American Journal of Computational and Applied Mathematics 2017, 7(4): 115-122
DOI: 10.5923/j.ajcam.20170704.03

Observer Design for a Class of Discrete-Time Takagi-Sugeno Implicit Models Subject to Unknown Inputs

Abdellah Louzimi[1,2], Abdellatif El Assoudi[1,2], Jalal Soulami[1,2,*], El Hassane El Yaagoubi[1,2]

[1]Laboratory of High Energy Physics and Condensed Matter, Faculty of Science, Hassan II University of Casablanca, Casablanca, Morocco
[2]ECPI, Department of Electrical Engineering, ENSEM, Hassan II University of Casablanca, Casablanca, Morocco

Abstract This paper investigates the problem of the fuzzy unknown inputs observer (FUIO) design for a class of discrete-time Takagi-Sugeno implicit models (DTSIMs) with unmeasurable premise variables which satisfying Lipschitz conditions. The unknown inputs (UIs) affect both state and output of the model. The idea of the proposed result is based on the separation between dynamic and static equations in the considered DTSIM. First, the method used to separate dynamic equations from static equations is developed. Next, based on the augmented system structure which contains the dynamic equations and the unknown inputs, a new observer design in explicit structure to estimate simultaneously the system state and the unknown inputs is established. The convergence of the state estimation error of the augmented system is studied by using the Lyapunov theory and the gain matrix of the FUIO is obtained by solving only one linear matrix inequality (LMI). At last, an illustrative example is given to show the effectiveness of the proposed technique.

Keywords Discrete-time Takagi-Sugeno implicit model, Fuzzy unknown input observer, Lipschitz condition, LMI

1. Introduction

It is well-known that to provide on-line estimation of unmeasurable states of various industrial processes, the approach consists in combining a priori knowledge about a process with experimental data. The algorithm permitting to realize a such estimation is called an observer. This last one is a dynamical system which combines a nominal model of the process with on-line measurements of the input and the output of the process to estimate the unmeasurable states. The field of the observer design for dynamic systems has attracted much attention of the researchers for a long time. This is due to its important role in the control and fault-tolerant areas.

Moreover, UIs can result either from uncertainty in the model or from the presence of unknown external excitation. Thus, due to the increasing demand for reliability and maintenability of the automatic control process, FUIO design is widely used in the area of fault detection and design of fault tolerant control strategy. This is one of the most attractive research areas in both theoretical and practical fields during these last two decades. Indeed, many works using different approaches can be found in the literature [1-3]. In this perspective, based on the Takagi-Sugeno (T-S) fuzzy approach, we study here the problem of observer design for a class of discrete-time nonlinear implicit models subject to UIs.

Currently, it is well-known that ordinary T-S fuzzy approach [4, 5] have been widely and successfully used in the nonlinear processes modelling to describe the behaviour of many chemical and physical processes. The idea of this approach is to apprehend the global behaviour of a process by a set of local models. The success of such approach relies on the fact that once the T-S fuzzy models are obtained, some analysis and design tools developed in the linear case can be used, which facilitates observer or/and controller synthesis for complex nonlinear systems, see for example [6, 7] and the references therein.

On the other hand, in reality, many industrial processes are naturally modelled as systems of differential and algebraic equations also called descriptor models or singular models or implicit models, see [8-10] for some real applications of implicit models. The numerical simulation of such models usually combines an ODE numerical method together with an optimization algorithm.

Moreover, notice that the ordinary T-S fuzzy model is a special case of the fuzzy implicit model. Indeed, in [11, 12], a fuzzy implicit model is defined by extending the T-S fuzzy model [4].

* Corresponding author:
jalal.soulami@gmail.com (Jalal Soulami)
Published online at http://journal.sapub.org/ajcam

The FUIO design problem for T-S explicit or implicit models has received considerable attention and is still an active area of research in both continuous-time and discrete-time cases during the two last decades. Indeed, concerning the case of continuous-time T-S explicit or implicit models subject to UIs, various developments on fuzzy observer and its application to fault detection exist in the literature. We may cite [13-17] for explicit models and [11, 12, 18-23] for implicit models. Likewise, in discrete-time T-S models case, several works exist for explicit or implicit structures see e.g. [24-28]. It should be noted that, generally an interesting way to solve the various FUIO raised previously is to write the convergence conditions on the LMI form [29].

