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Construction of Life Table and Some Mathematical Models for Male Population of Bangladesh

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Abstract The purpose of the present study is to estimate male adult mortality by Widowhood Method using the secondary data for female marital status of Bangladesh. The male life table has been constructed from the number of persons surviving at an exact age x (l_x) values obtained for linking male infant mortality as well as male adult mortality. Age specific death rates (ASDRs), crude death rate (CDR) and crude birth rate (CBR) for male has been estimated. Moreover, an effort has also been given concentration to fit some mathematical models to age structure, ASDRs and l_x values for male population. Model validation technique, cross validity prediction power (CVPP), is applied to those mathematical models to test out either they are suitable or not. It is seen that the life expectancy at birth and CDR for male in 1961 are 43.43 and 21.80 respectively. And ASDRs show traditional U shape pattern. It is found that age distribution of population follows modified negative exponential model containing three parameters. And, ASDRs and the l_x values follow 4^{th} degree polynomial model. Moreover, it is investigated that the stability of R^2 of these models is more than 95%.

Keywords Adult Mortality, Widowhood Method, Life Table, Modeling and Cross Validity Prediction Power (CVPP)

1. Introduction

In any country, the success and overall improvement of a national plan and researches fully depend on the correct information about the population parameters based on national data. There are numerous agencies, organizations and researchers who have been providing the information about the population parameters of Bangladesh. But, very few of them give information on national based. Since in developing countries like Bangladesh, complete vital registration system has not been started yet. So, the majority of the data sources of information have been providing indirect techniques. Consequently, demands of indirect techniques have been increasing to estimate population parameters from very few limited and defected data. In last few decades, a number of indirect techniques have been developed. Widowhood Method is one of them. Hill has devised Widowhood Method to guesstimate adult mortality from widowhood information[1, 2]. Many researchers have used Widowhood Method to estimate the demographic parameters of Bangladesh[3, 4].

In this study, an effort has been given attention to mull over the following objectives:

i) to estimate male adult mortality of Bangladesh by Widowhood Method,

ii) to construct an abridged life table for male from the calculated l_x values, and

iii) to fit some mathematical models for the estimated parameters of Bangladesh by means of census data.

This paper is structured as follows. Sources of data of the study are presented in section 2. Section 3 describes the methods and methodological issues. Results and discussion of this paper are reported in section 4. Finally section 5 concludes the conclusion of the manuscript.

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2. Sources of Data of This Study

The age data for male and marital status data for female of 1961 census[5] have been used as raw data in the present study. For linking infant mortality, child mortality and adult mortality at the time of life table construction, l_2 values have been taken from Statistical Year Book of Bangladesh[6].

3. Methods and Methodological Issues

3.1. Smoothing of Age Data for Male

To evaluate the data, U.N. age sex accuracy index has been calculated. It is found to be 74.1 for both sexes. It is to be mentioned here that Whipple's index and Myer's index have not been calculated because age structure for single age group is not found. This index indicates that male age distribution of 1961 census is highly inaccurate and for this reason this data need to be adjusted. That is why, age distribution of male of 1961 census population has been smoothed by latest smoothing method named 4253H, twice[7]. Smoothing method has been accomplished using the package Minitab Release 12.1. The observed and smoothed male population of Bangladesh have been presented in Table 1 and depicted in Fig. 1.

Table 1. Observed and Smoothed Population by Age and Their Percentage Distribution for Male of Bangladesh

Age Group in Years	Observed Population (000)	Smoothed Population (000)	Observed Percent	Smoothed Percent
0-4	4580	4580	17.38	17.89
5-9	4869	3922	18.48	15.32
10-14	2610	2928	9.91	11.43
15-19	1922	2127	7.30	8.31
20-24	1825	1833	6.93	7.16
25-29	2002	1770	7.60	6.91
30-34	1693	1672	6.43	6.53
35-39	1559	1485	5.91	5.80
40-44	1254	1245	4.76	4.86
45-49	1016	1056	3.86	4.13
50-54	947	996	3.60	3.89
55-59	611	996	2.32	3.89
60+	1463	996	5.55	3.89

