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Dengue in Tanzania - Vector Control and Vaccination

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Abstract In this paper a mathematical model is presented to examine the effect of treatment, careful and Careless Susceptibles with control on the transmission of Dengue fever in the society. A nonlinear mathematical model for the problem is proposed and analysed quantitatively using the stability theory of the differential equations. The results show that the disease-free equilibrium point is locally and globally asymptotically stable if the reproduction number ($\boldsymbol{R_0}$) is less than unity. Then the endemic equilibrium is locally and globally asymptotically stable under certain conditions, using the additive compound matrices approach and Lyapunov method respectively. However treatment, careful Susceptibles and the control on the transmission of dengue fever disease will have a positive effect on decreasing the growth rate of dengue fever disease. The numerical simulation shows that on the application of vaccination, the number of infected individual is reduced.

Keywords Dengue Fever Disease, treatment, Careful, Careless, Susceptible, Equilibrium, Control, infected, reproduction number, Vaccination

1. Introduction

Dengue fever is a severe infection, flu-like illness transmitted to humans through the bites of infected female Aedes mosquitoes. Four different serotypes can cause dengue fever. A human infected by one serotype, on recovery, gains total immunity to that serotype and only partial and transient immunity with respect to the other three. Dengue fever can vary from mild to severe; the more severe forms of dengue fever include dengue hemorrhagic fever and dengue shock syndrome. Dengue hemorrhagic fever occurs when a person get infected by different type of dengue virus after being infected by another one sometimes before. Dengue shock syndrome is the most severe form of dengue infection. Dengue is found in tropical and sub-tropical climates worldwide, mostly in urban and semi-urban areas [1].

Mathematical modelling of the population models continues to provide important insights into population behaviour and control. Over the years, it has also become an important tool in understanding the dynamics of diseases, and the decision making process regarding intervention programs for controlling population and disease problems in many countries [2].

Mathematical modelling also became considerable important tool in the study of epidemiology because it helps in understanding the observed epidemiological patterns, disease control and provides understanding of the underlying mechanisms which influence the spread of disease and may suggest control strategies [1-8]. Moreover [9], presented a dynamical model that studied the temporal model for dengue disease with treatment. So far no research has considered a dynamical system that incorporates the control strategies to reduce the spread of the dengue fever disease through the campaign to educate the careless human susceptible, control vector human contact, removing vector breeding areas, insecticides application and control maturation rate from larvae to adult. In this work, we present an extension of the model of [9] to include temporary immunity, control strategies and Susceptibles with different behaviour i.e. the dynamical system that incorporates the effects Careful and Careless Susceptibles on the transmission of Dengue fever in the society with vaccination. In this paper, data reported by the ministry of health in Tanzania is used. In July 2010 for the first time in Dar es Salaam region -Tanzania, an outbreak of dengue fever was reported, over 40 people were infected and then also between May and July 2013,172 were infected with this disease. Moreover in the year 2014, the government of Tanzania announced the dangers of the disease in which people were alerted about the disease and the precaution to be taken. In this 2014, 399 people were infected in which 2 died of the disease in Dar es Salaam region (http://www.wavuti.com/2014/05/wizara-ya-afya-kitengo-cha.html). Data will be obtained from the different literature and estimated since there is no enough data in Tanzania. The purpose of this study is to match the empirical data with the modal simulation. Hence we formulate the **SITRS** (susceptible, Infected, Treated, Recovered, susceptible) and

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SVITRS (Susceptible, Vaccinated, Infected, Recovered, Susceptible) models for transmission of dengue fever disease.

2. Formulation of the Model

In this section, we adopt the model presented in [10]. The model is based on two populations: humans and mosquitoes. Human population (N_h) is divided into five groups such as S_{h_1} - Careful human Susceptibles, S_{h_2} - Careless human Susceptibles, I_h - infected human, T_h - treated infected human, R_h - recovery infected human, so that we have $N = S_{h_1} + S_{h_2} + I_h + T_h + R_h$ and the population of female mosquitoes, indexed by m is divided into three groups that is A_m -Aquatic phase (that includes the egg, larva and pupa stages), S_m - Susceptibles (mosquitoes that are able to contract the disease), I_m -Infectives (mosquitoes capable of transmitting the disease to human). In formulating the model, the following assumptions are considered:

i. Total human population (N_h) is constant,
ii. The population is homogeneous, which means that every individual of a compartment is homogeneously mixed with the other individuals,
iii. Immigration and emigration are not considered,
iv. Each vector has an equal probability to bite any host,
v. Humans and mosquitoes are assumed to be born susceptible i.e. there is no natural protection,
vi. The coefficient of transmission of the disease is fixed and does not vary seasonally,
vii. For the mosquito there is no resistant phase, due to its short lifetime,
viii. The possibility of careless Susceptibles contracting dengue fever disease is higher than that for careful Susceptibles.

Considering the above assumptions, we then have the following schematic model flow diagram for dengue fever disease with control:

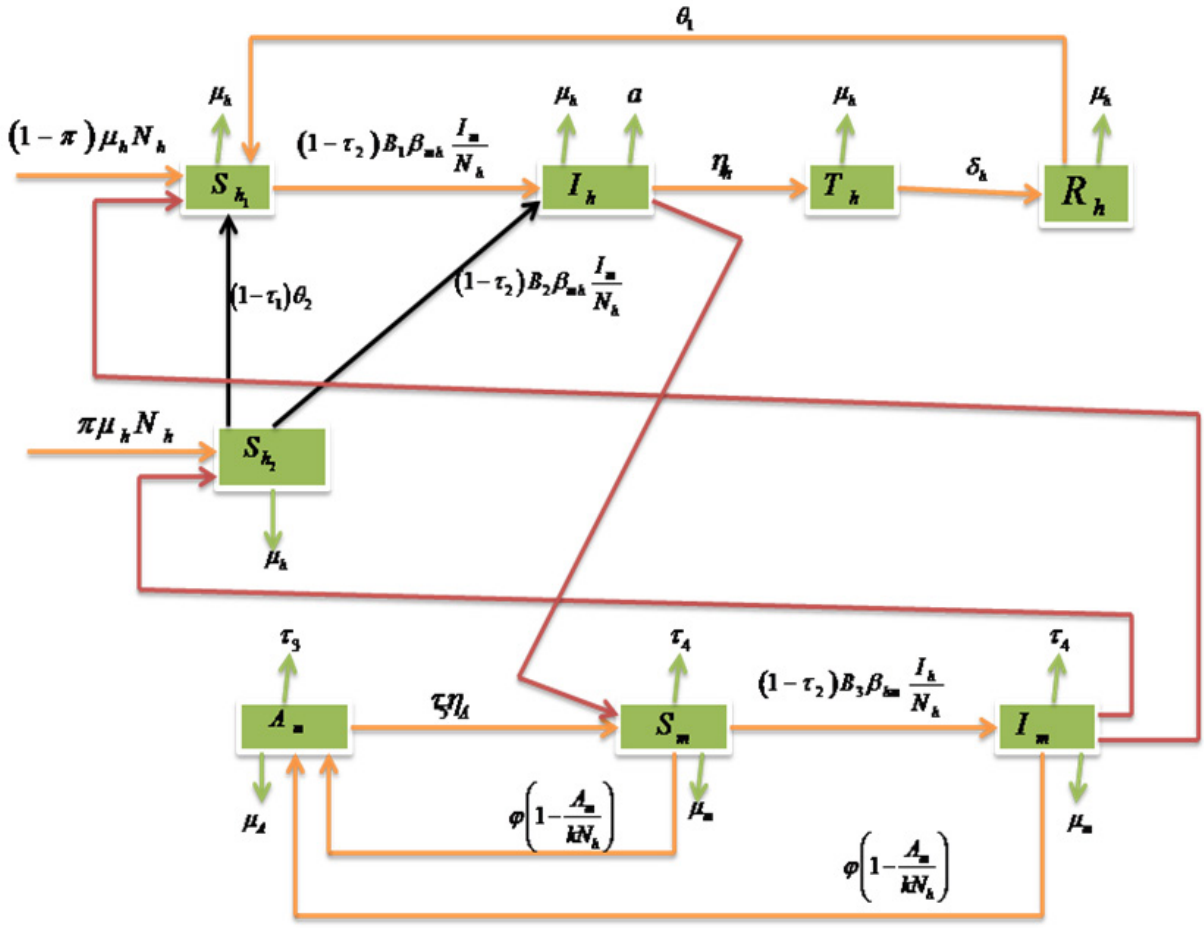


Figure 1. Model Flow diagram for dengue fever disease with control

From the above flow diagram, the model will be governed by the following equations [10]:

$$\frac{dS_{h_1}}{dt} = (1-\pi)\mu_h N_h - (1-\tau_2) B_1 \beta_{mh} \frac{I_m}{N_h} S_{h_1} - \mu_h S_{h_1} + \theta_1 R_h + (1-\tau_1)\theta_2 S_{h_2}$$

$$\frac{dS_{h_2}}{dt} = \pi\mu_h N_h - (1-\tau_2) B_2 \beta_{mh} \frac{I_m}{N_h} S_{h_2} - \mu_h S_{h_2} - (1-\tau_1)\theta_2 S_{h_2}$$

$$\frac{dI_h}{dt} = \left((1-\tau_2) B_1 S_{h_1} + (1-\tau_2) B_2 S_{h_2}\right)\beta_{mh} \frac{I_m}{N_h} - (\mu_h + \eta_h + a) I_h$$

$$\frac{dT_h}{dt} = \eta_h I_h - (\mu_h + \delta_h) T_h, \quad \frac{dR_h}{dt} = \delta_h T_h - (\mu_h + \theta_1) R_h$$

$$\frac{dA_m}{dt} = \varphi\left(1 - \frac{A_m}{kN_h}\right)(S_m + I_m) - (\mu_A + \tau_5\eta_A + \tau_3) A_m$$

$$\frac{dS_m}{dt} = \tau_5\eta_A A_m - \left((1-\tau_2) B_3 \beta_{hm} \frac{I_h}{N_h} + \mu_m\right) S_m - \tau_4 S_m$$

$$\frac{dI_m}{dt} = (1-\tau_2) B_3 \beta_{hm} \frac{I_h}{N_h} S_m - (\mu_m + \tau_4) I_m \tag{1}$$

where β_{mh} is the transmission probability from I_m (per bite), η_A is the maturation rate from larvae to adult (per day), τ_5 is the control maturation rate from larvae to adult, B_3 is the average daily biting (per day) for mosquito susceptible, β_{hm} is the transmission probability from I_h (per bite), k is the number of larvae per human, φ is the number of eggs at each deposit per capita (per day), π is the fraction of subpopulation recruited into the population, B_1 is the average daily biting (per day) for careful human susceptible, B_2 is the average daily biting (per day) for careless human susceptible, θ_2 is the Positive change in behaviour of Careless individuals, τ_1 is the campaign of educating Careless human susceptible, μ_h is the average lifespan of humans (per day), a is the per capita disease induced death rate for humans, μ_A is the natural mortality of larvae (per day), τ_2 is the control of vector human contact , τ_3 is the reducing vector breeding areas, η_h mean viremic period (per day), μ_m average lifespan of adult mosquitoes (per day), τ_4 insecticide application, θ_1 portion that moves from compartment R_h to S_{h_1} due to loss of immunity and δ_h treatment parameter.

3. Model Analysis

We study the solutions of System (1) in the closed set $\Omega = \left\{ \begin{array}{l} (S_{h_1}, S_{h_2}, I_h, T_h, R_h, A_m, S_m, I_m) \in \mathbb{R}_+^8 : \\ S_{h_1}, S_{h_2}, I_h, T_h, R_h, A_m, S_m, I_m \geq 0, A_m \leq kN_h, \\ S_m + I_m \leq mN_h, S_{h_1} + S_{h_2} + I_h + T_h + R_h \leq N_h \end{array} \right\}$

The Ω set is positively invariant with respect to Equation (1) [10].

