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Further Acceleration of the Jarratt Method for Solving Nonlinear Equations

R. Thukral

Padé Research Centre, 39 Deanswood Hill, Leeds, West Yorkshire, England

Abstract New four-point iterative methods for solving nonlinear equations are constructed using a suitable parametric function. It is proved that these methods have the convergence order of twelve. Per iteration the new method requires three evaluations of the function and two evaluations of its first derivative. We examine the effectiveness of the new Jarratt -type methods by approximating the simple root of a given nonlinear equation. Numerical comparisons are included to demonstrate exceptional convergence speed of the proposed methods using only a few function evaluations.

Keywords Jarratt-type methods, Nonlinear equations, Order of convergence, Computational efficiency, Iterative methods

1. Introduction

In this paper, we present Jarratt-type methods to find a simple root of the nonlinear equation

$$f(x)=0, \tag{1}$$

where $f: I \subset \mathbb{R} \rightarrow \mathbb{R}$ is a scalar function on an open interval I and it is sufficiently smooth in a neighbourhood of α. It is well known that the techniques to solve nonlinear equations have many applications in science and engineering. In order to construct the new twelfth-order method we combine two well-known methods, namely the original Jarratt-type method with fourth-order convergence [3] and the recently introduced Jarratt-type sixth-order method [1, 4, 7, 9-11]. The new family of the Jarratt-type methods requires three evaluations of the function and two evaluations of its first derivative. Hence the new methods have a better efficiency index than the well-known lower order methods [1, 4, 7, 9-11]. This paper is actually a continuation from the previous study [7]. The prime motive for presentation of the new twelfth-order method was to increase the sixth-order convergence method given in [1, 4, 7, 9-11]. Consequently, we find that the new Jarratt-type methods are efficient and robust.

2. Preliminaries α

In order to establish the order of convergence of the new twelfth-order methods, we state some of the definitions:

Definition 1 Let $f(x)$ be a real function with a simple root α and let $\{x_n\}$ be a sequence of real numbers that converge towards α. The order of convergence p is given by

$$\lim_{n\to\infty} \frac{x_{n+1}-\alpha}{\left(x_n-\alpha\right)^p}=\lambda \neq 0, \tag{2}$$

where and λ is the asymptotic error constant, [2,8].

Definition 2 Let $e_k = x_k - \alpha$ be the error in the kth iteration, then the relation

$$p \in \mathbb{R}^+ \; e_{k+1} = \zeta e_k^p + \mathrm{O}\left(e_k^{p+1}\right), \tag{3}$$

is the error equation. If the error equation exists then p is the order of convergence of the iterative method, [2,8].

Definition 3 Let r be the number of function evaluations of the new method. The efficiency of the new method is measured by the concept of efficiency index and defined as

$$\sqrt[r]{p}, \tag{4}$$

where p is the order of the method, [2,8].

Definition 4 Suppose that x_{n-1}, x_n and x_{n+1} are three successive iterations closer to the root α of (1). Then the computational order of convergence may be approximated by

$$\mathrm{COC} \approx \frac{\ln\left|\left(x_{n+1}-\alpha\right)\left(x_n-\alpha\right)^{-1}\right|}{\ln\left|\left(x_n-\alpha\right)\left(x_{n-1}-\alpha\right)^{-1}\right|}, \tag{5}$$

where, now and in the sequel, $n \in \mathbb{N}$, [12].

* Corresponding author:
rthukral@hotmail.co.uk (R. Thukral)
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3. Construction of the New Twelfth Order Methods

We construct new four-point Jarratt-type methods using five function evaluations. The first three steps are the same as those of the sixth-order method introduced in [1, 4, 7, 9-11], while the fourth step is constructed using parametric functions which are determined in such a way that the order of convergence of the four-step method is twelve. To obtain the solution of (1) by the new Jarrat-type methods, we must evaluate the first derivative of (1) and set a particular initial approximation x_0, ideally close to the simple root. To derive a higher efficiency index, we use divided difference, thus, the essential terms used in the methods are given by

