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A Triangular Fuzzy Model for Decision Making

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Abstract In the present paper we develop an improved version of the Triangular Fuzzy Model (TFM) for verifying the creditability of a chosen decision. The TFM is a variation of a special form of the Centre of Gravity (COG) defuzzification technique, which we have used in earlier papers as an assessment method in several human activities. The main idea of the TFM is the replacement of the rectangles appearing in the graph of the membership function of the COG method by isosceles triangles sharing common parts. In this way we cover the ambiguous cases of individuals' scores being at the limits between two successive categories. A real application is also presented illustrating our results in practice, in which the outcomes of the TFM are compared with those of the COG technique and of other traditional methods (calculation of means and GPA index).

Keywords Decision making, Verification of a decision, GPA assessment index, Fuzzy logic, Center of gravity (COG) deffuzzification technique, Triangular Fuzzy Model (TFM)

1. Introduction

Decision Making (DM) is the process of choosing a solution between two or more alternatives, aiming to achieve the best possible results for a given problem. Obviously the above process has sense if, and only if, there exist more than one *feasible solutions* and a suitable criterion that helps the decision maker (d-m) to choose the best among these solutions. We recall that a solution is characterized as feasible, if it satisfies all the restrictions imposed onto the real system by the statement of the problem as well as all the natural restrictions imposed onto the problem by the real system; e.g. if x denotes the quantity of stock of a product, it must be $x \geq 0$. The choice of the suitable criterion, especially when the results of DM are affected by random events, depends upon the d-m's desired goals; e.g. optimistic or conservative criterion, etc.

The rapid technological progress, the impressive development of the transport means, the globalization of the modern society, the enormous changes happened to the local and international economies and other similar reasons led during the last 50-60 years to a continuously increasing complexity of the our everyday life problems. As a result the DM process became in many cases a very difficult task, which is not possible to be based on the d-m's experience, intuition and skills only, as it usually happened in the past. Thus, from the beginning of the 1950's a progressive development started of a systematic methodology for the DM process, which is based on Probability Theory, Statistics, Economics, Psychology, etc and it is known as *Statistical Decision Theory*.

According to the nowadays existing standards the DM process involves the following steps:

- **d_1:** *Analysis* of the decision-problem, i.e. understanding, simplifying and reformulating the problem in a way permitting the application of the principles of DM on it.
- **d_2:** Collection from the real system and interpretation of all the necessary information related to the problem.
- **d_3:** Determination of all the alternative feasible solutions.
- **d_4:** Choice of the best solution in terms of the suitable (according to the d-m's goals and targets) criterion.

One could add one more step to the DM process, the *verification* (checking the creditability) *of the chosen decision* according to the results obtained by applying it in practice. However, this step is extended to areas, which due to their depth and importance for the administrative rationalism have become autonomous. Therefore, it is usually examined separately from the other steps of the DM process.

Notice that the first three steps of the DM process presented above are *continuous* in the sense that the completion of each one of them usually needs some time, during which the d-m's reasoning is characterized by transitions between hierarchically neighbouring steps. The flow-diagram of the DM process is represented in Figure 1 below:

$$\mathbf{d_1} \leftrightarrow \mathbf{d_2} \leftrightarrow \mathbf{d_3} \rightarrow \mathbf{d_4}$$

Figure 1. The flow-diagram of the DM process

Our target in the present paper is the development of an improved version of the Triangular Fuzzy Model (TFM) for verifying a taken decision. For this, the rest of the paper is

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organized as follows: In section 2 we present examples illustrating the process of DM under fuzzy conditions. In section 3 we apply, through a real example, two traditional methods for the verification of a taken decision (calculation of the means and GPA index), while in section 4 we use principles of *Fuzzy Logic* (FL) for presenting two alternative methods for the same purpose: A special form of the Center of Gravity (COG) defuzzification technique that has been used in earlier papers as a tool for assessing the individuals' performance in several human activities and an improved version of the TFM, which is a variation of the COG method. The outcomes of the above - four in total - methods are compared to each other and explanations are given for the differences appeared among them. The last section 5 is devoted to our final conclusion and to a brief discussion about our future plans for further research on the subject.