3.1. Disease Free Equilibrium (DFE)

For the disease free equilibrium, it is assumed that there is no infection for both populations of human and mosquitoes i.e. $I_h(t) = 0$ and $I_m(t) = 0$, denoted by ' E_0 '. Thus E_0 of the model system (1) is obtained as

$$E_0 = \left(S_{h_1}(t), S_{h_2}(t), 0, 0, 0, A_m(t), S_m(t), 0\right) = \left(\frac{(1-\pi)N_h\left(\mu_h + (1-\tau_1)\theta_2\right) + (1-\tau_1)\theta_2\pi N_h}{\mu_h + (1-\tau_1)\theta_2}, \right.$$

$$\left. \frac{\pi\mu_h N_h}{\mu_h + (1-\tau_1)\theta_2}, 0, 0, 0, \frac{qkN_h}{\varphi\tau_5\eta_A}, \frac{qkN_h}{\varphi\left(\mu_m + \tau_4\right)}, 0\right)$$

where $q = \left(\varphi\tau_5\eta_A - \left(\mu_A + \tau_3 + \tau_5\eta_A\right)\left(\mu_m + \tau_4\right)\right)$

3.2. The Basic Reproduction Number, R_0

The basic reproduction number, denoted by R_0, is defined as the average number of secondary infections that occurs when one infective individual is introduced into a completely susceptible population [11].

The basic reproduction number of the model (1) R_0 is calculated by using the next generation matrix of an ODE [11]. Using the approach of [11]. R_0 is obtaining by taking the largest (dominant) Eigen value (spectral radius) of

$$\left[\frac{\partial F_i(E_0)}{\partial X_j}\right]\left[\frac{\partial V_i(E_0)}{\partial X_j}\right]^{-1},$$

where, F_i is the rate of appearance of new infection in compartment i, V_i^+ is the transfer of individuals out of the compartment i by all other means and E_0 is the disease free equilibrium.

$$F_i = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \left((1-\tau_2)B_1 S_{h_1} + (1-\tau_2)B_2 S_{h_2}\right)\beta_{mh}\frac{I_m}{N_h} \\ (1-\tau_2)B_3\beta_{hm}\frac{I_h}{N_h}S_m \end{bmatrix}$$

Using the linearization method, the associated matrix at DFE is given by

$$\mathbf{F} = \begin{pmatrix} \frac{\partial F_1}{\partial I_h}(E_0) & \frac{\partial F_1}{\partial I_m}(E_0) \\ \frac{\partial F_2}{\partial I_h}(E_0) & \frac{\partial F_2}{\partial I_m}(E_0) \end{pmatrix}$$

This implies that

$$\mathbf{F} = \begin{pmatrix} 0 & \frac{\left((1-\tau_2)B_1 S_{h_1} + (1-\tau_2)B_2 S_{h_2}\right)\beta_{mh}}{N_h} \\ \frac{(1-\tau_2)B_3\beta_{hm}S_m}{N_h} & 0 \end{pmatrix}$$

With

$$S_{h_1} = \frac{(1-\pi)N_h\left(\mu_h + (1-\tau_1)\theta_2\right) + (1-\tau_1)\theta_2\pi N_h}{\mu_h + (1-\tau_1)\theta_2},$$

$$S_{h_2} = \frac{\pi\mu_h N_h}{\mu_h + (1-\tau_1)\theta_2}, \quad S_m = \frac{kN_h q}{\varphi\left(\mu_m + \tau_4\right)}$$

we have

$$\mathbf{F}=\begin{pmatrix} 0 & \left(\dfrac{(1-\tau_2)B_1(1-\pi)\left(\mu_h+(1-\tau_1)\theta_2\right)+(1-\tau_1)\theta_2\pi}{\mu_h+(1-\tau_1)\theta_2}+(1-\tau_2)B_2\dfrac{\pi\mu_h}{\mu_h+(1-\tau_1)\theta_2}\right)\beta_{mh} \\ \dfrac{(1-\tau_2)B_3\beta_{hm}kq}{\varphi\left(\mu_m+\tau_4\right)} & 0 \end{pmatrix}$$

The transfer of individuals out of the compartment i is given by

$$\mathbf{V}_i=\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} \left(\mu_h+\eta_h+a\right)I_h \\ \left(\mu_m+\tau_4\right)I_m \end{bmatrix}$$

Using the linearization method, the associated matrix at DFE is given by,

$$\mathbf{V}=\begin{pmatrix} \dfrac{\partial V_1}{\partial I_h}(E_0) & \dfrac{\partial V_1}{\partial I_m}(E_0) \\ \dfrac{\partial V_2}{\partial I_h}(E_0) & \dfrac{\partial V_2}{\partial I_m}(E_0) \end{pmatrix}$$

This gives

$$\mathbf{V}=\begin{pmatrix} \mu_h+\eta_h+a & 0 \\ 0 & \mu_m+\tau_4 \end{pmatrix}$$

With

$$\mathbf{V}^{-1}=\begin{pmatrix} \dfrac{1}{\mu_h+\eta_h+a} & 0 \\ 0 & \dfrac{1}{\mu_m+\tau_4} \end{pmatrix}$$

Therefore

$$\mathbf{FV}^{-1}=\begin{pmatrix} 0 & \left(\dfrac{(1-\tau_2)B_1(1-\pi)\left(\mu_h+(1-\tau_1)\theta_2\right)+\theta_2\pi}{\mu_h+(1-\tau_1)\theta_2}+\dfrac{(1-\tau_2)B_2\pi\mu_h}{\mu_h+(1-\tau_1)\theta_2}\right)\beta_{mh} \\ \dfrac{(1-\tau_2)B_3\beta_{hm}kq}{\varphi\left(\mu_m+\tau_4\right)} & 0 \end{pmatrix}\begin{pmatrix} \dfrac{1}{\mu_h+\eta_h+a} & 0 \\ 0 & \dfrac{1}{\mu_m+\tau_4} \end{pmatrix} \quad (2)$$

Then eigenvalues of the equation (2) is given by

$$\det\left(\mathbf{FV}^{-1}-\mathbf{I}\lambda\right)=\det\begin{pmatrix} 0-\lambda & \left(\dfrac{(1-\tau_2)B_1\beta_{mh}(1-\pi)\left(\mu_h+(1-\tau_1)\theta_2\right)+(1-\tau_1)\theta_2\pi\beta_{mh}+(1-\tau_2)B_2\beta_{mh}\pi\mu_h}{\left(\mu_h+(1-\tau_1)\theta_2\right)\mu_m}\right) \\ \dfrac{(1-\tau_2)kqB_3\beta_{hm}}{\varphi\left(\mu_m+\tau_4\right)\left(\mu_h+\eta_h+a\right)} & 0-\lambda \end{pmatrix}$$

This gives

$$\lambda^2=\frac{\sqrt{-kB_3\beta_{hm}\beta_{mh}\left(a+\eta_h+\mu_h\right)BH}}{\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4)}$$

where $B=\varphi\left(\varphi\tau_5\eta_A-\left(\mu_A+\tau_3+\tau_5\eta_A\right)\left(\mu_m+\tau_4\right)\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)$

$$H=\left(\theta_2\left(1-\tau_1\right)\left(\pi+\left(-1+\pi\right)B_1\left(-1+\tau_2\right)\right)+\left(\left(-1+\pi\right)B_1-\pi B_2\right)\mu_h\left(-1+\tau_2\right)\right)\left(-1+\tau_2\right)$$

$$q = -\left(\left(\mu_A + \tau_3 + \tau_5\eta_A\right)\left(\mu_m + \tau_4\right) - \tau_5\eta_A\varphi\right)$$

consequently

$$\lambda = \frac{\sqrt{-kB_3\beta_{hm}\beta_{mh}\left(a+\eta_h+\mu_h\right)BH}}{\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4)} \text{ or } \lambda = -\frac{\sqrt{-kB_3\beta_{hm}\beta_{mh}\left(a+\eta_h+\mu_h\right)BH}}{\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4)}$$

It follows that the Basic Reproductive number which is given by the largest Eigen value for model system (1) denoted by R_0 is given by

$$R_0 = \frac{\sqrt{-kB_3\beta_{hm}\beta_{mh}\left(a+\eta_h+\mu_h\right)BH}}{\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4)} \tag{3}$$

If $R_0 < 1$, the disease cannot invade the population and the infection will die out over a period of time, and also, if $R_0 > 1$, then an invasion is possible and infection can spread through the population. Generally, the larger the value of R_0, the more severe, and possibly widespread the epidemic will be, [10].

3.3. Local Stability of Disease Free Equilibrium Point

To determine the local stability of the disease free equilibrium, the variation matrix $\mathbf{J}_{E_0}$ of the model system (1) corresponding to the disease free E_0 is obtained as

$$\mathbf{J}_{E_0} = \begin{bmatrix} -\mu_h & \left(1-\tau_1\right)\theta_2 & 0 & 0 & \theta_1 & 0 & 0 & f \\ 0 & p & 0 & 0 & 0 & 0 & 0 & y \\ 0 & 0 & w & 0 & 0 & 0 & 0 & j \\ 0 & 0 & \eta_h & -\delta_h-\mu_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_h & -\theta_1-\mu_h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & t & \varphi\left(1-\dfrac{q}{\varphi\eta_A\tau_5}\right) & g \\ 0 & 0 & z & 0 & 0 & \eta_A\tau_5 & -\mu_m-\tau_4 & 0 \\ 0 & 0 & v & 0 & 0 & 0 & 0 & -\mu_m-\tau_4 \end{bmatrix} \tag{4}$$

where $t = -\mu_A - \tau_3 - \eta_A\tau_5 - \dfrac{q\varphi'\left(1-\dfrac{q}{\varphi\eta_A\tau_5}\right)}{\varphi(\mu_m+\tau_4)}$

$$f = -\frac{\left(1-\tau_2\right)B_1\beta_{mh}\left(\left(1-\tau_1\right)\theta_2 - (-1+\pi)\mu_h\right)}{\left(1-\tau_1\right)\theta_2+\mu_h}$$

$$y = -\frac{\left(1-\tau_2\right)\pi B_2\beta_{mh}\mu_h}{\left(1-\tau_2\right)\theta_2+\mu_h}, v = \frac{\left(1-\tau_2\right)kqB_3\beta_{hm}}{\varphi(\mu_m+\tau_4)}$$

$$j = \left(1-\tau_2\right)\beta_{mh}\left(\frac{\pi B_2\mu_h}{\left(1-\tau_2\right)\theta_2+\mu_h} + B_1\left(1-\pi+\frac{\left(1-\tau_1\right)\pi\theta_2}{\left(1-\tau_1\right)\theta_2+\mu_h}\right)\right)$$

$$g = \varphi\left(1-\frac{q}{\varphi\eta_A\tau_5}\right), \quad p = -\left(1-\tau_1\right)\theta_2 - \mu_h$$

$$w = -a - \eta_h - \mu_h \ , z = -\frac{\left(1-\tau_2\right) kqB_3\beta_{hm}}{\varphi(\mu_m + \tau_4)}$$

Therefore the stability of the disease free equilibrium point can be clarified by studying the behaviour of $\mathbf{J}_{E_0}$ in which for local stability of DFE we seek for its all eigenvalues to have negative real parts. It follows that, the characteristic function of the matrix (4) with λ being the eigenvalues of the Jacobian matrix, by using Mathematica software gives the following values:

$$\lambda_1 = -\mu_h, \ \ \lambda_2 = -\left(1-\tau_1\right)\theta_2 - \mu_h$$

The other eigenvalues are given as

$$\lambda_3 = -\frac{1}{2}\left(a + \eta_h + \mu_h + \mu_m + \tau_4 + \frac{1}{d}\sqrt{\sigma} \right),$$

when $\sqrt{\sigma}$ is not a real number,

$$\lambda_4 = -\delta_h - \mu_h, \ \ \lambda_5 = -\theta_1 - \mu_h$$

$$\lambda_6 = -\frac{1}{2}\left(a + \eta_h + \mu_h + \mu_m + \tau_4 - \frac{1}{d}\sqrt{\sigma} \right)$$

when $\sqrt{\sigma}$ is not a real number,

$$\lambda_7 = \frac{1}{2\varphi\left(\left(1-\tau_2\right)\theta_2 + \mu_h\right)\left(\left(1-\tau_1\right)\theta_2 + \mu_h\right)\left(\mu_m + \tau_4\right)}\left(-\alpha - \sqrt{\rho}\right)$$

when $\sqrt{\rho}$ is not a real number, and finally.