The main contribution of this paper consists to propose a new result of FUIO design for a class of DTSIMs satisfying the Lipschitz conditions allowing the simultaneous estimation of the unknown states and unknown inputs. The procedure is based on the separation between dynamic and static equations in the DTSIM in order to use the augmented system structure which contain the dynamic equations and the UIs. The global exponential stability of the state estimation error of the augmented system is studied by using the Lyapunov theory and the stability condition is given in term of only one LMI. Besides, the proposed result is given without the use of an optimization algorithm.

The rest of the paper is structured as follows. The considered class of DTSIMs subject to UIs is presented in Section 2. The main contribution about FUIO design permitting to estimate unknown states and UIs is stated in Section 3. Finally, a numerical example to show the good performance of the proposed technique is given in Section 4.

Throughout the paper, the following notations are adopted. Matrix $X>0$ (or $X<0$) means that X is symmetric and positive definite (or negative definite). X^T denotes the transpose of X. The symbol I (or 0) represents the identity matrix (or zero matrix) with appropriate dimension. R^n and $R^{n\times m}$ denote the spaces of n-dimensional real vectors and $n\times m$ real matrices, respectively.

2. System Description and Preliminaries

In this paper, the following class of DTSIMs subject to UIs is adopted:

$$\begin{cases} Ex_{k+1} = \sum_{i=1}^{q} h_i(x_k)(A_i x_k + B_i u_k + D_i d_k) \\ y_k = Cx_k + Fd_k \end{cases} \tag{1}$$

where $x_k^T = \begin{bmatrix} X_k^{1T} & X_k^{2T} \end{bmatrix} \in R^n$ denotes the state vector with $X_k^1 \in R^{n_1}$ is the vector of difference variables, $X_k^2 \in R^{n_2}$ is the vector of algebraic variables with $n_1 + n_2 = n$, $u_k \in R^m$ is the input vector which is known (measured), $d_k \in R^r$ is the unknown input vector, $y_k \in R^p$ is the measurement vector. $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C \in R^{p\times n}$, $D_i \in R^{n\times r}$, $E \in R^{n\times n}$ with $rank(E) = n_1$, $F \in R^{p\times r}$, are real known constant matrices with:

$$A_i = \begin{pmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{pmatrix}; \quad B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix}; \quad D_i = \begin{pmatrix} D_{1i} \\ D_{2i} \end{pmatrix} \tag{2}$$

$$E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}; \quad C = \begin{pmatrix} C_1 & 0 \end{pmatrix} \tag{3}$$

where A_{22i} are supposed invertible. q is the number of sub-models and $h_i(x_k)$ are the weighting functions that verify the so-called convex sum properties:

$$\sum_{i=1}^{q} h_i(x_k) = 1, \quad 0 \le h_i(x_k) \le 1 \quad i = 1,\dots,q \tag{4}$$

They ensure the transition between the contribution of each sub model:

$$\begin{cases} Ex_{k+1} = A_i x_k + B_i u_k + D_i d_k \\ y_k = Cx_k + Fd_k \end{cases} \tag{5}$$

Before giving the main result, let us make the following assumption [8, 19]:

Assumption 1: Suppose that:

- (E, A_i) is regular, i.e. $\det(zE - A_i) \neq 0 \ \ \forall\, z \in \mathbb{C}$
- All sub-models (5) are impulse observable and detectable.

In what follows, as mentioned above, we proceed to the separation of the dynamic equations from static equations of the model (1). Indeed, from (2)-(3), sub-model (5) can be rewritten as follows:

$$\begin{cases} X_{k+1}^1 = A_{11i}X_k^1 + A_{12i}X_k^2 + B_{1i}u_k + D_{1i}d_k \\ 0 = A_{21i}X_k^1 + A_{22i}X_k^2 + B_{2i}u_k + D_{2i}d_k \\ y_k = C_1 X_k^1 + Fd_k \end{cases} \tag{6}$$

From (6) and using the fact that A_{22i} is invertible, the algebraic equations can be solved directly to obtain:

$$X_k^2 = J_i X_k^1 + K_i u_k + L_i d_k \tag{7}$$

where

$$\begin{cases} J_i = -A_{22i}^{-1} A_{21i} \\ K_i = -A_{22i}^{-1} B_{2i} \\ L_i = -A_{22i}^{-1} D_{2i} \end{cases} \tag{8}$$