2. DM under Fuzzy Conditions

In our everyday life a DM problem is frequently expressed in an ambiguous way involving a degree of uncertainty. In such cases the classical Statistical Decision Theory based on principles of the traditional bivalent logic (yes-no) is proved inadequate for helping the d-m for choosing the correct decision. On the contrary, FL, based on the notion of fuzzy sets introduced by Zadeh in 1965, offers a rich field of resources for this purpose by allowing the d-m to frame the goals and constraints of the decision problem in vague, linguistic terms, which may reflect the real situation. For general facts on FL we refer to the book [2].

The following examples illustrate the standard process of DM under fuzzy conditions:

EXAMPLE 1: A company wants to employ as a sales manager the candidate with the best qualifications, provided that his/her request for salary is not very high and that his/her residence is in a close driving distance from the company's place. They are four candidates for the above position, say A, B, C and D, with annual salary requests 29050, 25000, 14050, and 6250 euros respectively. Who of them is the best choice for the company under the above (fuzzy) conditions?

In this DM problem we have the *fuzzy goal (G)* of employing the candidate with the best qualifications under the *fuzzy constraints* that his/her request for salary must not be very high (C_1) and that his/her residence must be in a close driving distance from the companies place (C_2). The steps of the DM process in such fuzzy situations are the following:

Step 1: Choice of the universal set of the discourse

In our case we must obviously consider as universal set the set U = {A, B, C, D} of the four candidates.

Step 2: Fuzzification of the decision problem's data

In this step the fuzzy goal and the fuzzy constraints of the problem are expressed as *fuzzy sets in U*. For this, we must define properly the corresponding for each case *membership function*.

For example, the membership function $m_{C_1}: U \to [0,1]$ for the fuzzy constraint C_1 can be defined by: $m_{C_1}(x)=1$ for $s(x) < 6000$, $m_{C_1}(x) = 1 - 2 * 10^{-5} * s(x)$ for $6000 \leq s(x) \leq 30000$ and $m_{C_1}(x) = 0$ for $s(x) > 30000$, where $s(x)$ denotes the salary of the candidate x, for all x in U. Then $m_{C_1}(A) = 1 - 2 * 0.2905 = 0.419$. Similarly we calculate the *membership degrees* of B, C and D and we write the constraint C_1 as a fuzzy set in U in the form of the symbolic sum C_1 = 0.419/A + 0.5/B + 0.719/C + 0.875/D.

In the same way (the relevant details are omitted here for reasons of brevity) we expressed the fuzzy goal G and the other fuzzy constraint C_2 as fuzzy sets in U in the form

G = 0.9/A + 0.6/B + 0.8/C + 0.6/D and

C_2 = 0.1/A + 0.9/B + 0.7/C + 1/D respectively[1].

Step 3: Evaluation of the fuzzy data

According to the *Bellman-Zadeh's criterion* for DM in a fuzzy environment [1], the *fuzzy decision F* expressed as a fuzzy set in U is the intersection of the fuzzy sets G, C_1 and C_2 of U and the solution of the problem corresponds to the element x of U having the highest membership degree in F. In fact, it is logical to define a fuzzy decision as the choice that satisfies both the goals and the constraints and, if we interpret this as a logical "and", we can model it with the intersection of all fuzzy goals and constraints of the decision problem. Finally we take the maximum of this set to obtain the best among the existing alternatives.

But, it is well known that the membership function of the intersection $G \cap C_1 \cap C_2$ is defined by $m_{G \cap C_1 \cap C_2}(x) = m_F = \min\{m_G(x), m_{C_1}(x), m_{C_2}(x)\}$ for all x in U. Therefore it is easy to check that F = 0.1/A + 0.5/B + 0.7/C + 0.6/D.

Step 4: Defuzzification

The highest membership degree in F is 0.7 and corresponds to the candidate C. Therefore the candidate C is the best choice for the company.

The fuzzy model of Bellman-Zadeh presented above can be further extended to accommodate the relative importance that could exist for the goal and constraints by using *weighting coefficients*. The following example illustrates this case:

EXAMPLE 2: Reconsider Example 1 and assume that the Management Council of the company, taking into account the existing company's budget, the results of the oral interviews of the four candidates and some other relevant factors, decided to attach weights 0.5, 0.2 and 0.3 to the goal G and to the constraints C_1 and C_2 respectively. Which will

1 We recall that the definition of the membership function is usually depending on empirical or statistical data collected from a sample of the population that we study. However a necessary condition for the creditability of the fuzzy model in representing the corresponding real situation is that the choice of the membership function is compatible with the common logic.

be the company's decision under these conditions?

