American Journal of Mathematics and Statistics 2013, 3(3): 143-152
DOI: 10.5923/j.ajms.20130303.07

Relations Connecting Eighth Order Mock Theta Functions

Roselin Antony

Department of Mathematics, College of Natural and Computational Sciences, Mekelle University, Mekelle, P.O. Box 231, Ethiopia

Abstract During the last years of his life, Ramanujan defined 17 functions F(q), where | q | < 1, and he named them as mock theta functions. The first detailed description of mock theta functions was given by Watson. In this paper, we obtain relations connecting mock theta functions, partial mock theta functions of order 8 and infinite products analogous to the identities of Ramanujan.

Keywords Mock Theta Functions, Partial Mock Theta Functions, Infinite Products

1. Introduction

Ramanujan gave a list of seventeen mock theta functions and labeled them as third, fifth and seventh orders without giving any reason for his classification[1, 2]. Ramanujan's general definition of a mock theta function is a function of $f(q)$ defined by a q-series convergent when | q | < 1 which satisfies the following two conditions,

(a)For every root ξ of unity, there exist a θ-function θ(q) such that difference between $f(q)$ and θ(q) is bounded as q $\rightarrow$ ξ, radially.

(b)There is no single theta function which works for all ξ, i.e. for every θ-function θ(q) there is some root of unity ξ for which $f(q)$ minus the theta function θ(q) is unbounded as q $\rightarrow$ ξ radially.

A study of these sums and expansions has been made by Watson[3], Agarwal[4] and Andrews[5]. Later on, Andrews and Hickerson[6], Choi[7] and Gordon and Mc Intosh[8] studied certain q-series in the Lost Notebook and named them as sixth, eighth and tenth order mock theta functions. Although Gordon and Mc Intosh[8] have given definitions of order of mock theta functions, and later Bringmann and Ono[9, 10] have given clarification for the order of the mock theta functions.

Also, relations connecting mock theta functions and partial mock theta functions are given by Srivastava[11] and Denis *et al.*[12]. Bhaskar Srivastava[13] provided relations connecting mock theta functions and partial mock theta functions of order 3, 5, 6 and 10 and relations connecting mock theta functions, partial mock theta functions of order 2, 3 and 6 and Ramanujan's function μ(q). Recently, Roselin Antony and Atakalti Araya[14] obtained relations connecting mock theta functions of order 2 and infinite products analogous to the identities of Ramanujan. Also, Roselin Antony and Hailemariam Fiseha[15] obtained relations connecting mock theta functions of order 10 and infinite products analogous to the identities of Ramanujan. Also, Roselin Antony[16] obtained relations connecting mock theta functions of order 6 and infinite products analogous to the identities of Ramanujan

$$\text{If } M(q)=\sum_{n=0}^{\infty}\Omega_n \tag{1.1}$$

is a mock theta function, then the corresponding partial mock theta function is denoted by the terminating series,

$$M_r(q)=\sum_{n=0}^{\infty}\Omega_n \tag{1.2}$$

2. Methodology

We shall make use of mock theta functions of order 8, their partial sums and infinite products in the known identity of Srivastava[11] to obtain new relations connecting mock theta functions of order 8.

Mock theta functions of order 8:

Gordon and McIntosh[8] found the following eight mock theta functions of order 8;

$$S_0(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}(-q;q^2)_n}{(-q^2;q^2)_n} \tag{1.3}$$

$$S_1(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+2)}(-q;q^2)_n}{(-q^2;q^2)_n} \tag{1.4}$$

