

Couette Flow of Third Grade Fluid between Parallel Porous Plates

A. Y. Gital^{1,2,*}, M. Abdulhameed³, C. Haruna⁴, M. S. Adamu², B. M. Abdulhamid², A. S. Aliyu⁵

¹Department of Computer Science, Universiti Teknologi Malaysia

²Mathematical Sciences Programme, Abubakar Tafawa University Bauchi-Nigeria

³Department of Mathematics, Universiti Tun Hussein Onn, Malaysia

⁴Artificial Intelligence Department, University of Malaya, Kuala Lumpur-Malaysia

⁵Department of Physics, Nassarawa State University, Keffi-Nigeria

Abstract In this Paper, the analytical solutions for the velocity and temperature profiles have been obtained in explicit forms for the Couette flow of a third grade fluid between two porous parallel plates. The obtained velocity and temperature profile are compared with the existing analytical solutions for a third grade fluid between parallel non-porous plates. It is found that the present results are in excellent agreement with the existing analytical solutions. Graphs representing the solutions are discussed, and appropriate conclusions drawn.

Keywords Third grade fluid, Porous plate, Analytical solution

1. Introduction

The present study deals with the problem of heat transfer for the Couette flow of a third grade fluid between two parallel porous plates. Analytical solutions for the velocity and temperature profiles have been obtained in explicit forms. The related studies in the recent years dealing with third grade fluids are [1-5].

Many models described by nonlinear differential equations of real life problems are difficult to solve either analytically or numerically. Much effort have been made in the literature for different techniques to make the nonlinear model solvable. Among the methods that have been employed to solve nonlinear models are related to Adomian decomposition method (ADM) [6], Variation iterative method (VIM) [7], spectral collocation method [8], homotopy analysis method (HAM) [9], homotopy perturbation method (HPM) [10], optimal homotopy asymptotic method (OHAM) [11], etc.

In the present investigation, we construct analytical approximate solutions of the governing problem of heat transfer for the Couette flow of a third grade between two parallel porous plates using optimal homotopy asymptotic method (OHAM). The results reveal that the OHAM is valid not only for weakly nonlinear equations, but also for strongly nonlinear ones. Effects of the viscoelastic parameter and the suction/injection on velocity and temperature profiles are

shown graphically.

2. Mathematical Model

For Couette flow problem, the lower plate is stationary and the upper plate is moving with a constant velocity U_0 and the effect constant pressure gradient is neglected. The movement of fluid is solely due to motion of the upper plate. The fluid is brought into motion through the action of viscous stress at the plate. The temperature of the lower plate is maintained at Θ_0 and that of the upper plate at Θ_1 . Mathematical model of the present flow problem are formulated as below:

$$-\rho W_0 \frac{du}{dy} = \mu \frac{d^2 u}{dy^2} - \alpha_1 W_0 \frac{d^3 u}{dy^3} + \beta_1 W_0^2 \frac{d^4 u}{dy^4} + 6\beta_2 \left(\frac{du}{dy} \right)^2 \frac{d^2 u}{dy^2}, \quad (1)$$

$$-\rho c_p W_0 \frac{d\Theta}{dy} = k \frac{d^2 \Theta}{dy^2} + \mu \left(\frac{du}{dy} \right)^2 - \alpha_1 W_0 \frac{d^2 u du}{dy^2 dy} + \beta_1 W_0^2 \frac{d^3 u du}{dy^3 dy} + 2\beta_2 \left(\frac{du}{dy} \right)^4, \quad (2)$$

subject to the boundary condition

* Corresponding author:

abdulsalamgital@yahoo.com (A. Y. Gital)

Published online at <http://journal.sapub.org/ajcam>

Copyright © 2014 Scientific & Academic Publishing. All Rights Reserved

$$u(-h) = 0, \quad u(h) = U_0, \quad (3)$$

$$\Theta(-h) = \Theta_0, \quad \Theta(h) = \Theta_1, \quad (4)$$

Where $\alpha_1, \beta_1, \beta_2$ are the material moduli, w_0 is the suction/injection parameter, u fluid velocity, Θ is the fluid temperature, μ is the fluid viscosity, ρ is the fluid density, c_p is the specific heat capacity and k is the thermal conductivity.

Introducing the dimensionless parameters

$$\begin{aligned} u^* &= \frac{u}{U_0}, \quad W_0^* = \frac{W_0}{U_0}, \quad y^* = \frac{U_0 y}{\nu}, \quad \alpha_1^* = \frac{\alpha_1 U_0^2}{\rho \nu^2}, \\ \Theta^* &= \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0}, \quad \beta_1^* = \frac{\beta_1 U_0^4}{\rho \nu^3}, \quad \beta^* = \frac{6(\beta_2 + \beta_3) U_0^4}{\rho \nu^3}. \end{aligned} \quad (5)$$

the non-dimensional problem, after dropping the asterisks, Eq. (1), (2) and boundary conditions (3) and (4) becomes

$$\frac{d^2 u}{dy^2} + W_0 \frac{du}{dy} - \alpha_1 W_0 \frac{d^3 u}{dy^3} + \beta_1 W_0^2 \frac{d^4 u}{dy^4} + \beta \left(\frac{du}{dy} \right)^2 \frac{d^2 u}{dy^2} = 0 \quad (6)$$

$$\frac{d^2 \Theta}{dy^2} + \text{Pr} W_0 \frac{d\Theta}{dy} + \left[\left(\frac{du}{dy} \right)^2 - \alpha_1 W_0 \frac{d^2 u du}{dy^2 dy} + \beta_1 W_0^2 \frac{d^3 u du}{dy^3 dy} + \beta \left(\frac{du}{dy} \right)^4 \right] = 0 \quad (7)$$

$$u(-1) = 0, \quad u(1) = 1, \quad (8)$$

$$\Theta(-1) = 0, \quad \Theta(1) = 1, \quad (9)$$

where $\lambda = \frac{\mu c_p}{k} \frac{U_0^2}{c_p (\Theta_1 - \Theta_0)} = \text{Pr} Ec$, Pr is the Prandtl number, Ec is the Eckert number and λ is the Brinkman number.