$$\lambda_8 = \frac{1}{2\varphi\left(\left(1-\tau_2\right)\theta_2 + \mu_h\right)\left(\left(1-\tau_1\right)\theta_2 + \mu_h\right)\left(\mu_m + \tau_4\right)}\left(-\alpha + \sqrt{\rho}\right)$$

when $\sqrt{\rho}$ is not a real number, where

$$d = \varphi\left(\left(1-\tau_2\right)\theta_2 + \mu_h\right)\left(\left(1-\tau_1\right)\theta_2 + \mu_h\right)\left(\mu_m + \tau_4\right)$$

$$\sigma = \Bigg(\varphi\left(\left(1-\tau_2\right)\theta_2 + \mu_h\right)\left(\left(1-\tau_1\right)\theta_2 + \mu_h\right)\left(\mu_m + \tau_4\right)\Bigg(4\left(1-\tau_2\right)^2$$
$$kqB_1B_3\beta_{hm}\beta_{mh}\left(\left(1-\tau_2\right)\theta_2 + \mu_h\right)\left(\left(1-\tau_1\right)\theta_2 - (-1+\pi)\mu_h\right) +$$
$$\left(\left(1-\tau_1\right)\theta_2 + \mu_h\right)\Bigg(4\left(1-\tau_2\right)^2 k\pi qB_2B_3\beta_{hm}\beta_{mh}\mu_h + \varphi\left(\left(1-\tau_2\right)\right.$$
$$\left.\theta_2 + \mu_h\right)\left(a + \eta_h + \mu_h - \mu_m - \tau_4\right)^2\left(\mu_m + \tau_4\right)\Bigg)\Bigg)\Bigg)$$

$$\rho = \left(\left(1-\tau_2\right)\theta_2 + \mu_h\right)^2\left(\left(1-\tau_1\right)\theta_2 + \mu_h\right)^2\Bigg(\varphi^2\mu_A^2\left(\mu_m + \tau_4\right)^2 +$$
$$\varphi^2\left(\mu_m + \tau_4\right)^2\left(\left(\mu_m - \tau_3 + \tau_4 - \eta_A\tau_5\right)^2 + 4\eta_A\tau_5\varphi\left(1 - \frac{q}{\varphi\eta_A\tau_5}\right)\right) - 2q\varphi$$
$$\left(\mu_m + \tau_4\right)\left(\mu_m - \tau_3 + \tau_4 - \eta_A\tau_5\right)\varphi'\left(1 - \frac{q}{\varphi\eta_A\tau_5}\right) + q^2\varphi'\left(1 - \frac{q}{\varphi\eta_A\tau_5}\right)^2 +$$

$$2\varphi\mu_A\left(\mu_m+\tau_4\right)\left(-\varphi\left(\mu_m+\tau_4\right)\left(\mu_m-\tau_3+\tau_4-\eta_A\tau_5\right)+q\varphi'\left(1-\frac{q}{\varphi\eta_A\tau_5}\right)\right)\Big)$$

$$\alpha=\left(\left(1-\tau_2\right)\theta_2+\mu_h\right)\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)\left(\varphi\left(\mu_m+\tau_4\right)\right.$$

$$\left.\left(\mu_A+\mu_m+\tau_3+\tau_4+\eta_A\tau_5\right)+q\varphi'\left(1-\frac{q}{\varphi\eta_A\tau_5}\right)\right).$$

Hence under certain conditions the system is stable since all the eight eigenvalues are negative. These imply that at $R_0<1$ the Disease Free Equilibrium point is locally asymptotically stable.

3.4. Global Stability of Disease Free Equilibrium Point

In this subsection, the global behaviour of the equilibria for system (1) is analysed. The following theorem provides the global property of the disease free equilibrium E_0 of the system. The results are obtained by means of Lyapunov function. In choosing the Lyapunov function, we adopt the idea of [12].

Theorem1: If $R_0\le 1$, then the infection-free equilibrium is globally asymptotically stable in the interior of Ω.

Proof: To establish the global stability of the disease-free equilibrium, we construct the following Lyapunov function:

$$L(t)=-k\varphi B_3\beta_{hm}(a+\eta_h+\mu_h)BHI_h(t)+\left(\varphi\left(a+\eta_h+\mu_h\right)\right. \tag{5}$$

$$\left.\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\right)^2\left(\mu_m+\tau_4\right)I_m(t)$$

Calculating the time derivative of L along (4), we obtain

$$L'(t)=-k\varphi B_3\beta_{hm}\left(a+\eta_h+\mu_h\right)BHI_h'(t)+\left(\varphi\left(a+\eta_h+\mu_h\right)\right.$$

$$\left.\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\right)^2\left(\mu_m+\tau_4\right)I_m'(t)$$

Then substituting $I_h'(t)\ \&\ I_m'(t)$ from system (1), we get

$$L'(t)=-k\varphi B_3\beta_{hm}\left(a+\eta_h+\mu_h\right)BH\left(\left(\left(1-u_2\right)B_1S_{h_1}+\left(1-u_2\right)B_2S_{h_2}\right)\beta_{mh}\frac{I_m}{N_h}-\left(\mu_h+\eta_h+a\right)I_h\right)+$$

$$\left(\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))\right)^2(\mu_m+\tau_4)\left(\left(1-\tau_2\right)B_3\beta_{hm}\frac{I_h}{N_h}S_m-\left(\mu_m+\tau_4\right)I_m\right)$$

With $R_0=\dfrac{\sqrt{\left(-kB_3\beta_{hm}\beta_{mh}(a+\eta_h+\mu_h)BH\right)}}{\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4)}$

it follows that

$$L'(t)=-\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\right)^2\left(\mu_m+\tau_4\right)^2\ I_m\left(\sqrt{f}R_0+1\right)\left(1-\sqrt{f}R_0\right)$$

$$-\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)(\mu_m+\tau_4)\right)^2\ R_0^2}{\beta_{mh}(a+\eta_h+\mu_h)k\varphi BH}$$

$$\left(k\varphi\left(a+\eta_h+\mu_h\right)BH\left(\mu_h+\eta_h+a\right)I_h+\left(\varphi\left(a+\eta_h+\mu_h\right)\right.\right.$$

$$\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\Big)^2\left(\mu_m+\tau_4\right)\left(1-\tau_2\right)\frac{I_h}{N_h}S_m\Bigg)$$

where $f=\dfrac{\left(\left(1-\tau_2\right)B_1S_{h_1}+\left(1-\tau_2\right)B_2S_{h_2}\right)}{N_h}$

Thus, $L'(t)$ is negative if $R_0<1$ and $L'=0$ if and only if $I_h=I_m=0$ is reduced to the DFE. Consequently, the largest compact invariant set in $\left\{\left(S_{h_1},S_{h_2},I_h,T_h,R_h,A_m,S_m,I_m\right)\ \varepsilon\ \Omega\ ,\ L'=0\right\}$ when $R_0<1$ is the singleton $\{E_0\}$. Hence, by LaSalle's invariance principle, it is implied that $''E_0''$ is globally asymptotically stable in Ω [13]. This completes the proof.

3.5. Existence of Local and Global Asymptotic Stability of Endemic Equilibrium

Since we are dealing with presence of dengue fever disease in human population, we can reduce system (1) to a 4-dimensional system by eliminating T_h, R_h, A_m & S_m respectively, in the feasible region Ω. The values of S_m can be determined by setting $S_m=mN_h-I_m$ to obtain

$$\frac{dS_{h_1}}{dt}=\left(1-\pi\right)\mu_hN_h-\left(1-\tau_2\right)B_1\beta_{mh}\frac{I_m}{N_h}S_{h_1}-\mu_hS_{h_1}+\theta_1R_h+\left(1-\tau_1\right)\theta_2S_{h_2}$$

$$\frac{dS_{h_2}}{dt}=\pi\mu_hN_h-\left(1-\tau_2\right)B_2\beta_{mh}\frac{I_m}{N_h}S_{h_2}-\mu_hS_{h_2}-\left(1-\tau_1\right)\theta_2S_{h_2}$$

$$\frac{dI_h}{dt}=\left(\left(1-\tau_2\right)B_1S_{h_1}+\left(1-\tau_2\right)B_2S_{h_2}\right)\beta_{mh}\frac{I_m}{N_h}-\left(\mu_h+\eta_h+a\right)I_h$$

$$\frac{dI_m}{dt}=\left(1-\tau_2\right)B_3\beta_{hm}\frac{I_h}{N_h}\left(mN_h-I_m\right)-\left(\mu_m+\tau_4\right)I_m \tag{6}$$

3.5.1. The Endemic Equilibrium and Its Stability

Here, we study the existence and stability of the endemic equilibrium points. If $R_0>1$, then the host-vector model system (6) has a unique endemic equilibrium given by

$E^*=\left(S_{h_1}^*,S_{h_2}^*,I_h^*,I_m^*\right)$ in Ω, with

$$S_{h_1}^*=\left(1-\tau_2\right)N_h\beta_{mh}\left(a+\eta_h+\mu_h\right)\Big(B_2\left(2\left(1-\tau_1\right)\theta_2+\mu_h\right)-$$

$$B_1\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)\Big)\left(\mu_m+\tau_4\right)+\left(1-\tau_2\right)^2m+B_3N_h\beta_{hm}\beta_{mh}$$

$$\Big(N_h\mu_h\Big(\left(1-\tau_1\right)\left(2B_2-B_1\right)\theta_2+\left((2-\pi)B_2\right)\mu_h-\left(1-\pi\right)B_1\Big)+$$

$$\left(2B_2-B_1\right)\theta_1\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)R_h^*\Big)+\sqrt{\Big(\left(1-\tau_2\right)^2N_h^2\beta_{mh}^2}$$

$$\Big(4B_1B_2\mu_h\left(a+\eta_h+\mu_h\right)\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)\left(\mu_m+\tau_4\right)$$

$$\Bigg(\frac{\Big(\varphi\left(a+\eta_h+\mu_h\right)\Big(\mu_h+\theta_2\left(1-\tau_1\right)\Big)\left(\mu_m+\tau_4\right)\Big)^2R_0^2}{B_3\beta_{hm}\beta_{mh}kBH}\left(\mu_m+\tau_4\right)$$

$$+\left(1-\tau_2\right)m+B_3\beta_{hm}\left(N_h\mu_h+\theta_1R_h^*\right)\Big)+\Big(B_2\mu_h\left(-\left(1-\tau_2\right)m\pi$$

$$-B_3N_h\beta_{hm}\mu_h+\left(a+\eta_h+\mu_h\right)\left(\mu_m+\tau_4\right)\Big)+B_1\Big(\left(a+\eta_h+\mu_h\right)$$

$$\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)\left(\mu_m+\tau_4\right)-\left(1-\tau_2\right)m-B_3\beta_{hm}\Big(N_h\mu_h\Big(\left(1-\tau_1\right)$$

$$\theta_2+\left(1-\pi\right)\mu_h\Big)+\theta_1\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)R_h^*\Big)\Big)^2\Big)\Big)\Big)/$$

$$\Big(2\left(1-\tau_2\right)^2m+\left(B_2-B_1\right)B_3N_h\beta_{hm}\beta_{mh}\mu_h\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)\Big),$$

$$S_{h_2}^*=\Big(\Big(\left(1-\tau_2\right)N_h\beta_{mh}\left(a+\eta_h+\mu\right)_hB_2\mu_h+$$

$$\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2R_0^2}{B_3\beta_{hm}kBH}$$

$$\left(1-\tau_2\right)N_hB_1\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)\Big)\left(\mu_m+\tau_4\right)+\left(1-\tau_2\right)^2m+$$

$$\Big(N_h\mu_h\Big(B_3N_h\beta_{hm}\beta_{mh}\pi B_2\mu_h+B_1\Big(\left(1-\tau_1\right)\theta_2+(1+\pi)\mu_h\Big)\Big)$$

$$+\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2R_0^2}{kBH\left(a+\eta_h+\mu_h\right)}$$

$$N_hB_1\theta_1\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)R_h^*\Big)+\sqrt{\Big(\left(1-\tau_2\right)^2}$$

$$N_h^2\beta_{mh}^2\Big(4B_1B_2\mu_h\left(a+\eta_h+\mu_h\right)\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)\left(\mu_m+\tau_4\right)$$

$$\Bigg(\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2R_0^2}{B_3\beta_{hm}\beta_{mh}kBH}$$

$$\left(\mu_m+\tau_4\right)+\left(1-\tau_2\right)m+B_3\beta_{hm}\left(N_h\mu_h+\theta_1R_h^*\right)\Big)+$$

$$\Big(B_2\mu_h\left(-\left(1-\tau_2\right)m\pi-B_3N_h\beta_{hm}\mu_h+\left(a+\eta_h+\mu_h\right)\left(\mu_m+\tau_4\right)\Big)+$$