$$f[v_n,x_n]=\frac{f(v_n)-f(x_n)}{v_n-x_n}, \tag{6}$$

$$f[w_n,v_n]=\frac{f(w_n)-f(v_n)}{w_n-v_n}, \tag{7}$$

$$f[v_n,x_n,x_n]=\frac{f[v_n,x_n]-f'(x_n)}{v_n-x_n}. \tag{8}$$

Method 1

$$u_n=x_n-\frac{2}{3}\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{9}$$

$$v_n=x_n-J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{10}$$

$$w_n=v_n-J(x_n)^2\left(\frac{f(v_n)}{f'(x_n)}\right), \tag{11}$$

$$x_{n+1}=w_n-\left(\frac{f(w_n)}{L(x_n)}\right), \tag{12}$$

where

$$\begin{aligned}L(x_n)=&2f[w_n,x_n]+f[w_n,v_n]\\&-2f[z_n,x_n]+(w_n-v_n)f[v_n,x_n,x_n],\end{aligned} \tag{13}$$

$$J(x_n)=\frac{3f'(u_n)+f'(x_n)}{6f'(u_n)-2f'(x_n)}, \tag{14}$$

provided that the denominators (9)-(12) are not equal to zero. It is well established that (10), (11) have an order of convergence 4, 6, respectively [7]. For the purpose of this paper, we construct new Jarratt-type methods which have the order of convergence twelve. In this section we establish the order of convergence of the new Jarratt-type method. In numerical mathematics it is essential to know the behaviour of an approximate method and therefore, we prove the order of convergence of the new twelfth-order method.

Theorem 1

Assume that the function $f:I\subset\mathbb{R}\to\mathbb{R}$ for an open interval I has a simple root $\alpha\in I$. Let $f(x)$ be sufficiently smooth in the interval I and the initial guess x_0 is sufficiently close to α, then the order of convergence of the new method defined by (12) is twelve.

Proof

Let α be a simple root of $f(x)$, i.e. $f(\alpha)=0$ and $f'(\alpha)\neq 0$, and the error is expressed as

$e=x-\alpha$.

Using Taylor series expansion, we get

$$f(x_n)=f'(\alpha)\left(e_n+\sum_{i=2}^{m}c_ie_n^i+\mathrm{O}\left(e_n^{m+1}\right)\right), \tag{15}$$

and

$$f'(x_n)=f'(\alpha)\left(1+\sum_{i=2}^{m}ic_ie_n^{i-1}+\mathrm{O}\left(e_n^{m}\right)\right), \tag{16}$$

where

$$c_k=\frac{f^{(k)}(\alpha)}{k!f'(\alpha)},\quad k\geq 2. \tag{17}$$

Dividing (15) by (16), we get

$$\frac{f(x_n)}{f'(x_n)}=e_n-c_2e_n^2+2\left(c_2^2-c_3\right)e_n^3+\cdots. \tag{18}$$

and hence, we have

$$x_n-\frac{2}{3}\frac{f(x_n)}{f'(x_n)}-\alpha=\frac{e_n}{3}+\frac{2}{3}c_2e_n^2-\frac{4}{3}\left(c_2^2-c_3\right)e_n^3+\cdots. \tag{19}$$

Expanding $f'(u_n)$ about α and from (19), we have

$$f'(u_n)=f'(\alpha)\left[1+\frac{2}{3}c_2e_n+\frac{1}{3}\left(c_2^2+c_3\right)e_n^2+\cdots\right]. \tag{20}$$

Using (16) and (20), we obtain

$$3f'(u_n)+f'(x_n)=4f'(\alpha)\left[1+c_2e_n+\left(c_2^2+c_3\right)e_n^2+\cdots\right]. \tag{21}$$

$$6f'(u_n)-2f'(x_n)=4f'(\alpha)\left[1+\left(2c_2^2-c_3\right)e_n^2+\cdots\right]. \tag{22}$$

Dividing (21) by (22) gives us

$$\begin{aligned}J(x_n)&=\frac{3f'(u_n)+f'(x_n)}{6f'(u_n)-2f'(x_n)}\\&=1+c_2e_n-\left(c_2^2-2c_3\right)e_n^2+\cdots\end{aligned} \tag{23}$$

Thus from (10), we have

$$\begin{aligned} v_n-\alpha &= e_n - J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right) \\ &= -\frac{1}{9}\left(9c_2c_3-9c_2^3-c_4\right)e_n^4+\cdots \end{aligned} \tag{24}$$

