An E(s²)-Optimal Supersaturated Designs Constructed from Bulutoglu and Cheng Theorem

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Abstract We constructed an $E(s^2)$-optimal supersaturated design (SSD) with an experimental run-size, n = 20 and number of factors, m = 57 (multiple of 19). The construction is based on balanced incomplete block designs using a theorem proposed by Bulutoglu and Cheng. This is achieved by constructing the initial blocks of the balanced incomplete block design using the theorem. Consequently, all other blocks are generated to constitute a balanced incomplete block design (BIBD), and the complete blocks that constitute the BIBD is transformed into the required supersaturated design(SSD).

Keywords $E(s^2)$-Optimal, Supersaturated, Design Matrix, Incomplete Block Designs, Run-Size

1. Introduction

Consider a two-level experimental design with n experimental runs and m factors. A two-level experimental design is called a supersaturated design(SSD) if m>n-1. According to[7], the analysis of supersaturated designs rely on the effect of sparsity principle in which the number of important factors is relatively small.

In[11], the $E(s^2)$ criterion is considered as the most popular criterion, and it measures the average correlation among the columns of the design matrix of an SSD. Suppose we have a design with n runs and m factors, and each of the factors has 2 levels, then $\frac{n}{2}$ of the entries in each column have values +1, and others -1 respectively. Let X denotes the $n \times m$ design matrix . Hence,

$$E(s^2) = \frac{\sum\limits_{i<j} s_{ij}^2}{\binom{m}{2}} \qquad (1.1)$$

where

s_{ij} denotes the element in the ith row and jth column of $X^T X$.

For the supersaturated design with n runs and m factors,

each column of the design matrix contains the same number of 1's and -1's. The number of possible factors, m that could be accommodated satisfies the inequality

$$n-1 < m \le \binom{n-1}{\frac{n}{2}-1} \qquad (1.2)$$

[8] showed that when n is even:

$$E(s^2) \ge \frac{m(n^2+n-1)-n^3}{n(m-1)} \qquad (1.3)$$

According to[7], the lower bound (1.3) is attainable when n is a multiple of 4 and m is a multiple of $n-1$, or $n=2$ (mod 4) or m is an even multiple of $n-1$.

Moreover,[6] and[8] independently derived the lower bound:

$$E(s^2) \ge \frac{n^2(m^2-n+1)}{(m-1)(n-1)} \qquad (1.4)$$

for any supersaturated design with m factors and n runs.

[1] proposed the $E(s^2)$ criterion for constructing two-level supersaturated designs, and[8] based the construction of supersaturated designs on combining q Hadamard matrices. This means that a simple construction of an SSD is to piece together q balanced incomplete balanced block designs(BIBD's) $\left(n-1, \ q(n-1), \frac{n}{2}-1\right)$. Thus, the resulting SSD will have distinct blocks.[9] presented a method of using difference family to construct BIBD's $\left(n-1, \ q(n-1), \frac{n}{2}-1\right)$ with distinct blocks, and from such a BIBD one could easily construct an $E(s^2)$-optimal SSD.

[10] introduced the construction of SSD's that satisfy specified lower bounds of $E(s^2)$, and [12] highlighted ways of constructing optimal mix-level supersaturated designs. In [4], a column wise-pairwise algorithm for constructing SSD's is discussed.

In this study, we shall construct an $E(s^2)$-optimal supersaturated design (SSD) with an even run-size, n, and number of factors, a multiple of $n-1$.

2. Materials and Method

Theorem [2]: Suppose $n-1$ is an odd prime power, q an even divisor of $n-2$ with $q \mid n-2$, x a primitive element of $GF(n-1)$, and T a subset of z_q of size $\frac{q}{2}$.

Let ℓ be the smallest positive integer such that $T + e = T$.

Then, the $e(n-1)$ sets:

$$\left\{ S_{r,a} : r = 0, \mathrm{K}, e-1, a \in GF(n-1) \right\}, \quad \text{where}$$

$$S_{r,a} = \left\{ x^{jq+i} + a : 0 \le j \le \frac{(n-2)}{q} - 1, i \in T + r \right\}$$

are distinct and constitute a BIBD $\left(n-1, e(n-1), \frac{n}{2}-1 \right)$.