* Corresponding author:
roselinmaths@gmail.com (Roselin Antony)
Published online at http://journal.sapub.org/ajms

$$T_0(q)=\sum_{n=0}^{\infty}\frac{q^{(n+1)(n+2)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}} \tag{1.5}$$

$$T_1(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}(-q^2;q^2)_n}{(-q;q^2)_{n+1}} \tag{1.6}$$

$$U_0(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}(-q;q^2)_n}{(-q^4;q^4)_n} \tag{1.7}$$

$$U_1(q)=\sum_{n=0}^{\infty}\frac{q^{(n+1)^2}(-q;q^2)_n}{(-q^2;q^4)_{n+1}} \tag{1.8}$$

$$V_0(q)=-1+2\sum_{n=0}^{\infty}\frac{q^{n^2}(-q;q^2)_n}{(q;q^2)_n}=-1+2\sum_{n=0}^{\infty}\frac{q^{2n^2}(-q^2;q^4)_n}{(q;q^2)_{2n+1}} \tag{1.9}$$

$$V_1(q)=\sum_{n=0}^{\infty}\frac{q^{(n+1)^2}(-q;q^2)_n}{(q;q^2)_{n+1}}=\sum_{n=0}^{\infty}\frac{q^{2n^2+2n+1}(-q^4;q^4)_n}{(q;q^2)_{2n+2}} \tag{1.10}$$

Ramanujan, in chapter 16 of his second notebook defined theta functions as follows;[17, 18]

$$A(q)=\sum_{n=0}^{\infty}q^{n(n+1)/2}=\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.11}$$

An identity due to Euler is,[19, 20]

$$\sum_{n=0}^{\infty}\frac{q^{(n+1)^2}}{(q;q^2)_{n+1}}=(-x;q)_\infty \tag{1.12}$$

The special cases of the above identity are

$$B(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q^2;q^2)_n}=\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty} \tag{1.13}$$

$$C(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}}{(q^2;q^2)_n}=\frac{(q,q^3,q^4;q^4)_\infty}{(q;q)_\infty} \tag{1.14}$$

The Famous Roger's –Ramanujan identity is,[21, 22]

$$D(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n}=\frac{1}{(q,q^4;q^5)_\infty} \tag{1.15}$$

$$E(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}}{(q;q)_n}=\frac{1}{(q^2,q^3;q^5)_\infty} \tag{1.16}$$

Hahn defined the septic analogue of the Rogers-Ramanujan functions as,[23, 24]

$$F(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(q^2;q^2)_n(-q;q)_{2n}}=\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty} \tag{1.17}$$

$$G(q)=\sum_{n=0}^{\infty}\frac{q^{2n(n+1)}}{(q^2;q^2)_n(-q;q)_{2n}}=\frac{(q^2,q^5,q^7;q^7)_\infty}{(q^2;q^2)_\infty} \tag{1.18}$$

$$H(q)=\sum_{n=0}^{\infty}\frac{q^{2n(n+1)}}{(q^2;q^2)_n(-q;q)_{2n+1}}=\frac{(q,q^6,q^7;q^7)_\infty}{(q^2;q^2)_\infty} \tag{1.19}$$

The Jackson – Slater identity;

Jackson[25] discovered the following identity;

$$I(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(q;q)_{2n}}=\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty} \tag{1.20}$$

This identity was independently rediscovered by Slater[26, Eqn.39] who also discovered its companion identity[Slater[26, Eqn.38]]

$$J(q)=\sum_{n=0}^{\infty}\frac{q^{2n(n+1)}}{(q;q)_{2n+1}}=\frac{(-q,-q^7,q^8;q^8)_\infty}{(q^2;q^2)_\infty} \tag{1.21}$$

The identity analogous to the Rogers-Ramanujan identity is the so-called Gollnitz – Gordon identity given by,[27, 28]

$$K(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n q^{n^2}}{(q^2;q^2)_n}=\frac{1}{(q,q^4,q^7;q^8)_\infty} \tag{1.22}$$

$$L(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n q^{n^2+2n}}{(q^2;q^2)_n}=\frac{1}{(q^3,q^4,q^5;q^8)_\infty} \tag{1.23}$$