Expanding $f(v_n)$ about α, we have

$$\begin{aligned} f(v_n) = f'(\alpha)\Bigg[&\frac{1}{9}\left(9c_2c_3-9c_2^3-c_4\right)e_n^4 \\ &+\frac{2}{27}\left(108c_2^2c_3-54c_2^4-27c_3^2-30c_2c_4-4c_5\right)e_n^5+\cdots\Bigg]. \end{aligned} \tag{25}$$

The expansion of the weight function used in the third step (11) is given as

$$J(x_n)^2 = 1+2c_2e_n+\left(4c_3-c_2^2\right)e_n^2+\cdots, \tag{26}$$

Thus from (11), we have

$$\begin{aligned} w_n-\alpha &= v_n - J(x_n)^2\left(\frac{f(v_n)}{f'(x_n)}\right) \\ &= -\frac{1}{9}\left(9c_2c_3-9c_2^3-c_4\right)e_n^6+\cdots \end{aligned} \tag{27}$$

The Taylor series expansion of $f(w_n)$ about α is given as

$$f(w_n) = f'(\alpha)\left[1+\frac{1}{9}\left(9c_2c_3-9c_2^3-c_4\right)e_n^6+\cdots\right]. \tag{28}$$

The expansion of the particular terms used in (13) are given as

$$\begin{aligned} f[w_n,x_n] &= \left(\frac{f(w_n)-f(x_n)}{w_n-x_n}\right) \\ &= f'(\alpha)\left[1+c_2e_n+c_3e_n^2+c_4e_n^3+\cdots\right]. \end{aligned} \tag{29}$$

$$\begin{aligned} f[w_n,v_n] &= \left(\frac{f(w_n)-f(v_n)}{w_n-v_n}\right) \\ &= f'(\alpha)\left[\frac{1}{9}\left(9c_2c_3-9c_2^3-c_4\right)e_n^4+\cdots\right]. \end{aligned} \tag{30}$$

$$\begin{aligned} f[v_n,x_n] &= \left(\frac{f(v_n)-f(x_n)}{v_n-x_n}\right) \\ &= f'(\alpha)\Big[1+c_2e_n+c_3e_n^2+c_4e_n^3 \\ &\quad +3^{-2}c_2\left(9c_2c_3-9c_2^3-c_4\right)e_n^4+\cdots\Big]. \end{aligned} \tag{31}$$

$$\begin{aligned} L(x_n) = f'(\alpha)\Big[&1+3^{-2}\left(c_4-2c_2c_3+2c_2^3\right) \\ &\times\left(c_4-9c_2c_3+9c_2^3\right)^2e_n^6+\cdots\Big]. \end{aligned} \tag{32}$$

Substituting appropriate expressions in (12), we obtain the error equation

$$\begin{aligned} e_{n+1} = 3^{-4}&\left(c_2^2-c_3\right)\left(c_4-c_2c_3+c_2^3\right) \\ &\times\left(c_4-9c_2c_3+9c_2^3\right)^2e_n^{12}. \end{aligned} \tag{33}$$

The expression (33) establishes the asymptotic error constant for the twelfth order of convergence for the Jarratt-type method defined by (12).

Method 2

$$u_n = x_n-\frac{2}{3}\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{34}$$

$$v_n = x_n-J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{35}$$

$$w_n = v_n-J(x_n)^2\left(\frac{f(v_n)}{f'(x_n)}\right), \tag{36}$$

$$x_{n+1} = w_n-P(x_n)f(w_n), \tag{37}$$

where

$$\begin{aligned} P(x_n) = &\left(1+f(w_n)f(x_n)^{-1}-\left(J(x_n)-1\right)^2f(v_n)f(x_n)^{-1}\right) \\ &\div\left(f[w_n,x_n]-f[v_n,x_n]+f[w_n,v_n]\right), \end{aligned} \tag{38}$$

provided that the denominators (34)-(37) are not equal to zero. Here also, in step two and three, it is well-known that (35), (36) have an order of convergence 4, 6, respectively [7].

Theorem 3

Assume that the function $f: I\subset\mathbb{R}\to\mathbb{R}$ for an open interval I has a simple root $\alpha\in I$. If $f(x)$ is sufficiently smooth in the neighbourhood of the root α, then the method defined by (37) is of order twelve.