3.2. Construction of Life Table

Widowhood method anticipated by Hill[2] has been used to estimate male adult mortality using 1961 census data on female marital status composition. For this, proportion not widowed π_x has been transformed to the probability of surviving ratios using some weighting factors given by Brass[8] corresponding to singulate mean age at marriage (SMAM) for female and weighted singulate mean age at marriage (WSMAM) for male. Ultimately these probabilities of surviving ratios have been rehabilitated to l_x values for male adult ages corresponding to the representative model life tables in the adult ages (Table 2).

The l_x values for male are estimated by linking l_2 values with the male adult mortality. In this case a linear function $Y_x = a + b\ Y_x^s$ is to be fitted by trial and error method in accordance with Sivamurthy and Sitharam[9], where Y_x is the logit of l_x and Y_x^s is the logit survival function of l_x^s of standard life table[10]. Then Y_x values have been rehabilitated to l_x values which are known as the number of persons surviving at an exact age x. The male life table has been constructed using the l_x values and presented in Table 3.

3.3. Estimation of Some Mortality Measures

In this section, some mortality measures are enumerated as follows:

ASDRs have been estimated from the life table using the formula ASDR= $\frac{{}_nd_x}{{}_nL_x}$ [11] and presented in Table 3 and depicted in Fig. 2. Male crude death rate (CDR) has been estimated using these ASDRs and smoothed male population applying the formula CDR = $\sum ASDR_xC_x$, where C_x is the age structure for male. It is found CDR for male is 21.80.

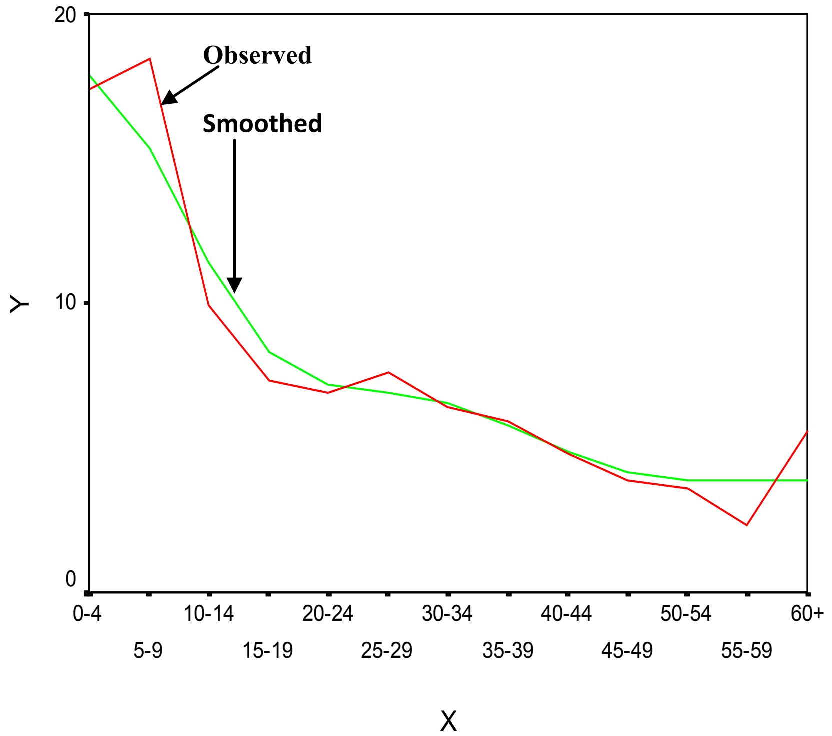

Figure 1. Observed and Smoothed Percentage Distribution for Male Population by Age of Bangladesh. X: Age Group in Years and Y: Percent Population