$$B_1\Big(\left(a+\eta_h+\mu_h\right)\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)\left(\mu_m+\tau_4\right)-\left(1-\tau_2\right)m-$$

$$B_3\beta_{hm}\Big(N_h\mu_h\Big(\left(1-\tau_1\right)\theta_2+\left(1-\pi\right)\mu_h\Big)+\theta_1\Big(\left(1-\tau_1\right)\theta_2$$

$$+\mu_h\Big)R_h^*\Big)\Big)\Big)^2\Big)\Big)\Big)/\Big(2\left(1-\tau_2\right)^2m+\left(B_2-B_1\right)$$

$$B_3N_h\beta_{hm}\beta_{mh}\mu_h\Big(\left(1-\tau_1\right)\theta_2+\mu_h\Big)\Big),$$

$$I_h^* = \frac{k\beta_{mh}BH\left(a+\eta_h+\mu_h\right)\left(\mu_m+\tau_4\right)}{\left(\begin{array}{l} k\beta_{mh}BH\left(a+\eta_h+\mu_h\right)\left(1-\tau_2\right)m+\left(\varphi\right. \\ \left.\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2 R_0^2 \end{array}\right)}$$

$$I_m^* = \left(\left(1-\tau_2\right)N_h\beta_{mh}(a+\eta_h+\mu_h)\left(\left(1-\tau_1\right)B_1\theta_2+(B_1+B_2)\mu_h\right)\right.$$

$$(\mu_m+\tau_4)+\left(1-\tau_2\right)^2 m+B_3N_h\beta_{hm}\beta_{mh}(N_h\mu_h(\pi B_2\mu_h+$$

$$B_1\left(\left(1-\tau_1\right)\theta_2+\left(1-\pi\right)\mu_h\right))+B_1\theta_1\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)R_h^*)+$$

$$\sqrt{\left(\left(1-\tau_2\right)^2 N_h^2\beta_{mh}^2\left(4B_1B_2\mu_h\left(a+\eta_h+\mu_h\right)\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)\right.\right.}$$

$$\left(\mu_m+\tau_4\right)\left(\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2 R_0^2}{B_3\beta_{hm}\beta_{mh}kBH}\right.$$

$$\left(\mu_m+\tau_4\right)+\left(1-\tau_2\right)m+B_3\beta_{hm}\left(N_h\mu_h+\theta_1R_h^*\right)\Big)+\Big(B_2\mu_h\left(-\left(1-\tau_2\right)\right.$$

$$m\pi-B_3N_h\beta_{hm}\mu_h+\left(a+\eta_h+\mu_h\right)\left(\mu_m+\tau_4\right)\Big)+B_1\Big(\left(a+\eta_h+\mu_h\right)$$

$$\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)\left(\mu_m+\tau_4\right)-\left(1-\tau_2\right)m-B_3\beta_{hm}\left(N_h\mu_h\left(\left(1-\tau_1\right)\right.\right.$$

$$\left.\left.\left.\left.\theta_2+\left(1-\pi\right)\mu_h\right)+\theta_1\left(\left(1-\tau_1\right)\theta_2+\mu_h\right)R_h^*\right)\right)^2\right)\Big)\Big)\Big)\Big/\left(2\left(1-\tau_2\right)^2 B_1\right.$$

$$\left.B_2\beta_{mh}^2\left(B_3\beta_{hm}\left(N_h\mu_h+\theta_1R_h^*\right)\left(\sqrt{f}R_0+1\right)\left(\sqrt{f}R_0-1\right)+\left(1-\tau_2\right)m\right)\right)$$

But from (3)

$$\left(a+\eta_h+\mu_h\right)=-\frac{(\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4))^2R_0^2}{B_3\beta_{hm}\beta_{mh}kBH}$$

$$B_3\beta_{hm}=-\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2R_0^2}{k\beta_{mh}BH\left(a+\eta_h+\mu_h\right)}$$

$$\beta_{mh}\left(a+\eta_h+\mu_h\right)=\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2R_0^2}{B_3\beta_{hm}kBH}$$

where

$$f=\frac{(\mu_m+\tau_4)(\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4))^2}{B_3\beta_{hm}\beta_{mh}kBHB_3\beta_1(N_h\mu_h+\theta_1(R_h)^*)}$$

3.5.2. Local Stability of the Endemic Equilibrium

In order to analyse the stability of the endemic equilibrium, the additive compound matrices approach is used, using the idea of [14]. Local stability of the endemic equilibrium point is determined by the variational matrix $\mathbf{J}(E^*)$ of the nonlinear system (6) corresponding to E^* to get

$$\mathbf{J}\left(E^*\right)=\begin{pmatrix} -\mu_h-\dfrac{B_1\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h} & \theta_2\left(1-\tau_1\right) & 0 & H \\ 0 & A & 0 & G \\ \dfrac{B_1\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h} & \dfrac{B_2\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h} & -a-\eta_h-\mu_h & B \\ 0 & 0 & C & F \end{pmatrix} \tag{7}$$

From (7) the second additive compound matrix is given by

$$\mathbf{J}^{[2]}(E^*)=\begin{pmatrix} a_{11} & 0 & a_{13} & 0 & \dfrac{B_1\left(1-\tau_2\right)\beta_{mh}S_{h_1}^*}{N_h} & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & a_{26} \\ 0 & a_{32} & a_{33} & 0 & \left(1-\tau_1\right)\theta_2 & 0 \\ a_{41} & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & a_{54} & a_{55} & 0 \\ 0 & 0 & a_{63} & 0 & \dfrac{B_2\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h} & a_{66} \end{pmatrix} \text{ in} \tag{9}$$

where $A=-\mu_h-\theta_2\left(1-\tau_1\right)-\dfrac{B_2\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h}$ $\quad B=\dfrac{\beta_{mh}\left(B_1\left(1-\tau_2\right)S_{h_1}^*+B_2\left(1-\tau_2\right)S_{h_2}^*\right)}{N_h}$

$C=mB_3\beta_{hm}\left(1-\tau_2\right)-\dfrac{B_3\beta_{hm}\left(1-\tau_2\right)I_m^*}{N_h}$ $\quad F=-\mu_m-\tau_4-\dfrac{B_3\beta_{hm}\left(1-\tau_2\right)I_h^*}{N_h}$

$G=-\dfrac{B_2\beta_{mh}\left(1-\tau_2\right)S_{h_2}^*}{N_h},$ $\quad H=-\dfrac{B_1\beta_{mh}\left(1-\tau_2\right)S_{h_1}^*}{N_h}$

$a_{11}=-2\mu_h-\dfrac{B_1\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}-\left(1-\tau_1\right)\theta_2-\dfrac{B_2\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}$

$a_{22}=-\dfrac{B_1\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}-a-\eta_h-2\mu_h$ $\quad a_{23}=\dfrac{\beta_{mh}\left(B_1\left(1-\tau_2\right)S_{h_1}^*+B_2\left(1-\tau_2\right)S_{h_2}^*\right)}{N_h},$

$a_{21}=\dfrac{B_2\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}$ $\quad a_{32}=mB_3\left(1-\tau_2\right)\beta_{hm}-\dfrac{B_3\left(1-\tau_2\right)\beta_{hm}I_m^*}{N_h},$

$a_{41}=-\dfrac{B_1\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}$ $\quad a_{33}=-\dfrac{B_1\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}-\left(\tau_4+\mu_h+\mu_m\right)-\dfrac{B_3\left(1-\tau_2\right)\beta_{hm}I_h^*}{N_h}$

$a_{44}=-2\mu_h-\left(1-\tau_1\right)\theta_2-\dfrac{B_2\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}-a-\eta_h$ $\quad a_{45}=\dfrac{\beta_{mh}\left(B_1\left(1-\tau_2\right)S_{h_1}^*+B_2\left(1-\tau_2\right)S_{h_2}^*\right)}{N_h}$

$a_{55}=-\mu_h-\theta_2\left(1-\tau_1\right)-\dfrac{B_2\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h}-\mu_m-\tau_4-\dfrac{B_3\beta_{hm}\left(1-\tau_2\right)I_h^*}{N_h}$

$$a_{54} = mB_3\left(1-\tau_2\right)\beta_{hm} - \frac{B_3\left(1-\tau_2\right)\beta_{hm}I_m^*}{N_h} \qquad a_{66} = -a-\eta_h-\mu_h-\mu_m-\tau_4 - \frac{B_3\beta_{hm}\left(1-\tau_2\right)I_h^*}{N_h}$$

$$a_{13} = -\frac{B_2\left(1-\tau_2\right)\beta_{mh}S_{h_2}^*}{N_h}, \qquad a_{63} = \frac{B_1\left(1-\tau_2\right)\beta_{mh}I_m^*}{N_h}$$

$$a_{26} = \frac{B_1\left(1-\tau_2\right)\beta_{mh}S_{h_1}^*}{N_h} \qquad a_{46} = \frac{B_2\left(1-\tau_2\right)\beta_{mh}S_{h_2}^*}{N_h}, a_{24} = \left(1-\tau_1\right)\theta_2$$

The following lemma is stated and proved by [15] to demonstrate the local stability of endemic equilibrium point E^*.

Lemma 1: Let $\mathbf{J}\left(E^*\right)$ be a 4×4 real matrix.

If $tr\left(\mathbf{J}\left(E^*\right)\right)$, $\det\left(\mathbf{J}\left(E^*\right)\right)$ and $\det\left(\mathbf{J}\left(E^*\right)^{[2]}\right)$ are all negative, then all eigenvalues of $\mathbf{J}\left(E^*\right)$ have negative real parts.

Using the above Lemma, we will study the stability of the endemic equilibrium.

Theorem 2: If $R_0 > 1$, the endemic equilibrium E^* of the model (1) is locally asymptotically stable in Ω.

Proof: From the Jacobian matrix $\mathbf{J}\left(E^*\right)$ in (7), we have

$$tr\left(\mathbf{J}\left(E^*\right)\right) = -\frac{B_1\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h} - 3\mu_h - \theta_2\left(1-\tau_1\right) - \frac{B_2\beta_{mh}\left(1-\tau_2\right)I_m^*}{N_h} \quad -a-\eta_h-\mu_m-\tau_4 - \frac{B_3\beta_{hm}\left(1-\tau_2\right)I_h^*}{N_h} < 0.$$

$$\begin{aligned}\operatorname{Det}\left(\mathbf{J}\left(E^*\right)\right) = -\frac{1}{N_h^3}\Bigg(&\frac{\left(\varphi\left(a+\eta_h+\mu_h\right)\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)\left(\mu_m+\tau_4\right)\right)^2}{kB_3\beta_{hm}\beta_{mh}BH}\\ &R_0^2\left(N_h\left(\mu_m+\tau_4\right)+B_3\beta_{hm}\left(1-\tau_2\right)I_h^*\right)\left(N_h\mu_h+B_1\beta_{mh}\left(1-\tau_2\right)I_m^*\right)\\ &\left(N_h\left(\mu_h+\theta_2\left(1-\tau_1\right)\right)+B_2\beta_{mh}\left(1-\tau_2\right)I_m^*\right)+B_3\beta_{hm}\beta_{mh}\mu_h\left(-1+\tau_2\right)^2\\ &\left(mN_h-I_m^*\right)\left(B_2N_h(\mu_h+\theta_2\left(1-\tau_1\right)\right)S_{h_2}^*+B_1\left(N_h\left(\theta_2\left(1-\tau_1\right)+\right.\right.\\ &\left.\left.\mu_h\right)S_{h_1}^*+B_2\beta_{mh}\left(1-\tau_2\right)I_m^*\left(S_{h_1}^*+S_{h_2}^*\right)\right)\Bigg)\Bigg)<0\end{aligned}$$

$$\begin{aligned}\det\left(J_M^{[2]}\left(E^*\right)\right) = -\frac{1}{N_h^6}\Bigg(&\left(1-\tau_2\right)^2B_3\beta_{hm}\beta_{mh}\left(mN_h-I_m^*\right)\\ &\left(\left(1-\tau_2\right)^2gR_0^2\left(mN_h-I_m^*\right)F+\left(N_h\left(a+\eta_h+2\mu_h\right)+\left(1-\tau_2\right)\right.\right.\\ &B_1\beta_{mh}I_m^*\Bigg)\Bigg(J\Bigg(V+\Bigg(-\left(1-\tau_2\right)B_1\beta_{mh}I_m^*B_1\left(N_h\left(\mu_h+\mu_m+\tau_4\right)+\right.\\ &\left(1-\tau_2\right)B_3\beta_1I_h^*)S_{h_1}^*+gR_0^2AI_m^*\left(1-\tau_2\right)B_1^3\beta_{mh}I_m^*S_{h_1}^*+\left(1-\tau_2\right)\\ &\left.\left.B_1\beta_{mh}I_m^*\left(1-\tau_1\right)B_2N_h\theta_2S_{h2}^*\right)\Bigg)\Bigg)+\left(N_h\left(\left(1-\tau_1\right)\theta_2+\mu_h+\mu_m+\tau_4\right)\right.\end{aligned}$$