3. Basic Idea of the Optimal Homotopy Asymptotic Method (OHAM)

We classifying the equations to solved for velocity u and temperature Θ :

$$\begin{aligned} L_1(u(y)) + N_1(u(y)) + g_1(y) &= 0, \\ L_2(\Theta(y)) + N_2(\Theta(y)) + g_2(y) &= 0, \\ B(u, \Theta) &= 0. \end{aligned} \quad (10)$$

where $L_j (j=1-2)$, are linear operator, $N_j (j=1-2)$ are non-linear operator, $g_j (j=1-2)$ are known function and B is a boundary operator.

By means of OHAM we first construct an optimal homotopy $\phi(y, q) : \mathbb{R} \times [0, 1] \rightarrow \mathbb{R}$ which satisfies the following equations:

$$\begin{aligned} (1-q) [L_1(\phi_1(y, q)) + g_1(y)] &= H(q) [L_1(\phi_1(y, q)) + g_1(y) + N_1(\phi_1(y, q))] \\ B_1(\phi_1(y, q)) &= 0, \end{aligned} \quad (11)$$

$$(1-q)\left[L_2(\phi_2(y,q)) + g_2(y)\right] = H(q)\left[L_2(\phi_2(y,q)) + g_2(y) + N_2(\phi_2(y,q))\right] \\ B_2(\phi_2(y,q)) = 0. \quad (12)$$

where $y \in \mathbb{R}$ and $0 \leq q \leq 1$ is an embedding parameter, $H(q)$ is a nonzero auxiliary function for $q \neq 0$ and $H(0) = 0$, and $\phi_j(y, q)$ ($j = 1-2$) are unknown functions. Clearly, when $q = 0$ and $q = 1$ it holds that

$$\phi_1(y, 0) = u_0(y), \quad \phi_2(y, 0) = \Theta_0(y), \quad (13)$$

$$\phi_1(y, 1) = u(y), \quad \phi_2(y, 1) = \Theta(y). \quad (14)$$

Choose the auxiliary function $H(y, q)$ in the form

$$H(q) = qC_1 + q^2C_2 + \dots \quad (15)$$

where $C_1, C_2 \dots C_n$ are constants to be determined. Construct a Taylor's series solution of equations ϕ_j in the form

$$\phi_j(y, q, C_i) = v_j(y) + \sum_{k=1}^{\infty} v_{jk}(y, C_i) q^k, \quad j = 1-2. \text{ and } i = 1, 2, \dots, n \quad (16)$$

where v_1 represent u and v_2 represent Θ . Substituting Eq. (16) into Eqs. (11) - (14) and collecting the same powers of q , and equating each coefficient of q to zero, we obtain

$$L_j(v_{j0}(y) + g_j(y)) = 0 \quad B(v_{j0}) = 0, \quad (17)$$

$$L_j(v_{j1}(y)) = C_1 N_{j0}(v_{j0}(y)), \quad B(v_{j1}) = 0, \quad (18)$$

$$L_j(v_k(y)) - L_j(v_{k-1}(y)) = C_k N_{j0}(v_{j0}(y)) \\ + \sum_{i=1}^{k-1} C_i \left[L_j(v_{k-i}(y)) + N_{j(k-i)}(v_{j0}(y), v_{j1}(y), \dots, v_{k-i}(y)) \right]. \quad (19)$$

$$B(v_k) = 0; \quad k = 2, 3, 4, \dots$$

where $N_{j(k-i)}(v_{j0}(y), v_{j1}(y), \dots, v_{k-i}(y))$ is the coefficient of q^{k-i} , obtained by expanding $N_j(v_j(y))$ in series with respect to the embedding parameter q .

$$N_j(u_j(y)) = N_{j0}(v_{j0}(y)) + qN_{j1}(v_{j0}(y), v_{j1}(y)) + q^2N_{j2}(v_{j0}(y), v_{j1}(y), v_{j2}(y)) \dots \\ = N_{j0}(v_{j0}(y)) + \sum_{k \geq 1} N_{jk}(v_{j0}(y), v_{j1}(y), v_{j2}(y), \dots, v_{jk}(y)) q^k \quad (20)$$

The solution of equations (31) and (32) can be approximately obtained in the form

$$u_j^m(y, q, C_i) = v_{j0}(y) + \sum_{k=1}^m v_{jk}(y, C_i), \quad (21)$$

$$\Theta_j^m(y, q, C_i) = v_{j0}(y) + \sum_{k=1}^m v_{jk}(y, C_i). \quad (22)$$

The residual $R_j(y, C_i)$ follows as

$$R_j(y, C_i) = L_j(u_j^m(y)) + g_j(y) + N_j(u_j^m(y), \Theta(y)). \quad (23)$$

If $R_j(y, C_i) = 0$, then $u_j^m(y, C_i)$ will be the exact solution. Generally such a case will not happen for nonlinear problems, but we can minimize the functional by the method of least squares

$$J_j(C_i) = \int_a^b R_j^2(y, C_i) dy. \quad (24)$$

where a and b belong to the domain of the problem. Finally, the unknown constants C_i ($i = 1, 2, \dots, m$) can be optimally identified from the conditions

$$\frac{\partial J_j(C_i)}{\partial C_1} = \frac{\partial J_j(C_i)}{\partial C_2} = \dots = \frac{\partial J_j(C_i)}{\partial C_m} = 0. \quad (25)$$

with these known values of C_i ($i = 1, 2, \dots, m$), the approximate solutions are well determined.

4. Solutions for Velocity and Temperature

Choosing the linear and nonlinear operators defined by

$$L_j(\phi_j(y, q)) = \frac{\partial \phi_j(y, q)}{\partial y^2}, \quad (26)$$

$$N_1(\phi_j(y, q)) = W_0 \frac{du}{dy} - \alpha_1 W_0 \frac{d^3 u}{dy^3} + \beta_1 W_0^2 \frac{d^4 u}{dy^4} + \beta \left(\frac{du}{dy} \right)^2 \frac{d^2 u}{dy^2}, \quad (27)$$

$$N_2(\phi_j(y, q)) = \text{Pr} W_0 \frac{d\Theta}{dy} + \lambda \left[\left(\frac{du}{dy} \right)^2 - \alpha_1 W_0 \frac{d^2 u}{dy^2} \frac{du}{dy} + \beta_1 W_0^2 \frac{d^3 u}{dy^3} \frac{du}{dy} + \beta \left(\frac{du}{dy} \right)^4 \right], \quad (28)$$