The nonic analogue of Rogers – Ramanujan functions is

$$M(q)=\sum_{n=0}^{\infty}\frac{(q;q)_{3n}.q^{3n^2}}{(q^3;q^3)_n(q^3;q^3)_{2n}}=\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty} \tag{1.24}$$

$$N(q)=\sum_{n=0}^{\infty}\frac{(q;q)_{3n}(1-q^{3n+2})q^{3n(n+1)}}{(q^3;q^3)_n(q^3;q^3)_{2n+1}}=\frac{(q^2,q^7,q^9;q^9)_\infty}{(q^3;q^3)_\infty} \tag{1.25}$$

$$P(q)=\sum_{n=0}^{\infty}\frac{(q;q)_{3n+1}.q^{3n(n+1)}}{(q^3;q^3)_n(q^3;q^3)_{2n+1}}=\frac{(q,q^8,q^9;q^9)_\infty}{(q^3;q^3)_\infty} \tag{1.26}$$

These equalities are due to Bailey[Bailey,[29]; Eqn.(1.6),(1.7) and (1.8)].

We shall make use of the following known identity of Srivastava[11];

$$\sum_{m=0}^{\infty}\delta_m\sum_{r=0}^{m}\alpha_r=\left(\sum_{r=0}^{\infty}\alpha_r\right)\left(\sum_{m=0}^{\infty}\delta_m\right)-\sum_{r=0}^{\infty}\alpha_{r+1}\sum_{m=0}^{r}\delta_m \tag{1.27}$$

3. Main Results and Discussion

We shall establish relations connecting mock theta functions, partial mock theta functions of order 8 and infinite products analogous to the identities of Ramanujan.

A) Taking $\delta_m=q^{m(m+1)/2}$ in (1.27) and by (1.11), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty} q^{m(m+1)/2}\sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}A_m(q). \tag{2.1}$$

i) Taking $\alpha_r = \frac{q^{r^2}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.1)

and making use of (1.3), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}S_0(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}S_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}A_m(q). \tag{2.2}$$

ii) Taking $\alpha_r = \frac{q^{r(r+2)}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.1)

and making use of (1.4), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}S_1(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}S_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+3)}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}A_m(q). \tag{2.3}$$

iii) Taking $\alpha_r = \frac{q^{(r+1)(r+2)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.1)

and making use of (1.5), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}T_0(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}T_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)(r+3)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}A_m(q). \tag{2.4}$$

iv) Taking $\alpha_r = \frac{q^{r(r+1)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.1)

and making use of (1.6), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}T_1(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}T_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}A_m(q). \tag{2.5}$$

v) Taking $\alpha_r = \frac{q^{r^2}(-q;q^2)_r}{(-q^4;q^4)_r}$ in (2.1)

and making use of (1.7), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}U_0(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}U_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^4;q^4)_{r+1}}A_m(q). \tag{2.6}$$

vi) Taking $\alpha_r = \frac{q^{(r+1)^2}(-q;q^2)_r}{(-q^2;q^4)_{r+1}}$ in (2.1)

and making use of (1.8), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}U_1(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}U_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(-q^2;q^4)_{r+2}}A_m(q). \tag{2.7}$$

vii) Taking $\alpha_r = \frac{2q^{2r^2}(-q^2;q^4)_r}{(q;q^2)_{2r+1}}$ in (2.1)

and making use of (1.9), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\left[V_0(q)+1\right] = \sum_{m=0}^{\infty} q^{m(m+1)/2}.\left[V_{0m}(q)+1\right] + \sum_{r=0}^{\infty}\frac{2q^{2(r+1)^2}(-q^2;q^4)_{r+1}}{(q;q^2)_{2(r+1)+1}}A_m(q). \tag{2.8}$$

viii) Taking $\alpha_r = \frac{q^{2r^2+2r+1}(-q^4;q^4)_r}{(q;q^2)_{2r+2}}$ in (2.1)

and making use of (1.10), we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}V_1(q) = \sum_{m=0}^{\infty} q^{m(m+1)/2}V_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2+2(r+1)+1}(-q^4;q^4)_{r+1}}{(q;q^2)_{2(r+1)+2}}A_m(q). \tag{2.9}$$