In this case the membership function of the fuzzy decision F is defined through a linear combination of the weighted goal and constraints of the form

$m_F(x) = w_1 * m_G(x) + w_2 * m_{C_1}(x) + w_3 * m_{C_2}(x)$,

where $m_G(x)$, $m_{C_1}(x)$, $m_{C_2}(x)$ are the membership degrees in G, C_1 and C_2 respectively of each x in U (see Example 1) and the coefficients w_1, w_2 and w_3 are the weights attached to the fuzzy goal and constraints respectively, with $w_1 + w_2 + w_3 = 1$ ([2], section 6.5). Therefore the membership degree of the candidate A in the fuzzy decision F in this case is $m_F(A) = 0.5 * 0.9 + 0.419 * 0.2 + 0.1 * 0.3 = 0.638$. In the same way we find that $m_F(B) = 0.67$, $m_F(C) = 0.7538$ and $m_F(D) = 0.775$. Therefore the candidate D will be the company's choice in this case.

3. Traditional Methods for the Verification of a Chosen Decision

In this section we shall apply two traditional methods for verifying a chosen decision, the *calculation of the means* and the *Grade Point Average (GPA) index*. For this, let us consider the following real example:

EXAMPLE: A car industry has decided to circulate its new model in the market in two different types, the luxury (L) Class and the regular (R) Class. Six months after the purchase of their cars the customers were asked to complete a written questionnaire concerning their degree of satisfaction for their new cars. Their answers were marked by the industry's marketing department within a climax from 0 to 100 and they were divided in the following five categories according to the corresponding scores: A (90-100) = Full satisfied customers, B (75-89) = Very satisfied customers, C (60-74) = Satisfied customers, D (50-59) = Rather satisfied customers and E (0-49) = Unsatisfied customers.

Table 1. Questionnaire's data

Customers' Categories	L Class	R Class
A	60	60
B	40	90
C	20	45
D	30	45
E	20	15
Total	170	255

The scores assigned to the customers' answers were the following:

L Class: 100(5 times), 99(3), 98(10), 95(15), 94(12), 93(1), 92 (8), 90(6), 89(3), 88(7), 85(13), 82(4), 80(6), 79(1), 78(1), 76(2), 75(3), 74(3), 73(1), 72(5), 70(4), 68(2), 63(2), 60(3), 59(5), 58(1), 57(2), 56(3), 55(4), 54(2), 53(1), 52(2), 51(2), 50(8), 48(7), 45(8), 42(1), 40(3), 35(1).

R Class: 100(7), 99(2), 98(3), 97(9), 95(18), 92(11), 91(4), 90(6), 88(12), 85(36), 82(8), 80(19), 78(9), 75(6), 70(17), 64(12), 60(16), 58(19), 56(3), 55(6), 50(17), 45(9), 40(6).

The above data can be summarized as it is shown in Table 1:

The evaluation of the above data (*verification* of the industry's decision about its new model) will be performed below using the above mentioned traditional methods.

3.1. Calculation of the Means

It is straightforward to calculate the means m_L and m_R of the scores of the customers' answers for the Luxury and the Regular Class respectively, which are $m_L \approx 76.006$ and $m_R \approx 75.09$. This means that all the customers were very satisfied with their new cars, with the customers who purchased the L Class being better satisfied than those who purchased the R Class.

3.2. Application of the GPA Index

We recall that the GPA index is a weighted mean where more importance is given to the higher scores achieved, to which greater coefficients (weights) are attached. In other words the GPA index focuses on the quality "performance" rather, than on the mean "performance" of a group of individuals.

For applying the GPA index on the data of our example let us denote by n_A, n_B, n_C, n_D and n_E the number of the industry's customers who belong to the above described categories A, B, C, D and E respectively and by n the total number of its customers. Then the GPA is calculated by the formula GPA = $\frac{n_D + 2n_C + 3n_B + 4n_A}{n}$. Obviously we have that $0 \leq$ GPA ≤ 4.