$$+\left(1-\tau_2\right)\left(B_3\beta_{hm}I_h^*+B_2\beta_{mh}I_m^*\right)\Big)\Big(Y\Big(\left(1-\tau_2\right)^3\left(1-\tau_1\right)B_1B_2B_3N_h$$

$$\beta_{hm}\beta_{mh}^2\theta_2I_m^*\left(mN_h-I_m^*\right)S_{h2}^*+\Big(N_h\left(a+\eta_h+\left(1-\tau_1\right)\theta_2+2\mu_h\right)$$

$$+\left(1-\tau_2\right)B_2\beta_{mh}I_m^*\Big)\Big(\Big(N_h\left(a+\eta_h+2\mu_h\right)\left(\sqrt{t}R_0+1\right)\left(\sqrt{t}R_0-1\right)\Big)$$

$$\Big(N_h\left(\left(1-\tau_1\right)\theta_2+2\mu_h\right)+\left(1-\tau_2\right)\left(B_1+B_2\right)\beta_{mh}I_m^*\Big)\Big(N_h\left(\mu_h+\mu_m\right.$$

$$\left.+\tau_4\right)+\left(1-\tau_2\right)\left(B_3\beta_{hm}I_h^*+B_1\beta_{mh}I_m^*\right)\Big)+\left(1-\tau_2\right)^2B_3\beta_{hm}\beta_{mh}$$

$$\left(mN_h-I_m^*\right)\Big(gR_0^2\left(1-\tau_2\right)B_2^2I_m^*S_{h2}^*+\Big(N_h\left(\left(1-\tau_1\right)\theta_2+2\mu_h\right)$$

$$+\left(1-\tau_2\right)\left(B_1+B_2\right)\beta_{mh}I_m^*\Big)\left(B_1S_{h1}^*+B_2S_{h2}^*\right)\Big)\Big)\Big)\Big)<0$$

But $\beta_{mh}=\dfrac{(\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4))^2}{k(a+\eta_h+\mu_h)B_3\beta_{hm}BH}$ from (3)

Let $g=\dfrac{(\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4))^2}{k(a+\eta_h+\mu_h)B_3\beta_{hm}BH}$

It follows that $\beta_{mh}=-gR_0^2$ where

$$t=\frac{AB_1I_m^*(\varphi(a+\eta_h+\mu_h)(\mu_h+\theta_2(1-\tau_1))(\mu_m+\tau_4))^2}{N_h(a+\eta_h+2\mu_h)k(a+\eta_h+\mu_h)B_3\beta_{hm}BH}$$

$$F=\Big(B_1\left(N_h\left(a+\eta_h+\mu_h+\mu_m+\tau_4\right)+\left(1-\tau_2\right)B_3\beta_{hm}I_h^*\right)S_{h1}^*+$$

$$\left(1-\tau_2\right)B_1^2\beta_{mh}I_m^*S_{h1}^*+B_2\left(N_h\left(a+\eta_h+\mu_h+\mu_m+\tau_4\right)+\left(1-\tau_2\right)\right.$$

$$\left.\left(B_3\beta_{hm}I_h^*+B_2\beta_{mh}I_m^*\right)\right)S_{h2}^*\Big)\Big(B_2N_h\left(\left(1-\tau_1\right)\theta_2+2\mu_h\right)S_{h2}^*+$$

$$B_1\left(N_h\left(\left(1-\tau_1\right)\theta_2+2\mu_h\right)S_{h1}^*+\left(1-\tau_2\right)B_2\beta_{mh}I_m^*\left(S_{h1}^*+S_{h2}^*\right)\right)\Big)$$

$$J=\left(1-\tau_2\right)B_2\beta_{mh}I_m^*\Big(N_h\left(\left(1-\tau_1\right)\theta_2+2\mu_h\right)+\left(1-\tau_2\right)\left(B_1+B_2\right)$$

$$\beta_{mh}I_m^*\Big)\Big(B_2\left(N_h\left(\mu_h+\mu_m+\tau_4\right)+\left(1-\tau_2\right)B_3\beta_{hm}I_h^*\right)+B_1\left(\left(1-\tau_1\right)\right.$$

$$\left.N_h\theta_2+\left(1-\tau_2\right)B_2\beta_2I_m^*\right)\Big)S_{h2}^*+\left(N_h\left(a+\eta_h+\mu_h+\mu_m+\tau_4\right)+\left(1-\tau_2\right)B_3\beta_{hm}I_h^*\right)$$

$$V=\Big(N_h\left(\left(1-\tau_1\right)\theta_2+2\mu_h\right)+\left(1-\tau_2\right)\left(B_1+B_2\right)\beta_{mh}I_m^*\Big)\Big(N_h\left(\mu_h+\right.$$

$$\left.\mu_m+\tau_4\right)+\left(1-\tau_2\right)\left(B_3\beta_{hm}I_h^*+B_1\beta_{mh}I_m^*\right)\Big)\left(B_1S_{h1}^*+B_2S_{h2}^*\right)$$

$$Y=\left(1-\tau_2\right)^3B_1B_3\beta_{hm}\beta_{mh}^2\left(mN_h-I_m^*\right)I_m^*\left(N_h\left(\left(1-\tau_1\right)\right.\right.$$

$$\theta_2 + 2\mu_h\big) + \big(1-\tau_2\big)\big(B_1 + B_2\big)\beta_{mh}I_m^*\Big)\Big(B_1\Big(N_h\big(a+\eta_h +$$

$$\big(1-\tau_1\big)\theta_2 + 2\mu_h\big) + \big(1-\tau_2\big)B_2\beta_{mh}I_m^*)S_{h_1}^* + \big(1-\tau_1\big)B_2N_h$$

$$\theta_2 S_{h2}^*\Big) + \Big(N_h\big(a+\eta_h+\mu_h+\mu_m+\tau_4\big) + \big(1-\tau_2\big)B_3\beta_{hm}I_h^*\Big)$$

Thus, from the lemma 1, the endemic equilibrium E^* of the model system (7) is locally asymptotically stable in Ω.

3.6. Global Stability of Endemic Equilibrium Point (EEP)

Theorem 3: If $R_0 > 1$ the endemic equilibrium E^* of the model system (1) is globally asymptotically stable.

Proof: To establish the global stability of endemic equilibrium E^* we construct the following positive Lyapunov function V as follows;

$$V\left(S_{h_1}^*, S_{h_2}^*, I_h^*, T_h^*, R_h^*, A_m^*, S_m^* \;\&\; \mathrm{I}_m^*\right) = \left(S_{h_1} - S_{h_1}^* - S_{h_1}^* \log\frac{S_{h_1}^*}{S_{h_1}}\right) + \left(S_{h_2} - S_{h_2}^* - S_{h_2}^* \log\frac{S_{h_2}^*}{S_{h_2}}\right) +$$

$$\left(I_h - I_h^* - I_h^* \log\frac{I_h^*}{I_h}\right) + \left(T_h - T_h^* - T_h^* \log\frac{T_h^*}{T_h}\right) + \left(R_h - R_h^* - R_h^* \log\frac{R_h^*}{R_h}\right) +$$

$$\left(A_m - A_m^* - A_m^* \log\frac{A_m^*}{A_m}\right) + \left(S_m - S_m^* - S_m^* \log\frac{S_m^*}{S_m}\right) + \left(I_m - I_m^* - I_m^* \log\frac{I_m^*}{I_m}\right)$$

Direct calculation of the derivative of V along the solutions of (1) gives,

$$\frac{dV}{dt} = \left(\frac{S_{h_1} - S_{h_1}^*}{S_{h_1}}\right)\frac{dS_{h_1}}{dt} + \left(\frac{S_{h_2} - S_{h_2}^*}{S_{h_2}}\right)\frac{dS_{h_2}}{dt} + \left(\frac{I_h - I_h^*}{I_h}\right)\frac{dI_h}{dt}$$

$$+ \left(\frac{T_h - T_h^*}{T_h}\right)\frac{dT_h}{dt} + \left(\frac{R_h - R_h^*}{R_h}\right)\frac{dR_h}{dt} + \left(\frac{A_m - A_m^*}{A_m}\right)\frac{dA_m}{dt}$$

$$+ \left(\frac{S_m - S_m^*}{S_m}\right)\frac{dS_m}{dt} + \left(\frac{I_m - I_m^*}{I_m}\right)\frac{dI_m}{dt}$$

$$\frac{dV}{dt} = X - Y \tag{8}$$

where

$$X = \mu_h N_h + \theta_1 R_h + \pi\mu_h N_h \frac{S_{h_1}^*}{S_{h_1}} + \theta_1 R_h^* \frac{S_{h_1}^*}{S_{h_1}} + \theta_2 \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_1}}$$

$$+ \frac{B_1\beta_{mh}I_m^*}{N_h}\frac{\left(S_{h_1} - S_{h_1}^*\right)^2}{S_{h_1}} + \tau_1\theta_2 \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} + \frac{\tau_2 B_1\beta_{mh}I_m}{N_h}\frac{\left(S_{h_1} - S_{h_1}^*\right)^2}{S_{h_1}} + \pi\mu_h N_h$$

$$+ \frac{B_2\beta_{mh}I_m^*}{N_h}\frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} + \frac{\tau_2 B_2\beta_{mh}I_m}{N_h}\frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} + B_1 S_{h_1}\frac{\beta_{mh}I_m}{N_h} + \tau_2 B_2 S_{h_2}\frac{\beta_{mh}I_m^*}{N_h} +$$

$$\tau_2 B_2 S_{h_2}^* \frac{\beta_{mh} I_m}{N_h} + B_1 S_{h_1} \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} + B_1 S_{h_1}^* \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} + B_1 S_{h_1}^* \frac{\beta_{mh} I_m^*}{N_h}$$

$$+\tau_2 B_1 S_{h_1} \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} + \tau_2 B_1 S_{h_1} \frac{\beta_{mh} I_m^*}{N_h} + \tau_2 B_1 S_{h_1}^* \frac{\beta_{mh} I_m}{N_h} +$$

$$\tau_2 B_1 S_{h_1}^* \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} + B_2 S_{h_2} \frac{\beta_{mh} I_m}{N_h} + B_2 S_{h_2}^* \frac{\beta_{mh} I_m^*}{N_h} + B_2 S_{h_2} \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h}$$

$$+B_2 S_{h_2}^* \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} + \tau_2 B_2 S_{h_2} \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} + \tau_2 B_2 S_{h_2}^* \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} + \eta_h I_h +$$

$$\eta_h I_h^* \frac{T_h^*}{T_h} + \delta_h T_h + \delta_h T_h^* \frac{R_h^*}{R_h} + \varphi S_m + \varphi I_m + \varphi S_m^* \frac{A_m^*}{A_m} + \varphi I_m^* \frac{A_m^*}{A_m}$$

$$+\varphi S_m^* \frac{\left(A_m - A_m^*\right)^2}{kN_h A_m} + \varphi I_m^* \frac{\left(A_m - A_m^*\right)^2}{kN_h A_m} + \tau_5 \eta_A A_m + \tau_5 \eta_A A_m^* \frac{S_m^*}{S_m}$$

$$+\frac{B_3 \beta_{hm} I_h^*}{N_h} \frac{\left(S_m - S_m^*\right)^2}{S_m} + \frac{\tau_2 B_3 \beta_{hm} I_h}{N_h} \frac{\left(S_m - S_m^*\right)^2}{S_m} + S_m B_3 \beta_{hm} \frac{I_h}{N_h} +$$

$$\tau_2 S_m^* B_3 \beta_{hm} \frac{I_h^*}{N_h} \frac{I_m^*}{I_m} + S_m B_3 \beta_{hm} \frac{I_h^*}{N_h} \frac{I_m^*}{I_m} + S_m^* B_3 \beta_{hm} \frac{I_h}{N_h} \frac{I_m^*}{I_m} + S_m^* B_3 \beta_{hm} \frac{I_h^*}{N_h}$$