B) Taking $\delta_m = \frac{q^{m^2}}{(q^2;q^2)_m}$ in (1.27)

and by (1.13), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}\sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}B_m(q). \tag{2.10}$$

i) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.10) and making use of (1.3), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}S_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}S_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}B_m(q). \tag{2.11}$$

ii) Taking $\alpha_r = \dfrac{q^{r(r+2)}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.10) and making use of (1.4), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}S_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}S_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+3)}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}B_m(q). \tag{2.12}$$

iii) Taking $\alpha_r = \dfrac{q^{(r+1)(r+2)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.10) and making use of (1.5), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}T_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}T_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)(r+3)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}B_m(q). \tag{2.13}$$

iv) Taking $\alpha_r = \dfrac{q^{r(r+1)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.10) and making use of (1.6), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}T_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}T_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}B_m(q). \tag{2.14}$$

v) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^4;q^4)_r}$ in (2.10) and making use of (1.7), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}U_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}U_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^4;q^4)_{r+1}}B_m(q). \tag{2.15}$$

vi) Taking $\alpha_r = \dfrac{q^{(r+1)^2}(-q;q^2)_r}{(-q^2;q^4)_{r+1}}$ in (2.10) and making use of (1.8), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}U_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}U_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(-q^2;q^4)_{r+2}}B_m(q). \tag{2.16}$$

vii) Taking $\alpha_r = \dfrac{2q^{2r^2}(-q^2;q^4)_r}{(q;q^2)_{2r+1}}$ in (2.10) and making use of (1.9), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}\left[V_0(q)+1\right] = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}.\left[V_{0m}(q)+1\right] + \sum_{r=0}^{\infty}\frac{2q^{2(r+1)^2}(-q^2;q^4)_{r+1}}{(q;q^2)_{2(r+1)+1}}B_m(q). \tag{2.17}$$

viii) Taking $\alpha_r = \dfrac{q^{2r^2+2r+1}(-q^4;q^4)_r}{(q;q^2)_{2r+2}}$ in (2.10)

and making use of (1.10), we get

$$\frac{(q^2,q^2,q^2;q^4)_\infty}{(q;q)_\infty}V_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}V_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2+2(r+1)+1}(-q^4;q^4)_{r+1}}{(q;q^2)_{2(r+1)+2}}B_m(q). \tag{2.18}$$

Similarly, by assuming $\delta_m = \dfrac{q^{m(m+1)}}{(q^2;q^2)_m}$, we can establish relations connecting mock theta functions of order 6 and the infinite product $C(q)$.

C) Taking $\delta_m = \dfrac{q^{m^2}}{(q;q)_m}$ in (1.27)

and by (1.15), we get

$$\frac{1}{(q,q^4;q^5)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}D_m(q). \tag{2.19}$$

i) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.19)

and making use of (1.3), we get

$$\frac{1}{(q,q^4;q^5)_\infty}S_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}S_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}D_m(q). \tag{2.20}$$

ii) Taking $\alpha_r = \dfrac{q^{r(r+2)}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.19)

and making use of (1.4), we get

$$\frac{1}{(q,q^4;q^5)_\infty}S_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}S_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+3)}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}D_m(q). \tag{2.21}$$

iii) Taking $\alpha_r = \dfrac{q^{(r+1)(r+2)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.19)

and making use of (1.5), we get

$$\frac{1}{(q,q^4;q^5)_\infty}T_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}T_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)(r+3)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}D_m(q). \tag{2.22}$$

iv) Taking $\alpha_r = \dfrac{q^{r(r+1)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.19)

and making use of (1.6), we get

$$\frac{1}{(q,q^4;q^5)_\infty}T_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}T_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}D_m(q). \tag{2.23}$$

v) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^4;q^4)_r}$ in (2.19)

and making use of (1.7), we get

$$\frac{1}{(q,q^4;q^5)_\infty}U_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}U_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^4;q^4)_{r+1}}D_m(q). \tag{2.24}$$

vi) Taking $\alpha_r = \dfrac{q^{(r+1)^2}(-q;q^2)_r}{(-q^2;q^4)_{r+1}}$ in (2.19)

and making use of (1.8), we get

$$\frac{1}{(q,q^4;q^5)_\infty}U_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}U_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(-q^2;q^4)_{r+2}}D_m(q). \tag{2.25}$$

vii) Taking $\alpha_r = \dfrac{2q^{2r^2}(-q^2;q^4)_r}{(q;q^2)_{2r+1}}$ in (2.19)