Table 2. Adult Mortality Male Population of Bangladesh by Widowhood Method

Age Group in Years	Smoothed Proportion not Widowed Female $\pi^f(x)$	Weights W(x)	Corrected Weights W/(x)	1-W/(x)	$l^m_{x+5} / l^m_{22.5}$
20	0.990019	0.549282	1.045282	-0.045282	0.990429074
25	0.980963	0.382252	0.878252	0.121748	0.978299032
30	0.959082	0.416888	0.912888	0.087112	0.955243671
35	0.915020	0.447954	0.943954	0.056046	0.910836390
40	0.840374	0.473682	0.969682	0.030318	0.837229932
45	0.736671	0.485786	0.981786	0.018214	0.734454975
50	0.615005	0.473770	0.969770	0.030230	0.611122047
55	0.486558	0.459920	0.955920	0.044080	0.480376441

Note: WSMAM (Male)=22.06 and SMAM (Female)=15.22

$$l_{x+5} / l_{22.5} = W^{/}(x)\ \pi(x-5) + [1 - W^{/}(x)]\pi(x)$$

$l^m_{22.5}$ = 0.6798

Table 3. Abridged Life Table and ASDRs for Male Population of Bangladesh

Age Group x	l_x	${}_nd_x$	${}_nq_x$	${}_np_x$	${}_nL_x$	T_x	e_x	Estimated ASDRs	Smoothed ASDRs
0	100000	16581	0.16581	0.83419	88393	4342518	43.42518	0.18758	0.18207
1	83419	5606	0.067203	0.932797	80055	4254125	50.99708	0.07003	0.09744
2	77813	2841	0.036511	0.963489	76393	4174070	53.64232	0.03719	0.04431
3	74972	1691	0.022555	0.977445	74127	4097677	54.6561	0.02281	0.02162
4	73281	1099	0.014997	0.985003	72732	4023550	54.90577	0.01511	0.01231
5	72182	2245	0.031102	0.968898	355298	3950818	54.73412	0.00632	0.00654
10	69937	904	0.012926	0.987074	347425	3595520	51.41084	0.0026	0.00334
15	69033	1036	0.015007	0.984993	342575	3248095	47.05134	0.00302	0.00285
20	67997	1202	0.017677	0.982323	336980	2905520	42.73012	0.00357	0.00337
25	66795	1428	0.021379	0.978621	330405	2568540	38.45408	0.00432	0.00404
30	65367	1734	0.026527	0.973473	322500	2238135	34.23952	0.00538	0.0052
35	63633	2233	0.035092	0.964908	312583	1915635	30.10443	0.00714	0.00707
40	61400	2984	0.048599	0.951401	299540	1603052	26.10834	0.00996	0.00992
45	58416	3986	0.068235	0.931765	282115	1303512	22.3143	0.01413	0.01437
50	54430	5402	0.099247	0.900753	258645	1021397	18.76533	0.02089	0.02114
55	49028	6731	0.137289	0.862711	228313	762752	15.55748	0.02948	0.03167
60	42297	8413	0.198903	0.801097	190453	534439	12.63539	0.04417	0.04489
65	33884	9313	0.274849	0.725151	146138	343986	10.15187	0.06373	0.05765
70	24571	9132	0.371658	0.628342	100025	197848	8.052094	0.0913	0.08806
75	15439	3594	0.232787	0.767213	68210	97823	6.336097	0.05269	0.17548
80+	11845	11845	1	0	29613	29613	2.500042	0.39999	0.32259

3.4. Model Fitting

From the scattered plot of population by ages (Fig. 3), it is observed that the population is modified negative exponentially distributed with regard to ages. Therefore, a modified negative exponential model is considered. The model is

$$y = c + e^{-(ax+b)} + u\ ; \tag{1}$$

where, x represents the middle value of the age group; y represents male population; a, b, c are constants and u is the stochastic error term of the model.

From the scattered plot of the number of persons surviving at an exact age x (l_x) by ages (Fig. 4), it appears that l_x can be fitted by polynomial for different ages. Hence, an n^{th} degree polynomial model is treated and the model is

$$y = a_0 + \sum_{i=1}^{n} a_i\ x^i + u\ ; \tag{2}$$

where, x is the mean value of the age group; y is l_x; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the disturbance term of the model. Here, a suitable n is found out for which the error sum of square is smallest amount.