$$+\tau_2 S_m B_3 \beta_{hm} \frac{I_h}{N_h} \frac{I_m^*}{I_m} + \tau_2 S_m B_3 \beta_{hm} \frac{I_h^*}{N_h} + \tau_2 S_m^* B_3 \beta_{hm} \frac{I_h}{N_h}$$

$$Y = -\pi\mu_h N_h - \mu_h N_h \frac{S_{h_1}^*}{S_{h_1}} - \frac{B_1 \beta_{mh} I_m}{N_h} \frac{\left(S_{h_1} - S_{h_1}^*\right)^2}{S_{h_1}} -$$

$$\frac{\tau_2 B_1 \beta_{mh} I_m^*}{N_h} \frac{\left(S_{h_1} - S_{h_1}^*\right)^2}{S_{h_1}} - \mu_h \frac{\left(S_{h_1} - S_{h_1}^*\right)^2}{S_{h_1}} - \theta_1 R_h^* - \theta_1 R_h \frac{S_{h_1}^*}{S_{h_1}} -$$

$$\tau_1 \theta_2 \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} - \frac{\tau_2 B_2 \beta_{mh} I_m^*}{N_h} \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} - \pi\mu_h N_h \frac{S_{h_2}^*}{S_{h_2}} -$$

$$\frac{B_2 \beta_{mh} I_m}{N_h} \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} - \mu_h \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}} - \theta_2 \frac{\left(S_{h_2} - S_{h_2}^*\right)^2}{S_{h_2}}$$

$$-B_1 S_{h_1} \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} - \tau_2 B_2 S_{h_2} \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} - \tau_2 B_2 S_{h_2}^* \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} -$$

$$B_1 S_{h_1} \frac{\beta_{mh} I_m^*}{N_h} - B_1 S_{h_1}^* \frac{\beta_{mh} I_m}{N_h} - B_1 S_{h_1}^* \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} - \tau_2 B_1 S_{h_1} \frac{\beta_{mh} I_m}{N_h}$$

$$-\tau_2 B_1 S_{h_1} \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} - \tau_2 B_1 S_{h_1}^* \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h} - \tau_2 B_1 S_{h_1}^* \frac{\beta_{mh} I_m^*}{N_h} - B_2 S_{h_2} \frac{\beta_{mh} I_m}{N_h} \frac{I_h^*}{I_h}$$

$$-B_2 S_{h_2} \frac{\beta_{mh} I_m^*}{N_h} - B_2 S_{h_2}^* \frac{\beta_{mh} I_m}{N_h} - B_2 S_{h_2}^* \frac{\beta_{mh} I_m^*}{N_h} \frac{I_h^*}{I_h} - \tau_2 B_2 S_{h_2} \frac{\beta_{mh} I_m}{N_h} -$$

$$\tau_2 B_2 S_{h_2}^* \frac{\beta_{mh} I_m^*}{N_h} - \left(\mu_h + \eta_h + a\right) \frac{\left(I_h - I_h^*\right)^2}{I_h} - \eta_h I_h^* - \eta_h I_h \frac{T_h^*}{T_h} - \left(\mu_h + \delta_h\right) \frac{\left(T_h - T_h^*\right)^2}{T_h}$$

$$-\delta_h T_h^* - \delta_h T_h \frac{R_h^*}{R_h} - \left(\mu_h + \theta_1\right) \frac{\left(R_h - R_h^*\right)^2}{R_h} - \varphi S_m \frac{A_m^*}{A_m} - \varphi S_m^* - \varphi S_m \frac{\left(A_m - A_m^*\right)^2}{k N_h A_m}$$

$$-\varphi S_m \frac{\left(A_m - A_m^*\right)^2}{k N_h A_m} - \varphi I_m \frac{A_m^*}{A_m} - \varphi I_m^* - \varphi I_m \frac{\left(A_m - A_m^*\right)^2}{k N_h A_m} - \left(\mu_A + \tau_5 \eta_A + \tau_3\right) \frac{\left(A_m - A_m^*\right)^2}{A_m}$$

$$-\tau_5 \eta_A A_m^* - \tau_5 \eta_A A_m \frac{S_m^*}{S_m} - \frac{B_3 \beta_{hm} I_h}{N_h} \frac{\left(S_m - S_m^*\right)^2}{S_m} - \frac{\tau_2 B_3 \beta_{hm} I_h^*}{N_h} \frac{\left(S_m - S_m^*\right)^2}{S_m}$$

$$-\left(\tau_4 + \mu_m\right) \frac{\left(S_m - S_m^*\right)^2}{S_m} - S_m B_3 \beta_{hm} \frac{I_h}{N_h} \frac{I_m^*}{I_m} - S_m B_3 \beta_{hm} \frac{I_h^*}{N_h} - S_m^* B_3 \beta_{hm} \frac{I_h}{N_h}$$

$$-S_m^* B_3 \beta_{hm} \frac{I_h^*}{N_h} \frac{I_m^*}{I_m} - \tau_2 S_m B_3 \beta_{hm} \frac{I_h}{N_h} - \tau_2 S_m B_3 \beta_{hm} \frac{I_h^*}{N_h} \frac{I_m^*}{I_m} -$$

$$\tau_2 S_m^* B_3 \beta_{hm} \frac{I_h}{N_h} \frac{I_m^*}{I_m} - \tau_2 S_m^* B_3 \beta_{hm} \frac{I_h^*}{N_h} - \left(\mu_m + \tau_4\right) \frac{\left(I_m - I_m^*\right)^2}{I_m}$$

Thus from equation (8), if $\mathrm{X} < \mathrm{Y}$. Then $\frac{dV}{dt}$ will be negative definite, implying that $\frac{dV}{dt} < 0$. It then follows that $\frac{dV}{dt} = 0$ if and only if $S_{h_1} = S_{h_1}^*, S_{h_2} = S_{h_2}^*, I_h = I_h^*, \ T_h = T_h^*, R_h = R_h^*, A_m = A_m^*, \ S_m = S_m^*$ and $I_m = I_m^*$. Therefore the largest compact invariant set in $\left\{S_{h_1}^*, S_{h_2}^*, I_h^*, T_h^*, R_h^*, A_m^*, S_m^*, I_m^* \in \Omega : \frac{dV}{dt} = 0\right\}$ is the singleton $\left\{E^*\right\}$ where E^* is the endemic equilibrium of the model system (1). By LaSalle's invariant principle, then it implies that E^* is globally asymptotically stable in Ω if $\mathrm{X} < \mathrm{Y}$. This completes the proof.

4. Numerical Simulations

Here, we illustrate the analytical results of the study by carrying out numerical simulations of the model system (1). Parameter values are obtained from the different literatures like (http://www.wavuti.com/2014/05/wizara-ya-afya- kitengo-cha.html), [9], [10] and [16]. Other parameter values are estimated to vary within realistic means and given as shown below.

$\beta_{hm} = 0.375$, $\beta_{mh} = 0.45$, $\pi = 0.96$, $B_1 = 0.5$, $B_2 = 0.9$, $B_3 = 0.7$, $\mu_m = \frac{1}{11}$, $k = 3$, $\eta_A = 0.35$, $\mu_A = 0.25$,

$\mu_h = \frac{1}{78 \times 365}$, $\eta_h = 1/3$, $\varphi = 5$, $\theta_2 = 0.6$, $a = 0.001$, $\tau_1 = 0.4$, $\tau_2 = 0.2$, $\tau_3 = 0.55$, $\tau_4 = 0.13$, $\tau_5 = 0.3$,

$\delta_h = 0.98$ and $\theta_1 = 0.01$ (9).

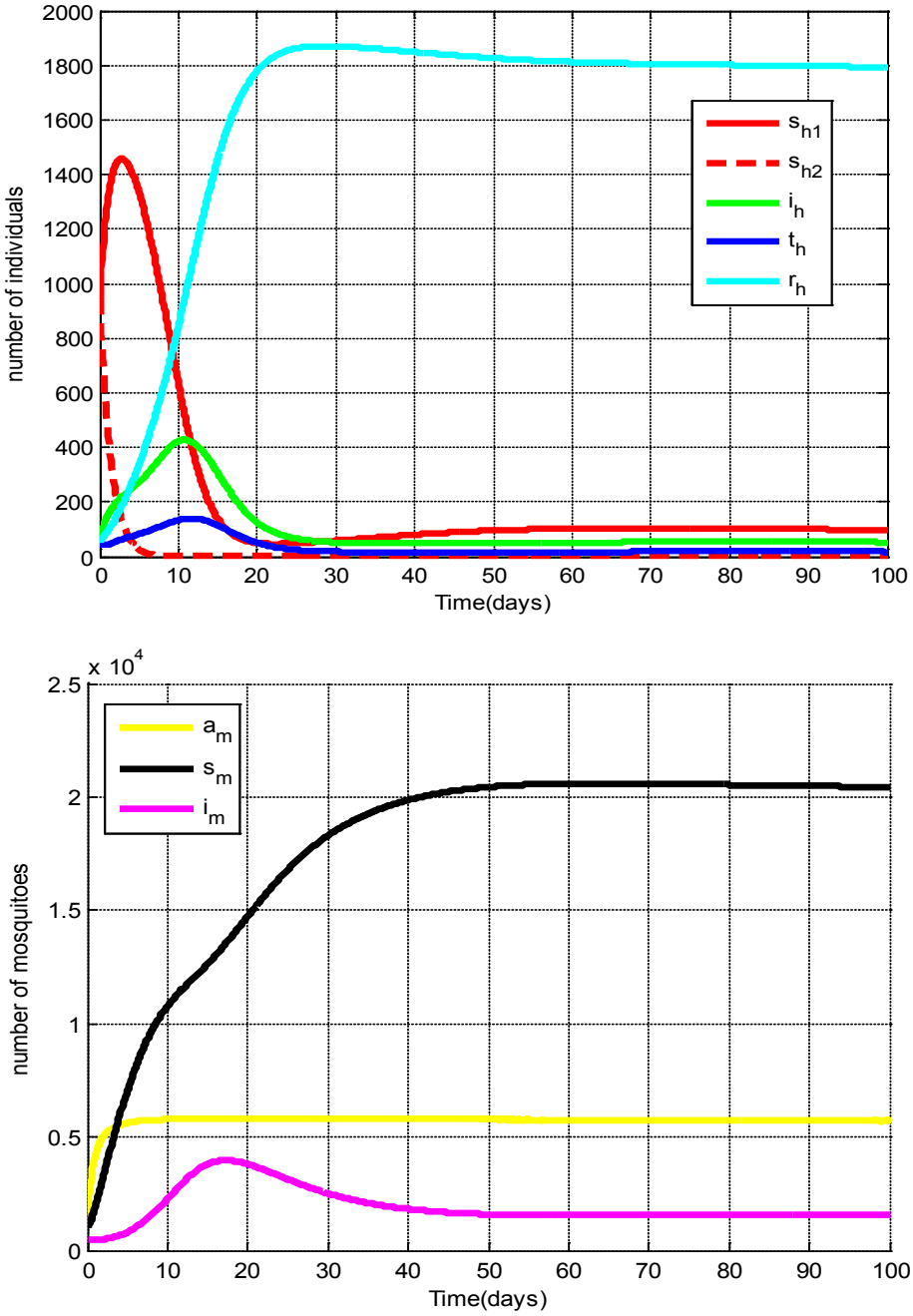


Figure 2. Distribution of population with time in all classes of human and mosquito when no control is applied i.e. $\tau_1 = 0$, $\tau_2 = 0$, $\tau_3 = 0$, $\tau_4 = 0$, $\tau_5 = 0$, $\delta_h = 1$

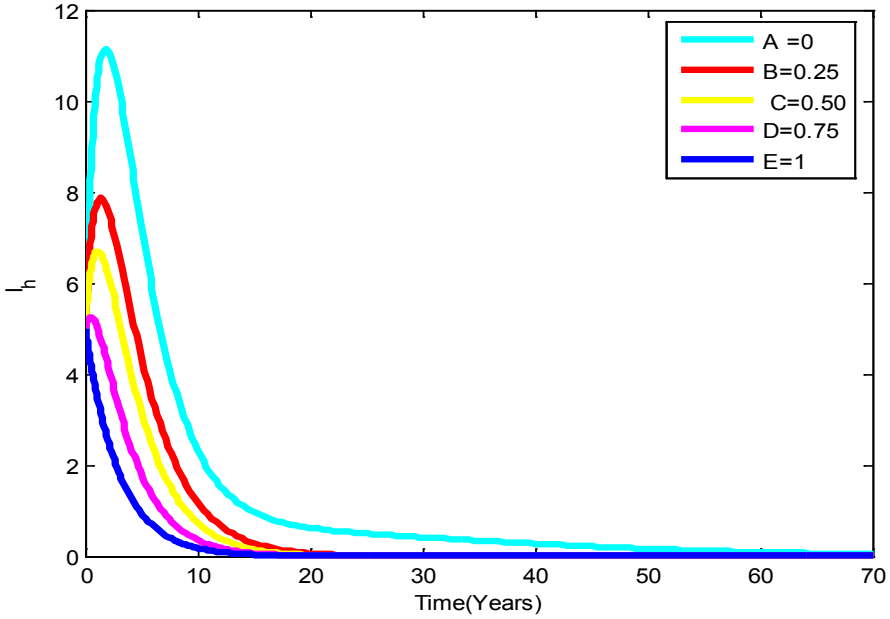
Figures 2 show the distribution of population with time in all classes of human and mosquito when no control is applied.