The correspond boundary conditions are

$$\phi_1(-1) = 0, \quad \phi_1(1) = 1, \quad (29)$$

$$\phi_2(-1) = 0, \quad \phi_2(1) = 1, \quad (30)$$

The zeroth order deformation:

$$\frac{d^2 u_0}{dy^2} = 0, \quad u_0(-1) = 0, \quad u_0(1) = 1, \quad (31)$$

$$\frac{d^2 \Theta_0}{dy^2} = 0, \quad \Theta_0(-1) = 0, \quad \Theta_0(1) = 1, \quad (32)$$

The first order deformation:

$$\frac{d^2 u_1}{dy^2} - \frac{d^2 u_0}{dy^2} = C_1 \left[\frac{d^2 u_0}{dy^2} + W_0 \frac{du_0}{dy} - \alpha_1 W_0 \frac{d^3 u_0}{dy^3} + \beta_1 W_0^2 \frac{d^4 u_0}{dy^4} + \beta \left(\frac{du_0}{dy} \right)^2 \frac{d^2 u_0}{dy^2} \right], \quad (33)$$

$$u_1(-1) = 0, \quad u_1(1) = 0,$$

$$\begin{aligned} & \frac{d^2 \Theta_1}{dy^2} - \frac{d^2 \Theta_0}{dy^2} \\ &= C_1 \left\{ \frac{d^2 \Theta_0}{dy^2} + W_0 \text{Pr} \frac{d\Theta_0}{dy} + \lambda \left[\left(\frac{du_0}{dy} \right)^2 - \alpha_1 W_0 \frac{d^2 u_0}{dy^2} \frac{du_0}{dy} + \beta_1 W_0^2 \frac{d^3 u_0}{dy^3} \frac{du_0}{dy} + \beta \left(\frac{du_0}{dy} \right)^4 \right] \right\}, \quad (34) \end{aligned}$$

$$\Theta_1(-1) = 0, \quad \Theta_1(1) = 0,$$

The second order deformation:

$$\begin{aligned} \frac{d^2 u_2}{dy^2} - \frac{d^2 u_1}{dy^2} &= C_1 \left\{ \frac{d^2 u_1}{dy^2} + W_0 \frac{du_1}{dy} - \alpha_1 W_0 \frac{d^3 u_1}{dy^3} + \beta \left[2 \frac{du_0}{dy} \frac{d^2 u_0}{dy^2} \frac{du_1}{dy} + \left(\frac{du_0}{dy} \right)^2 \frac{d^2 u_1}{dy^2} \right] \right\} \\ &+ C_2 \left\{ \frac{d^2 u_0}{dy^2} + W_0 \frac{du_0}{dy} - \alpha_1 W_0 \frac{d^3 u_0}{dy^3} + \beta_1 W_0^2 \frac{d^4 u_0}{dy^4} + \beta \left(\frac{du_0}{dy} \right)^2 \frac{d^2 u_0}{dy^2} \right\} \quad (35) \end{aligned}$$

$$u_2(-1) = 0, \quad u_2(1) = 0,$$

$$\begin{aligned} & \frac{d^2 \Theta_2}{dy^2} - \frac{d^2 \Theta_1}{dy^2} \\ &= C_1 \left\{ \frac{d^2 \Theta_1}{dy^2} + \text{Pr} W_0 \frac{d\Theta_1}{dy} + \lambda \left[2 \frac{du_0}{dy} \frac{du_1}{dy} - \alpha_1 W_0 \left(\frac{d^2 u_0}{dy^2} \frac{du_1}{dy} + \frac{d^2 u_1}{dy^2} \frac{du_0}{dy} \right) \right. \right. \\ & \quad \left. \left. + \beta_1 W_0^2 \left(\frac{d^3 u_0}{dy^3} \frac{du_1}{dy} + \frac{d^3 u_1}{dy^3} \frac{du_0}{dy} \right) + 4\beta \left(\frac{du_0}{dy} \right)^3 \frac{du_1}{dy} \right] \right\} \quad (36) \\ &+ C_2 \left\{ \frac{d^2 \Theta_0}{dy^2} + \text{Pr} W_0 \frac{d\Theta_0}{dy} + \lambda \left[\left(\frac{du_0}{dy} \right)^2 - \alpha_1 W_0 \frac{d^2 u_0}{dy^2} \frac{du_0}{dy} + \beta_1 W_0^2 \frac{d^3 u_0}{dy^3} \frac{du_0}{dy} + \beta \left(\frac{du_0}{dy} \right)^4 \right] \right\}, \end{aligned}$$

$$\Theta_2(-1) = 0, \quad \Theta_2(1) = 0.$$

By using the widely applied symbolic computational software MATHEMATICA. It is found that the solutions of Eqs. (31) - (36) are:

$$u_0 = \frac{1}{2}(1+y), \quad (37)$$

$$\Theta_0 = \frac{1}{2}(1+y), \quad (38)$$

$$u_1 = \frac{1}{4} C_1 W_0 (-1 + y^2), \quad (39)$$

$$\Theta_1 = \frac{1}{32} C_1 (-1 + y^2) (8 \text{Pr} + 4\lambda + \beta\lambda), \quad (40)$$

$$u_2 = \frac{1}{48} W_0 (-1 + y^2) (12C_1 + 12C_1^2 + 12C_2 + 4W_0 C_1^2 y + 3C_1^2 \beta), \quad (41)$$

$$\begin{aligned} \Theta_2 = & \frac{1}{96} (-1 + y^2) (24 \text{Pr} C_1 W_0 + 24 \text{Pr} C_1^2 W_0 + 24 \text{Pr} C_2 W_0 + 8 \text{Pr}^2 C_1 W_0^2 y \\ & + 12C_1 \lambda + 12C_1^2 \lambda + 12C_2 \lambda + 8C_1^2 W_0 \lambda y + 4C_1^2 \text{Pr} W_0 \lambda y - 12C_1^2 W_0^2 \alpha \lambda \\ & + 3C_1 \beta \lambda + 3C_1^2 \beta \lambda + 3C_2 \beta \lambda + 4C_1^2 W_0 \beta \lambda y + C_1^2 \text{Pr} W_0 \beta \lambda y). \end{aligned} \quad (42)$$