In our case, using the data of Table 1 it is easy to check that both the GPA's of the customers of the L Class and of the R Class are equal to $\frac{43}{17} \approx 2.529$. Thus, according to the GPA index, the industry's customers of the L Class and of the R Class were equally satisfied with their new cars.

4. Using Principles of FL for the Verification of a Chosen Decision

Here we refer again to the real example presented in the previous section. We shall use principles of FL to present the following two alternative methods for verifying the creditability of the chosen by the car industry decision:

4.1. The COG Method

This method is a special form of the commonly used in FL COG (or centroid) defuzzification technique. According to the COG technique the defuzzification of a fuzzy situation's data is succeeded through the calculation of the coordinates of the COG of the level's section contained between the graph of the membership function associated with this situation and the OX axis.

In earlier papers ([3], [6], etc) the COG technique has been properly adapted for use as an assessment method of the individuals' performance in several human activities. Here, using similar techniques, we shall adapt the COG technique for the verification of a chosen decision.

For this, let us consider as universal set of our discourse the set U = {A, B, C, D, E} of the industry's customers categories described in the previous section. We are going to represent the sets L and R of the customers who purchased the L Class and R Class respectively as fuzzy sets in U. For this, we define the membership function

$m: U \rightarrow [0, 1]$ for both sets L and R in terms of the frequencies, i.e. by $y = m(x) = \frac{n_x}{n}$, where n_x denotes the number of customers belonging to the category x in U and n denotes the total number of the customers of the corresponding set.

Then, from Table 1 it turns out that L and R can be written as fuzzy sets in U in the form[2]:

$$L=\{(A,\frac{6}{17}), (B,\frac{4}{17}), (C,\frac{2}{17}),(D,\frac{3}{17}), (E,\frac{2}{17})\} \quad (1)$$

and

$$R=\{(A,\frac{4}{17}), (B,\frac{6}{17}), (C,\frac{3}{17}), (D,\frac{3}{17}), (E,\frac{1}{17})\} \quad (2)$$

respectively.

Now, we correspond to each $x \in U$ an interval of values from a prefixed numerical distribution as follows: E $\rightarrow$ [0, 1), D $\rightarrow$ [1, 2), C $\rightarrow$ [2, 3), B $\rightarrow$ [3, 4), A $\rightarrow$ [4, 5]. This actually means that we replace U with a set of real intervals. Consequently, we have that $y_1 = m(x) = m(E)$ for all x in [0,1), $y_2 = m(x)$ = m(D} for all x in [1,2), y_3 = m(x) = m(C) for all x in [2, 3), y_4 = m(x) = m(B) for all x in [3, 4) and y_5 = m(x) = m(A) for all x in [4,5). Since the membership values of the elements of U *in* L and R have been defined in terms of the corresponding frequencies, we obviously have that

$$\sum_{i=1}^{5} y_i = m(A) + m(B) + m(C) + m(D) + m(E) = 1 \quad (3)$$

We are now in position to construct the graph of the membership function y = m(x), which has the form of the bar graph shown in Figure 2, wherefrom one can easily observe that the area of the level's section, say F, contained between the bar graph of y = m(x) and the OX axis is equal to the sum of the areas of the five rectangles F_i , i =1, 2, 3, 4, 5. The one side of each one of these rectangles has length 1 unit and lies on the OX axis.

From Mechanics it is well known that the coordinates (x_c, y_c) of the COG, say F_c, of the level's section F can be calculated by the formulas:

$$x_c = \frac{\iint_F x\,dxdy}{\iint_F dxdy}, \quad y_c = \frac{\iint_F y\,dxdy}{\iint_F dxdy} \quad (4)$$