Proof

Using appropriate expressions in the proof of the theorem 1 and substituting them into (37), we obtain the asymptotic error constant

$$\begin{aligned} e_{n+1} = 3^{-6}&\left(10c_2^2-3c_3\right)\left(3c_4-3c_2c_3+10c_2^3\right) \\ &\times\left(c_4-9c_2c_3+9c_2^3\right)^2e_n^{12}. \end{aligned} \tag{39}$$

The expression (38) establishes the asymptotic error constant for the twelfth order of convergence for the new Jarratt-type method defined by (37). □

Method 3

$$u_n = x_n-\frac{2}{3}\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{40}$$

$$v_n = x_n - J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{41}$$

$$w_n = v_n - J(x_n)^2\left(\frac{f(v_n)}{f'(x_n)}\right), \tag{42}$$

$$x_{n+1} = w_n - P(x_n) f(w_n), \tag{43}$$

where

$$P(x_n) = \left(\frac{f[v_n, x_n]}{f[w_n, x_n] f[w_n, v_n]}\right)\left(1 + \frac{f(w_n)}{f(x_n)}\right), \tag{44}$$

provided that the denominators (40)-(43) are not equal to zero. For the purpose of this paper we construct new Jarratt -type methods which have the order of convergence twelve.

Theorem 3

Assume that the function $f: I \subset \mathbb{R} \to \mathbb{R}$ for an open interval I has a simple root $\alpha \in I$. If $f(x)$ is sufficiently smooth in the neighbourhood of the root α, then the method defined by (43) is of order twelve.

Proof

Using appropriate expressions in the proof of the theorem 1 and substituting them into (43), we obtain the asymptotic error constant

$$\begin{aligned} e_{n+1} &= 3^{-6}\left(10c_2^2 - 3c_3\right)\left(3c_4 - 3c_2c_3 + 10c_2^3\right) \\ &\quad \times \left(c_4 - 9c_2c_3 + 9c_2^3\right)^2 e_n^{12}. \end{aligned} \tag{45}$$

The expression (45) establishes the asymptotic error constant for the twelfth order of convergence for the new Jarratt-type method defined by (43).

Method 4

$$u_n = x_n - \frac{2}{3}\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{46}$$

$$v_n = x_n - J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{47}$$

$$w_n = v_n - K(x_n) f(v_n), \tag{48}$$

$$x_{n+1} = w_n - P(x_n) f(w_n), \tag{49}$$

where

$$K(x_n) = \frac{3}{2f'(y_n)} - \frac{1}{2f(x_n)}, \tag{50}$$

$P(x_n)$ is given by (44) and provided that the denominators (46)-(49) are not equal to zero and it is well established that (47), (48) have an order of convergence 4, 6, respectively [11]. For the purpose of this paper we construct new Jarratt -type methods which have the order of convergence twelve.

Theorem 4

Assume that the function $f: I \subset \mathbb{R} \to \mathbb{R}$ for an open interval I has a simple root $\alpha \in I$. If $f(x)$ is sufficiently smooth in the neighbourhood of the root α, then the method defined by (49) is of order twelve.

Proof

Using appropriate expressions in the proof of the theorem 1 and substituting them into (49), we obtain the asymptotic error constant

$$\begin{aligned} e_{n+1} &= 3^{-6}\left(10c_2^2 - 3c_3\right)\left(3c_4 - 3c_2c_3 + 10c_2^3\right) \\ &\quad \times \left(c_4 - 9c_2c_3 + 9c_2^3\right)^2 e_n^{12}. \end{aligned} \tag{51}$$

The expression (51) establishes the asymptotic error constant for the twelfth order of convergence for the new Jarratt-type method defined by (49).

Method 5

$$u_n = x_n - \frac{2}{3}\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{52}$$

$$v_n = x_n - J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{53}$$

$$w_n = v_n - S(x_n)\left(\frac{f(v_n)}{f'(x_n)}\right), \tag{54}$$

$$x_{n+1} = w_n - \left(\frac{f(w_n)}{L(x_n)}\right), \tag{55}$$

where

$$S(x_n) = \left[f'(u_n) + 2f[v_n, x_n, x_n](v_n - u_n)\right]^{-1}, \tag{56}$$

$L(x_n)$ is given by (13) provided that the denominators (52) - (55) are not equal to zero and it is well established that (53), (54) have an order of convergence 4, 6, respectively [5]. For the purpose of this paper we construct new Jarratt-type methods which have the order of convergence twelve.