In view of Eq. (41), the second order approximation solution ($m = 2$) for equations (15) and (16):

$$\begin{aligned} u^{(2)} = & \frac{1}{48} (y+1) (24 - 24C_1 W_0 - 12C_1^2 W_0 - 12C_2 W_0 + 24C_1 W_0 y \\ & + 12C_1^2 W_0 y + 12C_2 W_0 y - 4C_1^2 W_0^2 y + 4C_1^2 W_0^2 y^2 - 3C_1^2 W_0 \beta + 3C_1^2 W_0 \beta y). \end{aligned} \quad (43)$$

$$\begin{aligned} \Theta^{(2)} = & \frac{1}{96} (1+y) (48 - 48C_1 \text{Pr} W_0 - 24C_1^2 \text{Pr} W_0 - 24C_2 \text{Pr} W_0 \\ & + 48C_1 \text{Pr} W_0 y + 24C_1^2 \text{Pr} W_0 y + 24C_2 \text{Pr} W_0 y - 8C_1^2 (\text{Pr})^2 W_0^2 y \\ & + 8C_1^2 \text{Pr}^2 W_0^2 y^2 - 24C_1 \lambda - 12C_1^2 \lambda - 12C_2 \lambda + 24C_1 \lambda y \\ & + 12C_1^2 \lambda y + 12C_2 \lambda y - 8C_1^2 W_0 \lambda y - 4C_1^2 \text{Pr} W_0 \lambda y + 8C_1^2 W_0 \lambda y^2 \\ & + 4C_1^2 \text{Pr} W_0 \lambda y^2 + 12C_1^2 W_0^2 \lambda \alpha - 12C_1^2 W_0^2 \lambda \alpha y - 6C_1 \beta \lambda - 3C_1^2 \beta \lambda \\ & - 3C_2 \beta \lambda + 6C_1 \beta \lambda y + 3C_1^2 \beta \lambda y + 3C_2 \beta \lambda y - 4C_1^2 \beta \lambda W_0 y \\ & - C_1^2 \text{Pr} W_0 \beta \lambda y + 4C_1^2 W_0 \beta \lambda y^2 + C_1^2 \text{Pr} W_0 \beta \lambda y^2). \end{aligned} \quad (44)$$

Using Eq. (3), these result into the following residual

$$R_1 = \frac{d^2 u^{(2)}}{dy^2} + W_0 \frac{du^{(2)}}{dy} - \alpha_1 \frac{d^3 u^{(2)}}{dy^3} + \beta_1 \frac{d^4 u^{(2)}}{dy^4} + \beta \left(\frac{du^{(2)}}{dy} \right)^2 \frac{d^2 u^{(2)}}{dy^2}, \quad (45)$$

$$\begin{aligned} R_2 = & \frac{d^2 \Theta^{(2)}}{dy^2} + \text{Pr} W_0 \frac{d\Theta^{(2)}}{dy} \\ & + \left[\lambda \left(\frac{du^{(2)}}{dy} \right)^2 - \alpha_1 W_0 \frac{d^2 u^{(2)}}{dy^2} \frac{du^{(2)}}{dy} + \beta_1 W_0^2 \frac{d^3 u^{(2)}}{dy^3} \frac{du^{(2)}}{dy} + \beta \left(\frac{du^{(2)}}{dy} \right)^4 \right]. \end{aligned} \quad (46)$$

and the functional $J(C_1, C_2)$ is defined as

$$J_1 = \int_{-1}^1 R_1^2 dy, \quad (47)$$

$$J_2 = \int_{-1}^1 R_2^2 dy. \quad (48)$$

The unknown constant C_1 and C_2 can be calculated from the conditions

$$\frac{dJ_1}{dC_1} = \frac{dJ_1}{dC_2} = 0, \quad (49)$$

$$\frac{dJ_2}{dC_1} = \frac{dJ_2}{dC_2} = 0. \quad (50)$$

The unknown constant C_1 and C_2 can be calculated from the conditions

$$\frac{dJ_1}{dC_1} = \frac{dJ_1}{dC_2} = 0, \quad (51)$$

$$\frac{dJ_2}{dC_1} = \frac{dJ_2}{dC_2} = 0. \quad (52)$$

(a) For fluid velocity, we take $\alpha = 0.5$. Using the condition (25), we obtain the values

$$C_1 = - \frac{0.0833333 [0.010097v_0^4 + 0.00252425v_0^4\beta + 0.1v_0^3(4+\beta)^2 + 0.00202132v_0^2(4+\beta)^3 + 0.1v_0(4+\beta)^4 + 0.0000732422(4+\beta)^5]}{[+0.0000243841v_0^6 + 0.1v_0^5 + 0.1v_0^5\beta + 0.000157766v_0^4(4+\beta)^2 + 0.1v_0^3(4+\beta)^3 + 0.0000631665v_0^2(4+\beta)^4 + 0.1v_0(4+\beta)^5 + 1.52588 \times 10^{-6}(4+\beta)^6]} \quad (53)$$

and

$$C_2 = C_1 [-2 + C_1 (-1 - 2.25\beta)] \quad (54)$$

For no transpiration rate, $v_0 = 0$, therefore from Eq. (43); the approximate solution of second order is found to be

$$u = \frac{1}{2}(1+y) \quad (55)$$

The above mentioned expression is of the same type that was obtained by Siddiqui *et al.*[5]:

For transpiration rate $v_0 \neq 0$. Therefore as a particular case, taking suction ($v_0 = 0.5$) and $\beta = 0.3$, we obtain $C_1 = -0.812845$, $C_2 = 0.915419$ and the approximate analytical solution as

$$u = \frac{1}{2}(1+y) + \frac{1}{4}(0.406423 - 0.406423y^2) - \frac{1}{48}(4.87707 + 0.660717y - 4.87707y^2 - 0.660717y^3). \quad (56)$$

For injection case ($v_0 = -0.5$) and $\beta = 0.3$ we obtain

$$u = \frac{1}{2}(1+y) + \frac{1}{4}(0.406423 - 0.406423y^2) + \frac{1}{48}(4.87707 - 0.660717y - 4.87707y + 0.660717y^3). \quad (57)$$

The general form is given by

$$u = \frac{1}{2}(1+y) - \frac{1}{4}(0.812845v_0 - 0.812845y^2) - \frac{1}{48}(9.15949v_0 + 2.64287v_0^2y - 9.15949v_0y^2 - 2.64287v_0^2y^3 + 1.98215v_0\beta - 1.98215v_0\beta y^2). \quad (58)$$