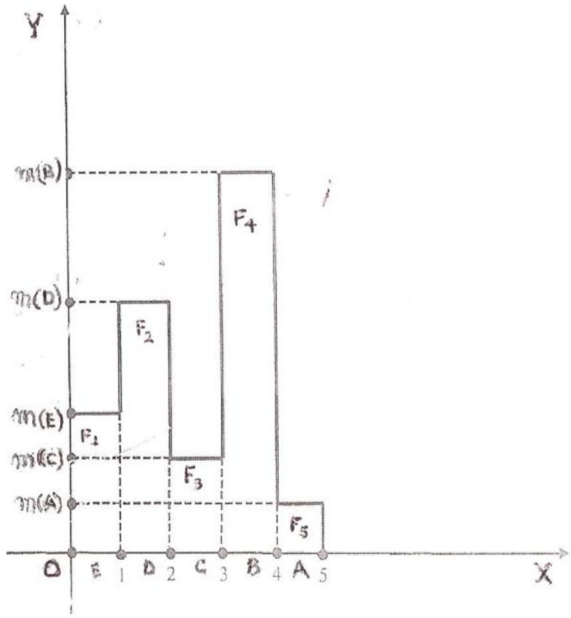

Figure 2. Bar graphical data representation

Taking into account the data represented by Figure 2 and equation (3) it is straightforward to check (e.g. see section 3 of [6]) that in this case formulas (4) can be transformed to the form:

$$x_c = \frac{1}{2}\left(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5\right), \qquad (5)$$
$$y_c = \frac{1}{2}\left(y_1^{\,2} + y_2^{\,2} + y_3^{\,2} + y_4^{\,2} + y_5^{\,2}\right)$$

Then, using elementary algebraic inequalities it is easy to check that there is a unique minimum for y_c corresponding to the COG $F_m(\frac{5}{2}, \frac{1}{10})$ ([6], section 3). Further, the ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from the first of formulas (5) we find that $x_c = \frac{9}{2}$ and $y_c = \frac{1}{2}$. Therefore the COG in this case is the point $F_i(\frac{9}{2}, \frac{1}{2})$. On the other hand the worst case is when $y_1=1$ and $y_2=y_3=y_4= y_5=0$. Then from the first of formulas (5) we find that the COG in this case is the point $F_w(\frac{1}{2}, \frac{1}{2})$. Therefore the COG F_c of the level's section F lies in the area of the triangle $F_w F_m F_i$.

Then by elementary geometric observations ([9], section 3) one can obtain the following criterion:

- *Between the two groups of the industry's customers the group with the biggest x_c corresponds to the customers who are better satisfied with their new cars.*

2 We recall that a fuzzy set can be symbolically written in several forms, e.g. as a symbolic sum (see section 2), as a set of ordered pairs (see above), etc.

- *If the two groups have the same $x_c \geq 2.5$, then the group with the higher y_c corresponds to the customers who are better satisfied with their new cars.*
- *If the two groups have the same $x_c < 2.5$, then the group with the lower y_c corresponds to the customers who are better satisfied with their new cars.*

Substituting in formulas (5) the values of y_i's taken from forms (1) and (2) of the fuzzy sets L and R respectively it is straightforward to check that the coordinate x_c of the COG for both L and R is equal to $\frac{103}{34} \approx 3.029 > 2.5$. However, the coordinate y_c is equal to $\frac{69}{578}$ for L and to $\frac{71}{578}$ for R. Therefore, according to the above stated criterion, the customers who purchased the R Class were better satisfied with their new cars than those who purchased the L class.

4.2. The Triangular Fuzzy Model (TFM)

The TFM is actually a variation of the above presented COG method. In the initial version of TFM developed in earlier papers [4-5] the individuals under assessment were divided in three assessment categories (A, B, C-E). In the improved version that we will develop here these categories are increased to five. In this way the model becomes more accurate.

The main idea of TFM is the replacement of the rectangles appearing in the graph of the membership function of the COG method (Figure 2) by isosceles triangles sharing common parts (Figure 3).

