimator $\tilde{\theta}$ w.r.t. the classical estimator ($\hat{\theta}$) is defined as below:-

$$\text{R.Eff}\,(\tilde{\theta} \mid \theta, R) = \frac{\text{MSE}(\hat{\theta})}{\text{MSE}(\tilde{\theta})} = \frac{\theta^2/n\alpha_0}{\text{MSE}(\tilde{\theta})} \tag{1.22}$$

See for example [1], [2], [3] and [19] .

4. Conclusions and Numerical Results

The computations of Relative Efficiency [R.Eff(·)] and Bias Ratio [B(·)] for the equations (1.18) and (1.19) were used for the estimators $\tilde{\theta}$. These computations (using Mat. LAB programs) were performed for Δ = 0.01, 0.05, 0.1, λ = 0.25(0.25)2 and n = 4,6,8.

These computations are given in three annexed tables. The observation mentioned in the tables lead to the following results:

1. R.Eff ($\tilde{\theta}$) were adversely proportional with small value of α_0 and n specially when θ_0 close to θ.

2. R.Eff ($\tilde{\theta}$) are maximum when $\theta = \theta_0(\lambda = 1)$ for all Δ, α_0 and n. This feature shown the important usefulness of prior knowledge which given higher effects of proposed estimator as well as the important role of shrinkage technique and its philosophy.

3. Bias Ratio [B ($\tilde{\theta}$)] $[B(\tilde{\theta}) = \frac{\text{Bias}(\tilde{\theta})}{\theta}]$ were reasonably small when $\theta \approx \theta_0$, otherwise start to be maximum for all Δ and n. This property shown that the proposed estimator $\tilde{\theta}$ is very closely to unbiased ness especially when $\theta = \theta_0$.

4. Bias Ratio [B ($\tilde{\theta}$)] were at most decreasing function with Δ for all n and λ

5. Effective Interval [the values of λ that makes R. Eff. greater than one] for $\tilde{\theta}$ was [0.25, 2] for all α_0, n and Δ. Here the pretest criterion is very important for guarantee that prior information is very closely to the actual value and prevent it faraway from it, which get optimal effect of the considered estimator to obtain high efficiency.

6. The suggested estimator $\tilde{\theta}$ is more efficient than the classical estimator $\hat{\theta}$ specially when θ_0 is very close to θ which is given the effective of $\tilde{\theta}$ when given an important weight of prior knowledge. And the augmentation of efficiency may be reach to tens times. Also $\tilde{\theta}$ more efficient than the estimator introduced by [19] in the sense of Mean Squared Error and Relative Efficiency.

7. When the shape parameter of Gamma distribution equal to one [α_0 = 1], the distribution become an Exponential distribution, thus the suggested estimator $\tilde{\theta}$ in this case is more efficient than the estimator introduced by [3].

Table (1). Shown Bias Ratio [B (·)] and Relative Efficiency [R. Eff. (·)] of $\tilde{\theta}$ when α_0 = 1

Δ	n \ λ	Bias R.Eff.	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	4	B(·)	0.75	0.5	0.25	2.868 E−010	0.25	0.5	0.75	1
		R.Eff.(·)	1.7778	4	16	5.7485 E+015	16	4	1.7778	1
	6	B(·)	0.74996	0.5	0.25	5.4372 E−010	0.24999	0.49997	0.74995	0.99993
		R.Eff.(·)	1.778	4	16	3.0405 E+009	16.002	4.0005	1.778	1.0001
	8	B(·)	0.74946	0.49999	0.25002	4.0446 E−010	0.24975	0.4995	0.74918	0.99884
		R.Eff.(·)	1.7803	4.0002	15.998	1.7522 E+007	16.032	4.008	1.7816	1.0023
0.05	4	B(·)	0.74985	0.49999	0.25001	3.4098 E−010	0.24993	0.4988	0.74981	0.99973
		R.Eff.(·)	1.7785	4.0002	15.999	1.464 E+008	16.009	4.002	1.7787	1.0005
	6	B(·)	0.74743	0.49981	0.25018	0.000618	0.24863	0.49761	0.74642	0.99516
		R.Eff.(·)	1.79	4.003	15.977	6.7747 E+005	16.177	4.0385	1.7948	1.0097
	8	B(·)	0.74141	0.49947	0.25052	0.0019752	0.24548	0.49217	0.73849	0.98467
		R.Eff.(·)	1.819	4.0083	15.934	84859	16.59	4.1278	1.8335	1.0313
0.1	4	B(·)	0.74816	0.49973	0.25017	0.000514	0.24898	0.49833	0.74757	0.99675
		R.Eff.(·)	1.7865	4.0042	15.978	1.0079 E+006	16.131	4.0268	1.7893	1.0065
	6	B(·)	0.73951	0.49876	0.25083	0.0027431	0.2443	0.49055	0.73636	0.98195
		R.Eff.(·)	1.8282	4.0194	15.893	47188	16.748	4.1549	1.8441	1.037
	8	B(·)	0.72733	0.49765	0.25138	0.0051264	0.2387	0.48087	0.7222	0.96319
		R.Eff.(·)	1.8891	4.0358	15.821	1.4534	17.523	4.3222	1.9167	1.0777