Thus, substituting (7) in (6) we obtain:

$$\begin{cases} X_{k+1}^1 = M_i X_k^1 + N_i u_k + P_i d_k \\ X_k^2 = J_i X_k^1 + K_i u_k + L_i d_k \\ y_k = C_1 X_k^1 + F d_k \end{cases} \tag{9}$$

where

$$\begin{cases} M_i = A_{11i} + A_{12i} J_i \\ N_i = B_{1i} + A_{12i} K_i \\ P_i = D_{1i} + A_{12i} L_i \end{cases} \tag{10}$$

The weighting functions $h_i(x_k),\ i = 1, \dots, q$ can be rewritten as:

$$h_i(x_k) = h_i(X_k^1, X_k^2 = J_i X_k^1 + K_i u_k + L_i d_k) = h_i(\eta_k) \tag{11}$$

with $\eta_k^T = \begin{bmatrix} X_k^{1T} & d_k^T & u_k^T \end{bmatrix}$.

So, by aggregation of the resulting sub-models (9), the following global fuzzy model is obtained:

$$\begin{cases} X_{k+1}^1 = \sum_{i=1}^{q} h_i(\eta_k) \left(M_i X_k^1 + N_i u_k + P_i d_k \right) \\ X_k^2 = \sum_{i=1}^{q} h_i(\eta_k) \left(J_i X_k^1 + K_i u_k + L_i d_k \right) \\ y_k = C_1 X_k^1 + F d_k \end{cases} \tag{12}$$

Assumption 2: Suppose that d_k is considered as a constant unknown control input per time interval i.e.:

$$d_{k+1} = d_k \quad k \in \begin{bmatrix} T_1 & T_2 \end{bmatrix}, \quad \forall T_1, T_2 \in R^+ \tag{13}$$

In order to take the main contribution, we rewrite the system (12) under the equivalent augmented state representation given by:

$$\begin{cases} \xi_{k+1}^1 = \sum_{i=1}^{q} h_i(\eta_k) \left(\tilde{M}_i \xi_k^1 + \tilde{N}_i u_k \right) \\ \xi_k^2 = \sum_{i=1}^{q} h_i(\eta_k) \left(\tilde{J}_i \xi_k^1 + K_i u_k \right) \\ y_k = \Lambda \xi_k^1 \end{cases} \tag{14}$$

where

$$\begin{cases} \xi_k^{1T} = \begin{bmatrix} X_k^{1T} & d_k^T \end{bmatrix}; \ \xi_k^2 = X_k^2 \\ \eta_k^T = \begin{bmatrix} \xi_k^{1T} & u_k^T \end{bmatrix}; \ \tilde{M}_i = \begin{pmatrix} M_i & P_i \\ 0 & I \end{pmatrix} \\ \tilde{N}_i = \begin{pmatrix} N_i \\ 0 \end{pmatrix}; \ \tilde{J}_i = \begin{pmatrix} J_i & L_i \end{pmatrix} \\ \Lambda = \begin{pmatrix} C_1 & F \end{pmatrix} \end{cases} \tag{15}$$

which is equivalent to the following state representation:

$$\begin{cases} \xi_{k+1}^1 = M_0 \xi_k^1 + N_0 u_k + \sum_{i=1}^{q} h_i(\eta_k) \left(\bar{M}_i \xi_k^1 + \bar{N}_i u_k \right) \\ \xi_k^2 = \sum_{i=1}^{q} h_i(\eta_k) \left(\tilde{J}_i \xi_k^1 + K_i u_k \right) \\ y_k = \Lambda \xi_k^1 \end{cases} \tag{16}$$

where

$$\begin{cases} M_0 = \frac{1}{q} \sum_{i=1}^{q} \tilde{M}_i \\ N_0 = \frac{1}{q} \sum_{i=1}^{q} \tilde{N}_i \\ \bar{M}_i = \tilde{M}_i - M_0 \\ \bar{N}_i = \tilde{N}_i - N_0 \end{cases} \tag{17}$$