Again from the dotted plot of ASDRs for male by ages (Fig. 5), it also seems that ASDRs can be fitted by polynomial for different ages. In this case, an n^{th} degree polynomial model is considered of the following form

$$y = a_0 + \sum_{i=1}^{n} a_i x^i + u \text{ [12];} \quad (3)$$

where, x is the average of the age group; y is ASDRs; a_0 is the constant; a_i is the coefficient of x^i (i =1, 2, 3, ..., n) and u is the error term of the model. At this juncture a suitable n is chosen such that the error sum of square is lowest amount.

Using the software STATISTICA, all these mathematical models have been estimated. The information on model fittings has been presented in Table 4.

Note that as usual models, i. e. Gompertz, Makeham and logistic models were also applied but seem to be worse fitted with respect to their shrinkages. Therefore, the findings of those models were not exposed in this study.

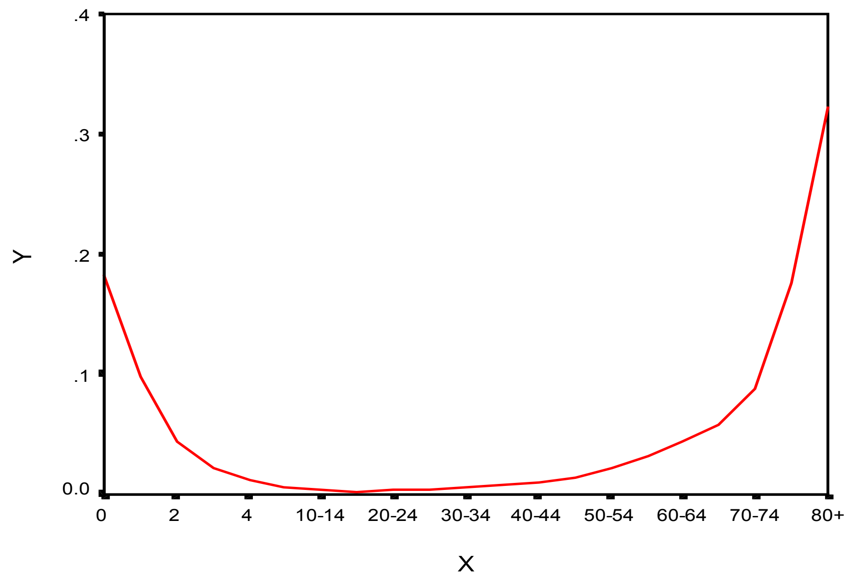

Figure 2. The ASDRs for Male Population of Bangladesh.