Furthermore, if $\frac{(n-2)}{q}$ is odd, and U a subset of

$\{0, \mathrm{L}, e-1\}$ *of size* $\frac{e}{2}$ such that $U^* = U + \left(\frac{q}{2} \right)$, where U^* is the complement of U in $\{0, \mathrm{K}, e-1\}$ and the addition is reduced modulo q, then the $\frac{e(n-1)}{q}$ set: $\left\{ S_{r,a} : r \in U, a \in GF(n-1) \right\}$ constitutes a BIBD with distinct blocks.

The theorem satisfies the lower bound (1.4), and shall be used in the construction of the supersaturated design (SSD). The construction of the SSD will be based on BIBD, which is more general and flexible [6]. According to [2], given a Hadamard matrix of order n, there exists a BIBD $\left(n-1, q(n-1), \frac{n}{2}-1 \right)$ for every $q > 0$.

The following definitions shall be used in this work.

Definition 1: Let $GF(s)$ denotes a finite field with s elements. An element in a group within a finite field from which other elements are obtained is called a generator of the group. The generator of a group is called a primitive element of the field.

Definition 2: Let $z_q = \{0, 1, 2, \mathrm{L}, q-1\}, q > 0$. Multiplication and addition in z_q are in reduced modulo q. Now, for each subset $T = \{a_1, a_2, \mathrm{K}, a_t\}$ of z_q, *and if* $b \in z_q$, then $T + b = \{a_1 + b, a_2 + b, \mathrm{K}, a_t + b\}$.

Definition 3: According to [5], an incomplete block design $D(v, b, k, r)$ is called a balanced incomplete block design (BIBD) if

(i) each block contains k different treatments

(ii) each treatment appears in r blocks, that is, each treatment is replicated r times.

(iii) every pair of treatments appears within blocks exactly λ times.

For a BIBD, according to [3]

(a) $b \ge v$ *or* $k \ge r$

(b) $bk = vr$

(c) $r(k-1) = \lambda (v-1); r > \lambda$

Definition 4: We define a BIBD by its incidence matrix $N = [n_{ij}]$, where

$$n_{ij} = \begin{cases} 1, & \text{if the ith treatment appears in the jth block} \\ 0, & \text{otherwise} \end{cases}$$

Construction of an $E(s^2)$-Optimal Supersaturated Design:

Let the primitive element $x = 2$. Then, we shall construct a BIBD (19, 57, 9) in the following way:

Let the value of integer q in the theorem be 6. $T = U = \{0, 1, 2\}$ since T is a subset of z_6 *of size* 3. There are many of such subsets of sizes 3 within $z_6 = \{0, 1, 2, 3, 4, 5\}$. The number of subsets of sizes 3 in z_6 is $^6C_3 = 20$. The 20 subsets of z_6 each of size 3 are: {0, 1, 2}, {0, 1, 3}, {0, 1, 4}, {0, 1, 5}, {0, 2, 3}, {0, 2, 4}, {0, 2, 5}, {0, 3, 4}, {0, 3, 5}, {0, 4, 5}, {1, 2, 3}, {1, 2, 4}, {1, 2, 5}, {1, 3, 4}, {1, 3, 5}, {1, 4, 5}, {2, 3, 4}, {2, 3, 5}, {2, 4, 5}, {3, 4, 5}. Hence, T + e = T + 3=T for all these subsets except {0, 1, 2}. For the subset {0, 1, 2}, $e = 3$.

We now obtain the initial blocks of the BIBD as follows: For $q = 6$, by the Theorem:

$$0 \le j \le \frac{(n-2)}{q} - 1$$

which reduces to $0 \le j \le 2$, for $n = 20$.

Since, $e = 3$ *and* $i \in T + r$, $(r = 0, 1, 2)$, then $a \in GF(n-1) = GF(19) = GF(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18)$.