3. Synthesis of the Observer

Based on the transformation of the DTSIM (1) into the equivalent form (16), the proposed FUIO for (1) is taken in the following form:

$$\begin{cases} \hat{\xi}_{k+1}^1 = M_0 \hat{\xi}_k^1 + N_0 u_k - G(\hat{y}_k - y_k) \\ \qquad + \sum_{i=1}^{q} h_i(\hat{\eta}_k) \left(\bar{M}_i \hat{\xi}_k^1 + \bar{N}_i u_k \right) \\ \hat{\xi}_k^2 = \sum_{i=1}^{q} h_i(\hat{\eta}_k) \left(\tilde{J}_i \hat{\xi}_k^1 + K_i u_k \right) \\ \hat{y}_k = \Lambda \hat{\xi}_k^1 \end{cases} \tag{18}$$

where $(\hat{\xi}_k^1,\ \hat{\xi}_k^2)$, $\hat{y}_k$ and $\hat{\eta}_k$ represent the estimate of $(\xi_k^1,\ \xi_k^2)$, y_k and η_k respectively. Matrix G is to be determined such that $(\hat{\xi}_k^1,\ \hat{\xi}_k^2)$ converges toward $(\xi_k^1,\ \xi_k^2)$ exponentially.

To give the condition for the exponential convergence of the observer (18), we define the state estimation error:

$$\varepsilon_k = \begin{pmatrix} \varepsilon_k^1 \\ \varepsilon_k^2 \end{pmatrix} = \begin{pmatrix} \hat{\xi}_k^1 - \xi_k^1 \\ \hat{\xi}_k^2 - \xi_k^2 \end{pmatrix} \tag{19}$$

From (16) and (18), the dynamic of the estimation error can be described by:

$$\begin{cases} \varepsilon_{k+1}^1 = \Gamma \varepsilon_k^1 + \sum_{i=1}^{q} \left(\bar{M}_i \delta_k^1 + \delta_k^2 u_k \right) \\ \varepsilon_k^2 = \sum_{i=1}^{q} h_i(\hat{\eta}_k) \tilde{J}_i \varepsilon_k^1 + \sum_{i=1}^{q} \left(h_i(\hat{\eta}_k) - h_i(\eta_k) \right) \left(\tilde{J}_i \hat{\xi}_k^1 + K_i u_k \right) \end{cases} \tag{20}$$

where

$$\Gamma = M_0 - G\Lambda \tag{21}$$

and

$$\begin{cases} \delta_k^1 &= h_i(\hat{\eta}_k)\hat{\xi}_k^1 - h_i(\eta_k)\xi_k^1 \\ \delta_k^2 &= \bar{N}_i\left(h_i(\hat{\eta}_k) - h_i(\eta_k)\right) \end{cases} \tag{22}$$

So, to prove the convergence of ε_k toward zero, it suffices to prove that ε_k^1 converges toward zero.

Assumption 3: Assume that the following conditions hold:

$$\begin{cases} |\delta_k^1| < \mu_i |\varepsilon_k^1| \\ |\delta_k^2| < \nu_i |\varepsilon_k^1| \\ |u_k| < \rho \end{cases} \tag{23}$$

where μ_i, ν_i are positive scalars Lipschitz constants and $\rho > 0$.

Using the Assumption 3, the term $\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right)$ can be bounded as follows:

$$\left|\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right)\right| < \alpha |\varepsilon_k^1| \tag{24}$$

where

$$\alpha = \sum_{i=1}^{q}\left(\sigma(\bar{M}_i)\mu_i + \nu_i\rho\right) \tag{25}$$

with $\sigma(\bar{M}_i)$ denotes the maximum singular value of the matrix $\bar{M}_i$.

The following Theorem provides the main result of this paper.

Theorem 1: Under above Assumption 3, the DTSIM (20) is globally exponentially stable if given $\theta > 0$ there exist matrices $P > 0$, $Q > 0$ and U verifying the following LMI:

$$\begin{pmatrix} \Sigma & * & * \\ U\Lambda & -P & * \\ PM_0 - U\Lambda & 0 & -Q \end{pmatrix} < 0 \tag{26}$$

where

$$\Sigma = M_0^T P M_0 - M_0^T U\Lambda - \Lambda^T U^T M_0 + \alpha^2 P + \alpha^2 Q - \theta^2 P \tag{27}$$