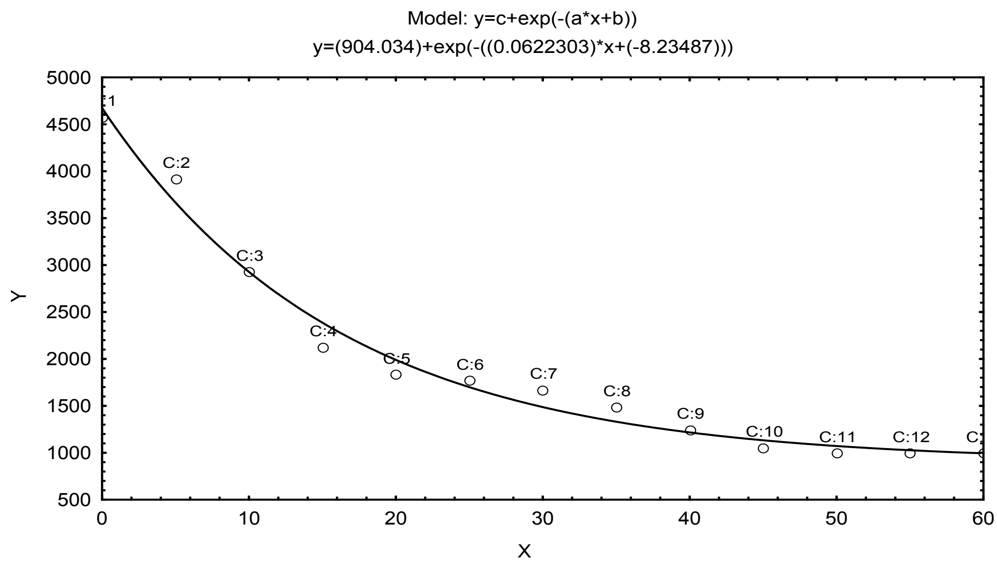

Figure 3. Observed and Fitted Model for Male Age Distribution of Bangladesh. X: Age Group in Years and Y: Male Population

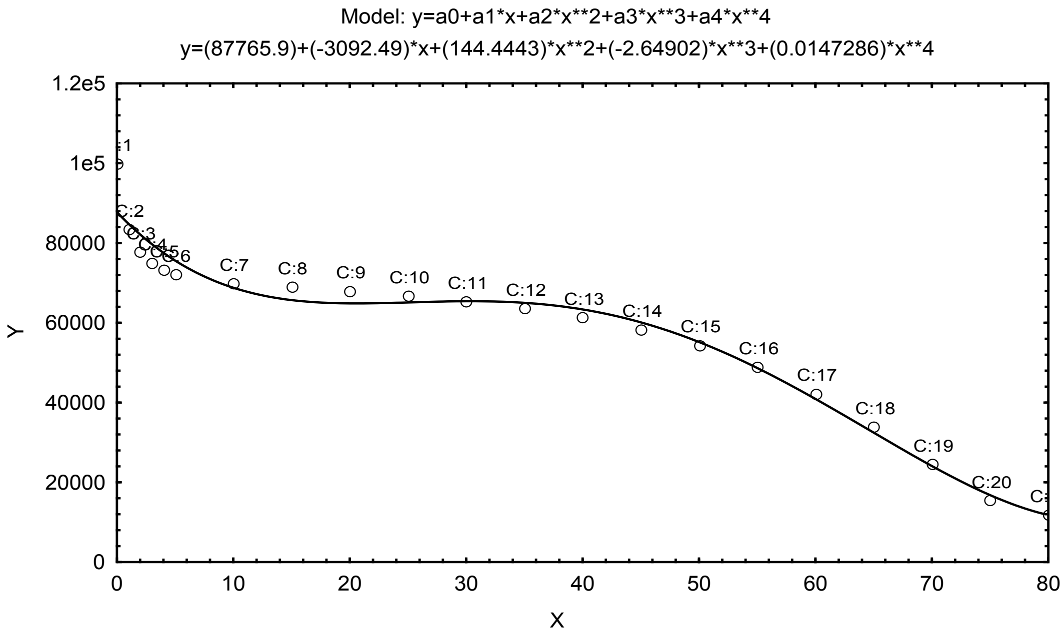

Figure 4. Observed and Fitted Model of l_x Values for Male of Bangladesh. X: Age Group in Years and Y: l_x