Considering   the   set;

$$S_{r,a} = \left\{ x^{jq+i} + a : 0 \leq j \leq \frac{n-2}{q} - 1, i \in T + r \right\} \quad , \quad \text{and}$$

evaluating x^{jq+i}, for $j = 0, 1, 2$; $i = 0, 1, 2$; $q = 6$; $e = 3$; $r = 0, 1, 2$, we obtain Table 1 .

Table 1. Blocks

x^{jq+i}	x^{jq+i}	x^{jq+i}
when $r = 0, i \in (0, 1, 2)$ $j = 0$ x^0, x^1, x^2 $j = 1$ x^6, x^7, x^8 $j = 2$ x^{12}, x^{13}, x^{14}	when $r = 1, i \in (1, 2, 3)$ $j = 0$ x^1, x^2, x^3 $j = 1$ x^7, x^8, x^9 $j = 2$ x^{13}, x^{14}, x^{15}	when $r = 2, i \in (2, 3, 4)$ $j = 0$ x^2, x^3, x^4 $j = 1$ x^8, x^9, x^{10} $j = 2$ x^{14}, x^{15}, x^{16}

3. Results and Discussion

With the primitive element $x = 2$, Table 1 contains three initial blocks namely, $\{2^0, 2^1, 2^2, 2^6, 2^7, 2^8, 2^{12}, 2^{13}, 2^{14}\}$, $\{2^1, 2^2, 2^3, 2^7, 2^8, 2^9, 2^{13}, 2^{14}, 2^{15}\}$ and $\{2^2, 2^3, 2^4, 2^8, 2^9, 2^{10}, 2^{14}, 2^{15}, 2^{16}\}$. As multiplication is in reduced modulo $(n-1)$, i.e., reduced modulo 19, the first initial block contains: $\{2^0, 2^1, 2^2, 2^6, 2^7, 2^8, 2^{12}, 2^{13}, 2^{14}\}$, where $2^0 = 1$, $2^1 = 2$, $2^2 = 4$,

$2^6 = 64 = 7 \pmod{19}$, $2^7 = 128 = 14 \pmod{19}$,

$2^8 = 256 = 9 \pmod{19}$, $2^{12} = 4096 = 11 \pmod{19}$,

$2^{13} = 8192 = 3 \pmod{19}$, $2^{14} = 16384 = 6 \pmod{19}$.

Therefore, the first initial block (FIB) is $\{1, 2, 4, 7, 14, 9, 11, 6\}$

From Table 1, the second initial block (SIB) contains: $\{2^1, 2^2, 2^3, 2^7, 2^8, 2^9, 2^{13}, 2^{14}, 2^{15}\}$, where $2^1 = 2$, $2^2 = 4$, $2^3 = 8$,

$2^7 = 128 = 14 \pmod{19}$, $2^8 = 256 = 9 \pmod{19}$,

$2^9 = 512 = 18 \pmod{19}$, $2^{13} = 8192 = 3 \pmod{19}$,

$2^{14} = 16384 = 6 \pmod{19}$, $2^{15} = 32768 = 12 \pmod{19}$.

Therefore, the second initial block is $\{2, 4, 8, 14, 9, 18, 3, 6, 12\}$

When the above procedure is repeated for the third initial block, we have the third initial block (TIB) as $\{4, 8, 16, 9, 18, 17, 6, 12, 5\}$.

We note that in the three initial blocks, the numbers indicate the treatments that occur in each block. In this case, the three initial blocks are shown in Table 2 .