Figures 2 show the human and mosquito populations in the absence of any control. The human infection reaches a peak between the 2th and the 20th day. The infection of the mosquitoes reaches a peak between the 10th and the 30th day. The total number of infected humans obtained from System (1) is higher than observations in Tanzania. The difference is due to the absence of the data in the whole country of Tanzania [17].

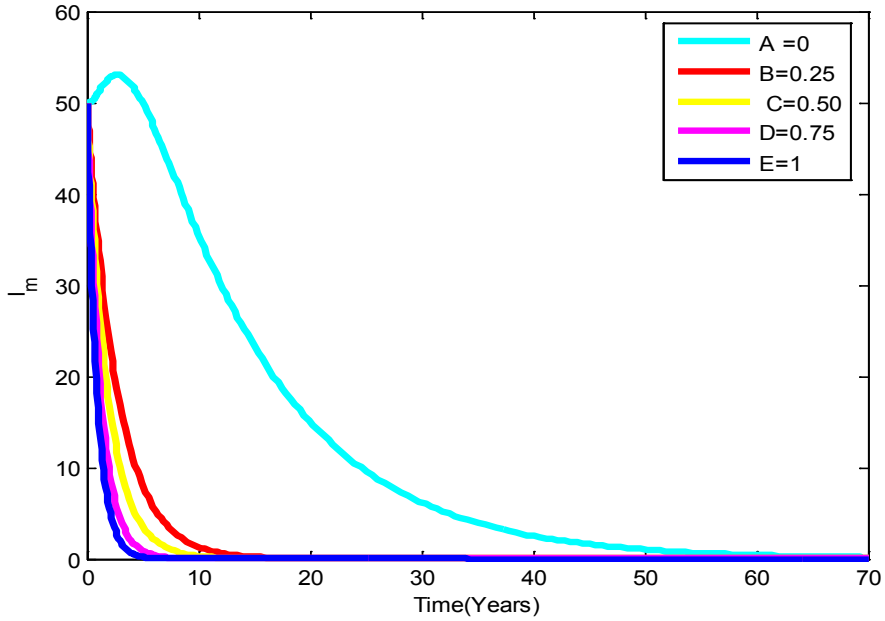
Figures 3 (i)-(ii) show the variation of infected human and mosquito populations with combine use of all five controls as shown: $A = \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0$, $B = \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0.25$,

$C = \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0.50$, $D = \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0.75$,

$$E = \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 1$$



(i)



(ii)

Figure 3. (i)-(ii): Variation of infected human and mosquito populations with combine use of all five controls

From figure 3 (i)-(ii), it is observed that when all the controls are used, the disease is eradicated.

5. Formulation of the Model with Vaccination

In this section, we develop a deterministic model that describes the dynamics of Dengue fever under application of Vaccination and treatment for humans where S_{h_1} is the careful human susceptible population, S_{h_2} is the careless human susceptible population, V_h is the vaccinated human population, I_h is the infected human population, T_h is the treated

human population and R_h is the recovered human population, ϑ is the fraction of the vaccinated careful human susceptible, χ is the proportional rate at which vaccinated careful human susceptible loses effect, ω is the reaction of the vaccinated careless human susceptible, ρ is the proportional rate at which vaccinated careless human susceptible loses effect, σ is the proportion of the vaccinated new born, ϕ is the infection rate of vaccinated careful human susceptible and ε is the infection rate of vaccinated careless human susceptible. Susceptible individuals acquire Dengue fever through the bite of female Aedes mosquito with force of infections given by $B_1\beta_{mh}\frac{I_m}{N_h}S_{h_1}$, and $B_2\beta_{mh}\frac{I_m}{N_h}S_{h_2}$ where $B_2>B_1$.

Considering the above clarification, we then have the following schematic model flow diagram for dengue fever disease with vaccination:

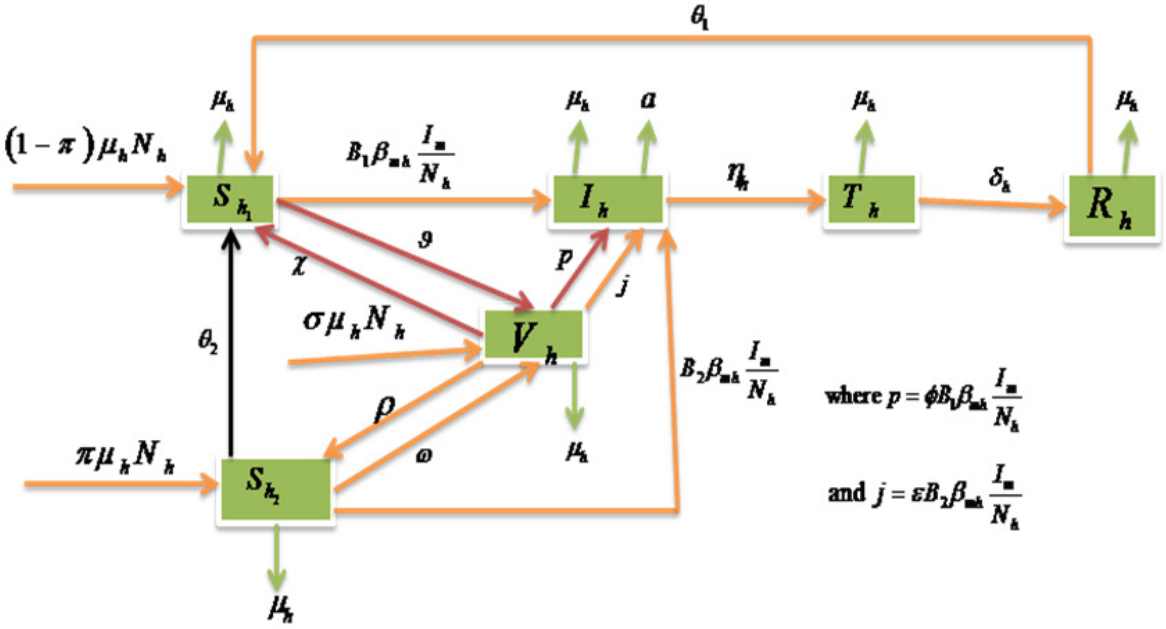


Figure 5. Model Flow diagram for dengue fever disease with vaccination

From the above flow diagram, the model will be governed by the following equations [17]:

$$\frac{dS_{h_1}}{dt}=(1-\pi)\mu_h N_h-B_1\beta_{mh}\frac{I_m}{N_h}S_{h_1}-(\mu_h+\vartheta)S_{h_1}+\theta_1 R_h+\theta_2 S_{h_2}+\chi V_h$$

$$\frac{dS_{h_2}}{dt}=\pi\mu_h N_h-B_2\beta_{mh}\frac{I_m}{N_h}S_{h_2}-(\mu_h+\theta_2+\omega)S_{h_2}+\rho V_h$$

$$\frac{dV_h}{dt}=\sigma\mu_h N_h+\vartheta S_{h_h}+\omega S_{h_2}-(\chi+\rho+\mu_h)V_h-(\phi B_1+\varepsilon B_2)\beta_{mh}\frac{I_m}{N_h}V_h$$

$$\frac{dI_h}{dt}=(B_1 S_{h_1}+B_2 S_{h_2})\beta_{mh}\frac{I_m}{N_h}+(\phi B_1+\varepsilon B_2)\beta_{mh}\frac{I_m}{N_h}V_h-(\mu_h+\eta_h+a)I_h$$

$$\frac{dT_h}{dt}=\eta_h I_h-(\mu_h+\delta_h)T_h,\quad \frac{dR_h}{dt}=\delta_h T_h-(\mu_h+\theta_1)R_h \tag{10}$$

Mass vaccination generates the possibility of eliminating or eradication the infectious disease [18]. The more vaccinated people, the less likely a susceptible person will come into contact with the infection. With the introduction of a vaccine, the SITRS model related to the human population changes to the SVITRS model. Vaccination is continuous with a constant proportion σ of vaccinated new born. A fraction ϑ and ω of careful and careless susceptible is vaccinated respectively. The vaccination reduces but does not eliminate susceptibility to infection. For this reason, we consider the

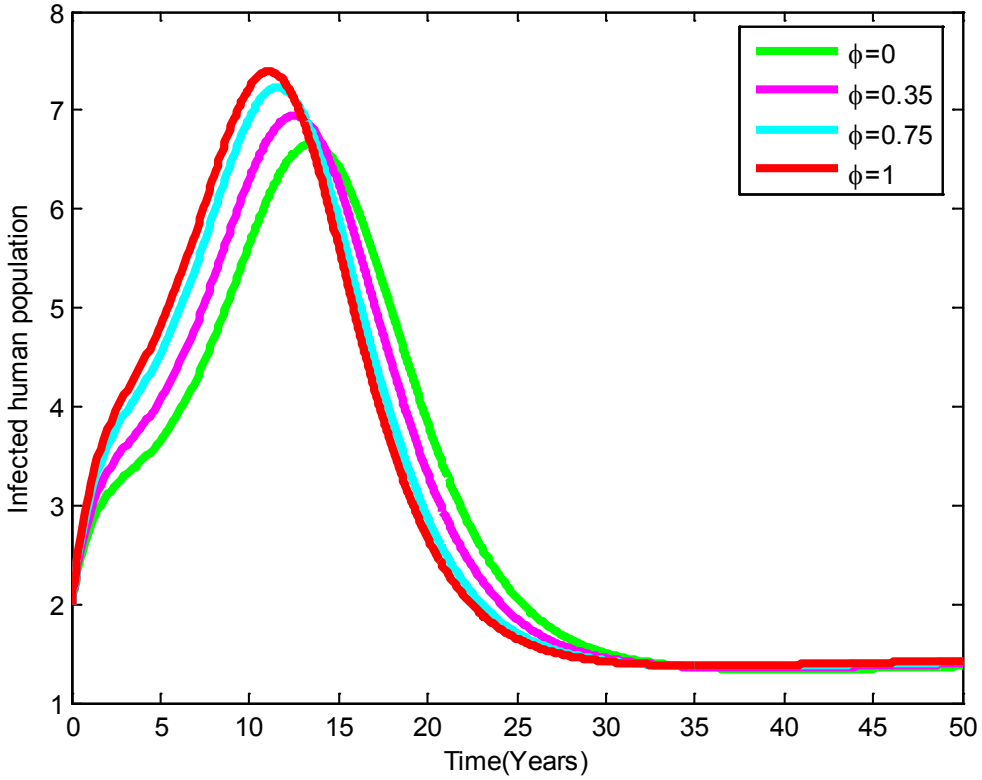
infection rate of vaccinated people: when $\phi = \varepsilon = 0$ the vaccine is perfect and when $\phi = \varepsilon = 1$ the vaccine has no effect at all. The vaccination loses effectiveness at a rate χ and ρ careful and careless susceptible respectively [17].