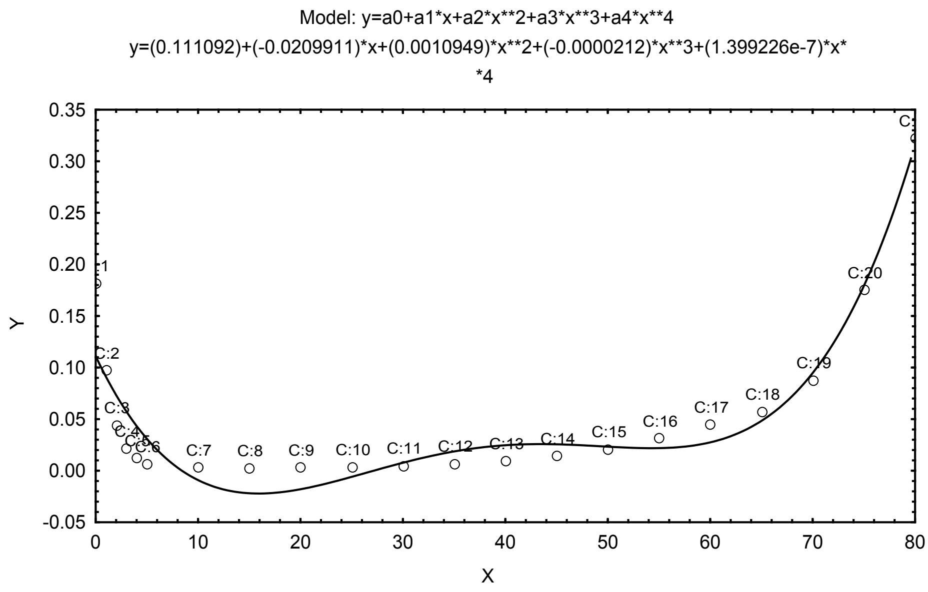

Figure 5. Observed and Fitted Model for ASDRs for Male of Bangladesh. X: Age Group in Years and Y: ASDRs

3.5. Model Validation Technique: Cross Validity Prediction Power (CVPP)

To check how much those models are stable over the population, the CVPP, ρ^2_{cv}, is applied. Here

$$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2) \;;$$

where, n is the sample size or number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable[13].

3.6. Shrinkage of the Model

The shrinkage of the model is Shrinkage = $| \rho^2_{cv} - R^2 |$; where ρ^2_{cv} is cross validity prediction power & R^2 is the coefficient of determination of the model. Moreover, the stability of R^2 of the model is equal to 1- shrinkage. The estimated CVPP, ρ^2_{cv}, corresponding to their R^2 and shrinkage are shown in Table 4. Cross validity prediction power is also employed as validation technique[14-31].

3.7. Crude Birth Rate (CBR) for Male Population

There are various types of measures in fertility. But, we estimate CBR only. Assuming intercensal growth rate between the intercensal period from 1951 to 1961 is equal to the growth rate (r) as in 1961. Now CBR has been estimated from the balancing equation CBR = CDR + r; assuming the net migration rate is zero. Intercensal growth rate has been estimated using the exponential growth rate formula. Note that the inter censual growth rate during 1951 to 1961 for male is 20.2 per thousand.

4. Results and Discussion

Life table for male presented in Table 3 exhibits the life expectancy at birth for male in 1961 is 43.43. But it is observed that the life expectancy at birth for male in 1961 was 44.22 (BBS, 1980) which is close to our estimate. Therefore the life expectancy at birth for male in 1961 around 43 or 44 is quite reasonable.

Again, the ASDRs for male of Bangladesh in 1961 presented in Table 3 and in Fig. 2 shows the traditional U shaped curve. It is seen that the curve is showing gradually decreasing in the continuous age interval[0, 20) but gradually increasing in the age interval[20, 80+].

The CDR for male of Bangladesh in 1961 is to be found in the present study as 21.80 and the CBR for male of Bangladesh is 42.00 in 1961.

The modified negative exponential model is assumed for male age structure of Bangladesh and the fitted equation is as follows:

$$y = (904.034)+\exp(-(0.0622303)x+(8.23487)) \quad (1)$$

with coefficient of determination R^2 is 0.98497 and ρ^2_{cv} =0.980577.

The polynomial model is assumed for number of persons surviving at an exact age x (l_x) for male of Bangladesh and the fitted equation is

$$y = (87765.9)+(-3092.49)x+(144.4443)x^2 +(-2.64902)x^3+(0.0147286)x^4 \quad (2)$$

giving R^2 = 0.97376 and ρ^2_{cv} is 0.956475.