and making use of (1.9), we get

$$\frac{1}{(q,q^4;q^5)_\infty}\left[V_0(q)+1\right] = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}.\left[V_{0m}(q)+1\right] + \sum_{r=0}^{\infty}\frac{2q^{2(r+1)^2}(-q^2;q^4)_{r+1}}{(q;q^2)_{2(r+1)+1}}D_m(q). \tag{2.26}$$

viii) Taking $\alpha_r = \dfrac{q^{2r^2+2r+1}(-q^4;q^4)_r}{(q;q^2)_{2r+2}}$ in (2.19)

and making use of (1.10), we get

$$\frac{1}{(q,q^4;q^5)_\infty}V_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}V_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2+2(r+1)+1}(-q^4;q^4)_{r+1}}{(q;q^2)_{2(r+1)+2}}D_m(q). \tag{2.27}$$

By taking $\delta_m = \dfrac{q^{m(m+1)}}{(q;q)_m}$, relations can be developed connecting mock theta functions and the infinite product $E(q)$.

D) Taking $\delta_m = \dfrac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}$ in (1.27)

and by (1.17), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}\sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}F_m(q). \tag{2.28}$$

i) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.28)

and by (1.3), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}S_0(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}S_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}F_m(q). \tag{2.29}$$

ii) Taking $\alpha_r = \dfrac{q^{r(r+2)}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.28)

and by (1.4), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}S_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}S_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+3)}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}F_m(q). \tag{2.30}$$

iii) Taking $\alpha_r = \dfrac{q^{(r+1)(r+2)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.28)

and by (1.5), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}T_0(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}T_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)(r+3)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}F_m(q). \tag{2.31}$$

iv) Taking $\alpha_r = \dfrac{q^{r(r+1)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.28)

and by (1.6), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}T_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}T_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}F_m(q). \tag{2.32}$$

v) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^4;q^4)_r}$ in (2.28)

and by (1.7), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}U_0(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}U_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^4;q^4)_{r+1}}F_m(q). \tag{2.33}$$

vi) Taking $\alpha_r = \dfrac{q^{(r+1)^2}(-q;q^2)_r}{(-q^2;q^4)_{r+1}}$ in (2.28)

and by (1.8), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}U_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}U_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(-q^2;q^4)_{r+2}}F_m(q). \tag{2.34}$$

vii) Taking $\alpha_r = \dfrac{2q^{2r^2}(-q^2;q^4)_r}{(q;q^2)_{2r+1}}$ in (2.28)

and by (1.9), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}\left[V_0(q)+1\right]$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}.\left[V_{0m}(q)+1\right] \quad (2.35)$$
$$+\sum_{r=0}^{\infty}\frac{2q^{2(r+1)^2}(-q^2;q^4)_{r+1}}{(q;q^2)_{2(r+1)+1}}F_m(q).$$

viii) Taking $\alpha_r = \dfrac{q^{2r^2+2r+1}(-q^4;q^4)_r}{(q;q^2)_{2r+2}}$ in (2.28)

and by (1.10), we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}V_1(q)$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}V_{1m}(q) \quad (2.36)$$
$$+\sum_{r=0}^{\infty}\frac{q^{2(r+1)^2+2(r+1)+1}(-q^4;q^4)_{r+1}}{(q;q^2)_{2(r+1)+2}}F_m(q).$$

In the similar way, by assuming

$\delta_m = \dfrac{q^{2m(m+1)}}{(q^2;q^2)_m(-q;q)_{2m}}$ and

$\delta_m = \dfrac{q^{2m(m+1)}}{(q^2;q^2)_m(-q;q)_{2m+1}}$, relations connecting mock theta functions of order six and the infinite products $G(q)$ and $H(q)$ can be obtained.