Table (2). Shown Bias Ratio [B (·)] and Relative Efficiency [R. Eff. (·)] of $\tilde{\theta}$ when $\alpha_0 = 2$

Δ	λ / n	Bias R.Eff.	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	4	B(·)	0.74995	0.5	0.25004	9.0614 E − 005	0.24986	0.49981	0.74977	0.99972
		R.Eff.(·)	1.778	4.0001	15.994	7.2228 E + 006	16.018	4.003	1.7789	1.0006
	6	B(·)	0.74875	0.49947	0.25019	0.000917	0.24836	0.49704	0.7469	0.99619
		R.Eff.(·)	1.7837	4.0084	15.975	2.8478 E + 005	16.21	4.0378	1.7924	1.0076
	8	B(·)	0.74426	0.49695	0.24965	0.0023461	0.24496	0.49226	0.73956	0.98687
		R.Eff.(·)	1.8052	4.049	16.044	62160	16.658	4.1254	1.8278	1.0266
0.05	4	B(·)	0.7475	0.499	0.2505	0.0019817	0.24653	0.49504	0.74355	0.99206
		R.Eff.(·)	1.7896	4.016	15.937	84749	16.448	4.0799	1.8085	1.016
	6	B(·)	0.73644	0.4925	0.24863	0.004725	0.23918	0.4831	0.72699	0.9709
		R.Eff.(·)	1.8433	4.1211	16.171	16658	17.448	4.2809	1.8907	1.0601
	8	B(·)	0.71884	0.48115	0.24345	0.00575	0.23195	0.46965	0.70734	0.94504
		R.Eff.(·)	1.9336	4.3156	16.8447	7583.1	18.517	4.525	1.9957	1.1182
0.1	4	B(·)	0.7402	0.49518	0.25017	0.0051498	0.23987	0.48488	0.7299	0.9749
		R.Eff.(·)	1.8248	4.0775	15.973	14213	17.348	4.2494	1.8758	1.0515
	6	B(·)	0.71797	0.48107	0.24418	0.0072827	0.2296	0.4665	0.7034	0.9403
		R.Eff.(·)	1.938	4.3162	16.735	5151.9	18.868	4.5836	2.0173	1.1292
	8	B(·)	0.69271	0.4639	0.23511	0.006307	0.2225	0.4513	0.6801	0.9089
		R.Eff.(·)	2.0803	4.6358	17.991	3167.2	20.037	4.8933	2.1567	1.2079

Table (3). Shown Bias Ratio [B (·)] and Relative Efficiency [R. Eff. (·)] of $\tilde{\theta}$ when $\alpha_0 = 3$

Δ	λ / n	Bias R.Eff.	0.25	0.50	0.75	1	1.25	1.50	1.75	2
0.01	4	B(·)	0.74875	0.49947	0.25019	0.0001714	0.24836	0.49764	0.74691	0.99619
		R.Eff.(·)	1.7837	4.0084	15.975	2.8478 E + 005	16.21	4.0378	1.7924	1.0076
	6	B(·)	0.74051	0.49468	0.24884	0.0030057	0.24283	0.8866	0.7345	0.98033
		R.Eff.(·)	1.8233	4.086	16.147	39830	16.943	4.1855	1.8528	1.0401
	8	B(·)	0.72729	0.48602	0.24475	0.0034775	0.23779	0.47906	0.72033	0.9616
		R.Eff.(·)	1.8897	4.2316	16.684	23596	17.658	4.3535	1.9258	1.0807
0.05	4	B(·)	0.73644	0.49254	0.24863	0.004725	0.23918	0.48309	0.72699	0.9709
		R.Eff.(·)	1.8433	4.1211	16.171	16658	17.448	4.2809	1.8907	1.0601
	6	B(·)	0.6876	0.46124	0.23484	0.004421	0.21796	0.44435	0.67075	0.89715
		R.Eff.(·)	2.1091	4.6857	18.02	2663.6	20.809	5.0368	2.2133	1.2378
	8	B(·)	0.68155	0.45564	0.22973	0.0038161	0.22209	0.44801	0.6739	0.89983
		R.Eff.(·)	2.1494	4.806	18.837	3467.3	20.13	4.9689	2.1977	1.2331
0.1	4	B(·)	0.71797	0.48107	0.24418	0.0072827	0.2296	0.4665	0.7034	0.9403
		R.Eff.(·)	1.938	4.3162	16.735	5151.9	18.868	4.5836	2.0173	1.1292
	6	B(·)	0.68106	0.45587	0.23067	0.0054748	0.21972	0.4449	0.67011	0.89531
		R.Eff.(·)	2.1514	4.7977	18.647	2460.6	20.506	5.0325	2.2211	1.2448
	8	B(·)	0.65249	0.43593	0.21936	0.002718	0.21378	0.43034	0.6469	0.86348
		R.Eff.(·)	2.3418	5.2336	20.4	151.8	21.451	5.3686	2.382	1.3382

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