Theorem 5

Assume that the function $f: I \subset \mathbb{R} \to \mathbb{R}$ for an open interval I has a simple root $\alpha \in I$. If $f(x)$ is sufficiently smooth in the neighbourhood of the root α, then the method defined by (55) is of order twelve.

Proof

Using appropriate expressions in the proof of the theorem 1 and substituting them into (55), we obtain the asymptotic error constant

$$\begin{aligned} e_{n+1} &= 3^{-6}\left(10c_2^2 - 3c_3\right)\left(3c_4 - 3c_2c_3 + 10c_2^3\right) \\ &\quad \times \left(c_4 - 9c_2c_3 + 9c_2^3\right)^2 e_n^{12}. \end{aligned} \tag{57}$$

The expression (57) establishes the asymptotic error constant for the twelfth order of convergence for the new Jarratt-type method defined by (55).

4. The Soleymani et al. Twelfth Order Method

For the purpose of comparison, we consider the Soleymani et al. method presented recently in [6]. Soleymani et al. developed a twelfth order efficient class of four-step root finding method and the scheme is given as

$$u_n = x_n - \frac{2}{3}\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{58}$$

$$v_n = x_n - J(x_n)\left(\frac{f(x_n)}{f'(x_n)}\right), \tag{59}$$

$$w_n = v_n - \left(\frac{f(v_n)}{g(x_n)}\right), \tag{60}$$

$$x_{n+1} = w_n - \left(\frac{f(w_n)}{h(x_n)}\right), \tag{61}$$

where

$$g(x_n) = 2f[v_n, x_n] - f'(x_n) + t(v_n - x_n), \tag{62}$$

$$\begin{aligned} h(x_n) = {} & 2f[w_n, x_n] + f[w_n, v_n] - 2f[u_n, x_n] \\ & + (v_n - w_n) f[v_n, x_n, x_n], \end{aligned} \tag{63}$$

where

$$\begin{aligned} t = {} & \left[2f[v_n, x_n] - (u_n - x_n) + (x_n - 2u_n + v_n) f'(x_n) \right. \\ & \left. + (x_n - v_n) f'(u_n)\right] \\ & \div \left[(x_n - u_n)(x_n - v_n)(x_n - 3u_n + 2v_n)\right]. \end{aligned} \tag{64}$$

Further details of the above method maybe found in [6].

5. Numerical Examples

Table 1. Test functions and their roots

Functions	Roots	Initial Guess
$f_1(x) = \exp(x)\sin(x) + \ln(1 + x^2)$	$\alpha = 0$	$x_0 = 0.1$
$f_2(x) = (x^2 - 1)^{-1} - 1$	$\alpha = \sqrt{2}$	$x_0 = 1.2$
$f_3(x) = (x - 2)(x^{10} + x + 1)\exp(-x - 1)$	$\alpha = 2$	$x_0 = 2.1$
$f_4(x) = (x + 1)\exp(\sin(x)) - x^2 \exp(\cos(x)) - 1$	$\alpha = 0$	$x_0 = 3^{-1}$
$f_5(x) = \sin(x)^2 - x^2 + 1$	$\alpha = 1.40449165...$	$x_0 = 1$
$f_6(x) = \exp(-x) - \cos(x),$	$\alpha = 0$	$x_0 = -0.2$
$f_7(x) = \ln(x^2 + x + 2) - x + 1$	$\alpha = 4.15259074...$	$x_0 = 3$
$f_8(x) = x^{10} - 2x^3 - x + 1$	$\alpha = 0.59144...$	$x_0 = 0.75$
$f_9(x) = \cos(x)^2 - 5^{-1}x$	$\alpha = 1.08598...$	$x_0 = 1.5$
$f_{10}(x) = 1 + x\exp(x^{-1}) + x\sin(x) - 3x^2\cos(x^{-2})$	$\alpha = -0.9806824....$	$x_0 = -1.5$

In this section, we apply the present methods defined by (12), (37), (43), (49), (55), to solve some nonlinear equations, which not only illustrate the method practically but also support the validity of theoretical results we have derived. For comparison we have considered the Soleymani et al. twelfth order method (61). To demonstrate the performance of the new higher order methods, we use ten particular nonlinear equations. We shall determine the consistency and stability of results by examining the convergence of the new iterative methods. The findings are generalised by illustrating the effectiveness of the higher order methods for determining the simple root of a nonlinear equation. Consequently, we give estimates of the approximate solution produced by the higher order methods and list the errors obtained by each of the methods. The numerical computations listed in the tables were performed on an algebraic system called Maple. In fact, the errors displayed are of absolute value and insignificant approximations by the various methods have been omitted in the following tables.