E) Taking $\delta_m = \dfrac{q^{2m^2}}{(q;q)_{2m}}$ in (1.27)

and by (1.20), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}\sum_{r=0}^{\infty}\alpha_r$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}\sum_{r=0}^{m}\alpha_r+\sum_{r=0}^{\infty}\alpha_{r+1}I_m(q). \quad (2.37)$$

i) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.37)

and by (1.3), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}S_0(q)$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}S_{0m}(q) \quad (2.38)$$
$$+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}I_m(q).$$

ii) Taking $\alpha_r = \dfrac{q^{r(r+2)}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.37)

and by (1.4), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}S_1(q)$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}S_{1m}(q) \quad (2.39)$$
$$+\sum_{r=0}^{\infty}\frac{q^{(r+1)(r+3)}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}I_m(q).$$

iii) Taking $\alpha_r = \dfrac{q^{(r+1)(r+2)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.37)

and by (1.5), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}T_0(q)$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}T_{0m}(q) \quad (2.40)$$
$$+\sum_{r=0}^{\infty}\frac{q^{(r+2)(r+3)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}I_m(q).$$

iv) Taking $\alpha_r = \dfrac{q^{r(r+1)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.37)

and by (1.6), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}T_1(q)$$
$$=\sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}T_{1m}(q) \quad (2.41)$$
$$+\sum_{r=0}^{\infty}\frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}I_m(q).$$

v) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^4;q^4)_r}$ in (2.37)

and by (1.7), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}U_0(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}U_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^4;q^4)_{r+1}}I_m(q). \tag{2.42}$$

vi) Taking $\alpha_r = \dfrac{q^{(r+1)^2}(-q;q^2)_r}{(-q^2;q^4)_{r+1}}$ in (2.37)

and by (1.8), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}U_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}U_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(-q^2;q^4)_{r+2}}I_m(q). \tag{2.43}$$

vii) Taking $\alpha_r = \dfrac{2q^{2r^2}(-q^2;q^4)_r}{(q;q^2)_{2r+1}}$ in (2.37)

and by (1.9), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}\left[V_0(q)+1\right] = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}.\left[V_{0m}(q)+1\right] + \sum_{r=0}^{\infty}\frac{2q^{2(r+1)^2}(-q^2;q^4)_{r+1}}{(q;q^2)_{2(r+1)+1}}I_m(q). \tag{2.44}$$

viii) Taking $\alpha_r = \dfrac{q^{2r^2+2r+1}(-q^4;q^4)_r}{(q;q^2)_{2r+2}}$ in (2.37)

and by (1.10), we get

$$\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty}V_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{(q;q)_{2m}}V_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2+2(r+1)+1}(-q^4;q^4)_{r+1}}{(q;q^2)_{2(r+1)+2}}I_m(q). \tag{2.45}$$

In the similar way, by taking $\delta_m = \dfrac{q^{2m(m+1)}}{(q;q)_{2m+1}}$,

$\delta_m = \dfrac{(-q;q^2)_m q^{m^2}}{(q^2;q^2)_m}$ and $\delta_m = \dfrac{(-q;q^2)_m q^{m^2+2m}}{(q^2;q^2)_m}$,

relations connecting mock theta functions of order six and the infinite products $J(q)$, $K(q)$ and $L(q)$ can be obtained.

F) Taking $\delta_m = \dfrac{(q;q)_{3n}q^{3m^2}}{(q^3:q^3)_m(q^3;q^3)_{2m}}$ in (1.27)

and by (1.24), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}\sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}M_m(q). \tag{2.46}$$

i) Taking $\alpha_r = \dfrac{q^{r^2}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.46)

and by (1.3), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}S_0(q) = \sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}S_{0m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}M_m(q). \tag{2.47}$$

ii) Taking $\alpha_r = \dfrac{q^{r(r+2)}(-q;q^2)_r}{(-q^2;q^2)_r}$ in (2.46)

and by (1.4), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}S_1(q) = \sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}S_{1m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)(r+3)}(-q;q^2)_{r+1}}{(-q^2;q^2)_{r+1}}M_m(q). \tag{2.48}$$

iii) Taking $\alpha_r = \dfrac{q^{(r+1)(r+2)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.46)

and by (1.5), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}T_0(q)$$

$$=\sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}T_{0m}(q) \tag{2.49}$$