Table 2. The Three Initial Blocks

		Blocks	
Treatments	FIB	SIB	TIB
1	1	0	0
2	1	1	0
3	1	1	0
4	1	1	1
5	0	0	1
6	1	1	1
7	1	0	0
8	0	1	1
9	1	1	1
10	0	0	0
11	1	0	0
12	0	1	1
13	0	0	0
14	1	1	0
15	0	0	0
16	0	0	1
17	0	0	1
18	0	1	1
19	0	0	0

Now, consider $x^{jq+i} + a$, where $a \in GF(19)$; that is, adding the integers $0, 1, \ldots, 18 \pmod{19}$ to all the elements in the three initial blocks produces a BIBD with 57 distinct blocks each of size 9 and 19 blocks. From the First Initial Blocks:

FIB + 0 = {1, 2, 4, 7, 14, 9, 11, 3, 6}, FIB +1 = {2, 3, 5, 8, 15, 10, 12, 4, 7},

FIB + 2 = {3, 4, 6, 9, 16, 11, 13, 5, 8}, FIB + 3 = {4, 5, 7, 10, 17, 12, 14, 6, 9},

FIB + 4 = {5, 6, 8, 11, 18, 13, 15, 7, 10}, FIB + 5 = {6, 7, 9, 12, 19, 14, 16, 8, 11},

FIB + 6 = {7, 8, 10, 13, 1, 15, 17, 9, 12}, FIB + 7 = {8, 9, 11, 14, 2, 16, 18, 10, 13},

FIB + 8 = {9, 10, 12, 15, 3, 17, 19, 11, 14}, FIB + 9 = {10, 11, 13, 16, 4, 18, 1, 12, 15},

FIB + 10 = {11, 12, 14, 17, 5, 19, 2, 13, 16}, FIB + 11 = {12, 13, 15, 18, 6, 1, 3, 14, 17},

FIB + 12 = {13, 14, 16, 19, 7, 2, 4, 15, 18}, FIB + 13 = {14, 15, 17, 1, 8, 3, 5, 16, 19},

FIB + 14 = {15, 16, 18, 2, 9, 4, 6, 17, 1}, FIB + 15 = {16, 17, 19, 3, 10, 5, 7, 18, 2},

FIB + 16 = {17, 18, 1, 4, 11, 6, 8, 19, 3}, FIB + 17 = {18, 19, 2, 5, 12, 7, 9, 1, 4},

FIB + 18 = {19, 1, 3, 6, 13, 8, 10, 2, 5}.

The above procedure is repeated on the Second Initial Block and the Third Initial Block respectively, altogether gives the BIBD (19, 57, 9). The First 19 blocks of the BIBD (19, 57, 9) obtained from the First Initial Block are shown in Table 3 below .

Adopting the BIBD(19, 57, 9), we obtain a 19 x 57 treatment-block incidence matrix for BIBD(19, 57, 9). Using SSD(n, m) = SSD(n, e(n-1)), we obtain an SSD(20, 57) from the BIBD(19, 57, 9) by converting all 0's in the incidence matrix of the BIBD(19, 57, 9) to -1's and adding a first row of all 1's to the treatment-block incidence matrix. This gives

an $E(s^2)$-optimal 20-run SSD for 57 factors shown in Table 4 below.

 Table 4 shows the complete 57 factors of the SSD(20, 57) constructed from the BIBD(19, 57, 9). We observe that all

the two level factors occur in the higher level for the first run (run 1) of the design.

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Table 3. First 19 blocks of the BIBD (19, 57, 9) obtained from the first initial block

Treatments	Blocks																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	+11	+12	+13	+14	+15	+16	+17	+18
1	1	0	0	0	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1
2	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1	1	0	1	1
3	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1	1	1	1
4	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1	1	0
5	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1	1
6	1	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1
7	1	1	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0	0	0
8	0	1	1	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1
9	1	0	1	1	0	1	1	1	1	0	0	0	0	0	1	0	0	1	0
10	0	1	0	1	1	0	1	1	1	1	0	0	0	0	0	1	0	0	1
11	1	0	1	0	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0
12	0	1	0	1	0	1	0	0	1	1	1	1	0	0	0	0	0	1	0
13	0	0	1	0	1	0	1	1	0	1	1	1	1	0	0	0	0	0	1
14	1	0	0	1	0	1	0	1	1	0	1	1	1	0	0	0	0	0	0
15	0	1	0	0	1	0	1	0	1	1	0	1	1	1	1	0	0	0	0
16	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1	1	0	0	0
17	0	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1	1	0	0
18	0	0	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1	1	0
19	0	0	0	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1	1