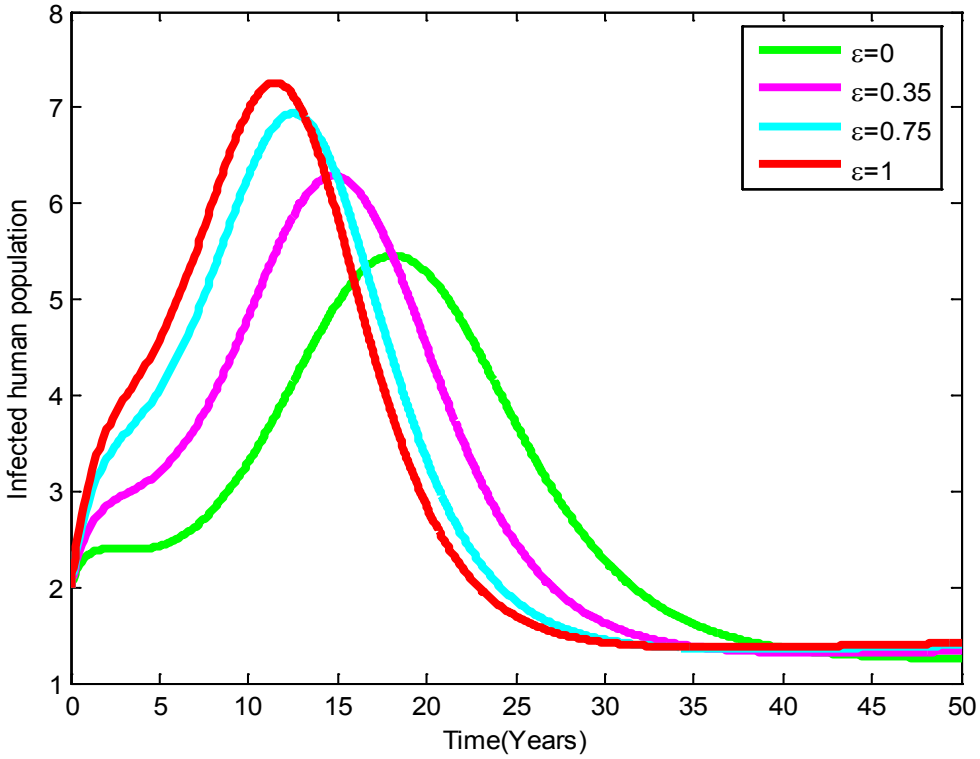
5.1. Model Simulation

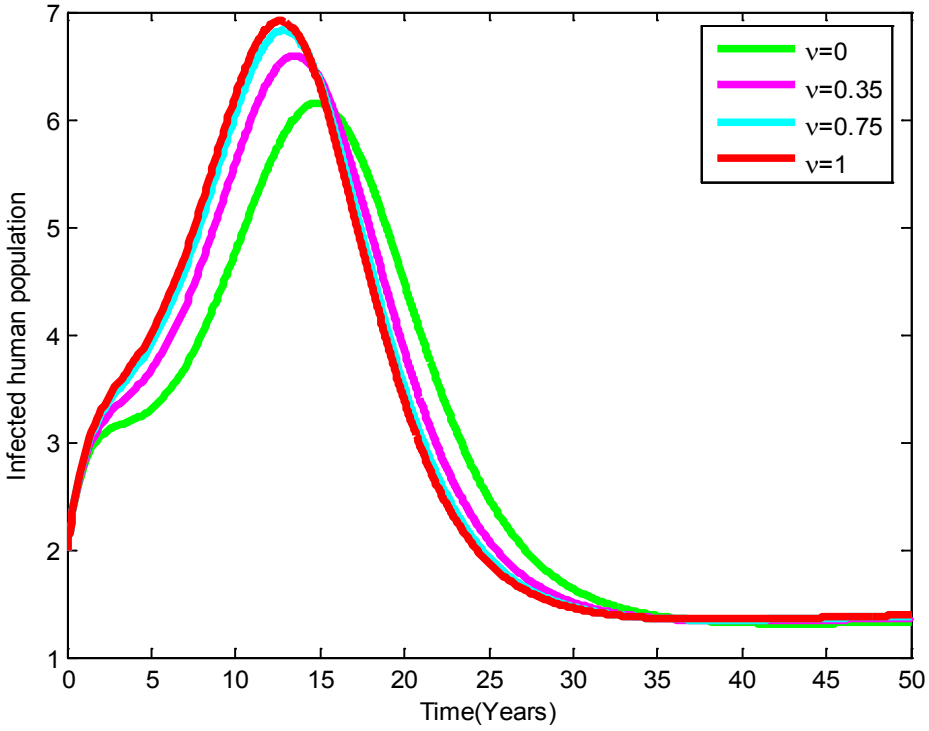
Here, we perform numerical simulations of the model system (11) using the set of estimated parameter values. Parameter values are obtained from the different literatures like (http:/e/www.wavuti.com/2014/05/wizara-ya-afya-kitengo-cha.html), [9], [10], [16], other parameter values are estimated to vary within realistic means and given as $\beta_{hm} = 0.375$, $\beta_{mh} = 0.45$, $\pi = 0.96$, $B_1 = 0.5$, $B_2 = 0.9$, $B_3 = 0.7$, $\mu_m = \frac{1}{11}$, $k = 3$, $\eta_A = 0.35$, $\mu_A = 0.25$, $\mu_h = \frac{1}{78 \times 365}$, $\eta_h = 1/3$, $\varphi = 5$, $\theta_2 = 0.6$, $a = 0.001$, $\vartheta = 0.85$, $\chi = 0.75$, $\omega = 0.82$, $\rho = 0.30$, $\sigma = 0.16$, $\phi = 0.25$, $\varepsilon = 0.70$, $\delta_h = 0.98$ and $\theta_1 = 0.01$.

Figures 4 show the variation of infected human populations with different levels of infection rate of vaccinated careful human susceptible ϕ, infection rate of vaccinated careless human susceptible ε, fraction of the vaccinated careful human susceptible ϑ and fraction of the vaccinated careless human susceptible ω.

From figure 4 we vary infection rate of vaccinated careful susceptible ϕ, infection rate of vaccinated careless human susceptible ε, fraction of the vaccinated careful human susceptible ϑ and fraction of the vaccinated careless human susceptible ω, and it is observed that the effectiveness of the vaccine reduces the disease spread.




ε=0
ε=0.35
ε=0.75
ε=1
Infected human population
Time(Years)


ν=0
ν=0.35
ν=0.75
ν=1
Infected human population
Time(Years)

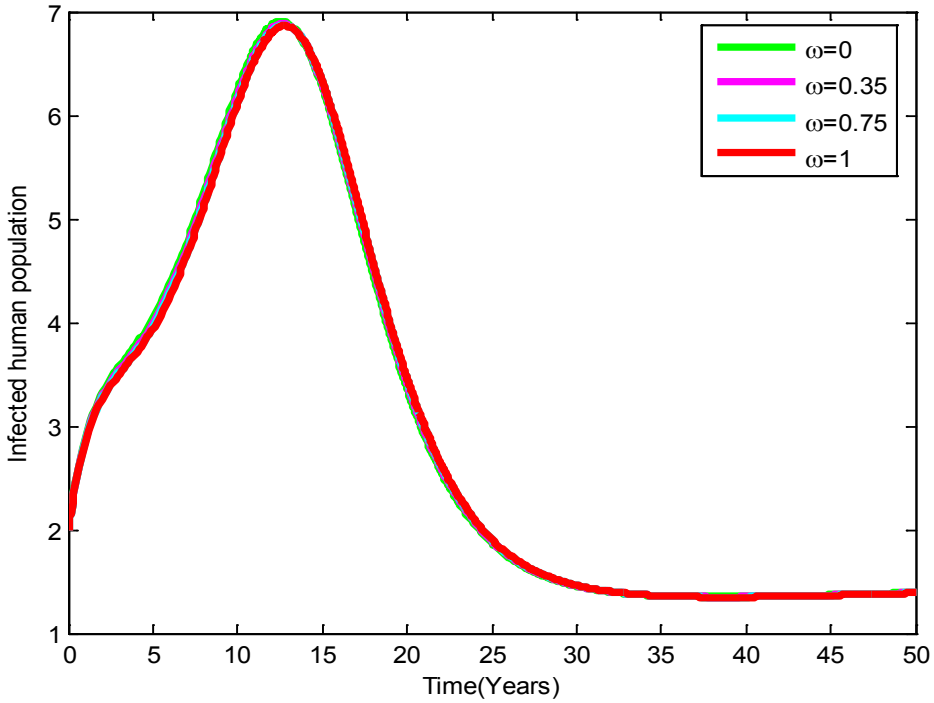


Figure 4. Variation of infected human populations with different levels infection rate of vaccinated careful human susceptible ϕ, infection rate of vaccinated careless human susceptible ε, fraction of the vaccinated careful human susceptible ϑ and fraction of the vaccinated careless human susceptible ω

6. Conclusions

A compartmental model for Dengue fever disease was presented, The was model based on campaign of educating Careless human susceptible τ_1, control vector human contact τ_2, reducing vector breeding areas τ_3, and insecticide application τ_4, control maturation rate from larvae to adult τ_5 and treatment. The results show that Treatment and the controls on the transmission of dengue fever disease will have a positive effect on decreasing the growth rate of dengue fever disease. Then also shows that when $\phi = \varepsilon = 0$ the infected human population decrease and when $\phi = \varepsilon = 1$ the infected human population increase.

REFERENCES

[1] Rodrigues, H.S., Monteiro, M.T.T., Torres, D.F.M., and Zinober, A., 2010, Control of dengue disease Computational and Mathematical Methods in Science and Engineering, 816–822.

[2] Okosun, K.O., and Makinde, O.D., 2013, Optimal Control Analysis of Malaria in the Presence of Non-Linear Incidence Rate, Appl. Comput. Math 12(1)20-32.

[3] Evans, T.P.O., and Bishop, S.R., 2014, A spatial model with pulsed releases to compare strategies for the sterile insect technique applied to the mosquito Aedes aegypti, Mathematical Biosciences 254, 6–27.

[4] Aldila, D., Götz, T., and Soewono, E., 2013, An optimal control problem arising from a dengue disease transmission model. Mathematical Biosciences 24, 9–16.

[5] Thome, R.C.A., Yang, H.M., and Esteva, L., 2010, Optimal control of Aedes aegypti mosquitoes by the sterile insect technique and insecticide. Math. Biosci. 223, 12-23.

[6] Wijayaa, K.P., Goetzb, T., Soewonoa, E., and Nurainia, N., 2013, Temephos spraying and thermal fogging efficacy on Aedes aegypti in homogeneous urban residences, ScienceAsia 39, 48–56.

[7] Lashari, A.A., Hattaf, K., Zaman, G., and Li, Xue-Zhi., 2013, Backward bifurcation and optimal control of a vector borne disease. Appl. Math. Inf. Sci. 7(1), 301-309.

[8] Fister, K.R., McCarthy, M.L., Oppenheimer, S.F., and Collins, C.,2013, Optimal control of insects through sterile insect release and habitat modification. Mathematical Biosciences 244, 201–212.

[9] Massawe, L.N., Massawe, E.S., and Makinde, O.D., 2015, Temporal model for dengue disease with treatment. Advances in Infectious Diseases, 5, 21-36.

[10] H.S. Rodrigues, M.T.T. Monteiro, and D.F.M. Torres,

Sensitivity Analysis in a Dengue Epidemiological Model. Conference Papers in Mathematics, 2013.

[11] Driessche, P van den., and Watmough J., 2002, "Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission," Mathematical Biosciences, 180, 29–48.

[12] Ozair, M., Lashari, A.A., Jung, Il Hyo., Seo, Y. Il., and Kim, B.N., 2013, Analysis of a Vector-Borne Disease with Variable Human Population. Research Article Stability, 1-12.

[13] J.P. LaSalle, the Stability of Dynamical Systems, Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, Pa, USA, 1976.

[14] Lee, K.S. and Lashari, A.A. (2014). Global Stability of a Host-Vector Model for Pine Wilt Disease with Nonlinear Incidence Rate Abstract and Applied Analysis, 1-11.

[15] McCluskey, C.C., and Driessche, P van den. (2004). Global analysis of tuberclosis models, *Journal of Differential Equations*, 16, 139–166.

[16] Dumont, Y., Chiroleu, F., and Domerg, C., 2008, "On a temporal model for the Chikungunya disease: modelling, theory and numerics," Mathematical Biosciences, 213(1), 80–91.

[17] H.S. Rodrigues, M.T.T. Monteiro and D.F.M Torres, Dengue in Cap e Verde: Vector control and vaccination, 2012.

[18] C. Farrington, on vaccine efficacy and reproduction numbers. Mathematical, 2003.