$$+\sum_{r=0}^{\infty}\frac{q^{(r+2)(r+3)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}M_m(q).$$

iv) Taking $\alpha_r=\frac{q^{r(r+1)}(-q^2;q^2)_r}{(-q;q^2)_{r+1}}$ in (2.46)

and by (1.6), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}T_1(q)$$

$$=\sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q;q)_{2m}}T_{1m}(q) \tag{2.50}$$

$$+\sum_{r=0}^{\infty}\frac{q^{(r+1)(r+2)}(-q^2;q^2)_{r+1}}{(-q;q^2)_{r+2}}M_m(q).$$

v) Taking $\alpha_r=\frac{q^{r^2}(-q;q^2)_r}{(-q^4;q^4)_r}$ in (2.46)

and by (1.7), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}U_0(q)$$

$$=\sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{2m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}U_{0m}(q) \tag{2.51}$$

$$+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q^2)_{r+1}}{(-q^4;q^4)_{r+1}}M_m(q).$$

vi) Taking $\alpha_r=\frac{q^{(r+1)^2}(-q;q^2)_r}{(-q^2;q^4)_{r+1}}$ in (2.46)

and by (1.8), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}U_1(q)$$

$$=\sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}U_{1m}(q) \tag{2.52}$$

$$+\sum_{r=0}^{\infty}\frac{q^{(r+2)^2}(-q;q^2)_{r+1}}{(-q^2;q^4)_{r+2}}M_m(q).$$

vii) Taking $\alpha_r=\frac{2q^{2r^2}(-q^2;q^4)_r}{(q;q^2)_{2r+1}}$ in (2.46)

and by (1.9), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}\left[V_0(q)+1\right]$$

$$=\sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}.\left[V_{0m}(q)+1\right] \tag{2.53}$$

$$+\sum_{r=0}^{\infty}\frac{2q^{2(r+1)^2}(-q^2;q^4)_{r+1}}{(q;q^2)_{2(r+1)+1}}M_m(q).$$

viii) Taking $\alpha_r=\frac{q^{2r^2+2r+1}(-q^4;q^4)_r}{(q;q^2)_{2r+2}}$ in (2.46)

and by (1.10), we get

$$\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty}V_1(q)$$

$$=\sum_{m=0}^{\infty}\frac{(q;q)_{3m}q^{3m^2}}{(q^3;q^3)_m(q^3;q^3)_{2m}}V_{1m}(q) \tag{2.54}$$

$$+\sum_{r=0}^{\infty}\frac{q^{2(r+1)^2+2(r+1)+1}(-q^4;q^4)_{r+1}}{(q;q^2)_{2(r+1)+2}}M_m(q).$$

By assuming $\delta_m=\frac{(q;q)_{3m}(1-q^{3m+2})q^{3m(m+1)}}{(q^3;q^3)_m(q^3;q^3)_{2m+1}}$ and $\delta_m=\frac{(q;q)_{3m+1}q^{3m+1}}{(q^3;q^3)_m(q^3;q^3)_{2m+1}}$, relations connecting mock theta functions of order six and the infinite products $N(q)$ and $P(q)$ can be obtained.

Similar works have been done on mock theta functions of order 8 by various mathematicians. Ahmad Ali[30] obtained relations connecting mock theta functions of order 8 and various mock theta functions of other orders. Maheshwar Pathak and Pankaj Srivastava[31] established relations connecting connecting partial mock theta functions and mock theta functions of order two, six, eight and ten.

4. Conclusions

In the similar way, many relations can be obtained using mock theta functions of different orders and infinite products analogous to Ramanujan's identities.

REFERENCES

[1] Srivastava, B., 2007, Mock theta functions and Theta functions, New Zealand J. Math. 36, 287-294.

[2] Srivastava, B., 2012, A study of Bilateral New Mock theta functions, American Journal of Mathematics and Statistics, 2(4), 64-69.

[3] Watson, G.N., 1936, The final problem: an account of the mock theta functions, Jour. London Math. Soc., 11, 55-80.

[4] Agarwal, R.P., 1969, Certain basic hypergeometric identities associated with mock theta functions, Quart. Jour. Math., 20, 121-128.