The gain stabilizing the estimation error is given by:

$$G = P^{-1}U \tag{28}$$

Proof of Theorem 1: Consider the following standard Lyapunov function:

$$V_k = \varepsilon_k^{1T} P \varepsilon_k^1, \qquad P > 0 \tag{29}$$

The variation of V_k along the trajectory of (20) is given by:

$$\Delta V_k = V_{k+1} - V_k \tag{30}$$

From (20), we have:

$$\begin{aligned} \Delta V_k &= \varepsilon_k^{1T}(\Gamma^T P\Gamma - P)\varepsilon_k^1 \\ &+ \left(\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right)\right)^T P\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right) \\ &+ \left(\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right)\right)^T P\Gamma\varepsilon_k^1 \\ &+ \varepsilon_k^{1T}\Gamma^T P\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right) \end{aligned} \tag{31}$$

Lemma 1: For any matrices X and Y with appropriate dimensions, the following property holds for any invertible matrix Z:

$$X^T Y + Y^T X \le X^T Z^{-1} X + Y^T Z Y \tag{32}$$

For $Q > 0$, by applying Lemma 1, (31) becomes:

$$\begin{aligned} \Delta V_k &= \varepsilon_k^{1T}(\Gamma^T P\Gamma - P)\varepsilon_k^1 \\ &+ \left(\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right)\right)^T P\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right) \\ &+ \varepsilon_k^{1T}\Gamma^T PQ^{-1}P\Gamma\varepsilon_k^1 \\ &+ \left(\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right)\right)^T Q\sum_{i=1}^{q}\left(\bar{M}_i\delta_k^1 + \delta_k^2 u_k\right) \end{aligned} \tag{33}$$

Taking into account (24), (33) becomes:

$$\Delta V_k = \varepsilon_k^{1T}(\Gamma^T P\Gamma - P + \Gamma^T PQ^{-1}P\Gamma + \alpha^2 \mathrm{P} + \alpha^2 Q)\varepsilon_k^1 \tag{34}$$

Thus, estimation error convergence is exponentially ensured if the following condition is guaranteed (see [30] as cited in [6]):

$$\begin{aligned} \Delta V_k &= \varepsilon_k^{1T}(\Gamma^T P\Gamma - P + \Gamma^T PQ^{-1}P\Gamma + \alpha^2 P + \alpha^2 Q)\varepsilon_k^1 \\ &< (\theta^2 - 1)V_k, \quad \theta < 1 \end{aligned} \tag{35}$$

which is equivalent to the following condition:

$$\Gamma^T P\Gamma + \Gamma^T PQ^{-1}P\Gamma + \alpha^2 P + \alpha^2 Q - \theta^2 P < 0, \quad \theta < 1 \tag{36}$$

Then, replacing Γ from (21) into (36), we can establish the LMI condition (26) of Theorem 1 by using the Schur complement [29] and the following change of variable:

$$U = PG \tag{37}$$

Thus, from the Lypunov stability theory, if the LMI condition (26) is satisfied, the system (20) is globally exponentially stable. This ends the proof of Theorem 1.

4. Illustrative Example

In this section, to illustrate the effectiveness of the proposed design method, let us consider the one-link flexible joint robot described by the dynamical model given in [22].

The discrete-time system (1) was obtained by using the Euler discretisation of a step size Te = 0.01 which lead to the following DTSIM with unmeasurable premise variable and subject to UI:

$$\begin{cases} Ex_{k+1} = \sum_{i=1}^{2} h_i(x_k)(A_i x_k + Bu_k + Dd_k) \\ y_k = Cx_k \end{cases} \tag{38}$$

where $x_k \in R^6$, $u_k \in R$, $d_k \in R$ and $y_k \in R^2$ are the state vector, known input, unknown input and measured output vector, respectively. The matrices numerical values are:

$$A_1 = \begin{pmatrix} 1 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0.01 \\ -0.49 & 0.49 & -0.12 & 0 & -0.01 & 0 \\ 0.19 & -0.12 & 0 & 0 & 0 & -0.01 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.22 \\ 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 0 & 0.01 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0.01 \\ -0.49 & 0.49 & -0.12 & 0 & -0.01 & 0 \\ 0.19 & -0.54 & 0 & 0 & 0 & -0.01 \end{pmatrix},$$

$$D = \begin{pmatrix} 0.01 \\ 0 \\ 0.01 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The weighting functions are:

$$\begin{cases} h_1(x_k) = 1 - \dfrac{\sin(x_{2k})}{x_{2k}} \\ h_2(x_k) = \dfrac{\sin(x_{2k})}{x_{2k}} \end{cases}$$

Therefore to apply the proposed FUIO (18) for the model (38), as stated in Theorem 1, it suffices to rewrite the model (38) into its equivalent form (16) as mentioned in Section 2.