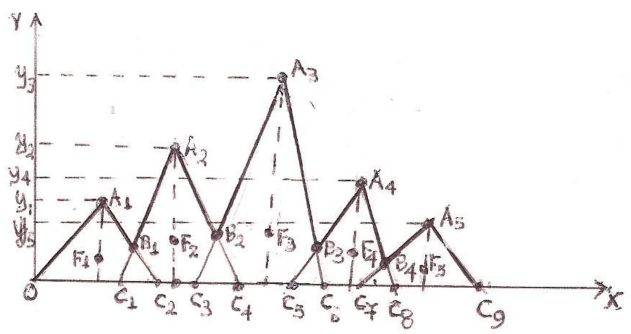

Figure 3. The TFM's scheme

Therefore, in the TFM's scheme (Figure 3) we have five such triangles, each one of them corresponding to a customer's category (E, D, C, B and A respectively). Without loss of generality and for making our calculations easier we consider isosceles triangles with bases of length 10 units lying on the OX axis. The height to the base of each triangle is equal to the percentage of the industry's customers who belong to the corresponding category. We allow for any two adjacent triangles to have 30% of their bases belonging to both of them. In this way we cover the ambiguous cases of the industry's customers being at the limits between two successive categories (e.g. between A and B).

The groups L and R of the customers who purchased the L Class and R Class respectively can be represented again, as we did in the COG method, as fuzzy set in *U*, whose membership function y=m(x) has as graph the line $OA_1B_1A_2B_2A_3$ B_3A_4 $B_4A_5C_9$ of Figure 3. It is easy to calculate the coordinates (b_{i1}, b_{i2}) of the points B_i, i = 1, 2, 3, 4, 5. In fact, B_1 is the intersection of the straight line segments A_1C_2, C_1A_2, B_2 is the intersection of C_3A_3, A_2C_4 and so on. Therefore, it is straightforward to determine the analytic form of y=m(x) consisting of 10 branches, corresponding to the equations of the straight lines OA_1, A_1B_1, B_1A_2, A_2B_2, B_2A_3, A_3B_3, B_3A_4, A_4B_4, B_4A_5 and A_5C_9 in the intervals [0, 5), [5, b_{11}), [b_{11}, 12), [12, b_{21}), [b_{21}, 19), [19, b_{31}), [b_{31}, 26), [26. b_{41}), [b_{41}, 33) and [33, 38] respectively.

However, in applying the TFM the use of the analytic form of y = m(x) is not needed (in contrast to the COG method) for the calculation of the COG of the resulting area. In fact, since the marginal cases of the customers' categories should be considered as common parts for any pair of the adjacent triangles, it is logical to not subtract the areas of the intersections from the area of the corresponding level's section, although in this way we count them twice; e.g. placing the ambiguous cases B+ and A- in both regions B and A. In other words, the COG method, which calculates the coordinates of the COG of the area between the graph of the membership function and the OX axis (see Figure 3), thus considering the areas of the "common" triangles $C_1B_1C_2$, $C_3B_2C_4$, $C_5B_3C_6$ and $C_7B_4C_8$ only once, is not the proper one to be applied in the above situation.

Indeed, in this case it is reasonable to represent each one of the five triangles OA_1C_2, $C_1A_2C_4$, $C_3A_3C_6$, $C_5A_4C_8$ and $C_7A_5C_9$ of Figure 3 by their COG's F_i, i=1, 2, 3, 4, 5 and to consider the entire area defined in this way as the system of these points-centers. More explicitly, the steps of the whole construction of the TFM are the following:

1. Let y_i, i=1,2,3,4,5 be the percentages of the industry's customers belonging to the categories E, D, C, B, and A respectively; then $\sum_{i=1}^{5} y_i$ =1 (100%).
2. We consider the isosceles triangles with bases having lengths of 10 units each and their heights being equal to y_i, i=1,2,3,4,5 in the way that has been illustrated in Figure 3. Each pair of adjacent triangles has common parts in the base with length 3 units.
3. We calculate the coordinates (x_{c_i}, y_{c_i}) of the COG F_i, i=1,2,3, 4, 5 of each triangle as follows: The COG of a triangle is the point of intersection of its medians, and since this point divides the median in proportion 2:1 from the vertex, we find, taking also into account that the triangles are isosceles, that $y_{c_i} = \frac{1}{3} y_i$. Also, since the triangles' bases have a length of 10 units, we observe that $x_{ci}=7i-2$.
4. We consider the system of the centers F_i, i=1, 2, 3 and we calculate the coordinates (X_c, Y_c) of the COG F of

the whole area S considered in Figure 3 by the following formulas, derived from the commonly used in such cases definition:

$$X_c = \frac{1}{S}\sum_{i=1}^{5} S_i x_{c_i}\ ,\ Y_c = \frac{1}{S}\sum_{i=1}^{5} S_i y_{c_i} \qquad (6)$$