[5] Andrews, G.E., 1981, Ramanujan's lost notebook – I : Partial (-) functions, Adv. Math., 41, 137-170.

[6] Andrews, G.E. and Hickerson, D., 1991, Ramanujan's lost notebook - III : The sixth order mock theta functions, Adv. Math., 89, 60-105.

[7] Choi, Y.S., 1999, Tenth order mock theta functions in Ramanujan's lost notebook, Invent. Math., 136, 497-596.

[8] Gordon, B. and Mc Intosh, R.J., 2000, Some eighth order mock theta functions, Jour. Lodon Math. Soc., 62, 321-335.

[9] Bringmann, K., Ono, K., The f(q) mock theta functions conjecture and partition ranks, Inven. Math., 165, 243.

[10] Bringmann, K., Ono, K., 2007, Lifting elliptic cups forms to mass form with an application to partitions, Proc. Math. Acad. Sci. (USA), 104, 3725.

[11] Srivastava, A.K., 1997, On Partial sums of mock theta functions of order three, Proc. Indian Aca. Sci. (Math. Sci.), 107 (1), 1-12.

[12] Denis, R.Y., Singh, S.N. and Singh, S.P., 2006, On certain relation connecting mock theta functions, Italian Jour. Pure & Appl. Math., 19, 55-60.

[13] Srivastava, B., 2005, Ramanujan's mock theta functions, Math. J. Okayama Univ. 47(1), 163-174.

[14] Roselin Antony and Araya, A., 2013, On partial sums of Mock theta functions of order 2, International Journal of Advanced Computer and Mathematical Sciences, 4(1), 586-603.

[15] Roselin Antony and Fiseha, H., 2012, On certain relations connecting mock theta functions of order 10, International Journal of Engineering and Mathematical Sciences, 2(1), 83-101.

[16] Roselin Antony, 2013, Some relations on sixth order mock theta functions, International Journal of Mathematics Research, 5(1), 65-70

[17] Ramanujan, S., 1957, Ramanujan's Notebooks (Vols. I and II), Tata Institute of Fundamental Research, Bombay.

[18] Berndt, B.C., 1991, Ramanujan's Notebooks - Part III, Springer, New York.

[19] Euler, L., 1748, Introductio in Analysin Infinitorum, Marcum – Michaelem Bousuet, Lausannae.

[20] Andrews, G.E., 1981, Ramanujan's lost notebook – I : Partial (-) functions, Adv. Math., 41, 137-170.

[21] Rogers, L.J., 1894, Second memoir on the expansion of certain infinite products, Proc. London Math. Soc., 25, 318-343.

[22] Ramanujan, S., 1919, Proof of certain identities in combinatory analysis, Proc. Camb. Philos. Soc., 19, 214-216.

[23] Hahn, H., 2003, Septic Analogues of the Rogers-Ramanujan functions, Acta Arith., 110, 38-399.

[24] Hahn, H., 2004, Einstein series, analogues of the Rogers-Ramanujan functions and partitions, Ph. D. thesis, University of Illinois at Urbana-champaign.

[25] Jackson, F.H., 1928, Examples of a generalization of Euler's transformation for power series, Messenger of Math., 57, 169-187.

[26] Slater, L.J., 1952, Further identities of the Rogers – Ramanujan type, Proc. London Math. Soc., 54(2), 147-167.

[27] Gollnitz, H., 1967,Partitionen mit Differenzenbedingunger, J. Reine Angew Math., 225, 154-190.

[28] Gordon, B., 1965, Some continued fractions of Rogers-Ramanujan type, Duke Math. J., 32, 741- 448.

[29] Bailey, W.N., 1947, Some identities in combinatory analysis, Proc. London Math. Soc., 49, 421-435.

[30] Ali, A., 2011, On Eight order Mock theta functions, Italian J. Pure and Applied Mathematics, 28, 327-332.

[31] Pathak, M., and Srivastava,P., 2009, Certain relations for Mock theta functions of order eight, Commun. Korean Math. Soc., 24(4), 629-640.