Thus, by Theorem 1 with $\theta = 0.95$ the following observer gain G is obtained:

$$G = \begin{pmatrix} 0.73 & -0.01 \\ 0.03 & 0.96 \\ -0.24 & 1.16 \\ 2.27 & 6.22 \\ 8.33 & 0.11 \end{pmatrix}$$

Simulation results with initial conditions:

$$\xi_k^1 = \begin{bmatrix} 0.00 & 0.31 & 0.00 & 0.00 & 4.00 \end{bmatrix}^T,$$

$$\xi_k^2 = \begin{bmatrix} 15.28 & -16.69 \end{bmatrix}^T$$

$$\hat{\xi}_k^1 = \begin{bmatrix} 0.00 & 0.47 & 0.00 & 0.01 & 0.00 \end{bmatrix}^T,$$

$$\hat{\xi}_k^2 = \begin{bmatrix} 22.93 & -24.71 \end{bmatrix}^T$$

are given in Figures 1, 2 and 3 where the input $u_k = \sin(k)$ and the expression of unknown input signal d_k is defined as in Figure 3. These simulation results show the performances of the proposed FUIO (18) with the gain G where the dashed lines denote the state variables and unknown input estimated by the FUIO. They show that the FUIO gives a good estimation of unknown states and unknown input of the considered DTSIM.