(b) For fluid temperature

Taking the values $\lambda = \text{Pr} = 1$, $\alpha_1 = v_0 = 0.5$, we obtained $C_1 = -0.870445$, $C_2 = -0.945332$, and the approximate solution of second order, in the form

$$\Theta = \frac{1}{2}(1+y) + \frac{1}{32}(7.13765 - 7.13765y^2) + \frac{1}{96}(-19.3445 - 6.44024y + 19.3445y^2 + 6.44024y^3). \quad (59)$$

and the general form

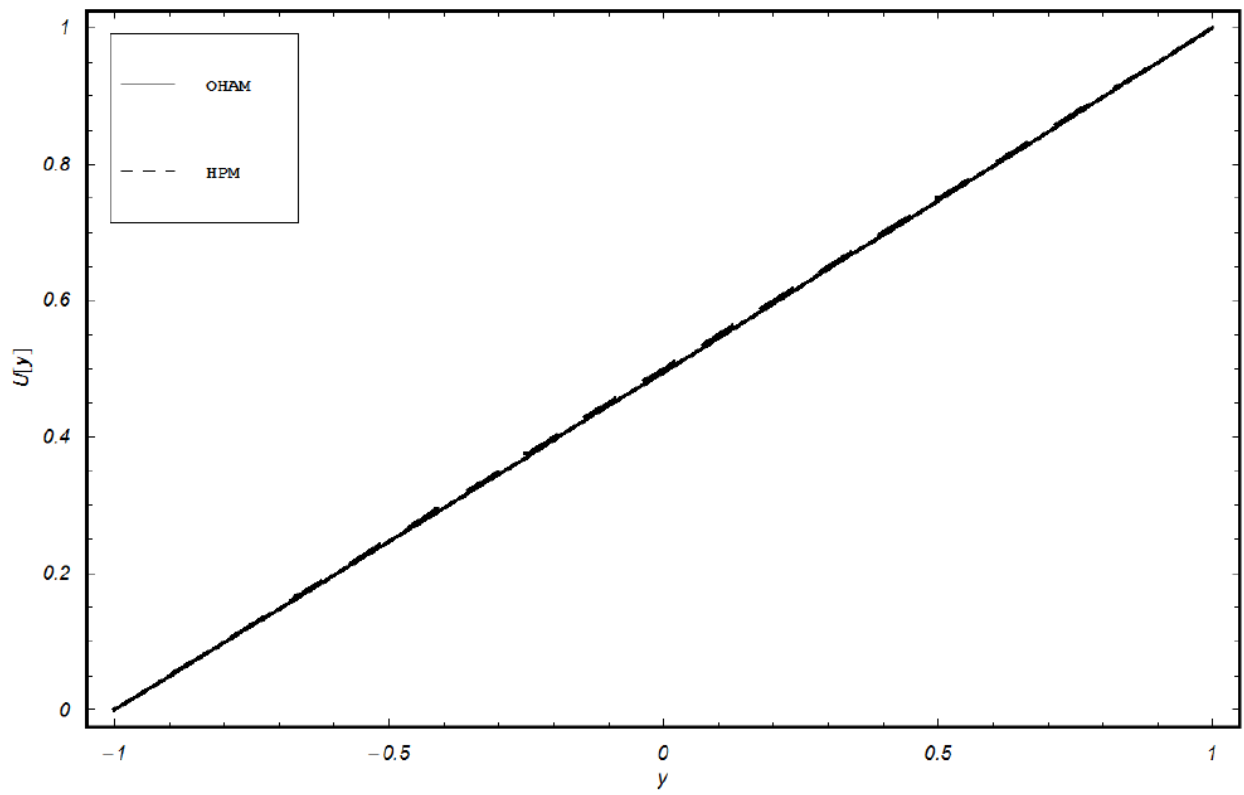
$$\begin{aligned} \Theta = & \frac{1}{2}(1+y) + \frac{1}{32}(6.82381 \text{Pr} v_0 - 6.82381 \text{Pr} v_0 y^2 + 3.41191\lambda \\ & + 0.852977\beta\lambda - 0.852977\beta\lambda y^2) + \frac{1}{96}(25.1814 \text{Pr} v_0 - 5.82055 \text{Pr}^2 v_0^2 y \\ & - 25.1814 \text{Pr} v_0 y^2 + 5.82055 \text{Pr}^2 v_0^2 y^2 + 12.5907\lambda - 5.82055\lambda v_0 y \\ & - 2.91028 \text{Pr} v_0 \lambda y - 12.5907\lambda y^2 + 5.82055v_0 \lambda y^3 + 2.91028 \text{Pr} v_0 \lambda y^3 \\ & + 8.73083v_0^2 \alpha \lambda - 8.73083v_0^2 \alpha \lambda y^2 + 3.14767\beta\lambda - 2.91028v_0 \lambda \beta - 0.727569 \text{Pr} v_0 \beta \lambda y \\ & - 3.14767\beta\lambda y^2 + 2.91028v_0 \beta \lambda y^3 + 0.727569 \text{Pr} v_0 \beta \lambda y^3). \end{aligned} \quad (60)$$