The new higher order method requires five function evaluations and has the order of convergence twelve. We have used definition 3 to determine the efficiency index of the new method. Hence, the efficiency index of the twelfth order methods given is $\sqrt[4]{12}$ which is identical to the Soleymani et al. method, given section 4. The test functions and their exact root α are displayed in table 1. The difference between the root α and the approximation x_n for test functions with initial guess x_0, are displayed in Table 2. In fact, x_n is calculated by using the same total number of function evaluations (TNFE) for all methods. Furthermore, the computational order of convergence (COC) is displayed in Table 3.

6. Remarks and Conclusions

In this paper, we have demonstrated the performance of the new twelfth order Jarratt-type iterative methods. The prime motive for presenting these new methods was to establish a higher order of convergence method than the existing sixth order methods [1, 4, 7, 9-11]. We have examined the effectiveness of the new methods by showing the accuracy of the simple root of a nonlinear equation. After an extensive experimentation we were not able to designate a specific iterative method, which always produces the best results, for all tested nonlinear equations. The main purpose of demonstrating the new Jarratt-type methods for several types of nonlinear equations was purely to illustrate the accuracy of the approximate solution, the stability of the convergence, the consistency of the results and to determine the efficiency of the new iterative method. We have shown numerically and verified that the new Jarratt-type methods converge by the order twelve. Finally, we conclude that the new four-point methods may be considered a very good alternative to the classical methods.

Table 2. Comparison of new iterative methods

f_i	**(12)**	**(49)**	**(55)**	**(61)**	**(37)**	**(43)**
f_1	0.36e-1331	0.395e-1210	0.564e-1653	0.113e-1639	0.298e-1452	0.426e-1307
f_2	0.164e-746	0.334e-574	0.343e-501	0.202e-536	0.400e-924	0.373e-1021
f_3	0.634e-922	0.205e-801	0.994e-941	0.824e-910	0.145e-1022	0.251e-925
f_4	0.232e-1170	0.812e-1388	0.142e-1135	0.442e-1145	0.150e-1190	0.698e-1156
f_5	0.748e-930	0.823e-524	0.249e-1156	0.396e-1150	0.731e-766	0.228e-786
f_6	0.139e-1341	0.529e-1213	0.290e-1546	0.711e-1532	0.242e-1437	0.201e-1325
f_7	0.690e-1537	0.561e-1431	0.461e-1588	0.151e-1594	0.132e-1564	0.798e-1553
f_8	0.114e-1465	0.748e-1469	0.203e-1473	0.304e-1473	0.104e-1482	0.278e-1657
f_9	0.223e-519	0.846e-324	0.136e-828	0.291e-800	0.101e-373	0.375e-380
f_{10}	0.226e-1258	0.602e-1396	0.958e-1244	0.102e-1240	0.284e-1235	0.793e-1312

Table 3. COC of various iterative methods

f_i	**(12)**	**(49)**	**(55)**	**(61)**	**(37)**	**(43)**
f_1	12.000	12.000	12.000	12.000	12.000	12.000
f_2	12.000	12.000	12.000	12.000	12.000	11.996
f_3	12.000	12.000	12.000	12.000	12.000	12.000
f_4	11.997	12.000	12.000	12.000	11.997	11.997
f_5	12.000	12.000	11.997	12.000	11.993	12.000
f_6	12.000	12.000	12.000	12.000	12.000	12.000
f_7	12.000	12.000	12.000	12.000	12.000	12.000
f_8	12.000	11.998	12.000	12.000	12.000	12.000
f_9	12.000	11.998	12.000	11.994	11.972	12.000
f_{10}	11.998	11.998	11.998	11.998	11.997	11.998

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