Table 4. The complete 57 factors of the SSD(20, 57) constructed from the BIBD(19, 57, 9)

Runs	Factors																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	1
3	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1
4	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	1	1
5	1	1	1	-1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1
6	-1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	-1	-1	1	1
7	1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1
8	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1
9	-1	1	1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1
10	1	-1	1	-1	-1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	1	-1
11	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	-1	-1
12	1	-1	1	1	1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1
13	-1	1	-1	-1	-1	1	1	-1	1	1	-1	-1	-1	-1	-1	-1	1	-1	-1
14	-1	-1	1	1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	1
15	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1	-1	-1
16	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	1	1	1	-1	-1	-1
17	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	1	1	-1	-1	-1
18	-1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	1	1	-1	-1
19	-1	-1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1	1	1	-1
20	-1	-1	-1	-1	-1	1	-1	-1	1	-1	1	1	1	1	-1	1	1	1	1

Table 4. Cont'd

Runs	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
									Factors											
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	-1	-1	1	-1	-1	-1	1	-1	-1	-1	-1	1	1	-1	1	-1	1	1	1	-1
3	-1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	1	1	1	-1
4	1	-1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	1	1	-1
5	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	1	1
6	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	1
7	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1
8	-1	-1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1
9	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	-1
10	1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1
11	-1	1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1
12	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1	-1
13	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	-1
14	-1	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1
15	1	1	1	-1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	-1	-1	1
16	-1	1	-1	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	1	-1	1	-1	-1
17	-1	-1	1	-1	1	-1	-1	1	1	-1	1	-1	1	-1	1	1	-1	-1	1	1
18	-1	-1	1	1	-1	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1	1	1
19	1	-1	1	-1	1	-1	-1	1	1	-1	1	-1	1	-1	1	1	1	-1	-1	1
20	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	-1	1	1	1	-1	-1

Table 4. Cont'd

Runs	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
								Factors										
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	1	1	1	-1	-1
3	-1	-1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	1	1	1	-1
4	-1	-1	-1	1	1	1	-1	-1	1	1	1	-1	1	1	-1	1	1	1
5	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	1	1
6	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	1
7	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	-1	-1	-1	1	-1	1
8	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	-1	1	-1	1	1
9	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	-1	-1	-1	1
10	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1	-1	-1	-1
11	1	-1	1	-1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1
12	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1
13	-1	-1	1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	-1
14	1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	1	1	-1	-1
15	-1	1	-1	1	1	1	1	-1	1	1	-1	-1	-1	-1	1	1	1	-1
16	-1	-1	1	-1	-1	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	1
17	-1	-1	-1	1	-1	-1	1	1	1	-1	1	1	-1	-1	-1	-1	1	1
18	1	-1	-1	-1	1	-1	-1	1	1	-1	1	1	-1	-1	-1	-1	-1	1
19	1	-1	-1	-1	1	-1	-1	1	1	-1	1	1	-1	-1	-1	-1	-1	-1
20	1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	1	1	1	-1	-1	-1

4. Conclusions

Using the primitive element of the field, GF(s) as defined in the work of[2], we constructed a supersaturated design, SSD(20, 57) with 57 factors and 20 experimental runs. We determine the $E(s^2)$-value for the SSD(20, 57) as 40.31 which satisfies the $E(s^2)$ lower bound proposed by[8]. This guarantees the validity of design, SSD(20, 57). In the literature, a number of $E(s^2)$-optimal supersaturated design construction techniques have been utilised, but oftentimes, most are very rigorous, not adaptable and flexible. In this respect, constructing an $E(s^2)$-optimal supersaturated design from balanced incomplete block design excels. All the details of this construction technique have been successfully demonstrated in this paper. Obviously, this technique would

be of great aid in the search for more E(s^2)-optimal supersaturated designs for factors screening in the preliminary stage of industrial or scientific experiments.

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