5. Analysis of Results

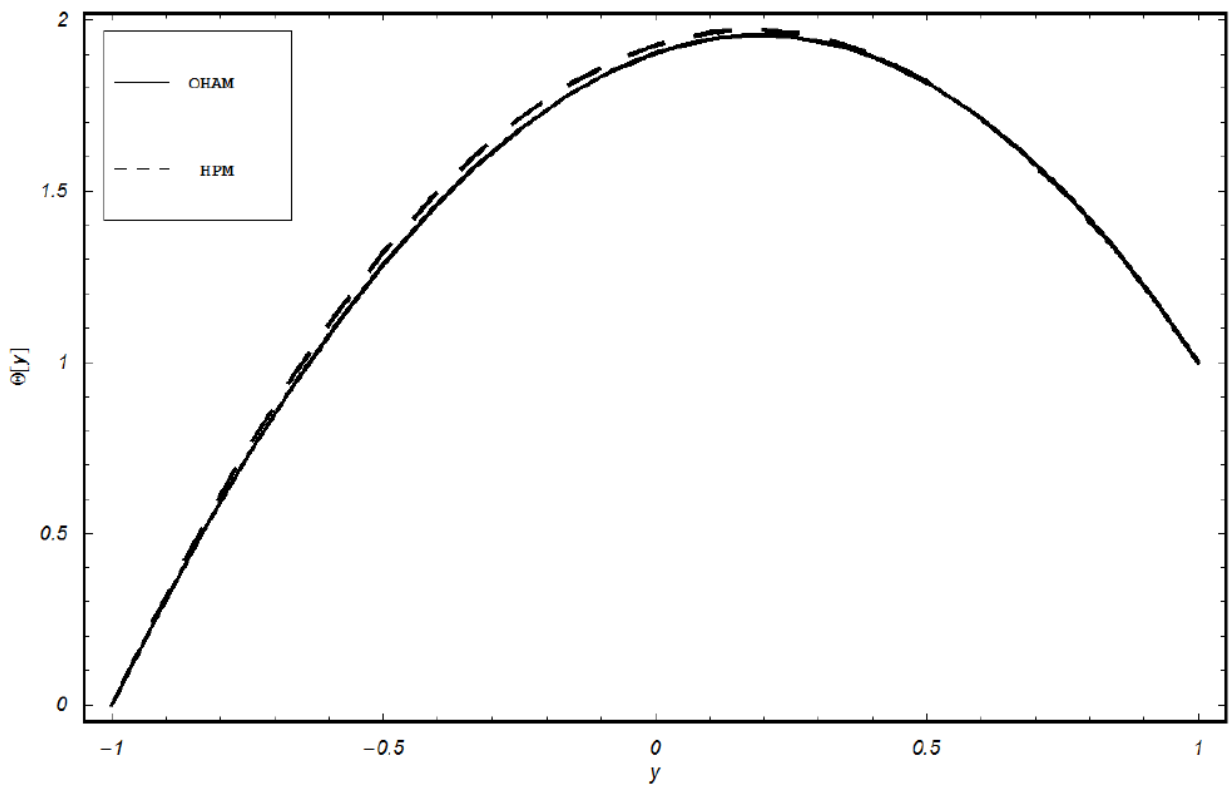
Fig. 1 shows the comparison between the present results, obtained by using OHAM and those Siddiqui et al. [5] obtained by employing HPM in non-porous media. We note that, the precision from the solution acquired through the current method, for velocity and temperature profiles are excellent.

Fig. 2(a) shows the effect of the material constant β on temperature profile, in case of suction, when $\beta = \lambda = \text{Pr} = 1$ and $\alpha = 0.1$. It is observed from this figure that the fluid temperature increases with increase in the value of β . From Fig. 2 (b), for injection case, it is observed that the behavior of the temperature profile is slow to that in Fig. 2 (a).

Fig. 3 (a) shows the effect of the material constant β for velocity profile in case of suction, when $v_0 = 0.3$ is fixed. It is observed from this figure that the fluid velocity decreases with increase in the value of β . However, in Fig. 3 (b) for injection case, the behavior of the velocity profile is reversed to that in Fig. 3 (a).

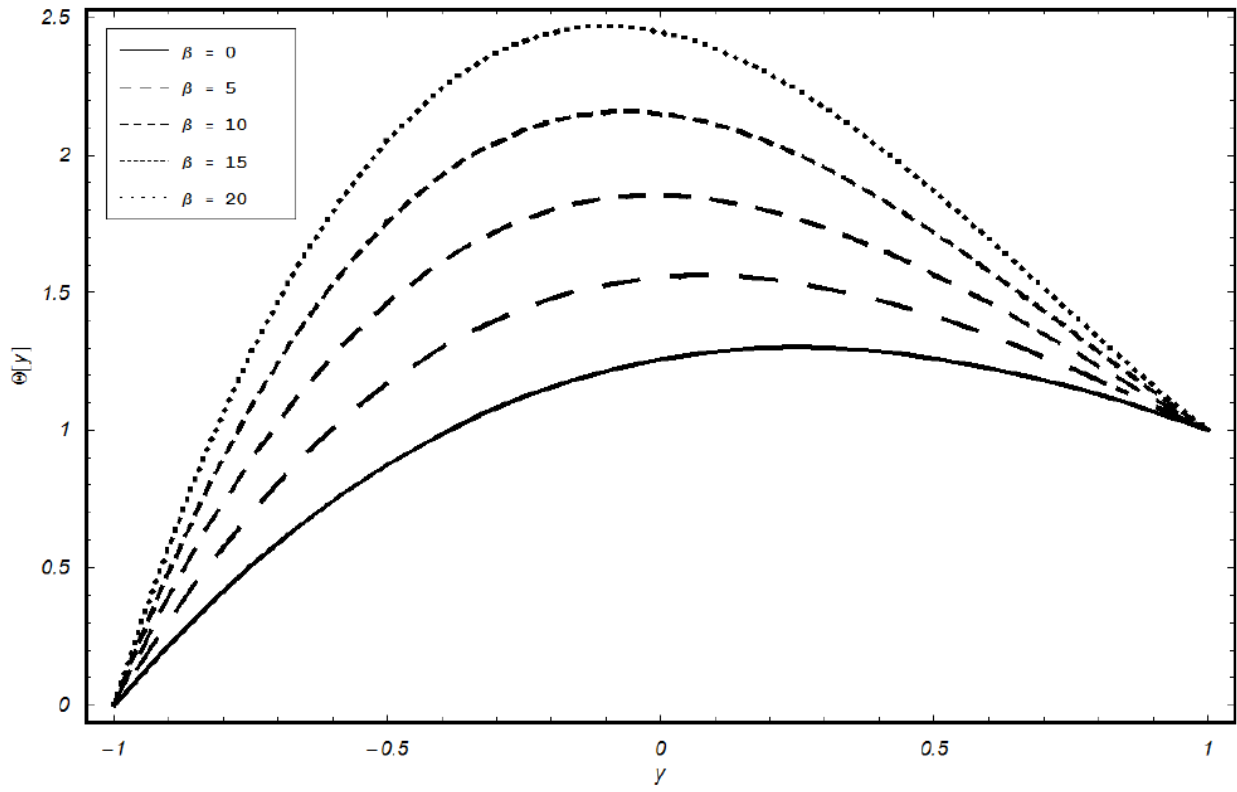


(a)