Again, another polynomial model is assumed for male ASDRs and the fitted equation is

$$y = (0.111092)+(-0.0209911)x+(0.0010949)x^2 +(-0.0000212)x^3+(1.39922e-7)x^4 \quad (3)$$

with coefficient of determination R^2 is 0.93199 and ρ^2_{cv} =0.88719.

From Table 4, it is shown that all the parameters of the fitted models are highly significant with significant proportion of variance explained. Moreover, Table 4 reveals that all the fitted models (1), (2) and (3) are highly cross-validated and their shrinkages are only 0.004393, 0.017285 and 0.0448, respectively. These imply that the fitted models (1), (2) and (3) will be stable more than 98%, 95% and 88%, respectively. Shrinkage coefficients show that the fit of these models are better well.

From the above equation (1), the rate of change of y with respect to x, i. e. the velocity curve for age structure has been estimated and shown in Fig. 6. From this figure it is moreover seen that age structure curve is smoothly and strictly decreasing in accordance with ages in years.

Table 4. Information on Model Fitting and Estimated CVPP of the Predicted Equations of Age Distribution, l_x and ASDRs for Male Population of Bangladesh

Models	n	k	R^2	ρ^2_{cv}	Parameters	Significant Probability (p)
(1)	13	1	0.98497	0.980577	a	0.000003
					b	0.00000
					c	0.00000
(2)	21	4	0.97376	0.956475	a_0	0.00000
					a_1	0.00000
					a_2	0.00050
					a_3	0.00084
					a_4	0.002204
(3)	21	4	0.93199	0.88719	a_0	0.000002
					a_1	0.00005
					a_2	0.000114
					a_3	0.00012
					a_4	0.000069

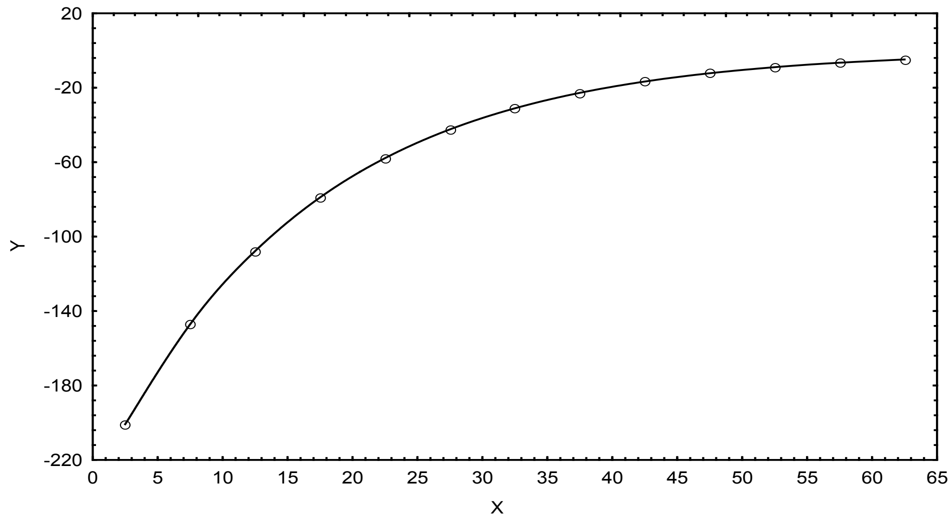

Figure 6. The Velocity Curve for Male Population of Bangladesh. X: Age Group, Y: Velocity

5. Conclusions

In this paper, abridged life table for male has been constructed using the female marital status by widowhood method. It is seen that the life expectancy at birth and CDR for male in 1961 are 43.43 and 21.80 respectively. And ASDRs exhibit traditional U shape pattern. It is found that age distribution for male population of Bangladesh follows modified negative exponential model three parameters in which all parameters are highly significant with large proportion of variation. On the other hand, ASDRs for male follow 4^{th} degree polynomial model containing five parameters in which all parameters are highly statistically significant. Moreover, the surviving function (l_x) for male of Bangladesh follows bi-quadratic polynomial model in which all parameters are highly statistically significant. The stability of R^2 of these models is more than 95%.

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