In formulas (6) Si, i= 1, 2, 3, 4, 5 denote the areas of the corresponding triangles. Therefore $Si = 5y_i$ and $S = \sum_{i=1}^{5} S_i = 5\sum_{i=1}^{5} y_i = 5$. Thus, from formulas (6) we finally get that

$$X_c = \frac{1}{5}\sum_{i=1}^{5} 5y_i(7i-2) = (7\sum_{i=1}^{5} iy_i) - 2 \qquad (7)$$

$$Y_c = \frac{1}{5}\sum_{i=1}^{5} 5y_i\left(\frac{1}{3}y_i\right) = \frac{1}{3}\sum_{i=1}^{5} y_i^2 .$$

5. We determine the area where the COG F lies as follows: For i, j=1, 2, 3, 4, 5, we have that $0 \le (y_i - y_j)^2 = y_i^2 + y_j^2 - 2y_iy_j$, therefore $y_i^2 + y_j^2 \ge 2y_iy_j$, with the equality holding if, and only if, $y_i = y_j$. Therefore

$$1 = (\sum_{i=1}^{5} y_i)^2 = \sum_{i=1}^{5} y_i^2 + 2\sum_{\substack{i,j=1,\\ i\neq j}}^{5} y_iy_j \le \sum_{i=1}^{5} y_i^2 + 2$$

$$\sum_{\substack{i,j=1,\\ i\neq j}}^{5} (y_i^2 + y_j^2) = 5\sum_{i=1}^{5} y_i^2 \text{ or } \sum_{i=1}^{5} y_i^2 \ge \frac{1}{5} \qquad (8)$$

with the equality holding if, and only if, $y_1 = y_2 = y_3 = y_4 = y_5 = \frac{1}{5}$. In the case of equality the first of formulas (7) gives that $X_c = 7(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5}) - 2 = 15$. Further, combining the inequality (8) with the second of formulas (7) one finds that $Y_c \ge \frac{1}{15}$ Therefore the unique minimum for Y_c corresponds to the COG $F_m(15, \frac{1}{15})$.

The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (7) we get that $X_c = 33$ and $Y_c = \frac{1}{3}$.Therefore the COG in this case is the point $F_i(33, \frac{1}{3})$. On the other hand, the worst case is when $y_1=1$ and $y_2=y_3=y_4=y_5=0$. Then from formulas (2), we find that the COG is the point $F_w(5, \frac{1}{3})$. Therefore the area where the centre of gravity F_c lies is the area of the triangle $F_w F_m F_i$. (Figure 4)

6. We formulate our assessment criterion as follows: From elementary geometric observations (Figure 4) it follows that for the two groups of the industry's customers the group having the greater X_c corresponds to the customers who are better satisfied with their new cars. Further, if the two groups have the same $X_c \ge 15$, then the group having the COG which is situated closer to Fi is the group with the higher Y_c. Also, if the two groups have the same $X_c<15$, *then* the group having the COG which is situated farther to Fw is the group with the lower Y_c.

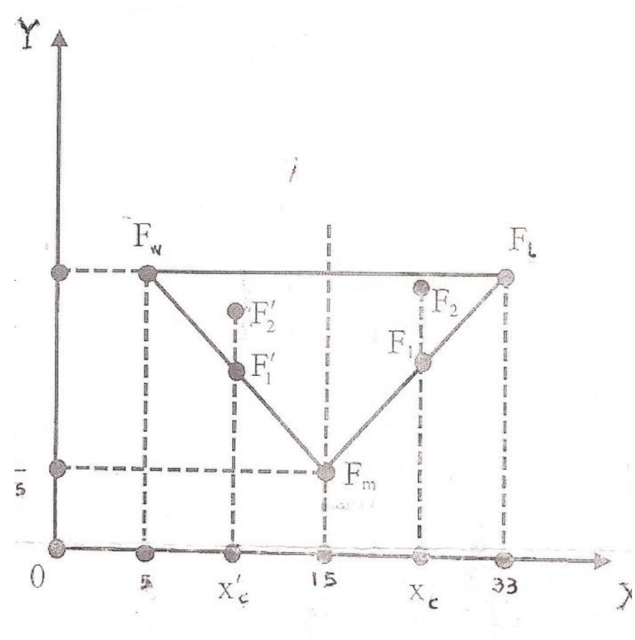

Figure 4. The area where the COG lies

Based on the above considerations it is logical to formulate our criterion for comparing the two groups in the following form:

- *Between the two groups of the industry's customers the group with the biggest X_c corresponds to the customers who are better satisfied with their new cars.*
- *If the two groups have the same $X_c \ge 15$, then the group with the higher Y_c corresponds to the customers who are better satisfied with their new cars.*
- *If the two groups have the same $X_c < 15$, then the group with the lower Y_c corresponds to the customers who are better satisfied with their new cars.*

Substituting in formulas (7) the values of y_i's taken from the forms (1) and (2) of the fuzzy sets L and R respectively (section 4.1) it is straightforward to check that the coordinate X_c of the COG for both L and R is equal to $\frac{396}{17} \approx 23.294>15$. However, the coordinate Y_c is equal to $\frac{69}{867}$ for L and to $\frac{71}{867}$ for R. Therefore, according to the above stated criterion, the customers who purchased the R Class were better satisfied with their new cars than those who purchased the L Class.

4.3. Comparison of the Methods Applied

In sections 3 and 4 we applied four different methods for verifying the creditability of the car industry's decision about its new model. According to the outcomes of all these methods, the above decision was proved to be satisfactory. In fact, the means in the first method were greater than 75 (very satisfied customers), while the value 2.529 of the GPA index in the second method is close enough to its maximum possible value 4. Also the values of the abscissa of the COG method and the TFR (3.029 and 23.294 respectively) are close enough to the corresponding values of the ideal case (4.5 and 33 respectively).

However, differences appeared among the outcomes of the above methods concerning the degree of satisfaction of the customers who purchased the L Class and the R Class respectively. In fact, the calculation of the means demonstrated a slightly higher degree of satisfaction for the customers of the L Class, while according to the GPA index the customers were equally satisfied in both cases. On the contrary, the two FL methods (COG and TFM) demonstrated a slightly higher degree of satisfaction for the customers of the R Class

The above differences are not embarrassing, because, in contrast to the calculation of the means which focuses on the mean behaviour (performance) of the two groups of customers, the other three methods (GPA, COG and TFM) focus on their quality behaviour by assigning greater weight coefficients to the higher scores achieved by the customers.

In fact, the formula calculating the GPA index (section 3.2) can be written in our case in the form

$$\text{GPA}= 0y_1+y_2+2y_3+3y_4+4y_5 \tag{9}$$

Further, since in COG and TFM the behaviour of a group is assessed by the value of the abscissa of the corresponding COG, observing the first of formulas (5) and (7) and formula (9) we can form the following Table:

Table 2. Weight coefficients of the y_i's

y_i	GPA	COG (x_c)	TFM (X_c)
y_1	0	1/2	7
y_2	1	3/2	14
y_3	2	5/2	21
y_4	3	7/2	28
y_5	4	9/2	35

From Table 2 becomes evident that TFM assigns greater coefficients to the higher with respect to the lower scores than COG and also COG does the same thing with respect to GPA. In other words TFM is more sensitive than COG, and COG is more sensitive than GPA for assessing the quality behaviour of a group.

In concluding, it is suggested to the user to choose among the above methods the one that fits better to its personal criteria of goals.

5. Discussion

The first two of the metods applied above for verifying the creditability of a chosen decision (calculation of the means and GPA index) are based on principles of the classical (bivalent) logic, while the other two methods (COG and TFM) are based on principles of FL. The TFM is a recently developed variation of the COG method. Consequently, there is a need to be applied in more real examples of decision problems in future for obtaining safer conclusions about its advantages/disadvantages with respect to the COG method. This is among the priorities of our future research plans.

On the other hand, since the TFM approach appears to have the potential of a general assessment method, our future research plans include also the effort of applying this approach in assessing the individuals' performance in various other human activities.

6. Conclusions

Two are the innovations of this paper: First we developed an improved version of the TFM for verifying the creditability of a chosen decision .and second it was shown that TFM is more sensitive to the higher scores than the other assessment methods used.

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