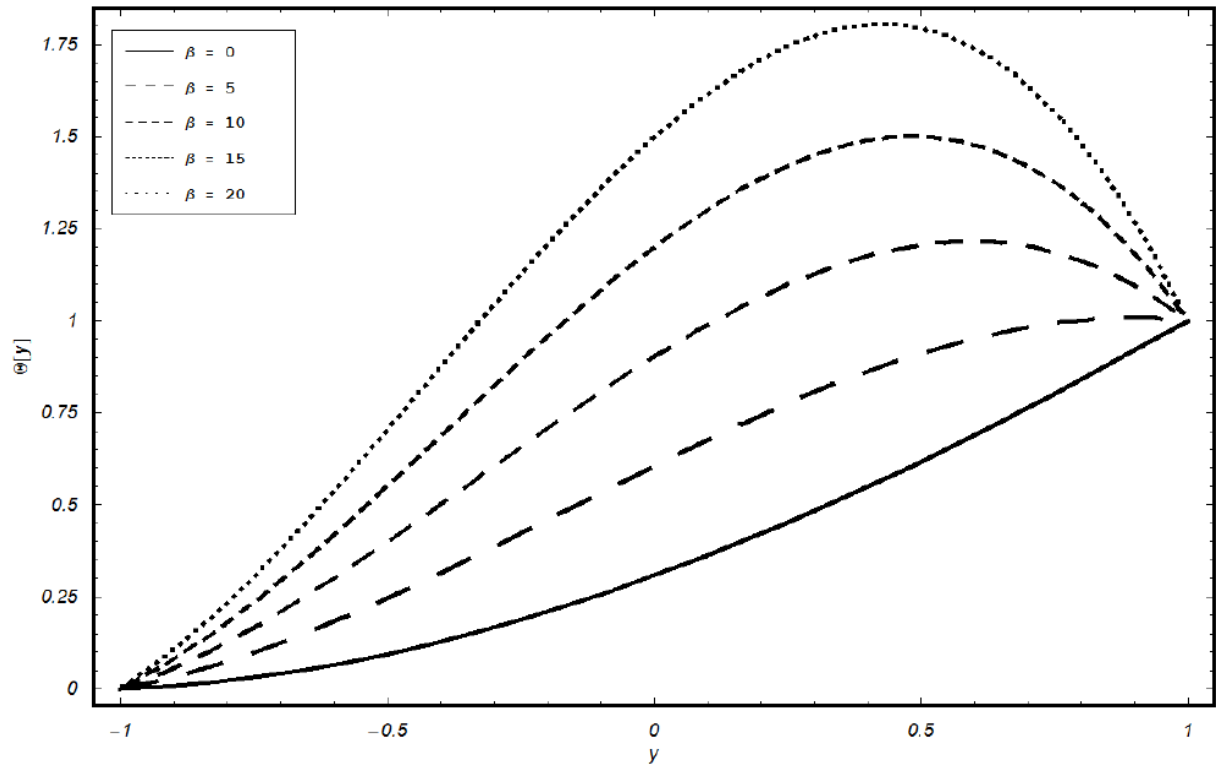


(b)

Figure 1. Comparison between the results obtained from OHAM and Siddiqui et al. results [5] obtained from HPM in case (a) velocity and case (b) temperature when $\lambda = \text{Pr} = \alpha = 1$ and $\beta = 20$

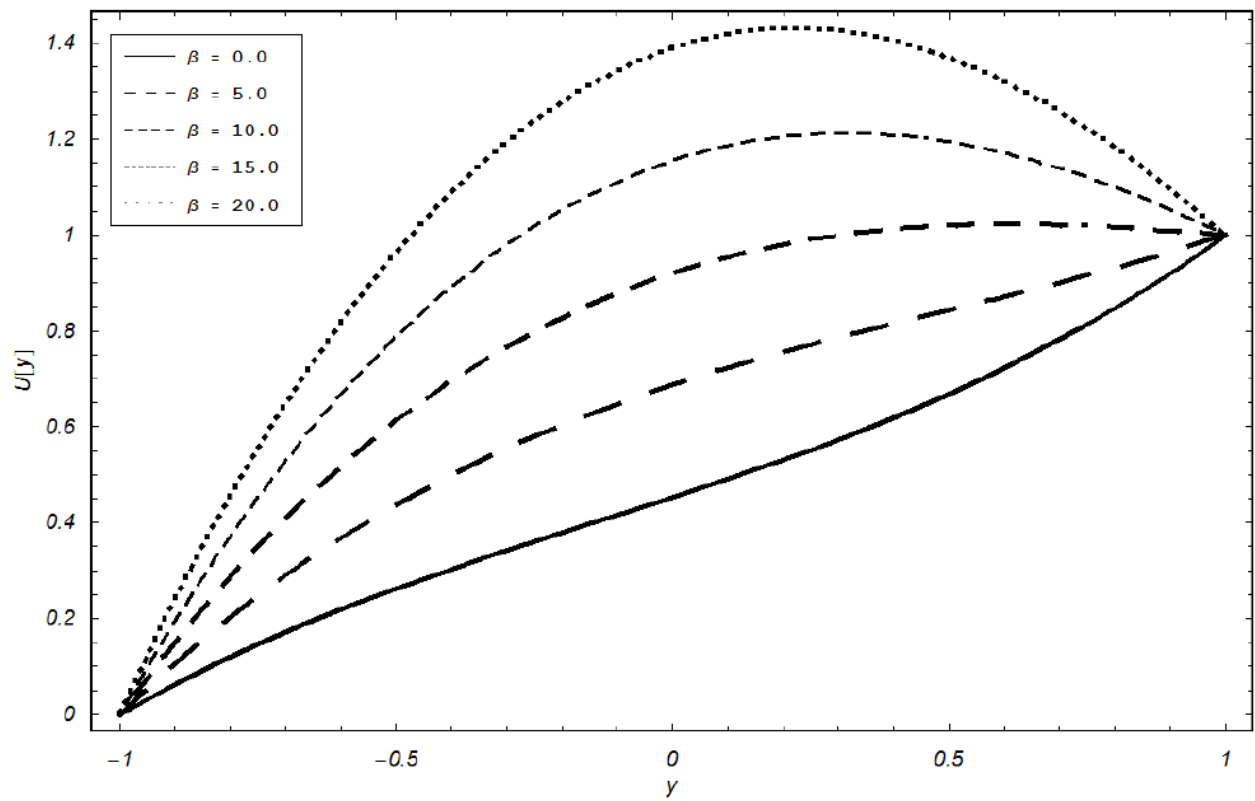


(a)