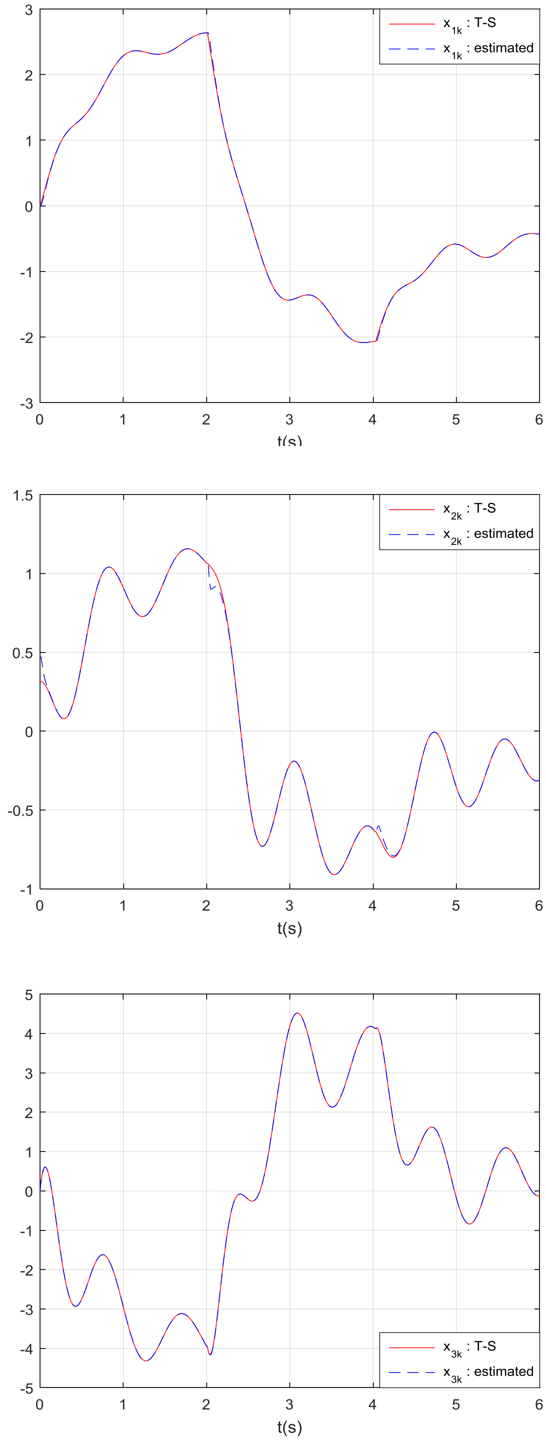

Figure 1. State variables x_{1k}, x_{2k}, x_{3k} and their estimates

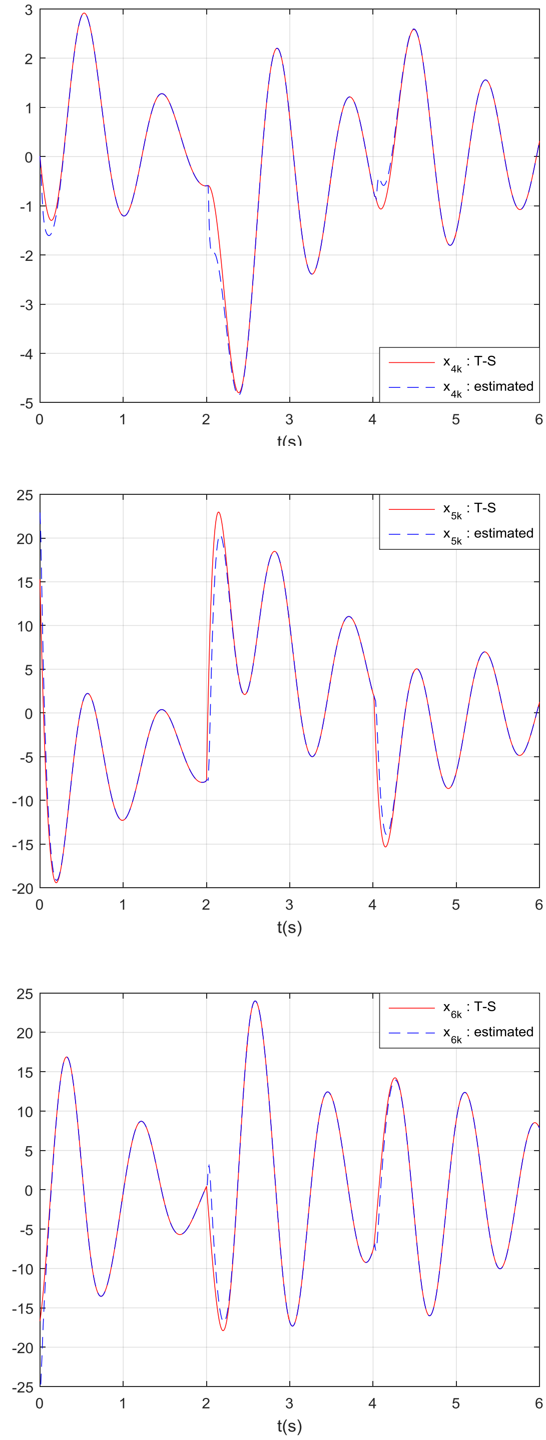

Figure 2. State variables x_{4k}, x_{5k}, x_{6k} and their estimates

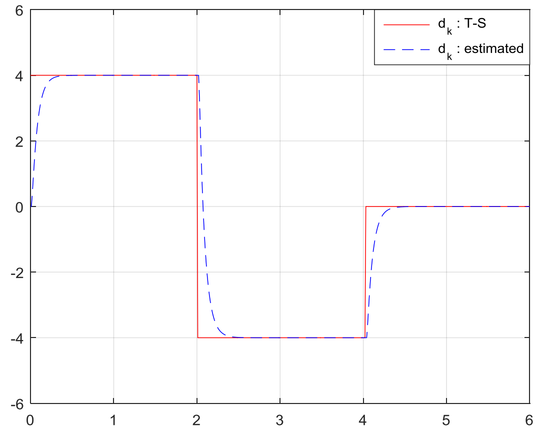

Figure 3. Unknown input d_k and its estimate

5. Conclusions

In this paper, we have presented a novel method of FUIO design for a class of DTSIMs with unmeasurable premise variables which satisfying Lipschitz conditions. The UIs affect both state and output of the model. The approach is based on the separation between dynamic and static equations in the considered fuzzy implicit model and on the use of an augmented system description formed by the dynamic equations and the UIs. Besides, the proposed result permitting to estimate simultaneously the system state and the UIs is given without the use of an optimization algorithm. The exponential convergence of the state estimation error is studied using the Lyapunov theory and the existence of the condition ensuring this convergence is expressed in term of only one LMI. In order to validate the proposed approach a numerical example is given. The proposed example shows the good performance of the proposed FUIO design, since both state and UIs are well estimated.

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