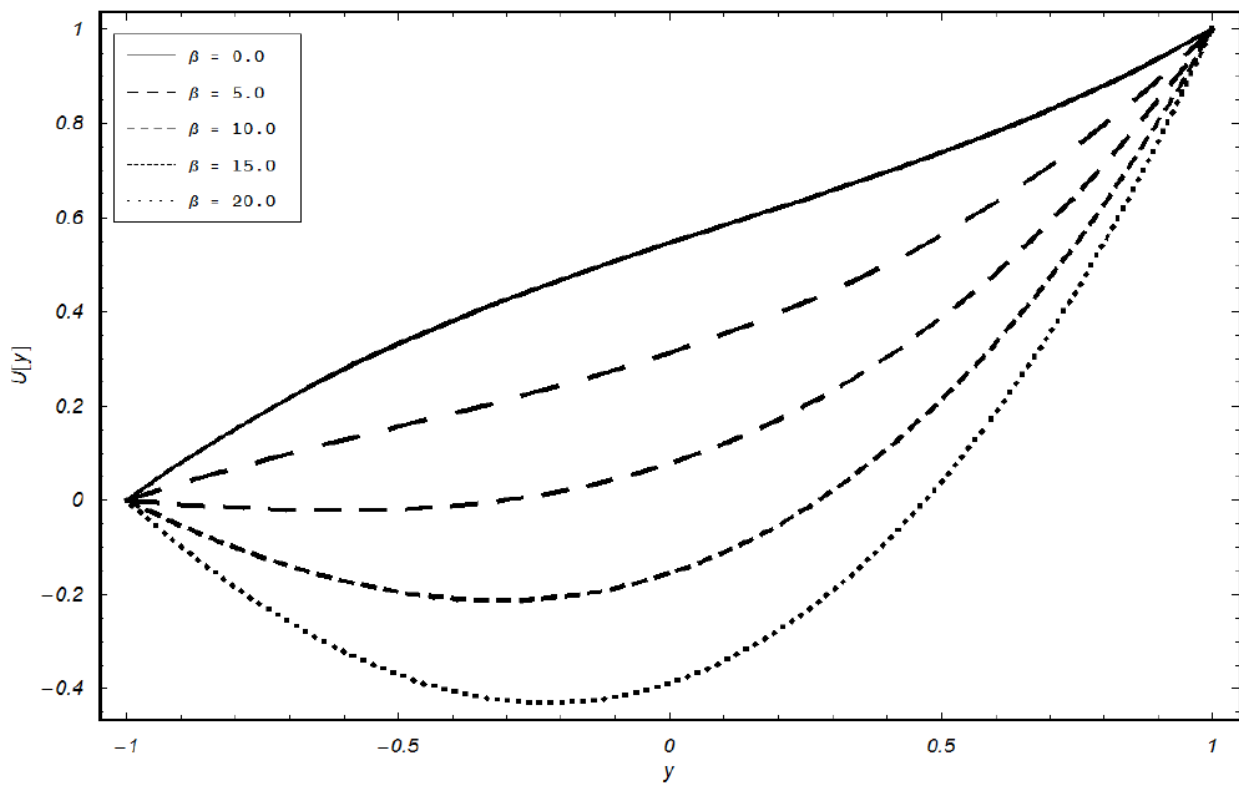


(b)

Figure 2. Effect of material constant β on the temperature profiles for the case of suction (a) and injection (b) when $\alpha = 0.5$, $\lambda = \text{Pr} = 1$ and $v_0 = \pm 1$



(a)



(b)

Figure 3. Effect of material parameter β on the velocity profiles for the case of suction (a) and injection (b) when $v_0 = \pm 1$

6. Conclusions

Analytical explicit expressions for the velocity and temperature profiles have been obtained for the Couette flow of a third grade fluid between two parallel porous plates, which match well with the previous analytical solutions the HPM solutions [5]. It is found that the Couette flow features dominate as viscoelastic parameter is increased. It is also observed that the effects of the fluid/temperature injection or suction exert a great influence on the general flow pattern, by enhancing or suppressing the influence of the wall into the flow domain.

REFERENCES

- [1] S. Okoya Samuel, Disappearance of criticality for reaction third-grade fluid with Reynold's model viscosity in a fiat channel International Journal of Non-Linear Mechanics, 46 (2011) 1110-1115.
- [2] S. Abelman, E. Momoniat and T. Hayat, Couette flow of a third grade fluid with rotating frame and slip condition, Nonlinear analysis: Real world applications 10 (2009) 3329-3334.
- [3] T. Hayat, S. Hina, A. Awatif and S. Asghar. Effect of wall properties on the peristaltic flow of a third grade fluid in a curved channel with heat and mass transfer, International Journal of Heat and mass transfer 54 (2011) 5126-5136.
- [4] D. Mohammad, K. Shashi and K. Surendra, Exact analytical solutions for Poiseuille and Couette-Poiseuille flow of third grade fluid between parallel plates, Communication Nonlinear Science Numerical Simulation 17 (2012) 1089-1097.
- [5] A. M. Siddiqui, A. Zeb, Q. K. Ghori and A. M Benharbit, Homotopy analysis method for heat transfer flow of a third grade fluid between parallel plates, Chaos, Solitons and Fractals 36 (2008)182-192.
- [6] G. Adomian, Solving frontier problems of physics: the decomposition method. Boston: Kluwer Academic; 1994.
- [7] J. H. He, Variational iterative method- a kind of non-linear analytical technique: some examples. International Journal of Non-Linear Mechanics 34 (1999) 699-708.
- [8] M. Javidi, A numerical solution of the generalized Burgers-Huxley equation by spectral collocation method. Applied Mathematics and Computation 178 (2006) 338-344.
- [9] L. Shijun, Notes on the homotopy analysis method: Some definitions and theorems 14 (2009) 983-997.
- [10] J.H. He, The homotopy perturbation method for nonlinear oscillators with discontinuities, Applied Mathematics and Computation 151 (2004) 287-292.
- [11] V. Marinca, and N. Herisanu, Application of Optimal Homotopy Asymptotic Method for Solving Nonlinear Equations Arising in Heat Transfer, International Communications in Heat and Mass Transfer 35 (2008) 710-715.